

A HYBRID 0D–2D WHOLE-ENGINE MATCHING FRAMEWORK FOR TURBOJET ENGINE PERFORMANCE SIMULATION

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Abstract

Accurate whole-engine matching is essential for reliable performance prediction of turbojet engines at the early design stage. This paper presents a hybrid 0D–2D whole-engine matching framework that balances computational efficiency and physical fidelity. Two-dimensional through-flow models are employed for the compressor and turbine, while the inlet, combustor, and nozzle are represented using simplified zero-dimensional performance models. To robustly determine the matched operating point, a hybrid global–local optimization strategy is adopted, combining Monte Carlo sampling for global exploration with the Nelder–Mead algorithm for local refinement. The proposed method is validated against experimental data under both design-point and off-design conditions. The results demonstrate good agreement across key performance parameters, with differences remaining within 5%. These findings indicate that the proposed framework provides an effective and reliable tool for preliminary turbojet engine performance analysis and early-stage design applications.

Keywords: Turbojet engine; Whole-engine matching; Hybrid 0D–2D modeling; Global–local optimization

1. Introduction

The performance of a gas turbine engine depends not only on the performance of individual components, but also on the mechanical and thermodynamic interactions between all components operating under matched conditions [1]. Therefore, accurately evaluating engine performance at any operating point requires an integrated whole-engine analysis. Conventional aero-engine development has relied heavily on empirical testing and iterative design–fabrication–integration–validation–refinement cycles to evaluate engine performance. Although this trial-and-error approach provides reliable results, it often results in long development times, high costs, and increased technical risks, particularly for complex engine systems.

To overcome the limitations of conventional development approaches, numerical simulation methods have been increasingly adopted for gas turbine engine performance prediction during the early stages of design. Among these methods, whole-engine matching has emerged as a fundamental approach for evaluating the performance of integrated propulsion systems [2, 3, 4]. In general, whole-engine matching involves the modeling of major engine components, the representation of their nonlinear interactions, and the determination of an equilibrium operating point at which all components operate compatibly while satisfying the overall requirements of the engine under a given operating condition. By accounting for component interactions across the entire engine system, whole-engine matching enables system-level performance evaluation and facilitates the early identification of potential performance limitations and compatibility issues.

The whole-engine matching methods can be classified according to the spatial dimensionality of the underlying component models, including zero-dimensional (0D), two-dimensional (2D), and three-dimensional (3D) approaches. Among these, 0D methods remain the most widely adopted for engine performance prediction due to their computational efficiency and ease of implementation. By

representing component behavior through performance maps and characteristic relationships, they enable rapid evaluation of engine operating points and overall performance over a wide operating envelope [2, 5]. However, their simplified representation of component physics limits their ability to capture detailed aerodynamic and flow-field characteristics within the engine. In contrast, 3D approaches based on computational fluid dynamics (CFD) provide detailed resolution of complex flow structures and component aerodynamics [4], but their computational cost generally limits their applicability to iterative whole-engine matching and preliminary design studies. To bridge the gap between 0D and 3D methods, 2D throughflow methods, particularly those based on streamline-curvature formulations, offer an effective compromise between computational cost and physical fidelity [**?**, 3]. Unlike conventional 0D models, they can capture meridional flow behavior and radial distributions of aerodynamic quantities within turbomachinery components, while remaining significantly more computationally efficient than 3D methods.

At the early stage of design, not all engine components require the same level of modeling fidelity. Turbomachinery components such as compressors and turbines have a strong influence on mass-flow compatibility and work balance, and therefore benefit from higher-fidelity two-dimensional (2D) throughflow modeling. In contrast, components including the inlet, combustor, and nozzle can often be adequately represented using simplified 0D performance models without significantly compromising overall matching accuracy. This strategy provides an effective balance between physical fidelity and computational efficiency. Motivated by this observation, the present study proposes a hybrid 0D–2D whole-engine matching framework for turbojet engines.

In addition, most existing whole-engine matching frameworks employ Newton–Raphson-based iterative schemes to determine steady-state equilibrium operating conditions [6, 3]. Although these methods are efficient and widely used, their convergence strongly depends on the quality of the initial guess and may deteriorate in the presence of strong nonlinearities and complex interactions between engine components [7, 8]. To improve robustness, this study proposes a hybrid optimization strategy that combines Monte Carlo [9] sampling for global exploration with the Nelder–Mead algorithm [10] for local convergence refinement. The Monte Carlo stage enhances the likelihood of identifying a feasible initial region, while the Nelder–Mead method ensures stable and efficient convergence to the final solution.

The main contributions of this work are summarized as follows:

- A hybrid 0D–2D whole-engine matching framework is developed for turbojet engines, combining 0D modeling with 2D throughflow modeling to improve both accuracy and computational efficiency.
- A robust solution strategy combining Monte Carlo sampling and Nelder–Mead optimization is proposed to improve convergence of the whole-engine matching process.

The remainder of the paper is organized as follows: Section 2 presents the proposed method, Section 3 discusses the results, and Section 4 concludes the paper.

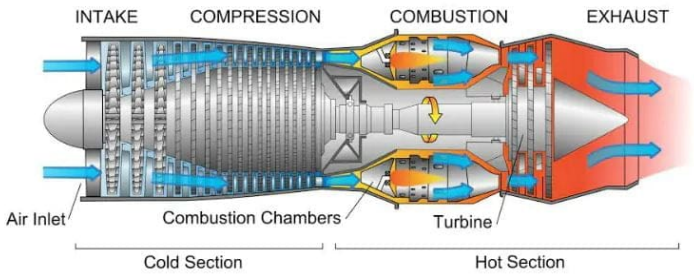


Figure 1 – Schematic of a turbojet engine

2. Proposed Method

2.1 Component-level modeling of turbojet engine

As mentioned previously, this study focuses on a turbojet engine configuration (as illustrated in Figure 1), consisting of five main components: inlet, compressor, combustor, turbine, and nozzle. The modeling approach for each component is presented in the following subsections, where the compressor and turbine are treated using two-dimensional (2D) throughflow analysis, while the remaining components are modeled using simplified zero-dimensional (0D) performance representations.

2.1.1 0D Inlet Model

In this study, the inlet is modeled using a zero-dimensional (0D) approach. The inlet process is assumed to be adiabatic and reversible, while total temperature is preserved. A pressure recovery factor is introduced to account for total pressure losses in the inlet duct.
The inlet total temperature and total pressure are given by

$$T_2 = T_{\text{amb}} \left(1 + \frac{\gamma - 1}{2} M^2\right) \tag{1}$$

$$P_2 = \sigma_i P_{\text{amb}} \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}} \tag{2}$$

where σ_i is the inlet pressure recovery coefficient, typically less than unity and dependent on inlet design and operating conditions.
Although simplified, this formulation captures the dominant thermodynamic effect of inlet compression while maintaining computational efficiency suitable for whole-engine matching analysis. In the present framework, the inlet serves as the upstream boundary condition provider, supplying total temperature and total pressure to the compressor module. This ensures consistent thermodynamic coupling between the inlet and downstream engine components, while avoiding the need for detailed aerodynamic inlet flow modeling.

2.1.2 Turbomachinery Throughflow Model

The compressor and turbine are modeled using a two-dimensional streamline-curvature throughflow approach, which provides an intermediate-fidelity representation between simple mean-line models and full three-dimensional CFD simulations. In this formulation, the flow field is described on a meridional plane extending from hub to shroud, where the governing equations are solved in a quasi-steady manner along stream surfaces.
The governing equations include the continuity equation and radial equilibrium equation, which together define the balance between axial flow development and radial pressure gradients within the turbomachine. These equations are discretized on a structured computational grid composed of streamlines and quasi-orthogonal lines derived directly from the blade geometry, as illustrated in Fig. 2. The streamline-curvature method enables the iterative correction of flow angles and meridional velocity components until consistency is achieved between the assumed and computed flow fields.

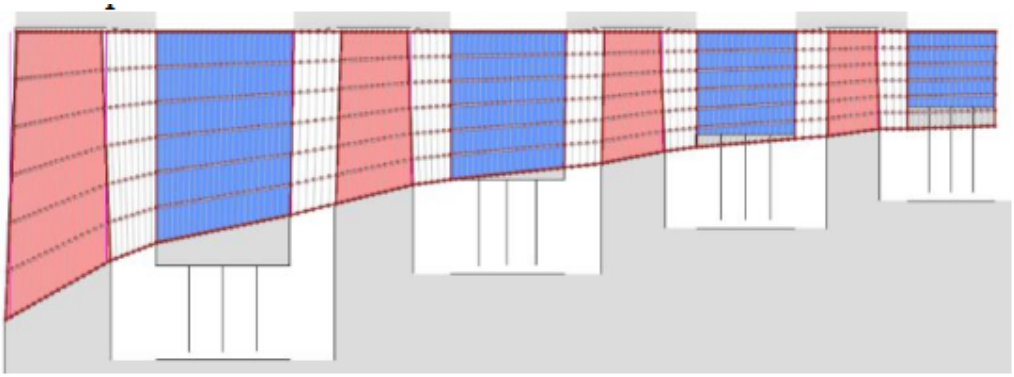


Figure 2 – Streamline curvature computational grid for the compressor

Since the inviscid formulation alone is insufficient to capture real-flow effects, aerodynamic losses, flow deviation (incidence and deviation), and blockage effects are incorporated through empirical correlations. These correlations provide a lumped representation of viscous dissipation, secondary flow structures, endwall effects, and tip-leakage phenomena, which are otherwise not resolved in the throughflow framework. In this sense, the empirical models serve as closure relations for the inviscid core equations and play a critical role in determining the overall accuracy of the predicted performance.

Within the computational procedure, the solver iteratively predicts the radial distributions of key flow variables, including flow angle, Mach number, static and total pressure, and thermodynamic properties. This allows the model to capture essential aerodynamic phenomena such as radial flow redistribution, spanwise loading variation, and stage-to-stage matching effects, which are particularly important for multistage turbomachinery analysis.

For turbine modeling, the throughflow solver is operated under prescribed inlet total conditions, rotational speed, and outlet static pressure. Based on these boundary conditions, the solver computes the corresponding flow field and generates global performance characteristics, including mass flow rate, total-to-total pressure ratio, and isentropic efficiency over a wide operating range.

The compressor model is formulated in a similar manner; however, it exhibits higher sensitivity to loss modeling and empirical closure assumptions. Therefore, multiple loss correlations are implemented and tested within the streamline-curvature framework to evaluate their influence on predicted compressor performance. This comparison highlights the importance of accurate loss representation in compressor analysis, particularly under off-design operating conditions where incidence and secondary flow effects become more pronounced.

2.1.3 0D Combustor Chamber Model

The combustor is represented as a zero-dimensional (0D), well-mixed control volume located between the inner and outer liner radii along the combustor length. This simplified representation assumes uniform thermodynamic properties within the chamber, thereby neglecting spatial variations in temperature, pressure, and species concentration. Despite this simplification, the 0D formulation is widely adopted in cycle-level engine analysis due to its ability to capture the dominant thermodynamic effects of heat addition while maintaining computational efficiency.

The model takes as inputs the compressor exit total pressure and temperature, air and fuel mass flow rates, combustor volume, and a prescribed total pressure loss coefficient. The total pressure loss accounts for irreversible processes such as wall friction, mixing losses, and flow non-uniformities associated with fuel injection and dilution air addition. In this formulation, the combustor is assumed to operate under an approximately constant-pressure condition, which is a reasonable approximation for gas turbine combustors where pressure variations are relatively small compared to temperature rise.

The total pressure at the combustor exit is obtained by applying the specified pressure loss coefficient, which effectively links inlet and outlet stagnation conditions without resolving the internal flow physics in detail. This approach allows the combustor to be consistently coupled with upstream (compressor) and downstream (turbine) components in the overall engine matching framework.

Combustion efficiency is evaluated using an empirical correlation expressed as a function of the chamber loading parameter, which characterizes the aerodynamic loading and residence time effects within the combustor. The chamber loading parameter is defined as:

$$K_v = 1.735 \times 10^6 \frac{W_{\text{air}}}{P_3^{1.25} T_3 V_{\text{cc}}} \tag{3}$$

where W_{air} is the air mass flow rate, P_3 and T_3 are the combustor inlet total pressure and temperature, and V_{cc} is the combustor volume. This parameter provides a lumped measure of combustor operating intensity and is commonly used to correlate combustion efficiency under different operating conditions.

The outlet total temperature is determined from an overall energy balance based on the conservation of enthalpy:

$$(1+f)\,c_p^-\,T_4 = c_p^-\,T_3 + f\,\eta_{\mathrm{cb}}\,Q \tag{4}$$

where c_p^- denotes the mean specific heat of the gas mixture, f is the fuel–air ratio, η_{cb} is the combustion efficiency, and Q is the fuel lower heating value. This formulation assumes complete mixing of reactants and products, allowing the combustor to be treated as an open thermodynamic system with uniform exit conditions.

To improve accuracy, temperature-dependent specific heat properties are obtained from an in-house thermodynamic property table using interpolation. Since both c_p and the combustor exit temperature T_4 are interdependent, an iterative procedure is employed to ensure thermodynamic consistency. The iteration is continued until the relative change in T_4 between successive iterations falls below a prescribed tolerance.

2.1.4 1D Nozzle Model

The nozzle module models the one-dimensional expansion of exhaust gases downstream of the turbine based on the conservation of mass, momentum, and energy, under the assumptions of adiabatic and isentropic flow. In this formulation, the nozzle is treated as a steady, quasi-one-dimensional device in which total enthalpy is converted into kinetic energy through gas expansion. Although simplified, this representation captures the dominant thermodynamic transformation governing thrust generation in turbojet engines.

Given the total temperature T_{t8}, total pressure P_{t8}, and mass flow rate $\dot{m}_8$ at the turbine exit, the solver determines the nozzle exit Mach number M_8 in an iterative manner. The need for iteration arises from the nonlinear coupling between Mach number and static flow properties, which cannot be expressed explicitly when mass flow continuity and isentropic relations are enforced simultaneously.

Once the Mach number is obtained, the corresponding static temperature and pressure are computed from standard isentropic relations:

$$T_8 = \frac{T_{t8}}{1+\frac{\gamma-1}{2}M_8^2} \tag{5}$$

$$P_8 = P_{t8}\left(1+\frac{\gamma-1}{2}M_8^2\right)^{-\frac{\gamma}{\gamma-1}} \tag{6}$$

These relations ensure consistency between stagnation and static thermodynamic states along the nozzle expansion process.

The exit velocity is then evaluated from the definition of Mach number as:

$$V_8 = M_8\sqrt{\gamma R T_8} \tag{7}$$

where γ and R are the specific heat ratio and gas constant of the exhaust gas mixture, respectively.

The net thrust is computed by combining momentum and pressure thrust contributions:

$$F_N = \dot{m}_8\,(V_8 - V_0) + (P_8 - P_{\mathrm{amb}})A_8 \tag{8}$$

where V_0 is the flight velocity, P_{amb} is the ambient pressure, and A_8 is the nozzle exit area. This formulation accounts for both momentum exchange and pressure mismatch effects at the nozzle exit plane.

The iterative procedure for determining M_8 is repeated until convergence is achieved within a prescribed tolerance. In cases where the mass flow exceeds the critical choking condition, the nozzle operates in a choked regime and the solution is enforced with $M_8 = 1$. This ensures physically consistent predictions of mass flow rate, exit conditions, and thrust under both subsonic and choked operating states.

The overall engine is modeled within a hybrid 0D–2D simulation framework, as illustrated in Fig. 3. In this framework, each engine component is represented as a control volume bounded by well-defined inlet and outlet cross-sections. This modular representation allows the gas turbine system to

be decomposed into interacting subsystems while preserving the conservation of mass, momentum, and energy at the interfaces between components.

The internal behavior of each component is evaluated using either performance maps or physics-based models, depending on the selected level of modeling fidelity. In particular, turbomachinery components are modeled using higher-fidelity two-dimensional throughflow formulations, whereas the inlet, combustor, and nozzle are represented using simplified zero-dimensional models. This selective modeling strategy enables a consistent trade-off between computational efficiency and physical fidelity across the entire engine system.

The state of each component is described by a set of thermodynamic variables, including mass flow rate, total pressure, total temperature, fuel–air ratio, and Mach number. These variables are continuously propagated through the engine flowpath to ensure thermodynamic consistency between adjacent components. As a result, the engine performance prediction is obtained by enforcing compatibility conditions at all component interfaces rather than solving each component independently.

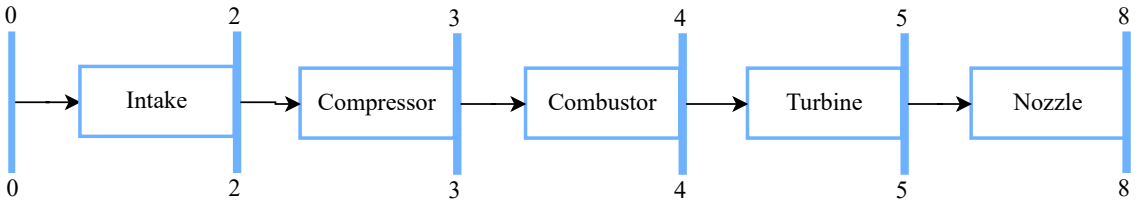


Figure 3 – Engine flowpath and component arrangement

The whole-engine matching problem is formulated as the determination of a self-consistent set of operating variables that simultaneously satisfies all component-level constraints. At the design point, the solution can be obtained in a sequential manner by propagating flow variables along the engine flowpath, as illustrated in Fig. 4. However, for off-design conditions, the problem becomes strongly coupled and nonlinear, since the operating states of the compressor, combustor, turbine, and nozzle are interdependent and must satisfy multiple balance constraints simultaneously.

Let the unknown vector of operating variables be defined as

$$\mathbf{x} = [G_{cp},\, WF,\, P_5]^T,$$

where G_{cp} is the compressor inlet mass flow rate, WF is the fuel–air ratio, and P_5 is the turbine outlet total pressure. These variables are selected as primary matching parameters because they directly govern mass conservation, energy addition, and turbine–nozzle pressure coupling within the engine system.

The equilibrium operating point is obtained by enforcing the following nonlinear balance equations:

$$f_1(\mathbf{x}) = G_{cp} + WF - G_{tb} = 0 \tag{9}$$

$$f_2(\mathbf{x}) = N_{cp} - N_{tb} = 0 \tag{10}$$

$$f_3(\mathbf{x}) = P_5 - P_8 = 0 \tag{11}$$

where $f_i(\mathbf{x})$ denotes the residual of each matching condition. Physically, these equations enforce mass conservation across the core flow path, mechanical power balance between compressor and turbine, and pressure compatibility between turbine exit and nozzle inlet, respectively. The whole-engine matching problem is therefore cast as a system of nonlinear algebraic equations of the form:

$$\mathbf{f}(\mathbf{x}) = \mathbf{0}.$$

Since the residual equations are strongly nonlinear and tightly coupled through component interactions, the resulting system does not admit a closed-form analytical solution. This is primarily due

to the strong interdependence between component operating points, where variations in one module propagate nonlinearly to downstream and upstream components. As a result, the problem is formulated as a nonlinear root-finding problem. In this study, the solution is equivalently obtained by minimizing the residual norm $\|\mathbf{f}(\mathbf{x})\|$, which represents the overall mismatch among mass flow, power, and pressure balance constraints. The iterative process is continued until the residual norm satisfies a prescribed convergence tolerance.

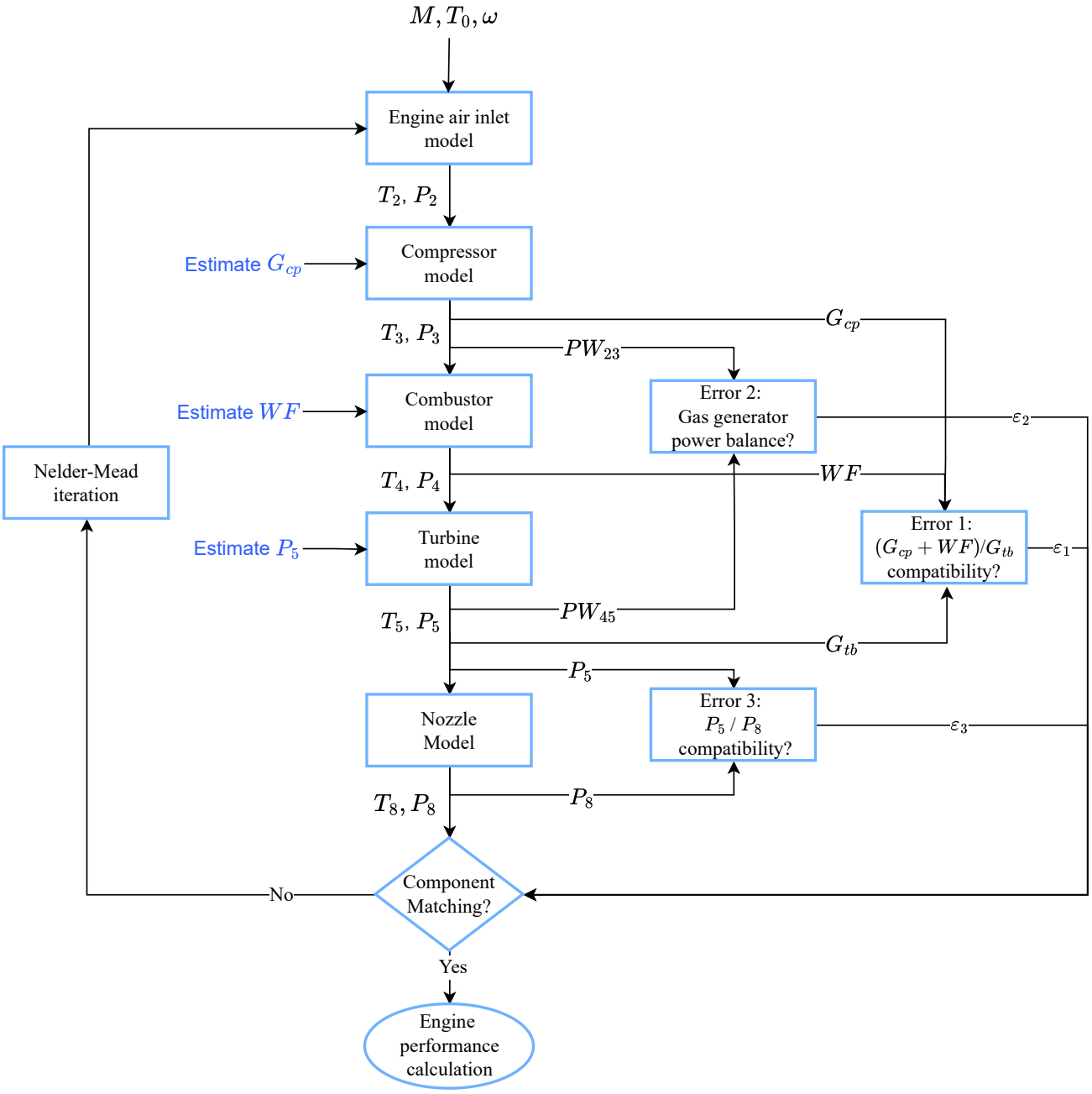


Figure 4 – Flow chart of a hybrid optimization strategy for whole-engine matching

Among various nonlinear optimization techniques, the Nelder–Mead simplex algorithm is adopted as the primary local search method due to its robustness and derivative-free nature. This characteristic is particularly advantageous for whole-engine matching problems, where the objective function is non-smooth and may exhibit discontinuities caused by the use of empirical component maps and piecewise-defined correlations. The method updates a simplex in the solution space through reflec-

tion, expansion, contraction, and shrinkage operations, enabling efficient local convergence without requiring gradient information.

For comparison, the Newton–Raphson method was also evaluated. Although it is known for its quadratic convergence near the solution, its performance in the present application was found to be sensitive to the quality of the initial guess. In addition, the need for Jacobian evaluation and inversion increases computational complexity, while strong nonlinear coupling between components may lead to ill-conditioned Jacobian matrices, resulting in convergence instability or divergence in certain operating regimes.

To overcome these limitations, a hybrid global–local optimization strategy is introduced. In the global exploration stage, Monte Carlo sampling is employed to generate a diverse set of initial candidate solutions across the feasible operating domain. This step enhances the probability of identifying regions close to physically meaningful equilibrium states and reduces the dependency on manually tuned initial guesses. Subsequently, the Nelder–Mead algorithm is applied as a local refinement tool to efficiently converge toward the final equilibrium solution. This two-stage strategy significantly improves both the robustness and convergence reliability of the overall whole-engine matching procedure. The overall solution procedure is summarized in Fig. 4.

3. Experiments and Analysis

To validate the proposed whole-engine matching framework, a small-scale turbojet engine configuration was considered. The engine consists of an axial compressor, an annular combustor, and a single-stage turbine, followed by a convergent nozzle. This configuration is representative of typical turbojet architectures and is suitable for evaluating the capability of the proposed hybrid 0D–2D modeling approach.

Performance predictions obtained from the proposed method were compared against available experimental data under both design-point and off-design operating conditions. The comparisons are summarized in Table 1 and Table 2, respectively.

At the design point, the proposed model shows good agreement with experimental measurements. All key performance parameters, including compressor mass flow rate, fuel mass flow rate, compressor outlet pressure, and net thrust, are predicted with deviations within approximately $\pm 5\%$. In particular, the overall mass flow balance and thrust prediction exhibit very good accuracy, indicating that the proposed matching strategy successfully captures the dominant thermodynamic and momentum exchange mechanisms within the engine system. Minor discrepancies are mainly attributed to simplified combustor modeling assumptions and empirical loss correlations used in the component-level models.

Table 1 – Comparison of the proposed method with experimental data at the design point

Parameter	Unit	Experiment data	Proposed Method	Deviation
Gin_{cp}	kg/s	1.000	0.977	-2.3
W_f	kg/s	1.000	0.957	-4.3%
P_3	atm	1.000	1.009	0.9 %
F_N	kgf	1.000	0.994	-0.6%

Under off-design operating conditions, the proposed framework maintains consistent predictive capability. Although the deviation in some parameters slightly increases compared to the design point, the overall trends remain well captured. In particular, the compressor outlet pressure shows a larger deviation, which can be associated with the increased sensitivity of off-design operation to component matching and nonlinear coupling effects. Nevertheless, the fuel consumption prediction remains within acceptable accuracy, demonstrating the robustness of the proposed hybrid modeling and solution strategy across different operating regimes.

Overall, the results confirm that the proposed hybrid 0D–2D whole-engine matching framework is capable of accurately predicting both design-point and off-design performance of a turbojet engine, while maintaining computational efficiency suitable for early-stage design and analysis.

Table 2 – Comparison of the proposed method with experimental data under off-design operating conditions

Parameter	Unit	Experiment data	Proposed Method	Deviation
W_f	kg/s	1.000	0.976	-2.4%
P_3	atm	1.000	0.952	-4.8 %

4. Conclusion

A hybrid 0D–2D whole-engine matching framework for turbojet engines has been developed to achieve a balance between physical fidelity and computational efficiency. The proposed approach integrates two-dimensional throughflow modeling for key turbomachinery components, namely the compressor and turbine, with simplified zero-dimensional performance models for the inlet, combustor, and nozzle. In addition, a hybrid global–local optimization strategy combining Monte Carlo sampling and Nelder–Mead optimization is implemented to improve the robustness and convergence reliability of the equilibrium solution procedure. The performance of the proposed framework has been evaluated against experimental data under both design-point and off-design operating conditions. The results demonstrate good agreement, with all key performance parameters exhibiting deviations within approximately 5%. In particular, the model is able to accurately capture the overall trends of mass flow, pressure ratio, and thrust across different operating regimes, confirming its capability to represent the dominant thermodynamic and aerodynamic interactions within the engine system. Overall, the proposed method provides a computationally efficient and sufficiently accurate tool for turbojet engine performance prediction. It is therefore well suited for preliminary design studies and early-stage performance assessment, where a trade-off between model fidelity and computational cost is required.

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References

[1] Qusai Al-Hamdan and Munzer Ebaid. A new approach of gas turbine component matching for electrical power generation. *International Journal of Mechanical Engineering and Applications*, 5(4):214–222, 2017.

[2] SM Hosseinimaab and AM Tousi. A new approach to off-design performance analysis of gas turbine engines and its application. *Energy Conversion and Management*, 243:114411, 2021.

[3] Shuo Zhang, Aotian Ma, Teng Zhang, Ning Ge, and Xing Huang. A performance simulation methodology for a whole turboshaft engine based on throughflow modelling. *Energies*, 17(2):494, 2024.

[4] Mateus Teixeira, Luigi Romagnosi, Mohamed Mezine, Yannick Baux, Jan Anker, Kilian Claramunt, and Charles Hirsch. A methodology for fully-coupled cfd engine simulations, applied to a micro gas turbine engine. In *Turbo Expo: Power for Land, Sea, and Air*, volume 51012, page V02CT42A047. American Society of Mechanical Engineers, 2018.

[5] Jeffryes W Chapman, Thomas M Lavelle, Ryan D May, Jonathan S Litt, and Ten-Huei Guo. Toolbox for the modeling and analysis of thermodynamic systems (t-mats) user's guide. Technical report, 2014.

[6] Ye Wang, Zepeng Wang, Xizhen Wang, Bokun Zhao, and Yongjun Zhao. A novel performance adaptation method for aero-engine matching over a wide operating range. *Journal of the Global Power and Propulsion Society*, 8:154–165, 2024.

[7] G Venkat Narayanan. A comparative study of numerical methods for solving nonlinear equations. *International Journal of Applied Mathematics*, 38(5s):898–912, 2025.

[8] Huda Farag Alspihe. A mathematical analysis of the newton-raphson method for solving nonlinear equations: Local convergence properties and the impact of initial values. pages 770–774, 2025.

[9] Nicholas Metropolis and Stanislaw Ulam. The monte carlo method. *Journal of the American statistical association*, 44(247):335–341, 1949.

[10] John A Nelder and Roger Mead. A simplex method for function minimization. *The computer journal*, 7(4):308–313, 1965.