

EFFECT OF ENGINE RESPONSE TIME IN AIRCRAFT CONCEPTUAL DESIGN

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Abstract

Engine response time is not something explicitly considered in the conventional power loading versus wing loading constraint diagram of aircraft conceptual design. In the constraint diagram, a constant power/thrust steady-state assumption is used for the constraint curves, including the emergency case. Due to the compliance of currently operational engines with EASA CS-E or FAR 33, the current method works well. However, aircraft prototypes with new engine architectures (such as Solid Oxide Fuel Cell-Gas Turbine hybrid), which potentially have slower response, require a method to consider engine response time in the constraint diagram. This is especially important in emergency cases, such as the go-around, where engine response time plays a dominant role. In this study, by incorporating engine response time, we propose a method of constructing a go-around constraint curve that also addresses the operational aspect. Sizing an aircraft with an engine that has a 20-second response time for the OEI climb gradient constraint requires a 36% greater thrust-to-weight ratio compared to a conventional engine with a nominal response time, while sizing it to perform a safe OEI go-around requires 5% increase in thrust-to-weight ratio.

Keywords: Aircraft conceptual design, Constraint diagram, Performance sizing, Engine response time, Go-around

1. Introduction

To determine a design point of an aircraft that satisfies a set of top-level requirements in the conceptual design phase, it is common to use a constraint diagram/matching chart [1, 2, 3]. In this constraint diagram, each line represents a unique requirement from the top-level aircraft requirements or regulatory documents, such as the Certification Specifications (CS) by EASA, to be checked against. Most of these lines are a steady-state approximation of each flight phase in a normal operation, with a certain performance metric as the representative target for each phase (take-off distance at take-off, rate-of-climb at climb, cruise speed/Mach number at cruise, and landing distance for landing).

Some emergency cases, such as the go-around, are also covered in the constraint diagram. The current method of the go-around constraint is solving for the required climb gradient as the performance metric, referring to a regulation quoted below.

> CS-25.119 Landing climb: all engines operating
>
> "In the landing configuration, the steady gradient of climb may not be less than 3.2%, with the engines **at the power or thrust that is available 8 seconds** after initiation of movement of the power or thrust controls from the minimum flight idle to the go-around power or thrust setting;..." [4]

However, the current method does not yet consider the power/thrust available at a certain time after the movement of the power/thrust lever, which depends on the transient characteristics of the propulsion system, as described by CS-E 745(a)(3).

> CS-E 745 Engine Acceleration
>
> "(a)It must be demonstrated, on a test bed, that:
>
> ... (3) For all Engines, an increase can be made from 15% of the rated Take-off Power or thrust to 95% of the rated Take-off Power or thrust in a time **not greater than 5 seconds**. A longer time may be accepted if **properly justified**. This power or thrust response must occur from a stabilised static condition using only the bleed air and accessories loads necessary to run the Engine." [5]

With emerging options of propulsion systems architecture being considered to minimise the climate impact of aviation, such as the Solid Oxide Fuel Cell - Gas Turbine (SOFC-GT) studied in the HYLENA project[1], a justification for the transient behaviour of the propulsion system could become an increasingly important point to be considered early in the design process, especially if it is slower compared to a conventional propulsion system. Intuitively, one would expect a propulsion system that takes longer to provide the full rated power/thrust compared to a reference to perform worse in a go-around, which is not yet captured in the conceptual design stage.

To ensure the transient behaviour of the propulsion system is considered, in this study, we showed a derivation of an analytical one-engine-inoperative climb gradient constraint equation. Following the climb gradient constraint, a derivation for a constraint that uses a suitable performance metric for the go-around manoeuvre (go-around height loss) as a function of engine response time is proposed in this study.

2. Methodology

Examples of constraint diagrams for a propeller, as well as a turbofan aircraft, are shown in Figure 1 and Figure 2.

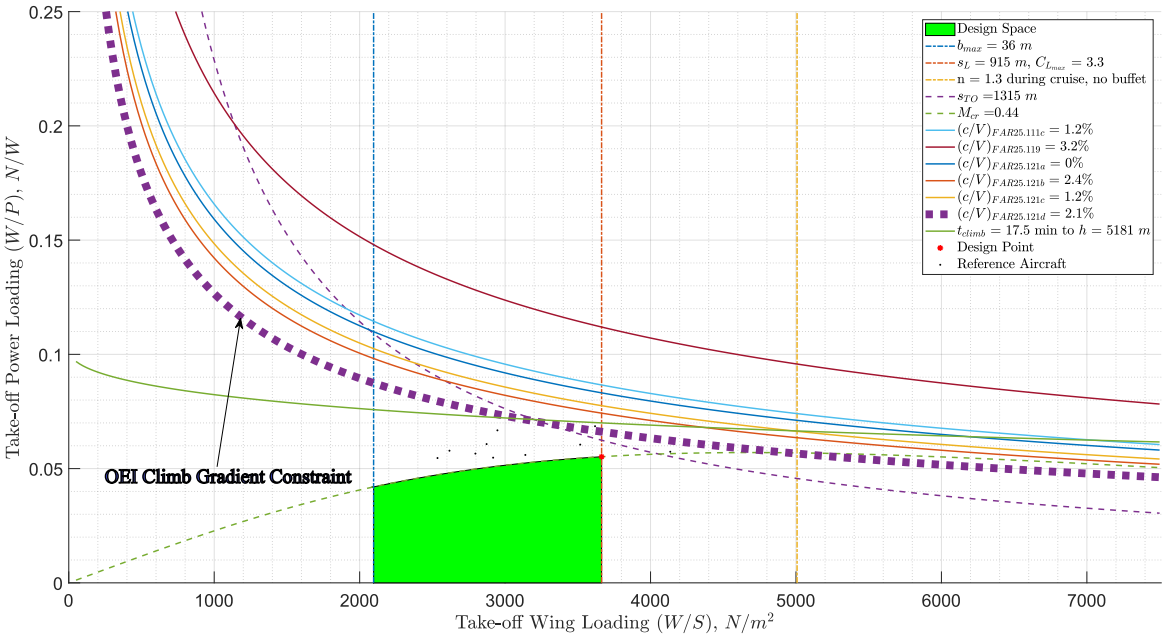


Figure 1 – Example constraint diagram of ATR 72

The formative difference between the two is mainly in how the engine sizing is represented. Turbofan engines list the performance specifications in terms of thrust, leading to the use of thrust-to-weight ratio as a variable, while turboprop engines use shaft power and therefore use power loading as a variable. In addition to that, the two aircraft have their own performance requirements, leading to different constraint curves and therefore a different feasible design space. Most of the constraint

[1] HYdrogen eLectrical Engine Novel Architecture, https://cordis.europa.eu/project/id/101137583

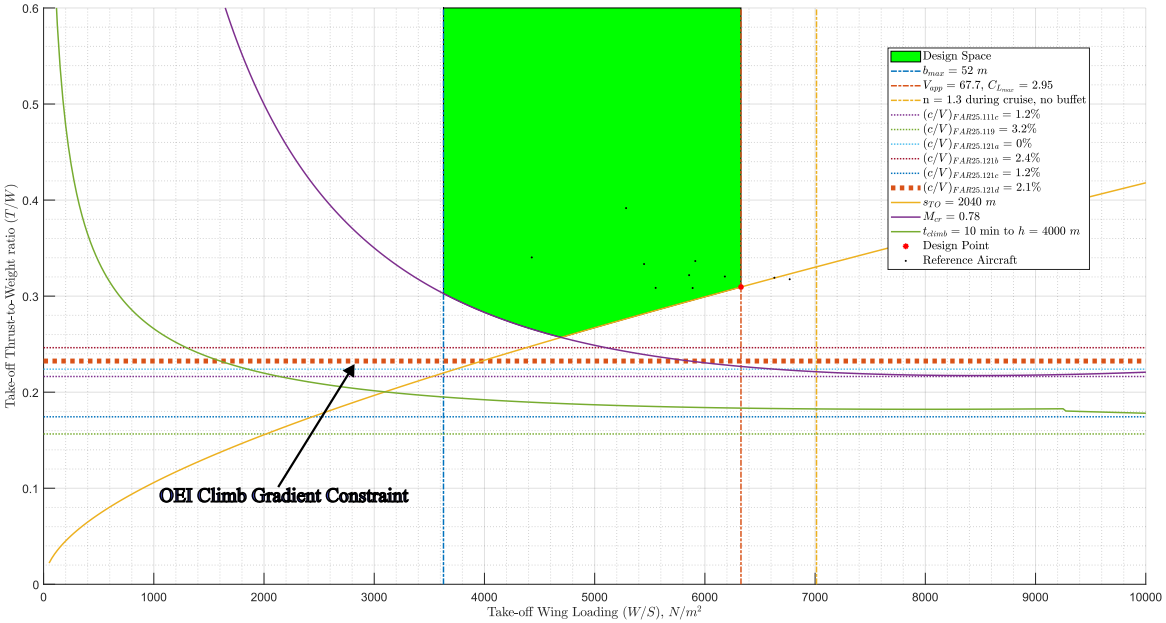


Figure 2 – Example constraint diagram of A320

curves in these diagrams are constructed with a steady-state assumption, simplifying the general performance equation to be used early in the conceptual process.

2.1 Engine response time in constraint diagram

The current method to construct the one-engine inoperative (OEI) go-around constraint is also based on the general flight performance for the longitudinal equation of motion. Balancing the thrust and drag forces and dividing them by the weight of the aircraft, we arrive at the following equation.

$$\frac{T}{W} = \frac{qC_{D_0}}{\frac{W}{S}} + n^2 \frac{k\frac{W}{S}}{q} + \frac{c}{V} + \frac{1}{g}\frac{dV}{dt} \tag{1}$$

In this equation, the first and second term on the right-hand side represents the drag required to maintain flight, where lift equals weight. This depends on the zero-lift drag coefficient C_{D_0}, the dynamic pressure q, and the induced drag factor k.

$$k = \frac{1}{\pi ARe} \tag{2}$$

$$q = \frac{1}{2}\rho V^2 \tag{3}$$

The third term represents the climb gradient $\frac{c}{V}$, which depends on the required rate of climb c and flight speed V. The last term represents the acceleration of the aircraft.

For a certain aircraft configuration and flight condition, appropriate values for Oswald's efficiency factor e and propeller efficiency η_p for the flight phase, as well as the intended aspect ratio AR of the wing, can be estimated. Their relationship is captured in the value for k. Assuming quasi-steady flight (i.e. no acceleration), the last term on the right-hand side of Equation 1 becomes zero. With an initial estimation of the minimum drag coefficient C_{D_0} and load factor $n = 1$ (1-g), the equation can be solved for the required climb gradient $\frac{c}{V}$, resulting in a curve as shown in Figure 1.

However, the certification specifications (CS) specify that the power value used in this approach should be the power value at 8 seconds after the movement of the power lever to the go-around setting by the pilot [4]. To determine the power value at 8 seconds, a simplified first-order rise thrust model, due to engine dynamics, has been proposed in a study to solve the optimum control input for a go-around manoeuvre to minimise height loss [6].

$$T(t) = T_{\max} + [T(t_0) - T_{\max}]\, e^{-t/\tau} \tag{4}$$

For a propeller-driven aircraft, the thrust of the propeller can be approximated by

$$T = \frac{\eta_p P}{V} \tag{5}$$

Assuming constant propeller efficiency η_p and constant airspeed V, thrust T is then directly proportional to power P, and therefore, how power varies over time t for a different engine time constant τ can be computed. In the following figure, the nominal engine shown in the green line is an engine that accelerates from 15% rated power to 95% in 5 seconds, as per engine acceleration requirements of CS-E 745 [5].

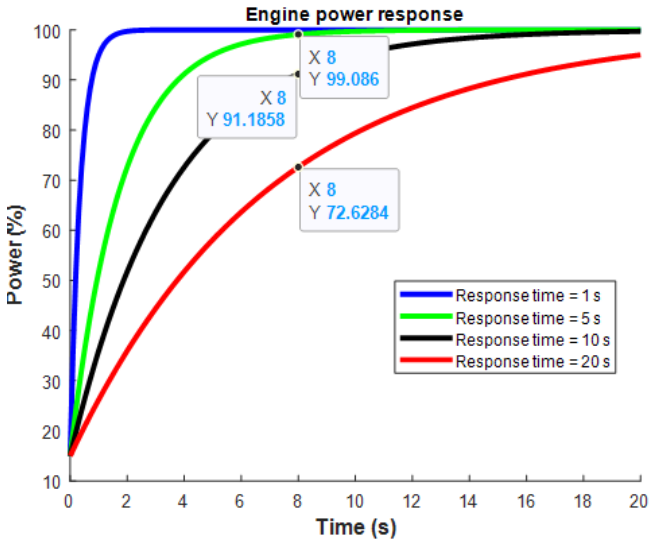


Figure 3 – Power vs time for different engine responses

By substituting the power value for the constraint with the estimated value using the first-order rise model, the constraint curve of the OEI go-around in Figure 1 could move downwards, further limiting the feasible design space.

2.2 Go around height loss

In practice, a successful go-around also depends on the initial altitude of the manoeuvre (so-called decision height), as well as pilot technique (pitch control). With the assumption that a well-trained pilot behaves optimally, the pilot technique can be represented by an optimal control input, providing a way to generalise the pitch control input [7].

Solving the simplified flight dynamics with this assumption has been studied in a previous paper. There, the go-around problem is solved by employing an optimal control strategy that optimises for a different objective compared to Buell [6], which is the time to reach the required climb gradient while minimising control input to ensure realistic control behaviour [8]. Using this approach, the height loss experienced during a go-around can be computed for different engine response times, as depicted in Figure 4.

However, to perform a detailed go-around study requires detailed information about the airfoil and the type of flaps, as well as how it is deflected over time. To limit the type of variables to suit the conceptual design phase, this study derives a height loss constraint with two approaches. The first approach assumes that the flight path angle will be updated as the engine provides increasing power over time. The height loss is then computed by integrating the vertical speed with the varying flight path angle until a time when the thrust is equal to the drag.

The second, more conservative assumption is that the aircraft is approaching the runway with a certain glideslope and continues to follow this flight path until the engine ramps up to enough power/thrust to provide a certain climb gradient. The time required to provide sufficient excess power/thrust for climb is then converted into height loss by multiplying it by the sink rate from the glideslope.

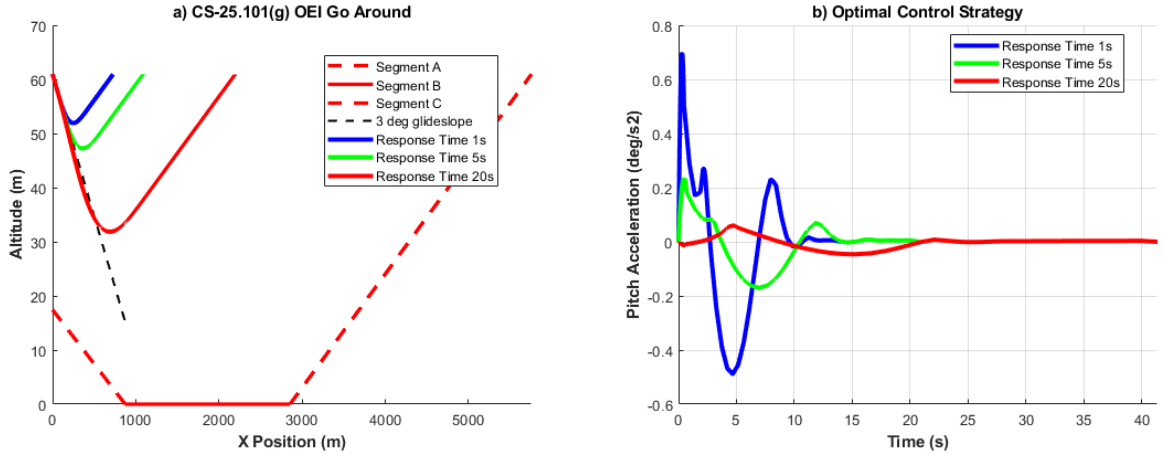


Figure 4 – (a) CS-25.101(g) Go-around trajectories for different engine response time (b) Optimal control strategy for different engine response times [8]

3. Transient-aware OEI climb gradient constraint

Here we start our derivation by substituting the thrust or power element in the steady-state general flight performance equation for the longitudinal equation of motion. To simplify the derivation, thrust is used instead of power in this study. For propeller-driven aircraft, the shaft power required can then be approximated by assuming a certain propeller efficiency η_p alongside a constant speed of interest.

First, we define engine response time as the time required to accelerate from 15% to 95% rated thrust or power $t_{15\to95}$, based on CS-E 745 [5]. With the first-order rise thrust model Equation 4, a derivation for the value of engine time constant τ as a function of engine response time $t_{15\to95}$ is described below.

$$
\begin{aligned}
0.95 &= 1 + [0.15 - 1]\, e^{-t_{15\to95}/\tau} \\
e^{-t_{15\to95}/\tau} &= \frac{0.95-1}{0.15-1} = \frac{1}{17} \\
\frac{-t_{15\to95}}{\tau} &= \ln\frac{1}{17} \Rightarrow \frac{t_{15\to95}}{\tau} = \ln 17 \\
\tau &= \frac{t_{15\to95}}{\ln 17} \\
\tau &\approx 0.3530 \cdot t_{15\to95}
\end{aligned}
\tag{6}
$$

At a time of 8 seconds, the thrust available as a function of engine response time is the following.

$$T(8) = T_{\max} + [T(t_0) - T_{\max}]\, e^{-\frac{8}{\tau}}$$

Here we assume $T(t_0)$ to be the flight idle thrust level during landing approach as 15% of the maximum thrust of the engine $T_{\max}$. Thus,

$$
\begin{aligned}
T(8) &= T_{\max} + [0.15T_{\max} - T_{\max}]\, e^{-\frac{8}{\tau}} \\
T(8) &= T_{\max} - 0.85 \cdot T_{\max} e^{-\frac{8}{\tau}}
\end{aligned}
$$

The thrust value at 8 seconds will be a fraction of the maximum thrust. Here, we define the 8-second thrust fraction as follows.

$$\Gamma_8 \equiv \frac{T(8)}{T_{\max}} = 1 - 0.85 \cdot e^{-\frac{8}{\tau}} \tag{7}$$

This study assumes a two-engined aircraft, and the following derivations are suitable as long as this limitation is adhered to. For aircraft with three or more engines, the derivation method can be followed while ensuring that the one-engine-inoperative criteria, such as the number of engines remaining and the required climb gradient, are accounted for. For this study, the one-engine-inoperative thrust is half of the total thrust of the aircraft.

$$\begin{aligned} N_{eng} &= 2 \\ T_{max,OEI} &= \frac{N_{eng} - 1}{N_{eng}} \cdot T \\ T_{max,OEI} &= \frac{1}{2} \cdot T \end{aligned}$$

Therefore, the one-engine-inoperative thrust at a time of 8 seconds is simply the thrust available of the remaining engine multiplied by the 8-second thrust fraction that we defined earlier in Equation 7.

$$T_{OEI}(8) = T_{max,OEI} \cdot \Gamma_8$$

From Equation 1, assuming the aircraft is flying in a 1-g no-acceleration flight, we can describe an equation for a one-engine-inoperative climb gradient requirement of 2.1% to be fulfilled with the thrust available from one engine at 8 seconds. Note that the climb gradient requirement value used in this example is only suitable for a two-engined aircraft.

$$\begin{aligned} \frac{T_{OEI}(8)}{W} &= \frac{qC_{D_0}}{\frac{W}{S}} + \frac{k\frac{W}{S}}{q} + 0.021 \\ \frac{T_{max,OEI} \cdot \Gamma_8}{W} &= \frac{qC_{D_0}}{\frac{W}{S}} + \frac{k\frac{W}{S}}{q} + 0.021 \\ \frac{1}{2}\frac{T}{W}\Gamma_8 &= \frac{qC_{D_0}}{\frac{W}{S}} + \frac{k\frac{W}{S}}{q} + 0.021 \end{aligned}$$

Hence, the analytical form of the one-engine-inoperative go-around requirement based on climb gradient for a two-engined aircraft is the following.

$$\frac{T}{W} = \frac{2}{\Gamma_8}\left(\frac{qC_{D_0}}{\frac{W}{S}} + \frac{k\frac{W}{S}}{q} + 0.021\right) \tag{8}$$

3.1 Example constraint curve with known approach speed

To provide an example, consider an experimental two-engined regional turboprop aircraft with an approach speed of 115 knots and aspect ratio AR of 12, with an estimated zero-lift drag coefficient C_{D_0} of 250 drag counts (0.0250). The engines fitted to this experimental aircraft are prototype engines with a response time $t_{15\to95}$ of 20 seconds. The one-engine-inoperative climb gradient constraint curve of this experimental aircraft, compared to an aircraft fitted with a serial production engine with a response time of 5 seconds, is shown on Figure 5.

For the wing loading value 377 kg/m^2 representing ATR72-600, with a maximum takeoff mass of 23000 kg and wing area of 61 m^2, the thrust-to-weight ratio of the 20-second response time engine is around 36% higher compared to the nominal engine. This supports the notion that engine transient behaviour is a significant driver of the engine sizing, and it is important for the aircraft designer to consider this early in the design process.

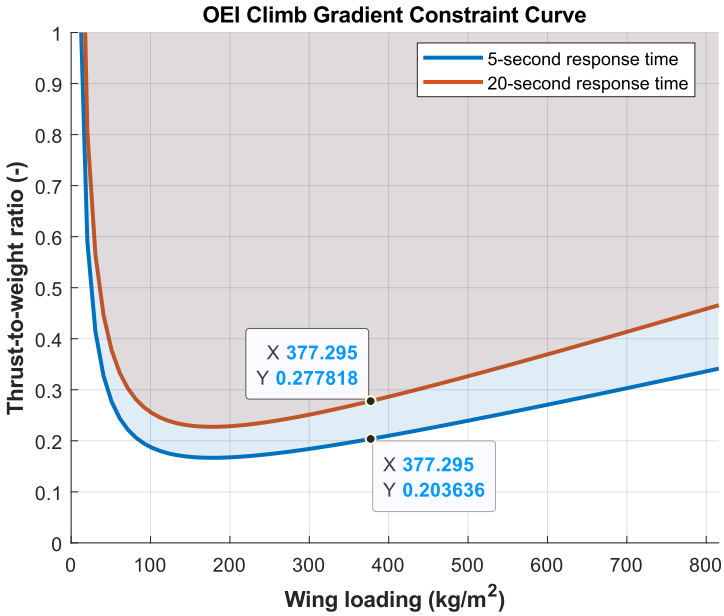


Figure 5 – Constraint curve of different engine response times

3.2 Example constraint curve with unknown approach speed

The approach speed of the aircraft is highly dependent on the wing geometry, including the flap configuration, which is often undefined in the early phase of the conceptual design. Due to this, another approach to solving the one-engine-inoperative climb gradient constraint is required.

In CS25, the speed of the aircraft during approach is the reference landing speed V_{REF}, which may not be less than 1.23 times the reference stall speed V_{SR}. For the reference stall speed, we can define it in terms of wing loading using the balance of forces on the vertical axis.

$$\begin{aligned} L &= W \\ \frac{1}{2} \cdot \rho \cdot V_{SR}^2 \cdot S \cdot C_{L_{max}} &= W \\ V_{SR}^2 &= \frac{2\frac{W}{S}}{\rho C_{L_{max}}} \end{aligned}$$

Using the CS25 reference landing speed criteria,

$$\begin{aligned} V_{REF} &\approx 1.23 \cdot V_{SR} \\ V_{REF}^2 &\approx 1.23^2 \frac{2\frac{W}{S} C_{L_{max}}}{\rho} \Rightarrow q = \frac{1}{2}\rho V_{REF}^2 \approx \frac{1.23^2 (W/S)}{C_{L_{max}}} \end{aligned}$$

Substituting the dynamic pressure into Equation 8, the thrust-to-weight ratio for one-engine-inoperative climb gradient constraint for a two-engined aircraft becomes the following.

$$\begin{aligned} \frac{T}{W} &= \frac{2\left(\frac{qC_{D_0}}{\frac{W}{S}} + \frac{k\frac{W}{S}}{q} + 0.021\right)}{\Gamma_8} \qquad , \frac{q}{W/S} = \frac{1.23^2}{C_{L_{MAX}}} \\ \frac{T}{W} &= \frac{2\left(C_{D_0}\frac{1.23^2}{C_{Lmax}} + k\frac{C_{Lmax}}{1.23^2} + 0.021\right)}{\Gamma_8} \end{aligned} \tag{9}$$

The thrust-to-weight ratio is now independent of the wing loading. Using the same example assumptions as previous, replacing the approach speed with an assumed maximum lift coefficient $C_{L_{max}}$ value

of 2.5, the comparison between the two engines is shown on Figure 6.

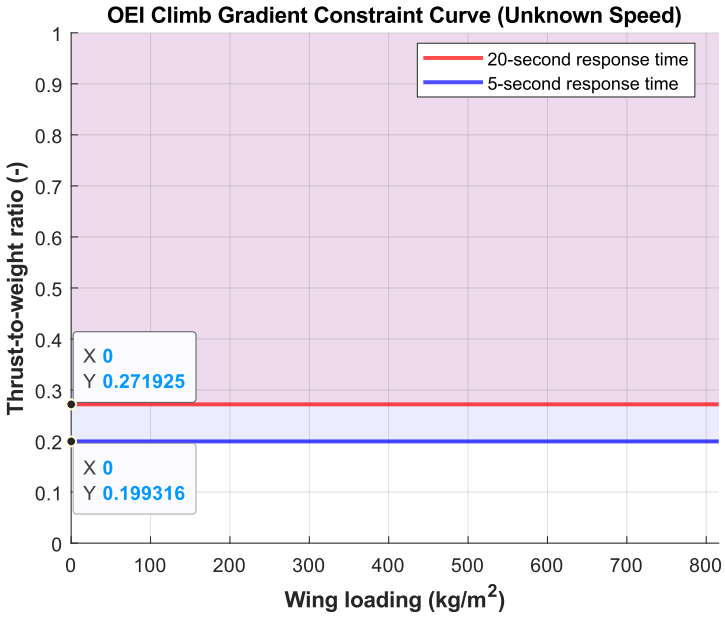


Figure 6 – Constraint curve of different engine response times, with known $C_{L_{max}}$ and unknown approach speed

4. OEI go-around height loss constraint

In practice, demonstrating the safety of the aircraft in the event of a one-engine-inoperative go-around involves conducting a test from an altitude above the decision height (for test safety) and recording the height loss during the manoeuvre to show that ground contact would not occur if it is performed from the decision height (AMC 25.101(g) Go-around). In this section, we show the analytical model of AMC25.101(g), with some simplified assumptions. We use this analytical model to derive a conceptual design constraint curve that considers the effect of increasing engine response time on the go-around height loss.

To model this test, we define the go-around decision time as $t = 0$, and the aircraft initially flies on a 3-degree glidepath at an airspeed of V_{REF}. This lets us define height loss as the maximum drop in altitude relative to the initial altitude.

$$h_{loss} \equiv -\min_{t \geq 0} \Delta h(t)$$

$$\Delta h(t) = h(t) - h(0)$$

A safe go-around from a decision height DH is possible if $h_{loss} \leq DH$.

The go-around is inherently a transient operation that depends on the state of many parameters over time. In this study, to analytically derive the height loss constraint, we use similar simplification assumptions to the climb gradient constraint derivation. The aircraft is modelled with these assumptions:

- Point mass model
- Longitudinal motion only
- 1-g flight
- No acceleration

This set of assumptions implies that the aircraft is controlled by the flight crew to maintain the speed at a constant V_{REF} or a quasi-steady flight. Small angles approximation (radians) is used as the flight-path angle variation is between -3 degrees (-0.0524 radians) and 2.1% climb gradient (1.2 degrees or 0.0210 radians). The flap deflection and landing gear extension are assumed to be unchanging during the manoeuvre.

Two approaches to the height loss estimation are derived in this study. The first approach is an optimistic case, where the flight path angle changes from the initial approach glideslope following the increase in thrust provided by the engine and the minimum altitude occurs when there is enough thrust for a level flight. The second approach is a conservative case, where it is assumed that the aircraft will stay at the current glideslope until such a time when the engine provides enough power to climb at a 2.1% climb gradient, resulting in more time compared to the optimistic case and therefore more height loss.

4.1 Optimistic height loss

The following derivations use the optimistic minimum altitude case. The flight path angle of the aircraft can be approximated with the following equation.

$$\gamma(t) \approx \frac{T(t) - D}{W}$$

Where the flight path angle γ is in radians. With the constant speed V_{REF}, the rate of climb or sink rate (if negative) approximately follows the following.

$$\dot{h}(t) = V_{REF} \sin\gamma \approx V_{REF} \cdot \gamma(t) \approx V_{REF} \left(\frac{T(t)}{W} - \frac{D}{W} \right)$$

We can integrate from $t = 0$ to $t = t_f$ to obtain the change in altitude.

$$\Delta h(t) = V_{REF} \int_0^{t_f} \left(\frac{T(t)}{W} - \frac{D}{W} \right) dt$$

Recall that we use a first-order rise as the model of thrust variations over time.

$$\frac{T(t)}{W} = \frac{T_{max}}{W} + \left(\frac{T_0}{W} - \frac{T_{max}}{W} \right) e^{-\frac{t}{\tau}}$$

Substituting this thrust variation results in the following for the change in altitude.

$$\Delta h(t) = V_{REF} \int_0^{t} \left(\frac{T_{max}}{W} - \frac{D}{W} + \left(\frac{T_0}{W} - \frac{T_{max}}{W} \right) e^{-\frac{t}{\tau}} \right) dt$$

To simplify the following derivation, let us define helper variables.

$$\begin{aligned} A &\equiv \left(\frac{T_{max}}{W} - \frac{D}{W} \right) \\ B &\equiv \left(\frac{T_{max} - T_0}{W} \right) \end{aligned} \tag{10}$$

For a two-engined aircraft under a one-engine-inoperative condition, $T_{max,OEI}$ represents the thrust of the remaining engine.

$$\frac{T_{max,OEI}}{W} = \frac{1}{2} \frac{T}{W} \tag{11}$$

The first helper variable is the specific excess thrust of the aircraft, while the second helper variable is proposed to be the specific go-around thrust step (change in thrust during go-around divided by aircraft weight). Using these variables, the change of altitude as a function of time becomes the following.

$$\Delta h(t) = V_{REF} \left[At - B\tau \left(1 - e^{-\frac{t}{\tau}} \right) \right] \tag{12}$$

Now we move forward with defining the time at which the altitude is minimum. As the aircraft is initially descending, the initial thrust is lower than the drag, and the initial rate of climb is negative.

$$T_0 < D$$
$$\dot{h}(0) < 0$$

In an optimistic scenario, the altitude reaches a minimum at a time t_1 when the rate of climb is equal to zero, and the thrust is equal to the drag.

$$T(t_1) = D$$

Using Equation 4, we can solve for the thrust at the minimum altitude time t_1.

$$\begin{aligned} T(t_1) &= T_{max} - (T_{max} - T_0)\, e^{-\frac{t_1}{\tau}} \\ D &= T_{max} - (T_{max} - T_0)\, e^{-\frac{t_1}{\tau}} \\ e^{-\frac{t_1}{\tau}} &= \frac{T_{max} - D}{T_{max} - T_0} = \frac{AW}{BW} = \frac{A}{B} \\ -\frac{t_1}{\tau} &= \ln\frac{A}{B} \\ t_1 &= \tau \ln\frac{B}{A} \end{aligned}$$

With the time to minimum altitude obtained, we can now perform substitutions to evaluate the altitude change using Equation 12.

$$\begin{aligned} \Delta h(t_1) &= V_{REF}\left[A t_1 - B\tau\left(1 - e^{-\frac{t_1}{\tau}}\right)\right] \\ \Delta h(t_1) &= V_{REF}\left[A\left(\tau \ln\frac{B}{A}\right) - B\tau\left(1 - \frac{A}{B}\right)\right] \\ \Delta h(t_1) &= V_{REF}\cdot\tau\left[A\left(\ln\frac{B}{A}\right) - B\left(1 - \frac{A}{B}\right)\right] \end{aligned}$$

The decision height DH must be larger than the height loss, which is simply the negative of this altitude change.

$$\begin{aligned} DH &\geq h_{loss} \\ DH &\geq V_{REF}\cdot\tau\left[B - A\left(\ln\frac{B}{A} + 1\right)\right] \end{aligned} \tag{13}$$

In this height loss model, the initial stabilised approach glide slope of 3 degrees (0.0524 radians) is taken into account, so that we can have a better approximation of the initial thrust of the aircraft. According to AMC25.101(g), the application of go-around thrust is simultaneous with the failure of the critical engine, which means the thrust immediately after $t = 0$ is lower than the thrust required to maintain a 3-degree glideslope.

$$\begin{aligned} \gamma_0 &\approx \frac{T_{approach} - D}{W} \\ \frac{T_{approach}}{W} &\approx \frac{D}{W} - 0.0524 \\ T_0 \equiv T(0^+) &\approx \frac{1}{2} T_{approach} \\ \frac{T_0}{W} &\approx \frac{1}{2}\left(\frac{D}{W} - 0.0524\right) \end{aligned}$$

Note that here we neglect the effect of feathering or windmilling drag associated with the inoperative engine, leading to a slightly optimistic approximation. The consideration of initial glideslope updates the specific go-around thrust step variable that we defined earlier.

$$B = \frac{T_{max}}{W} - \frac{1}{2}\left(\frac{D}{W} - 0.0524\right)$$
$$B = A + \frac{1}{2}\frac{D}{W} + \frac{1}{2}0.0524$$

For the initial condition of one-engine-inoperative thrust, $\frac{T_0}{W} \approx \frac{1}{2}\left(\frac{D}{W} - 0.0524\right)$, the helper variables can be rewritten as the following.

$$A = \frac{1}{2}\frac{T}{W} - \frac{D}{W}, \qquad B = A + \frac{1}{2}\left(\frac{D}{W} + 0.0524\right) \tag{14}$$

Substituting these expressions into the optimistic height-loss model yields the following implicit constraint equation:

$$DH = V_{\text{REF}} \cdot \tau \left[\frac{1}{2}\left(\frac{D}{W} + 0.0524\right) - \left(\frac{1}{2}\frac{T}{W} - \frac{D}{W}\right)\ln\left(1 + \frac{\frac{1}{2}\left(\frac{D}{W} + 0.0524\right)}{\frac{1}{2}\frac{T}{W} - \frac{D}{W}}\right)\right] \tag{15}$$

where

$$\frac{D}{W} = \frac{qC_{D0}}{W/S} + \frac{k(W/S)}{q}, \qquad \tau = \frac{t_{15\to 95}}{\ln 17} \tag{16}$$

This equation defines the optimistic OEI go-around height-loss boundary in terms of variables suitable for the conceptual design phase.

4.2 Conservative height loss

In the conservative approach of estimating height loss, it is assumed that the aircraft will stay at the current glideslope until a time t_2 when the available excess thrust is enough to overcome the drag and climb at a gradient of 2.1% for a two-engined aircraft in a one-engine-inoperative condition. Using Equation 1 with a 1-g load factor and no acceleration, the thrust required is the following.

$$T(t_2) = D + 0.021W$$

Using the first-order rise model and helper variables A and B as defined in Equation 10, the time to minimum altitude is the following.

$$\begin{aligned}
T(t_2) &= T_{\max} - (T_{\max} - T_0)\, e^{\frac{-t_2}{\tau}} \\
D + 0.021W &= T_{\max} - (T_{\max} - T_0)\, e^{\frac{-t_2}{\tau}} \\
(T_{\max} - T_0)\, e^{\frac{-t_2}{\tau}} &= T_{\max} - D - 0.021W \\
e^{\frac{-t_2}{\tau}} &= \frac{T_{\max} - D - 0.021W}{T_{\max} - T_0} \\
e^{\frac{-t_2}{\tau}} &= \frac{AW - 0.021W}{BW} \\
e^{-\frac{t_2}{\tau}} &= \frac{A - 0.021}{B} \\
t_2 &= \tau \ln\left(\frac{B}{A - 0.021}\right)
\end{aligned}$$

Since the vertical speed of the aircraft is approximately the 3° glidepath sink rate, the conservative height loss becomes the following.

$$h_{loss_{cons}} \approx V_{REF} \sin 3^\circ \cdot t_2 \approx V_{REF} \cdot 0.0524 \cdot \tau \ln\left(\frac{B}{A - 0.021}\right)$$

Setting the conservative height loss to be equal to the decision height, we can define the term inside the exponent to aid with derivation.

$$
\begin{aligned}
DH &= V_{REF} \cdot 0.0524 \cdot \tau \ln\left(\frac{B}{A-0.021}\right) \\
C &= \exp\frac{DH}{V_{REF} \cdot 0.0524 \cdot \tau} = \frac{B}{A-0.021} \\
B &= C(A-0.021)
\end{aligned}
$$

From the definition of helper variables in Equation 14, we can substitute B and solve for A.

$$
\begin{aligned}
B &= A + \frac{1}{2}\left(\frac{D}{W} + 0.0524\right) \\
A + \frac{1}{2}\left(\frac{D}{W} + 0.0524\right) &= C(A-0.021) \\
A + \frac{1}{2}\left(\frac{D}{W} + 0.0524\right) &= A \cdot C - 0.021C \\
A - A \cdot C &= -0.021C - \frac{1}{2}\left(\frac{D}{W} + 0.0524\right) \\
A(1-C) &= -0.021C - \frac{1}{2}\left(\frac{D}{W} + 0.0524\right) \\
A(C-1) &= 0.021C + \frac{1}{2}\left(\frac{D}{W} + 0.0524\right) \\
A &= \frac{0.021C + \frac{1}{2}\left(\frac{D}{W} + 0.0524\right)}{C-1}
\end{aligned}
$$

Unlike the implicit optimistic height loss constraint, which has to be solved either graphically or numerically, this equation can be rearranged to solve for $\frac{T}{W}$ from the definition of A in Equation 10.

$$
\begin{aligned}
A &= \frac{1}{2}\frac{T}{W} - \frac{D}{W} \qquad \Rightarrow \frac{T}{W} = 2\left(A + \frac{D}{W}\right) \\
\frac{T}{W} &= 2\left[\frac{0.021C + \frac{1}{2}\left(\frac{D}{W} + 0.0524\right)}{C-1} + \frac{D}{W}\right]
\end{aligned} \tag{17}
$$

where

$$
C = \exp\frac{DH}{V_{REF} \cdot 0.0524 \cdot \tau}, \qquad \frac{D}{W} = \frac{qC_{D0}}{W/S} + \frac{k(W/S)}{q}
$$

4.3 Example height loss constraint curves

Based on previous derivations, we can now begin constructing the constraint curve for the go-around height loss. In this example, we will use 115 knots as the V_{REF}, 0.0250 as the C_{D_0}, an aspect ratio of 12, and an Oswald efficiency factor of 0.7. These assumptions will be used to evaluate the one-engine-inoperative go-around height loss constraint for cases with engine response times of 1 second, 5 seconds, and 20 seconds for a certain decision height. The decision height follows the definition of ICAO Annex 6 for Category I precision approach, which is 200 feet [9].

As mentioned in the derivation, an optimistic approach is to assume the minimum altitude is reached when the thrust equals the drag, leading to a zero climb gradient. Using Equation 15, we can generate the constraint curve as shown in Figure 7.

From this figure, we can see that the lines of different engine response times coincide with each other. This implies that the height loss computed using the optimistic approach is well under the decision height of 200 feet, even with a 20-second engine response time. This is supported by Figure 8, where

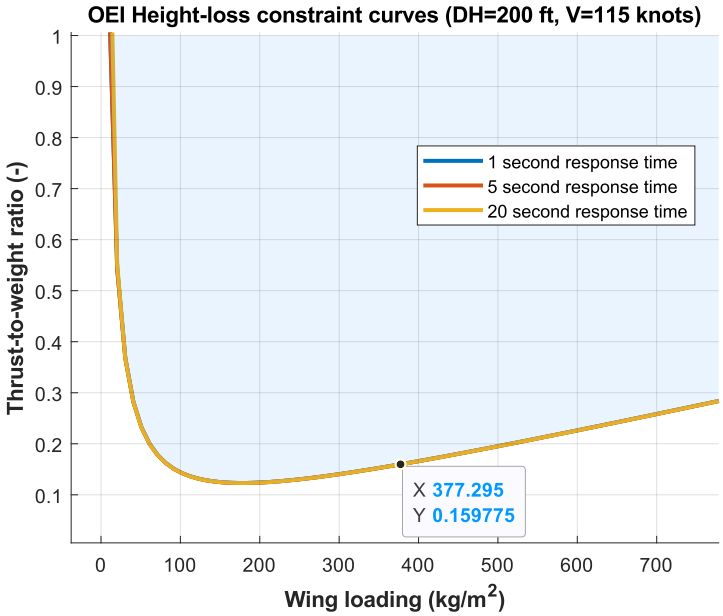


Figure 7 – One-engine-inoperative go-around constraint, optimistic

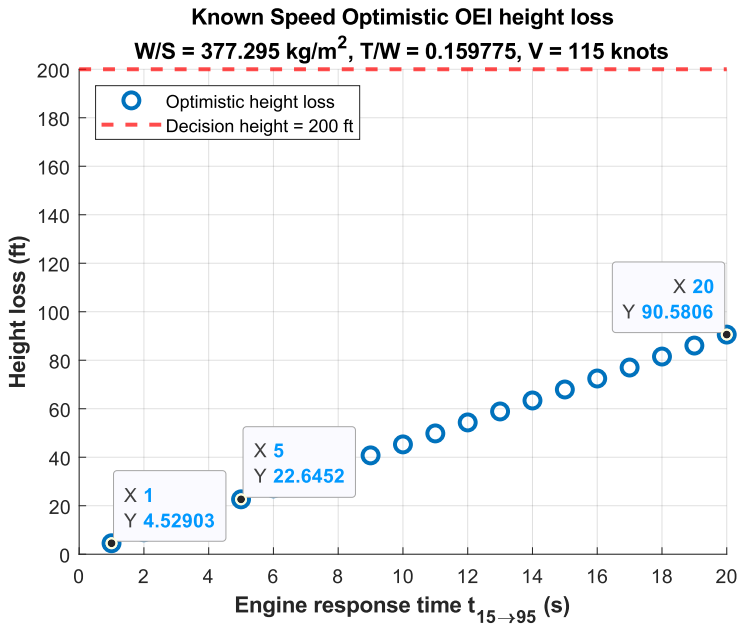


Figure 8 – Height loss variations to response time for one design point

we show the effect of response time on the height loss for a certain wing loading and thrust-to-weight ratio.

From Figure 8, we see that there is a significant margin for height loss if 200 feet is used as the decision height. However, using a stricter precision approach category will decrease the margin, and if the margin is nonexistent, then the design point is shifted to a higher thrust-to-weight ratio. The trend from the height loss result is similar to the trend reported in a more detailed simulation study [8], although the difference is greater in lower response times.

For the more conservative approach, the aircraft is assumed to stay at its current flight path angle until the thrust available is enough to climb in a 2.1% gradient, leading to more height loss. The constraint curve results are shown in Figure 9.

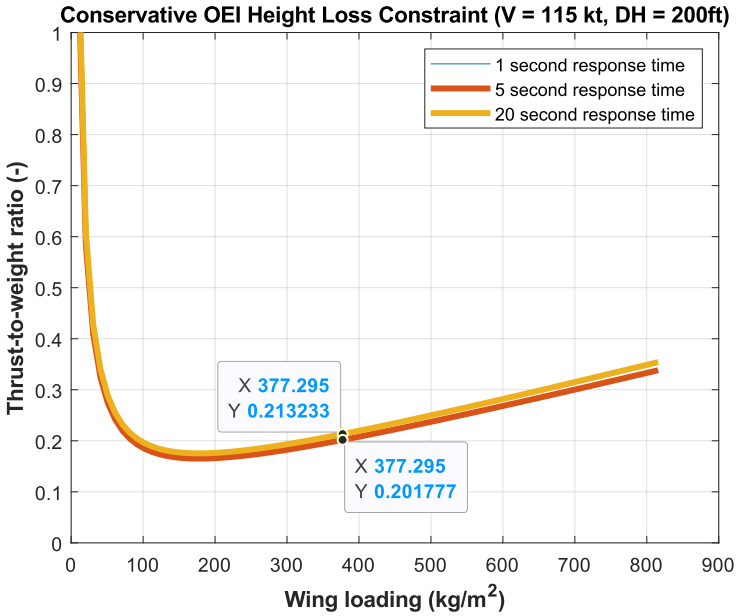


Figure 9 – One-engine-inoperative go-around constraint, conservative

From the results, it is observed that the 1-second response time engine coincides with the 5-second response time, while the 20-second response time engine requires a thrust-to-weight ratio value of around 5% higher. This result indicates that a decision height of 200 feet has ample margin to prevent the aircraft from touching the ground in the event of a one-engine-inoperative go-around for the normal response time engine.

If we compare with the one-engine-inoperative climb gradient constraint in Figure 5, the 200 ft height loss thrust-to-weight ratio values, even in the conservative approach, are lower. This implies that considering the climb gradient constraint is enough if the aircraft is designed to operate using the Category I decision height, as the height loss constraint is less constraining on the design space.

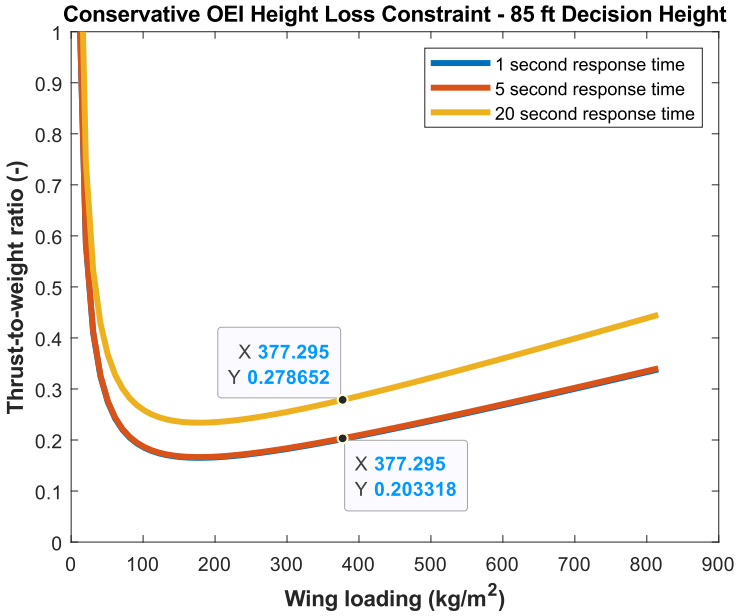


Figure 10 – Conservative OEI go-around height loss constraint, 85 feet decision height

By varying the decision height to observe the boundary, it is found that a decision height of 85 feet

using the conservative height loss assumption results in a similar constraint curve as the OEI climb gradient constraint, as shown in Figure 10.

This decision height falls into a stricter requirement of Category IIIA precision approach, which is defined as lower than 100 feet [9]. Compared to an operational decision height for a Category I precision approach of 200 feet, the lower decision height constraint implies that the engine of this particular aircraft is sized conservatively with respect to the go-around constraint. Note that this result is limited to the example aircraft with the assumed approach speed, aspect ratio, and zero-lift drag coefficient. However, if the aircraft is designed to operate with Category IIIB precision approach or higher, then the OEI height loss constraint may be more limiting compared to the OEI climb gradient constraint, and the benefit of using an engine with a faster response time becomes apparent.

Nevertheless, this method provides a workable estimation to size the engine of the aircraft to perform the one-engine-inoperative go-around manoeuvre in the conceptual design phase. A higher fidelity simulation taking into account the pitch schedule, flap schedule, ground effect, and others is still required to determine with certainty the go-around capability of the aircraft.

5. Conclusion

We have shown that an analytical derivation of one-engine-inoperative go-around constraints that captures the effect of engine response time while maintaining the use of variables to a limited amount of information available in the conceptual design phase is possible, with some assumptions. With a first-order rise transient behaviour assumption, it is observed that an engine with a response time of 20 seconds needs to be sized around 36% greater compared to an engine with a nominal response time, for the one-engine-inoperative go-around climb gradient criteria.

We have also shown the derivation of an operationally suitable one-engine-inoperative go-around constraint, which takes into account the height loss during the go-around manoeuvre. From the study example, a Category I decision height of 200 feet is less limiting on the design space compared to the one-engine-inoperative climb gradient constraint. Within a set of assumptions, there exists a height loss-driven decision height that results in an equivalent one-engine-inoperative climb gradient constraint curve. For this study, a decision height of 85 feet is equivalent to the climb gradient constraint. However, the height loss constraint is expected to be more limiting for slower engines if the aircraft is designed to operate within a stricter precision approach category.

Overall, it is quantitatively shown that it is required to fit the aircraft with a more powerful engine if a slower-than-nominal response time is expected, with respect to the go-around scenario. However, it is important to note that if other performance requirements are considered, the go-around constraints may not be limiting the design space as much as other performance constraints, such as takeoff field length or cruise speed.

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