

A MPC–MHE STRATEGY FOR FREQUENCY TRACKING AND PARAMETERS ESTIMATION OF RESONANT DE-ICING SYSTEMS

Aleksander Suti[1], Jason Pothin[1,2], Marc Budinger[1], Valérie Pommier-Budinger[2] & Aurélien Reysset[1]

[1]Dep. of Mechanical Engineering, ICA, Université de Toulouse, UPS, INSA, ISAE-SUPAERO, MINES-ALBI, CNRS, ICA, 3 rue Caroline Aigle, Toulouse, 31400, France
[2]Fédération ENAC ISAE-SUPAERO ONERA, Université de Toulouse, France

Abstract

Resonance-based de-icing systems that use electromechanical actuators are emerging as a more energy-efficient alternative to conventional anti-icing technologies, whether thermal or pneumatic. These systems use resonance excitation to effectively remove ice while significantly reducing energy consumption. Vector control schemes combined with proportional-integral controllers have been successfully implemented to achieve resonance frequency tracking during ice deposition and de-icing. However, these controllers cannot estimate the modal parameters that are characteristic of ice accumulation. This accumulation progressively alters the modal mass and stiffness of the structure, leading to a drift in the resonance frequency during icing. Monitoring these parameters would enable tracking of the icing conditions and provide the necessary information to trigger de-icing. To this end, this paper proposes an adaptive control framework combining Model Predictive Control (MPC) and Moving Horizon Estimation (MHE). The MHE algorithm continuously estimates the equivalent modal mass and stiffness of the iced structure from the system output responses. These estimates are used to update the prediction model employed by the MPC, which can be used to trigger the actuation system for de-icing. As a reference application, a structure equipped with a piezoelectric resonant de-icing system is considered. Model-in-the-loop simulations demonstrate the ability to accurately track resonance and estimate parameters under gradual variations representative of ice accretion events, as well as under abrupt variations representative of de-icing events. This highlights the potential of the proposed adaptive approach for improving the operation of resonant ice protection systems.

Keywords: Model Predictive Control; Moving Horizon Estimation; Ice Protection System; Piezoelectric actuator; frequency tracking; parameters estimation

1. Introduction

In-flight ice accretion remains a safety-critical challenge for airborne vehicles affecting lifting surfaces, sensors and propulsion components and potentially leading to loss of lift, control degradation or structural damage, causing several severe aircraft accidents over the past three decades [1]. The drive toward decarbonisation is reshaping ice-protection systems (IPSs) toward more-electric, low-energy solutions, such as resonant piezoelectric (PZT) or electromechanical de-icing systems [2, 3]. Conventional ice protection systems, such as hot-air bleed, electrothermal heaters or pneumatic boots, are mature but suffer from high energy consumption, added mass and integration constraints. On the other hand, resonant de-icing systems generate the local stress required to initiate cohesive and adhesive ice fracture by driving the host iced structure at one of its natural frequencies, while keeping the injected power orders of magnitude below that of thermal solutions [3, 4, 5]. The effectiveness of resonant de-icing on NACA airfoil leading edges has been experimentally demonstrated in [6], where complete de-icing was achieved within a few hundred milliseconds. However, the same study highlighted that the efficiency of such systems is conditioned by the accuracy of the resonance

tracking, as frequency detuning of only a few hertz can significantly reduce vibration amplitudes and prevent fracture initiation. This challenge is compounded by the fact that the resonance frequency is inherently time-varying. During ice accretion, the additional ice mass and the associated changes in structural stiffness progressively modify the modal properties of the system, causing a shift in the resonance frequency. Conversely, ice fracture and shedding produce abrupt changes in the modal parameters, resulting in rapid variations in the resonant dynamics. Tracking these dynamics in real time is therefore essential to ensure reliable closed-loop operation.

Many resonance tracking methods have been proposed, from equivalent-circuit admittance identification [7] and perturb-and-observe algorithms [8] to Kalman-filter-based backbone trackers [9] and phase-locked-loop schemes [10]. A particularly attractive solution for ultrasonic actuators and high-frequency PZT systems is the vector control technique [11]. A vector control technique transforms a sinusoidal quantity $(x(t))$ into two equivalent DC components (x_q, x_d) in a rotating d-q frame. In this frame, perfect resonance corresponds to regulating the direct component to zero $(x_d = 0)$, providing a simple and robust regulation objective [11]. To enforce this condition, proportional–integral (PI) controllers are commonly employed. However, these controllers are typically designed around fixed system parameters and may experience performance degradation when the resonance frequency varies significantly as a consequence of ice accretion and shedding. Since these phenomena continuously modify the modal properties of the structure, maintaining accurate resonance tracking requires both online estimation of the system parameters and continuous adaptation of the control action. Monitoring these modal parameters can also help to track icing conditions, providing the necessary information to trigger de-icing procedures.

To address this challenge, this paper proposes an adaptive control framework based on the combination of Model Predictive Control (MPC) [12] and Moving Horizon Estimation (MHE) [13]. MPC is an optimization-based control technique that predicts the future system evolution over a finite horizon and computes the control action by minimizing a cost function while explicitly accounting for system constraints. To ensure consistency between the prediction model and the actual iced structure, the MHE algorithm estimates the time-varying modal parameters of the system by solving an optimization problem over a finite window of past measurements. Compared with recursive estimation methods, MHE naturally accommodates system constraints and provides improved robustness in the presence of disturbances and measurement noise. In the proposed framework, the estimated natural frequency and modal stiffness are used to continuously update the prediction model, enabling accurate resonance tracking despite the evolving dynamics induced by ice accretion and fracture events.

The proposed approach is evaluated through Model-in-the-Loop (MIL) simulations of a PZT-based de-icing system integrated on a flat structure. The performance of the adaptive MPC–MHE framework is assessed against a non-adaptive MPC baseline. The results demonstrate accurate resonance tracking and reliable estimation of the modal parameters under both gradual parameter variations associated with ice accretion and abrupt changes representative of ice fracture and shedding events. The Optimal Control Problems (OCPs) associated with both the MPC and the MHE are formulated and solved using the CasADi framework [14, 15], which provides an efficient symbolic environment for the formulation and numerical solution of nonlinear optimization problems. The continuous-time optimization problems are transcribed into nonlinear programming (NLP) problems using a direct multiple-shooting formulation, enabling efficient solution while explicitly accounting for the system dynamics and operational constraints.

The proposed framework offers a twofold contribution to the development of resonant ice protection systems:

- Adaptive resonance tracking: The MHE produces online estimates of parameters that are fed back to the MPC prediction model, yielding an adaptive controller that remains tuned to the actual resonance even when the modal parameters drift due to ice accretion.
- Ice-accretion monitoring: The estimates provide a physical observable directly related to the ice mass and adhesion. The monitoring of these parameters can be used as a decision signal to trigger the ice protection system.

The paper is organised as follows: Section 2. describes the system under study and provides the model in the d–q frame. This section also formulates the adaptive MPC and the MHE problems. Section 3. presents the frequency tracking and parameter estimation results under various drift scenarios and analyses the performance of the closed-loop framework. Section 4. concludes and outlines future work.

2. Materials and Methods

2.1 System description

While the methodology proposed by the authors in [16] targets PZT-based ice protection systems for aeronautical structures and can be applied to geometries such as composite NACA 0024 leading edges, the objective of the present work is different. Rather than focusing on the optimization of a particular structural design, this paper aims to develop and assess an adaptive control framework for online modal parameter identification valuable for resonance tracking.

For this purpose, the methodology is demonstrated on a simplified test article previously employed by the authors in [17] to investigate the open-loop dynamics of resonant de-icing systems. The test article consists of a titanium plate ($130 \times 50 \times 1$ mm^3) equipped with two PIC 181 piezoelectric patches of dimensions $70 \times 17 \times 1$ mm^3 each, bonded on the clean face of the plate, as shown in Figure 1. This configuration maximizes the coupling with the targeted flexural vibration mode while providing a representative platform for evaluating the proposed control and estimation strategy. One piezoelectric patch is used for actuation, whereas the second is employed for sensing.

The plate is in free boundary conditions on its four edges and its surface is covered with a 3 mm-thick ice layer. The ice is modeled as an isotropic material with a Young's modulus of 9 GPa [18], a density of 900 kg/m^3, and a Poisson's ratio of 0.3 [19], which corresponds to the properties of the freezer ice used in the tests. The plate is made of Ti-6Al-4V, with a Young's modulus of 110 GPa, a density of 4510 kg/m^3, and a Poisson's ratio of 0.33 [20].

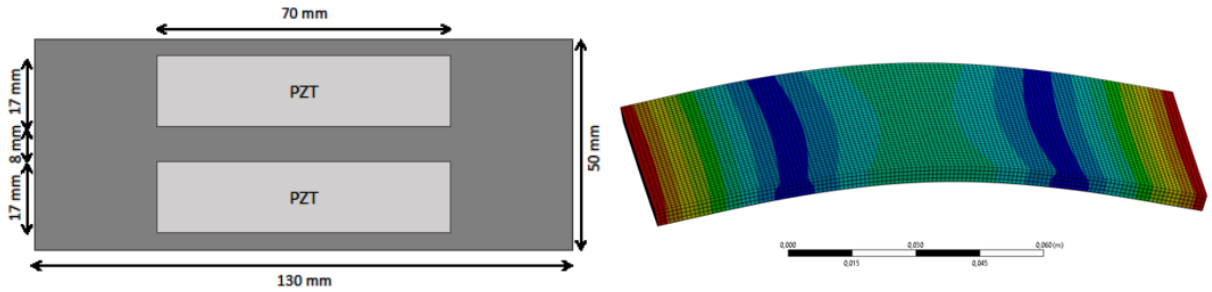


Figure 1 – Left to right, Geometry of the plate and First flexural mode of the titanium plate ([17])

As justified in [17], the first flexural mode of the plate is selected as the actuation mode, as it shows significant variation with ice accretion, representing a challenging case for frequency tracking. The corresponding mode shape, identified by finite-element modal analysis, is reported in Figure 1.

2.2 System modelling

The dynamics of the flexural mode of the reference structure equipped with the piezoelectric actuator (Figure 2) and the short-circuited piezoelectric sensor can be described using a lumped parameter spring–mass–damper model, through the following set of equations, [17]:

$$\begin{cases} M\ddot{x} + D_s\dot{x} + Kx = f_p = N_a u_a \\ i_a = i_m + i_c = N_a\dot{x} + C_0\dot{u}_a \\ i_s = N_s\dot{x} \end{cases} \tag{1}$$

where x is the modal displacement, u_a the actuator driving voltage, i_a the actuator current and i_s the sensor current. i_m and i_c are the motional and capacitive currents in the actuator. Since the sensor is short-circuited, the capacitive current in the sensor is null, meaning that the sensor current is only composed of the motional current. The mechanical parameters M, D_s and K are the modal

mass, damping and stiffness, respectively. N_a and N_s are the electromechanical coupling factors of the actuator and sensor. C_0 is the actuator static capacitance. The mechanical quality factor of the system is defined as $Q_m = \sqrt{KM}/D_s$.

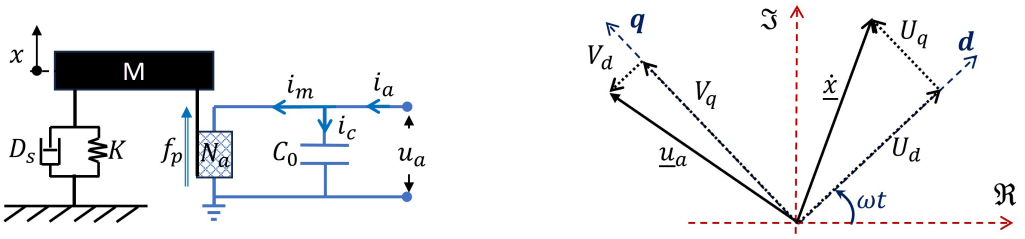


Figure 2 – System modelling: (left) Equivalent electromechanical scheme of the PZT actuators integrated with the structure; (right) Representation of the voltage and vibration phasors in a rotating reference frame.

Model in (1) is useful for describing the instantaneous evolution of the vibration. However, for the targeted application, describing the amplitude of the vibration speed is more relevant than the vibration speed itself. As the system is excited by a sinusoidal voltage at the angular frequency ω, the vibration speed $\dot{x}$ and the actuator voltage u_a can be projected into a rotating reference frame, Figure 2 (right), and written as follows:

$$\begin{aligned} \underline{\dot{x}} &= (V_d + jV_q)\, e^{j\omega t} \\ \underline{u_a} &= (U_d + jU_q)\, e^{j\omega t} \end{aligned} \tag{2}$$

where the underline denotes a complex quantity, and the corresponding physical signal is obtained as the real part of the complex representation, i.e., $\dot{x} = \Re\{\underline{\dot{x}}\}$. By substituting the complex representation of (2) into the first of (1), and under the assumptions reported in [11], the vibration velocity dynamics in the rotating d–q reference frame can be expressed in vector form as:

$$\dot{\mathbf{x}}(t) = \mathbf{f}_c\big(\mathbf{x}(t),\, \omega(t)\big), \tag{3}$$

$$\mathbf{f}_c(\mathbf{x},\, \omega) = \begin{bmatrix} \big(N_a \overline{U}_q - 2M\,(\omega - \omega_0)\, V_d - D_s V_q\big)/(2M) \\ \big(2M\,(\omega - \omega_0)\, V_q - D_s V_d\big)/(2M) \end{bmatrix}, \tag{4}$$

where $\mathbf{x} = [V_q\ V_d]^\top$ defines the state vector, with V_q and V_d the in-phase and quadrature components of the modal velocity, $\omega_0 = \sqrt{K/M}$ is the natural angular frequency and $\overline{U}_q$ is the quadrature voltage amplitude imposed by the inverter. During the resonance-tracking phase, $\overline{U}_q$ is assumed to be constant and set at a low level to excite the structure without initiating de-icing. Furthermore, the direct voltage component is set to $U_d = 0$, such that the excitation angular frequency ω constitutes the sole control input of the system.
In steady-state conditions, (4) implies that $V_d = 0$ if the system is excited at the resonance frequency ($\omega = \omega_0$).Consequently, tracking the resonance can be achieved by driving the system to the steady-state setpoint $V_d^{\#} = 0$. For these conditions, the quadrature component converges to $V_q^{\#} = N_a \overline{U}_q / D_s$, i.e. a value determined by the quadrature voltage amplitude $\overline{U}_q$.

Ice accretion progressively modifies the equivalent mass M, the damping D_s, the stiffness K and, consequently, the resonance frequency ω_0. In this work the ice accretion process is simulated by assuming that: (i) the ice layer can reach a thickness of approximately 4 mm per minute, corresponding to a typical threshold for de-icing system activation; (ii) the environmental and flight conditions, including temperature, airspeed, and liquid water content, remain constant during the considered icing period; (iii) over the prediction horizon of the controller (approximately one second), the model parameters associated with the ice layer can be assumed constant so that to the time-invariant modelling is to be extended with the following:

$$\dot{\mathbf{p}} \approx 0 \tag{5}$$

where $\mathbf{p} = [\omega_0, K]^\top$ is the parameter vector.

2.3 MPC for resonance frequency tracking

The resonance frequency tracking problem for the PZT-based de-icing system is addressed with the objective of maintaining accurate resonance tracking despite the time-varying dynamics induced by ice accretion and shedding, while explicitly accounting for system constraints. The block of the proposed MPC-MHE strategy for frequency tracking and parameter estimation of resonant de-icing systems is shown in Figure 3, where the block labeled AC/DC represents the shift to the d-q frame, which transforms AC components into DC components through a lock-in amplifier, while the DC/AC block performs the inverse transformation by means of a Voltage-Controlled Oscillator (VCO), followed by a voltage amplifier.

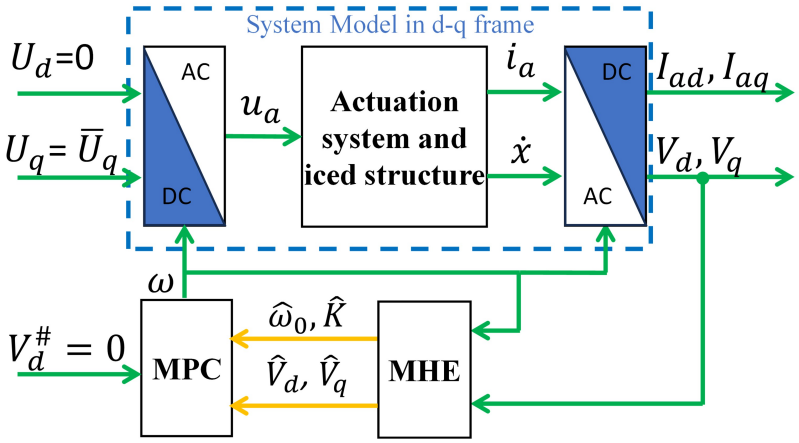


Figure 3 – Functional block diagram of the proposed MPC-MHE strategy for frequency tracking and parameter estimation of resonant de-icing systems.

Based on these objectives, the resonance frequency tracking problem is formulated as a finite-horizon optimal control problem and solved within an MPC framework. This optimal control problem is formulated in (OCP) and is based on a cost function comprising two main terms: one for tracking the reference state vector, and another for penalising variations in the excitation frequency and reducing aggressive control actions, thereby ensuring smoother actuator behaviour. The optimization problem is solved subject to equality constraints describing the system dynamics, together with inequality constraints enforcing operational limits on both the system states and the control inputs. In this case, the model (3) is discretised with the forward-Euler scheme with sampling period Δt and the adaptive MPC predicts the state trajectory over a horizon of N_h samples using the *current estimates* of the modal parameters supplied by the MHE (Section 2.4).

$$
\begin{aligned}
\min_{\{\omega(k),\mathbf{x}(k)\}} \quad & Q_{V_d} \sum_{k=1}^{N_h} \left(V_d(k+1) - V_d^{\#}\right)^2 + R_{\Delta u} \sum_{k=1}^{N_h-1} \left(\omega(k+1) - \omega(k)\right)^2 \\
\text{s.t.} \quad & \mathbf{x}(k+1) = \mathbf{x}(k) + \Delta t\, \mathbf{f}_c\big(\mathbf{x}(k), \omega(k), \hat{\mathbf{p}}(k)\big), && k = 1,\dots,N_h \\
& \mathbf{x}(1) = \mathbf{x}_0, \quad \omega(1) = \omega_{\text{in}}, \\
& \mathbf{x}^{\min} \le \mathbf{x}(k) \le \mathbf{x}^{\max}, && k = 1,\dots,N_h+1 \\
& \omega^{\min} \le \omega(k) \le \omega^{\max}, && k = 1,\dots,N_h
\end{aligned}
\tag{OCP}
$$

In (OCP1) the state tracking and the control action contributions are smoothed and weighted by Q_{V_d} and $R_{\Delta u}$, respectively. The reference state is setpoint $V_d^{\#} = 0$, while ω_{in} is the frequency applied at the initial instant.

2.4 MHE for states and parameters estimation

To estimate the variations of the modal parameters induced by ice accretion, a MHE technique is here employed. Differently from recursive estimators, where the current state estimate is based only on the most recent measurement and the previous state estimate, the MHE considers the evolution of a constrained and possibly nonlinear model on a fixed time horizon, and minimizes the deviation of the

model from a past window of measurements. Although MHE does not generally rely on any specific error distribution, tuning the estimator becomes easier when a probabilistic insight is considered [21]. To this end, a disturbed form of (3) is defined, together with a measurement output, as follows:

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}_c\big(\mathbf{x}(t),\, \omega(t), \mathbf{p}(t)\big) + \mathbf{v}_x, \\ \mathbf{y}(t) = \mathbf{h}(\mathbf{x}(t)) + \mathbf{v}_y, \end{cases} \tag{6}$$

where $\mathbf{v}_x$ and $\mathbf{v}_y$ are zero-mean Gaussian noises for states and outputs, respectively. Then, the MHE observer jointly estimates the state and the parameters over a sliding window of the last N_e+1 measurements. This optimal estimation problem is formulated in (OEP) as a constrained least-squares (LSQ) problem in which the parameters are treated as augmented decision variables alongside the state trajectory [22, 23].

$$\begin{aligned} \min_{\{\mathbf{x}(k),\, \omega(k),\, \mathbf{p}(k)\}} \quad & \sum_{k=i-N_e}^{i} \|\mathbf{h}(\hat{\mathbf{x}}(k)) - \bar{\mathbf{y}}(k)\|^2_{\mathbf{V}} \;+\; W \sum_{k=i-N_e}^{i-1} \big(\omega(k) - \bar{\omega}(k)\big)^2 \\ & + \sum_{k=i-N_e}^{i-2} \|\hat{\mathbf{p}}(k+1) - \hat{\mathbf{p}}(k)\|^2_{\mathbf{W_P}} \\ \text{s.t.} \quad & \mathbf{x}(k+1) = \hat{\mathbf{x}}(k) + \Delta t\, \mathbf{f}_c\big(\hat{\mathbf{x}}(k), \omega(k); \hat{\mathbf{p}}(k)\big) \qquad k = i-N_e, \ldots, i-1 \\ & \mathbf{x}^{\min} \le \hat{\mathbf{x}}(k) \le \mathbf{x}^{\max}, \qquad k = i-N_e, \ldots, i \\ & \boldsymbol{\omega}^{\min} \le \boldsymbol{\omega}(k) \le \boldsymbol{\omega}^{\max}, \qquad k = i-N_e, \ldots, i-1 \\ & \mathbf{p}^{\min} \le \hat{\mathbf{p}}(k) \le \mathbf{p}^{\max}, \qquad k = i-N_e, \ldots, i \end{aligned} \tag{OEP}$$

In (OEP), the cost function penalizes the change in the system predicted outputs $\mathbf{h}(\mathbf{x})$ from the actually measured outputs $\bar{\mathbf{y}}$ by the diagonal positive-definite matrix $\mathbf{V}$. Similarly, a weight W penalizes the change in the applied control inputs ω from the measured inputs $\bar{\omega}$. This latter accounts for actuator noise and/or inaccuracy. These weights are chosen to match the measurements and the applied control input noise covariances. Finally, the cost function includes a smoothness prior on the estimated parameters weighted by $\mathbf{W_P}$, which encodes the slow-drift assumption. In (OEP) $\|*\|^2_{\blacksquare}$ denotes the Euclidian norm with diagonal positive-definite matrices $\blacksquare(=\mathbf{V}, \mathbf{W_P})$. Here $\mathbf{V}$ and W are set as the inverse of the assumed measurement covariances, while elements of $\mathbf{W_P}$ are normalized weights $\mathbf{W}_P = \mathrm{diag}\left(\dfrac{\rho_{\omega_0}}{\omega_{0,\mathrm{nom}}}, \dfrac{\rho_K}{K_{\mathrm{nom}}}\right)$ penalise relative variations in the modal parameter estimation within the window. This regularisation is essential when measurements are noisy and the control action is close to the resonance. In this case, the parameters are weakly identifiable because the system dynamics depend mostly on the combination $\omega - \omega_0$, and without regularisation the optimiser tends to trade off the parameters against each other. The solution of the (OEP) leads mainly to the relative-state and parameters estimate sequence $\hat{\mathbf{x}}(k), \hat{\mathbf{p}}(k)$, with $k = (i-N_e, ..., i)$, where $\hat{\mathbf{x}}(i), \hat{\mathbf{p}}(i)$, denotes the current state and parameter vector estimate that can be used as a measurement feedback for the MPC.

2.5 Numerical solution and implementation

The (OCP) and the (OEP) are transcribed into nonlinear programs (NLPs) via direct multiple shooting [24]. Both problems are formulated in the CasADi framework [14] and solved by the IPOPT interior-point solver [15]. At each sampling point, the optimiser is warm-started using the solution from the previous step, shifted by one sample. This reduces the number of inner iterations, bringing the solve time per step down to just a few tens of milliseconds on a standard laptop CPU.

The two problems are solved sequentially in a receding-horizon fashion: at each instant t_k, the (OEP) is solved on the last N_e+1 samples to update $\hat{\mathbf{x}}(i), \hat{\mathbf{p}}(i)$; the (OCP) is then solved with the updated prediction model and only the first control element $\omega(1)$ is applied.

The numerical parameters used in the rest of the paper are summarized in Table 1.

Table 1 – Parameters used in the simulations.

Symbol	Value	Description
M_{nom}	0.074 kg	Nominal modal mass (with reference ice layer)
K_{nom}	3.554×10^6 N/m	Nominal modal stiffness
Q_m	60	Modal quality factor
N_a	0.0201 N/V	Actuator coupling coefficient
$\overline{U}_q$	40 V	Constant quadrature drive voltage amplitude
$\omega_{0_{\text{nom}}}$	6930.4 rad/s ($f_0 \approx 1103$ Hz)	Nominal resonance
$V_q^{\#}$	4.68 m/s	Quadrature velocity reference (at resonance)
Δt	1×10^{-3} s	Sampling period
N_h	50 samples	MPC horizon
N_e	10 samples	MHE window length
Q_{V_d}	5	State tracking weights (MPC)
$R_{\Delta u}$	0.1	Control smoothness weight (MPC)
ρ_{ω_0}	10^3	Smoothness prior on ω_0 (MHE)
ρ_K	10^4	Smoothness prior on K (MHE)
$[\omega^{\min}, \omega^{\max}]$	$[0.5, 1.5]\,\omega_{0_{\text{nom}}}$	Drive frequency bounds
$[\omega_0^{\min}, \omega_0^{\max}]$	$[0.85, 1.15]\,\omega_{0_{\text{nom}}}$	MHE bounds on ω_0
$[K^{\min}, K^{\max}]$	$[0.6, 1.4]\,K_{\text{nom}}$	MHE bounds on K

3. Results

The closed-loop performance of the proposed adaptive MPC–MHE framework is assessed through Model-in-the-Loop (MIL) simulations. The following test scenarios are considered:

1. Constant-parameter test: the modal parameters are kept constant throughout the simulation. This scenario is used as a baseline case to validate the MPC performance under nominal operating conditions.

2. Ice accretion and de-icing test: the modal parameters are subjected to a linear drift, representing the progressive accumulation of ice, followed by abrupt variations that emulate ice fracture and shedding. After the de-icing event, the parameters return to their clean-structure values. This scenario is used to evaluate the ability of the MHE to track variations in parameters and the ability of the adaptive MPC to maintain resonance conditions in the face of evolving system dynamics. In addition, a test with non-adaptive MPC that uses the nominal parameters throughout the simulation is proposed to highlight the benefits of the proposed MPC-MHE strategy.

3.1 Constant parameter test

The first test confirms that, with a time-invariant system, the MPC drives the excitation angular frequency towards resonance from an initial low-frequency condition. As shown in Figure 4, the results show that the pulsation frequency starts from $\omega_{\text{init}} = 0.5\,\omega_0$ and converges to ω_0 within approximately 20 ms. Once ω reaches its steady-state value, the coupling term between the V_d and V_q dynamics becomes negligible, and the two states evolve approximately as independent linear systems. Consequently, V_d returns to zero while V_q rises to its equilibrium point $V_q^{\#} = N_a, \overline{U}_q / D_s \approx 4.66$ m/s, with a characteristic time of approximately $2M/D_s$.

3.2 Linear-variant parameter test representative of ice accretion and de-icing test

To evaluate the proposed adaptive control framework under representative operating conditions, a simplified ice accretion and de-icing scenario is considered. Ice accretion is modeled as a linear drift of the modal parameters, with the natural frequency (ω_0) increasing by 1% per second and the modal stiffness (K) increasing by 2% per second. This drift is applied for one second, corresponding to a representative ice accretion phase. At the end of the accretion period, at one second, a de-icing event is simulated by abruptly restoring the modal parameters to their clean-structure values. Subsequently, the ice accretion process is restarted for one second following the same parameter growth rates. This scenario allows the assessment of the MHE capability to track both gradual parameter variations

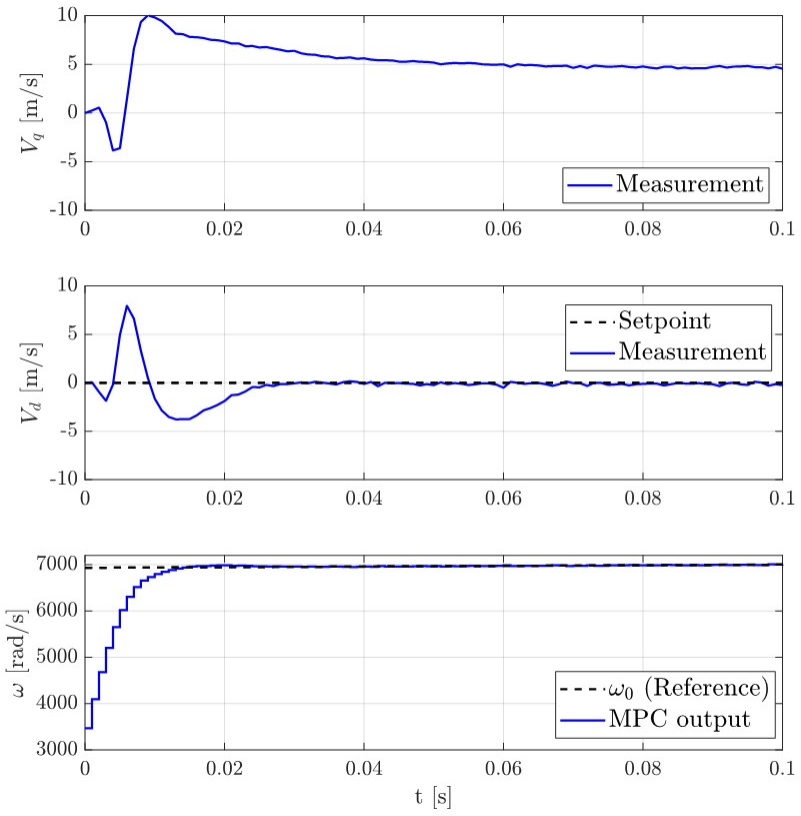


Figure 4 – Constant-parameter simulation results: (top) quadrature velocity V_q; (center) direct velocity V_d; (bottom) angular frequency ω.

associated with ice accumulation and abrupt changes resulting from ice fracture and shedding. It also assesses the adaptive MPC's capacity to maintain resonance tracking under evolving system dynamics. The results of this test are proposed in Figures 5 and 6, showing the system states V_q and V_d and the control action ω, and the parameter estimation, respectively.
Figure 5 shows that, after the initial transient, the direct velocity (V_d) component converges to zero, indicating that resonance conditions are successfully maintained. Consistently, the MPC progressively increases the excitation frequency to compensate for the resonance drift induced by the evolving modal parameters, while the quadrature component (V_q) converges to its equilibrium value.

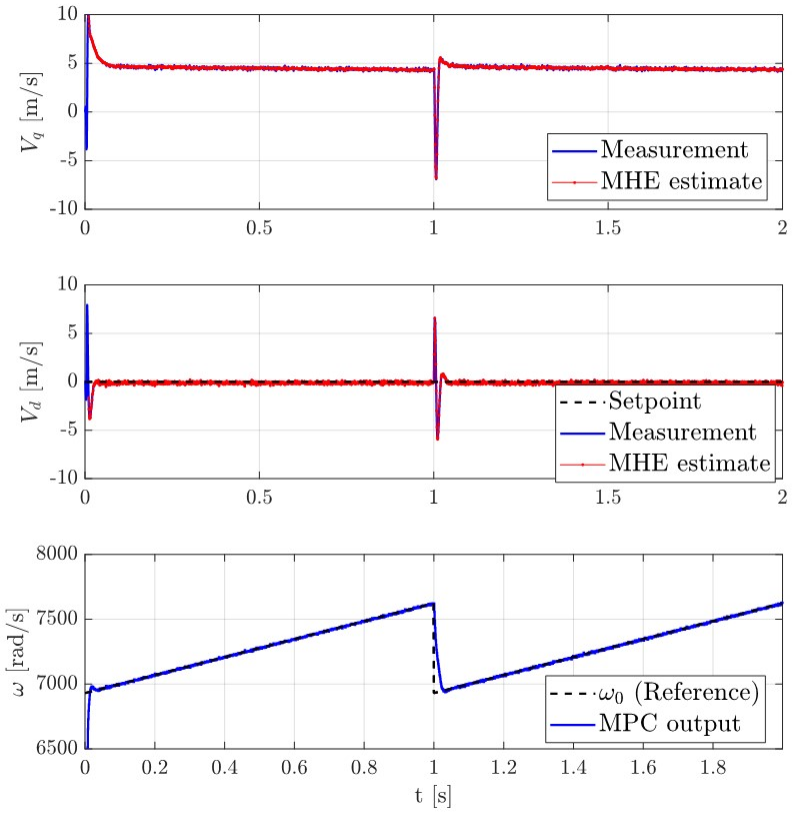


Figure 5 – Linear-variant parameter simulation results with MHE: (top) quadrature velocity V_q; (center) direct velocity V_d; (bottom) angular frequency ω.

The parameter estimation results reported in Figure 6 demonstrate that the MHE can accurately reconstruct the time-varying modal parameters. To achieve more realistic results, the simulations are run with noise injected into the signals (V_d and V_q), which are representative of the type of noise that could be measured. A close agreement between the reference and estimated values is observed during both the gradual parameter drift associated with ice accretion and the abrupt variations representative of de-icing events. Moreover, the parameter estimation dynamics exhibit a response time comparable to that of the resonance-tracking variable (V_d), confirming the effectiveness of the proposed adaptive estimation framework.
Finally, to highlight the role of the MHE feedback, the same simplified ice accretion and de-icing scenario is considered with a non-adaptive MPC that uses the nominal conditions (structure with no ice) throughout the simulation. As shown in Figure 7, satisfactory resonance tracking is only achieved when the system parameters remain close to the nominal conditions (i.e. the plate without ice), which occurs at the start of the simulation and immediately after the de-icing event. As the modal parameters progressively drift due to ice accretion, the mismatch between the prediction model and the actual system increases, leading to a degradation of the resonance-tracking performance.

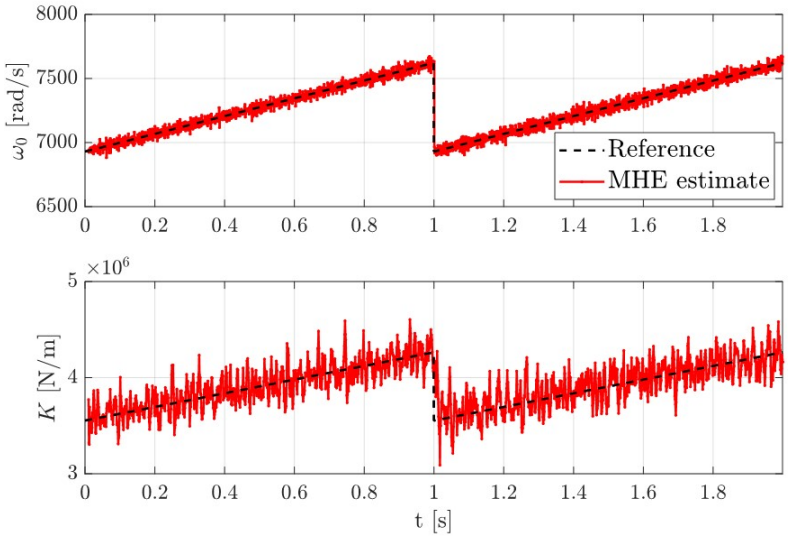


Figure 6 – Linear-variant parameter simulation results with MHE: (top) resonance frequency; (bottom) stiffness.

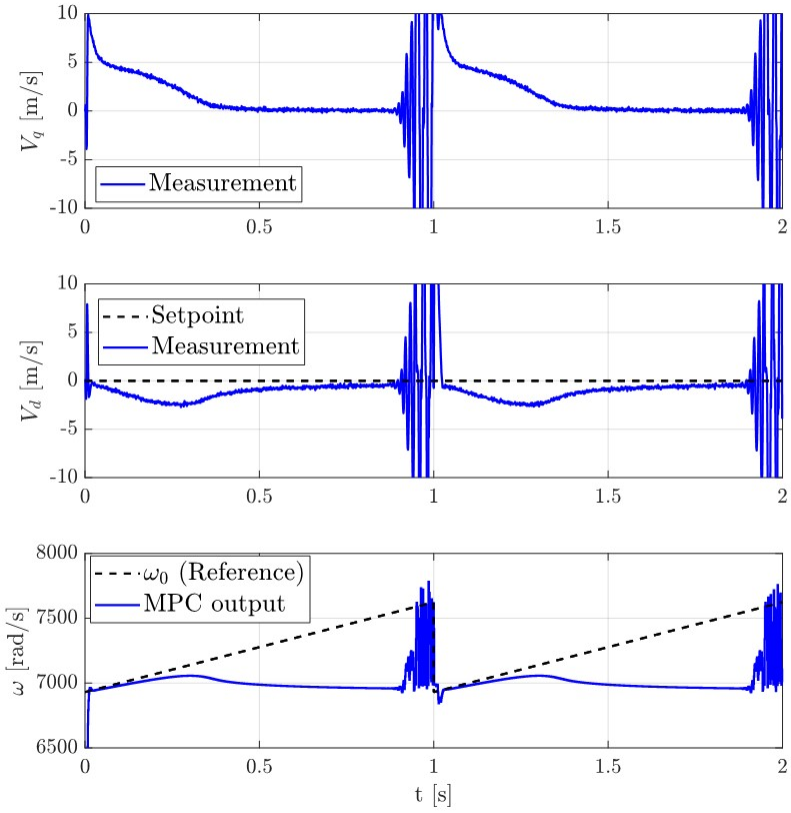


Figure 7 – Linear-variant parameter simulation results without MHE: (top) quadrature velocity V_q; (center) direct velocity V_d; (bottom) angular frequency ω.

4. Conclusions

This paper presents a preliminary adaptive resonance-tracking framework for piezoelectric electromechanical de-icing systems based on the combination of Model Predictive Control (MPC) and Moving Horizon Estimation (MHE). The proposed approach exploits the rotating (d)-(q) representation of the PZT–structure dynamics, while the MHE continuously estimates the modal parameters of the iced structure and updates the prediction model employed by the MPC. In this way, the controller is able to adapt to the evolving structural dynamics induced by ice accretion and shedding while explicitly accounting for system constraints. The proposed framework was assessed through Model-in-the-Loop simulations under representative ice accretion and de-icing scenarios. The results showed that the adaptive controller successfully tracked the resonance during both gradual parameter variations associated with ice growth and abrupt changes representative of ice fracture and shedding. The MHE was able to accurately reconstruct the time-varying modal parameters, providing estimates with dynamics compatible with the resonance-tracking requirements. In contrast, a non-adaptive MPC based on fixed nominal parameters exhibited a progressive degradation of tracking performance as the mismatch between the prediction model and the actual system increased. Beyond the resonance-tracking capability, the estimated modal parameters constitute physically meaningful observables related to the icing state of the structure. Their evolution may therefore provide valuable information for online ice accretion monitoring and, in future developments, could support the triggering and supervision of de-icing actions within a fully closed-loop ice protection system. Future work will focus on experimental validation of the proposed MPC–MHE framework on representative PZT-based de-icing demonstrators, the inclusion of additional parameters, such as damping, and uncertainties within the estimation process, and a systematic comparison with conventional resonance-tracking approaches and state observers to quantify the benefits and limitations of the proposed adaptive strategy. Finally, the switching strategy from monitoring (adaptive MPC-MHE) to de-icing (high-amplitude resonance tracking) will be designed and validated under realistic icing conditions.

5. Contact Author Email Address

Mailto: jason.pothin@isae-supaero.fr

6. Copyright Statement

The authors confirm that they, and/or their company or organization, hold copyright on all of the original material included in this paper. The authors also confirm that they have obtained permission from the copyright holder of any third party material included in this paper, to publish it as part of their paper. The authors confirm that they give permission, or have obtained permission from the copyright holder of this paper, for the publication and distribution of this paper as part of the ICAS proceedings or as individual off-prints from the proceedings.

Funding

This research was supported by the ANR – FRANCE (French National Research Agency) through the project SmartICE (Grant Agreement No. ANR-25-CE51-5794-01).

References

[1] Darren G. Jackson and Joshua I. Goldberg. Ice detection systems: A historical perspective. 9 2007.

[2] Valérie Pommier-Budinger, Marc Budinger, Pierre Rouset, Fabien Dezitter, François Huet, Marc Wetterwald, and Elmar Bonaccurso. Electromechanical resonant ice protection systems: initiation of fractures with piezoelectric actuators. *AIAA Journal*, 56(11):4400–4411, 2018.

[3] Valerian Palanque. *Design of low consumption electro-mechanical de-icing systems*. PhD thesis, Université de Toulouse; ISAE-Supaero, 2022.

[4] Dominique Leguillon. Strength or toughness? a criterion for crack onset at a notch. *European Journal of Mechanics A/Solids*, 21:61–72, 2002.

[5] Giulia Gastaldo, Valerian Palanque, Marc Budinger, and Valerie Pommier-Budinger. Stress and energy release rate influence on ice shedding with resonant electro-mechanical de-icing systems. In *ICAS 2022-33rd Congress of the International Council of the Aeronautical Sciences*, 2022.

[6] Valerian Palanque, Jason Pothin, Valerie Pommier-Budinger, and Marc Budinger. Piezoelectric resonant ice protection systems–part 1/2: Prediction of power requirement for de-icing a naca 0024 leading edge. *Chinese Journal of Aeronautics*, 37(3):92–103, 2024.

[7] Jin-dong Wang, Jia-jia Jiang, Fa-jie Duan, Fu-min Zhang, Xing-hua Qu, et al. A novel fast resonance frequency tracking method based on the admittance circle for ultrasonic transducers. *IEEE Transactions on Industrial Electronics*, 67(8):6864–6873, 2019.

[8] Zhengyu Lin, Jiabin Wang, and David Howe. A resonant frequency tracking technique for linear vapor compressors. In *2007 IEEE International Electric Machines & Drives Conference*, volume 1, pages 370–375. IEEE, 2007.

[9] Nicolas Peyret, Jean-Luc Dion, and Gael Chevallier. A framework for backbone experimental tracking: Piezoelectric actuators, stop-sine signal and kalman filtering. *Mechanical Systems and Signal Processing*, 78:28–42, 2016.

[10] Vincent Denis, Maxime Jossic, Christophe Giraud-Audine, Benjamin Chomette, Alexandre Renault, and Olivier Thomas. Identification of nonlinear modes using phase-locked-loop experimental continuation and normal form. *Mechanical Systems and Signal Processing*, 106:430–452, 2018.

[11] Sofiane Ghenna, Frédéric Giraud, Christophe Giraud-Audine, and Michel Amberg. Vector control of piezoelectric transducers and ultrasonic actuators. *IEEE Transactions on Industrial Electronics*, 65(6):4880–4888, 2017.

[12] Eduardo F. Camacho, Carlos Bordons, and José M. Maestre. *Model Predictive Control*. Springer Nature Switzerland, 2026.

[13] James B. Rawlings and Douglas A. Allan. *Moving Horizon Estimation*, pages 1–7. Springer London, 2021.

[14] Joel A. E. Andersson, Joris Gillis, Greg Horn, James B. Rawlings, and Moritz Diehl. CasADi: a software framework for nonlinear optimization and optimal control. *Mathematical Programming Computation*, 11(1):1–36, 2019.

[15] Andreas Wächter and Lorenz T. Biegler. On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming. *Mathematical Programming*, 106(1):25–57, 2006.

[16] Younes Rafik, Valerian Palanque, Marc Budinger, Valerie Pommier-Budinger, and Philippe Olivier. Improving resonant ice protection systems with substrate optimization. *Smart Materials and Structures*, 2024.

[17] Jason Pothin, Aurélien Reysset, Marc Budinger, Giovanni Boscoletto, and Valerie Pommier-Budinger. Dynamic model of an electromechanical de-icing system coupling fracture and resonance. *Smart Materials and Structures*, 2025.

[18] Erland M Schulson. The structure and mechanical behavior of ice. *Jom*, 51:21–27, 1999.

[19] Malcolm Mellor. Mechanical properties of polycrystalline ice. In *Physics and Mechanics of Ice: Symposium Copenhagen, August 6–10, 1979, Technical University of Denmark*, pages 217–245. Springer, 1980.

[20] Celestino Veiga, J Paulo Davim, AJR Loureiro, et al. Properties and applications of titanium alloys: a brief review. *Rev. Adv. Mater. Sci*, 32(2):133–148, 2012.

[21] Mario Zanon, Janick V. Frasch, and Moritz Diehl. Nonlinear moving horizon estimation for combined state and friction coefficient estimation in autonomous driving. In *2013 European Control Conference (ECC)*, pages 4130–4135. IEEE, 7 2013.

[22] Christopher V. Rao, James B. Rawlings, and David Q. Mayne. Constrained state estimation for nonlinear discrete-time systems: stability and moving horizon approximations. *IEEE Transactions on Automatic Control*, 48(2):246–258, 2003.

[23] James B. Rawlings, David Q. Mayne, and Moritz M. Diehl. *Model Predictive Control: Theory, Computation and Design*. Nob Hill Publishing, 2nd edition, 2017.

[24] M. Diehl, H. J. Ferreau, and N. Haverbeke. Efficient numerical methods for nonlinear mpc and moving horizon estimation. *Nonlinear Model Predictive Control*, pages 391–417, 2009.