NONLINEAR BEHAVIOR IN AIRCRAFT FUEL GAUGE READINGS: AN EXPLORING ANALYSIS

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Abstract

Fuel level gauge in aircraft is a challenging matter, once the aircraft's movements affect its measurement. In this context, this study addresses a detailed analysis of simulated results from a computational tool specifically developed to study aircraft fuel systems. The tool simulates fuel sensor readings across various aircraft attitudes and fuel levels. This analysis highlights the high nonlinearity of these signals in different scenarios. The objective was to explore the relationship between the sensor readings, aircraft orientation, and fuel volume. The dataset consisted of simulated readings from eight capacitive sensors placed at different locations inside an aircraft fuel tank. Each row in the dataset represented a specific combination of aircraft attitude (yaw, pitch, roll) and a "cut" (or cross-section) of the fuel tank, simulating a particular fuel volume. The sensors' readings at each fuel level (cut), correlated with the simulated fuel volume and aircraft attitude, provided valuable insights into the relationship between these variables, revealing a complex nonlinear interaction. Contour plots were generated to visualize each sensor's readings at different pitches and rolls to understand the trends in the sensor readings. Statistical analysis was conducted to quantify the coefficients and significance of pitch and roll angles and their interactions, as well as a Normalized Sensitivity Matrix (NSM) was used to analyze the sensitivity of the multiple sensors to changes in attitude angles, providing insights into how sensors respond to the attitude variations and their relative sensitivity. Each sensor responded differently to pitch and roll variations, providing essential information for sensor positioning and system design. This analysis lays the foundation for further research in aircraft fuel measurement, volume estimation, and sensor optimization.

Keywords: Fuel Data Analysis, Aircraft Sensor Data, Sensor Sign Non-linear Behavior.

1. Introduction

The aircraft fuel measurement system is a critical component in aviation, ensuring that pilots and crew have accurate and real-time data on the fuel status, which is essential for flight planning and safety [1]. Traditional methods of fuel measurement have their limitations. However, advancements in technology have introduced new computational resources and programming software, offering reliable solutions to study and understand aircraft fuel behavior [2, 3]. These innovations enable the development of a more accurate fuel measurement even during various aircraft maneuvers involving different attitudes.

Accurate measurement of fuel levels in aircraft remains a challenging task due to the dynamic nature of flight. The constant changes in pitch, roll, and yaw affect the readings from fuel sensors, complicating the task of determining precise fuel quantities. This issue is further exacerbated by the need to account for fuel distribution across multiple tanks of irregular geometries. The non-linear behavior of sensor readings, influenced by the aircraft's attitude, adds complexity to fuel level gauging. Recent advancements have introduced computational tools that simulate fuel sensor data under various flight conditions, offering improved accuracy in fuel level estimation and enhancing flight safety and fuel management practices [4]

To gain deeper insights into these challenges, this study conducts an exploratory data analysis (EDA) of simulated fuel sensor data. This research aims to explore and describe the complex interactions between fuel sensor readings and aircraft attitudes using a comprehensive dataset generated by a simulation tool. For a more detailed understanding of the fuel simulation computational tool and its applications, please refer to the full article by Di et al. (2024) [4].
The findings from this analysis will contribute to a better understanding of the non-linear behavior of sensor signals, ultimately leading to enhanced fuel gauging accuracy and improved flight safety and fuel management practices.
The dataset consists of simulated readings from eight capacitive sensors placed strategically within an aircraft fuel tank. Each data point represents a unique combination of aircraft attitude (including yaw, pitch, and roll) and a specific fuel tank cross-section, simulating different fuel volumes. Through detailed descriptive and exploratory analyses, this research seeks to uncover patterns, correlations, and insights into the sensor readings under varying conditions.
To achieve this, the study employs a range of visualization techniques, including contour plots and correlation matrices, to examine the relationships between sensor readings and aircraft attitudes. These visualizations help to identify trends, anomalies, and the degree of nonlinearity in the data. Furthermore, polynomial regression surfaces are fitted to the data to understand the overarching trends in sensor behavior.
A statistical analysis is performed to quantify the influence of pitch and roll angles and their interactions on sensor readings. Additionally, a Normalized Sensitivity Matrix (NSM) is used to evaluate the sensitivity of the sensors to changes in attitude angles, providing insights into their relative responsiveness and the implications for sensor placement and system design.
The results of this exploratory analysis highlight the complex, nonlinear nature of fuel sensor responses and provide valuable insights for several key areas: modeling fuel systems, improving volume estimation models, optimizing sensor placement, and enhancing overall fuel measurement accuracy. This work is crucial for understanding these relationships and sets the stage for further research into more precise fuel volume estimation and advanced sensor technologies in aviation.

2. Tank and Sensors Configuration

Measuring the fuel volume inside the aircraft tank during flight is challenging due to the dynamic nature of flight, which causes fuel distribution within the tank to change with the aircraft's attitude.
Accurately measuring fuel levels in aircraft fuel tanks presents several challenges as:

- **Dynamic Attitudes:** The aircraft's pitch, and roll constantly change during flight, causing fuel to slosh and redistribute within the tank. This affects the readings from fuel sensors, making it difficult to obtain accurate measurements.
- **Tank Geometry:** Aircraft fuel tanks are often irregularly shaped to maximize space utilization within the aircraft structure. This complexity adds to the difficulty of accurately measuring fuel volume and requires precise sensor placement and calibration.
- **Combining Sensor Readings:** To achieve a comprehensive understanding of fuel levels, data from multiple sensors must be combined. This process involves sophisticated algorithms to interpret the sensor data accurately and account for the tank's geometry and the aircraft's attitude.

The aircraft fuel tank simulated in this study is depicted in Figure 1. The tank features a complex geometry, designed to closely mimic real-world aircraft fuel tanks. It has a curved shape to fit within the aircraft's wing, as seen in the figure. Its total volume is approximately 4.52 m^3.
The tank's internal configuration includes eight capacitive fuel sensors, placed to provide comprehensive coverage of the fuel volume. Each sensor is designed to detect the fuel level at its specific location within the tank. The data from these sensors is used to construct a detailed map of the fuel distribution, which is critical for accurate fuel volume estimation.
These sensors are identified as **sensor_1** to **sensor_8** in the dataset and are positioned at various locations and orientations within the tank to capture the fuel levels under different aircraft attitudes.

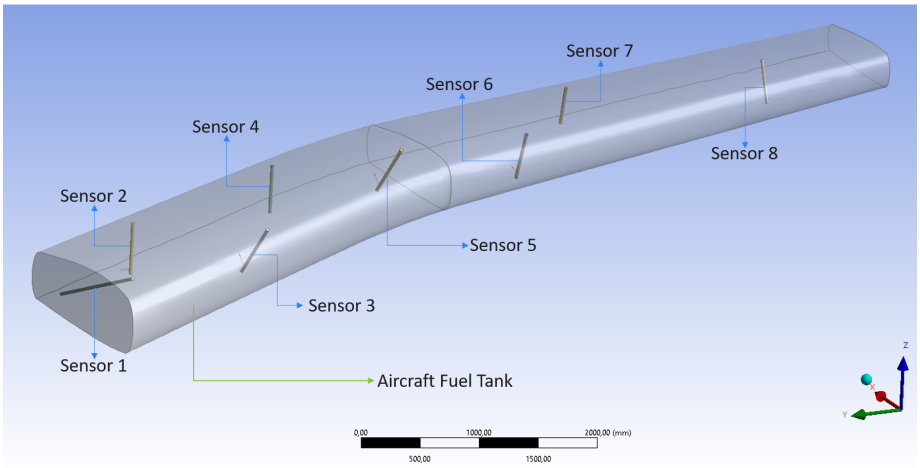


Figure 1 – Schematic of the aircraft fuel tank with sensor placements.

The positions of the sensors are also a target of further investigation and optimization, where the understanding of the relationship between sensors' readings behavior and their placements influences the accuracy of the final volume estimation.
The simulation setup and dataset is explained in the section 3. This detailed setup allows for an in-depth analysis of the fuel sensor data, providing insights into the behavior of the sensors in response to the dynamic conditions within the tank during flight. The simulation results are used to develop better models for accurate fuel volume estimation, enhancing the reliability of fuel gauging systems in aviation.

3. Data Explanation

The dataset used in this study is a simulation of aircraft fuel tank sensor readings, specifically for capacitive sensors. Each row in the dataset represents a unique combination of aircraft attitude and fuel volume. The sensor readings indicate the response of the sensors under these specific conditions. In the depicted table (1 and 2), the sensor readings are zero because the volume in the displayed lines is also close to zero.
A snippet of the dataset is provided below.

Table 1 – Snippet of the Dataset (Part 1)

Yaw	Pitch	Roll	Height_mm	Volume_mm^3	Cut	Simulation
0	-10	-15	-887.35	44.31	1	0
0	-10	-15	-884.38	611.17	2	0
0	-10	-15	-881.42	3491.88	3	0
0	-10	-15	-878.45	9972.61	4	0
0	-10	-15	-875.49	21415.61	5	0

Table 2 – Snippet of the Dataset (Part 2)

S1	S2	S3	S4	S5	S6	S7	S8
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

The dataset columns are described as follows:

- **Yaw_degree, Pitch_degree, Roll_degree:** The orientation of the aircraft in three-dimensional space during the simulation.

- **Height_mm:** The height at which the simulated sensor measurements were taken.
- **Volume_mm^3:** The total volume of fuel corresponding to each specific cut plane in the simulation, correlated with the sensor measurements.
- **Cut:** Indicates the sequence of the cut planes generated during the simulation.
- **Simulation:** A counter indicating the number of simulations, each representing a unique aircraft attitude.
- **S1 to S8:** The readings from eight capacitive sensors placed at different locations inside the tank.

Typical Pitch and Roll Angles

- **Pitch Angles**
  - *Cruise Flight:* Slightly positive, around 2-5 degrees. This helps maintain a level flight attitude and efficient aerodynamic performance.
  - *Climb:* Typically between 10-20 degrees, allowing for an effective ascent rate while maintaining control.
  - *Descent:* Often between -3 to -10 degrees, ensuring a stable descent rate and good visibility for the pilot.
- **Roll Angles**
  - *Straight and Level Flight:* Near 0 degrees. The wings are kept level to maintain a steady, straight flight path.
  - *Standard Turns:* Between 15 to 30 degrees, providing a balance between turn rate and passenger comfort.
  - *Steep Turns:* Can exceed 30 degrees, sometimes reaching up to 45 degrees or more, typically used in more aggressive maneuvers or specific training scenarios.

According to the above, the following angles were chosen, giving a dataset with a total of 961 attitudes analyzed. So, the data frame has the following characteristics:

- **Pitch_degree:** The data varies between -10 and 20 degrees.
- **Roll_degree:** The roll ranges from -15 to 15 degrees.
- **Volume_mm^3:** The volume of the analyzed tank is approximately 4.52 m^3.
- **Cut:** The dataset contains 500 unique cross-sections for each attitude.
- **Simulation:** There are 961 distinct simulations, each one representing a unique aircraft attitude.
- **sensor_1** to **sensor_8**: The readings from these sensors range between 0 and 1.

For the analysis, the variables that are going to be used are **Pitch_degree**, **Roll_degree**, **Volume_mm**^{3}, and the readings from **sensor_1** to **sensor_8.**

4. Initial Data Exploration

An initial exploration of the dataset was conducted to understand its structure, characteristics, and potential patterns. Descriptive statistics were computed to summarize the central tendency, dispersion, and shape of the distribution of the dataset.

The initial exploration of the dataset revealed the presence of negative values, around zero, in the `Volume_mm3` column, which represents the volume of fuel. These errors occurred due a really small values in the mesh of the tank geometry. Since the volume cannot be negative, these values were replaced as zeros to clean the dataset. The cleaned dataset ensures that the subsequent analysis is based on accurate and meaningful data.

Histograms of the sensor readings (as shown in Figure 2) already reveal several interesting patterns and behaviors.

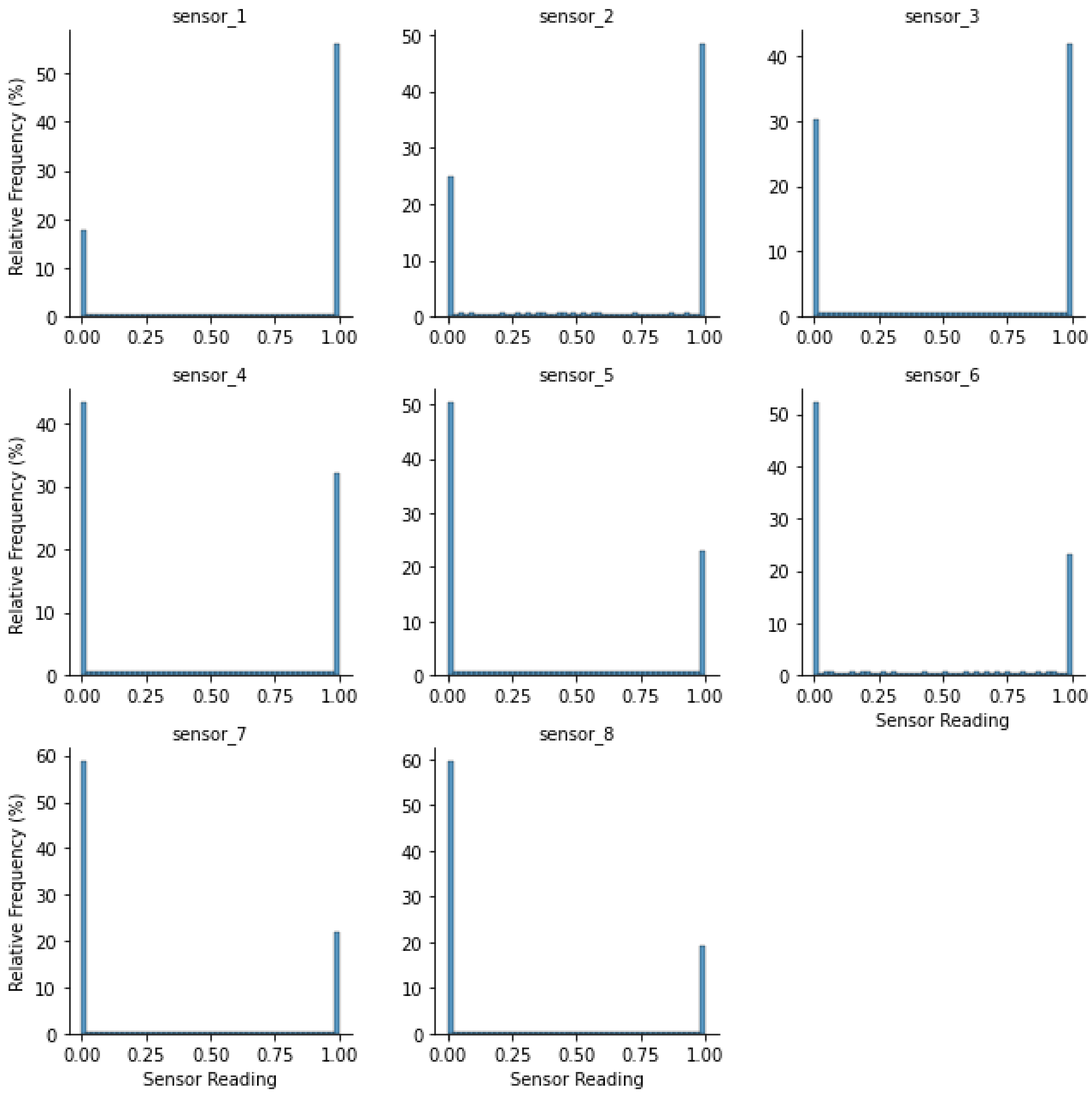


Figure 2 – Sensor Readings Histogram.

Saturation at 0 and 1

Most of the sensors have significant peaks at 0 and 1. This indicates that the sensors often read the minimum (0) and maximum (1) values. The difference percentages and the kind of saturation suggest where the sensors are acting to detect fuel at their respective positions.

Variation in Sensor Readings

Despite the peaks at 0 and 1, there are variations and readings across other values, which also provide insights into the fuel distribution and the sensor positions in the tank.

Differences Between Sensors

The degree of saturation and the distribution of readings vary between sensors. Some sensors show a higher frequency of intermediate values compared to others. This difference can be attributed to their positions in the tank. For instance, sensors located at the bottom might read one more often, as the tank empties (from being full), while those at the top might read zero more frequently, once the top will be quicker emptier than the bottom.

Detailed Observations for Each Sensor

- **Sensor 1:**
  - Shows a strong peak at 1 (around 60%), indicating it often reads the maximum value.
  - Likely positioned near the bottom of the tank, hence it senses full fuel more often.
- **Sensor 2:**
  - Displays a more even distribution with a small peak at 0 and 1.
  - It starts to be positioned not so close to the bottom anymore, also going in direction of the middle, experiencing various fuel levels.
- **Sensor 3:**
  - Going to a more even distribution between the peaks at 0 (around 30%) and 1 (around 40%), and with a slightly higher variation in intermediate values.
  - It Could be slightly lower than Sensor 1 but still towards the top.
- **Sensor 4:**
  - Peaks at 0 and 1 with intermediate values.
  - Likely positioned towards the middle of the tank. At this stage, it is seen the data inversion. At this point, this sensor sees more empty phases (around 45%) than full ones (around 35%)
- **Sensor 5:**
  - Similar to Sensor 4 with peaks at 0 and 1, but increasing empty readings.
  - Slightly different in height than Sensor 4, but been around the middle of the tank.
- **Sensor 6:**
  - Similar frequencies of the sensor 5. Shows peaks at 0 and 1 with slightly more variation in intermediate values compared to Sensor 5.
  - It is in a mid-upper position.
- **Sensor 7:**
  - Follows the pattern of sensors 5 and 6, but still increasing in 0 readings.
  - Positioned almost at the top of the tank.
- **Sensor 8:**
  - It has the highest peak at 0 (60%), suggesting it almost always reads empty.

– It is positioned at the very top of the tank.

The sensors show clear saturation at 0 and 1, indicating positions that frequently read empty or full fuel states. Sensors in the middle of the tank tend to have more variation in readings, reflecting the gradual change in fuel levels. The position of each sensor in the tank influences the distribution of its readings, with sensors at the top and bottom showing more saturation. Finding these patterns can help optimize sensor placement and interpret the fuel levels more accurately in the tank.
A scatterplot (Figure in full page on page 8) was also created to visualize the relationships between sensor readings and fuel volume. The plot provides insights into the distribution and correlations between these variables, aiding in the identification of patterns and trends in the data.
In order to get a better visualization, some attitudes were selected, and the scatterplot was reported and shown in Figure 3.
Exploring the interaction effects between pitch and roll angles and sensor readings, the analysis revealed complex nonlinear interactions, which can be seen in these two Figures (3 and 4).

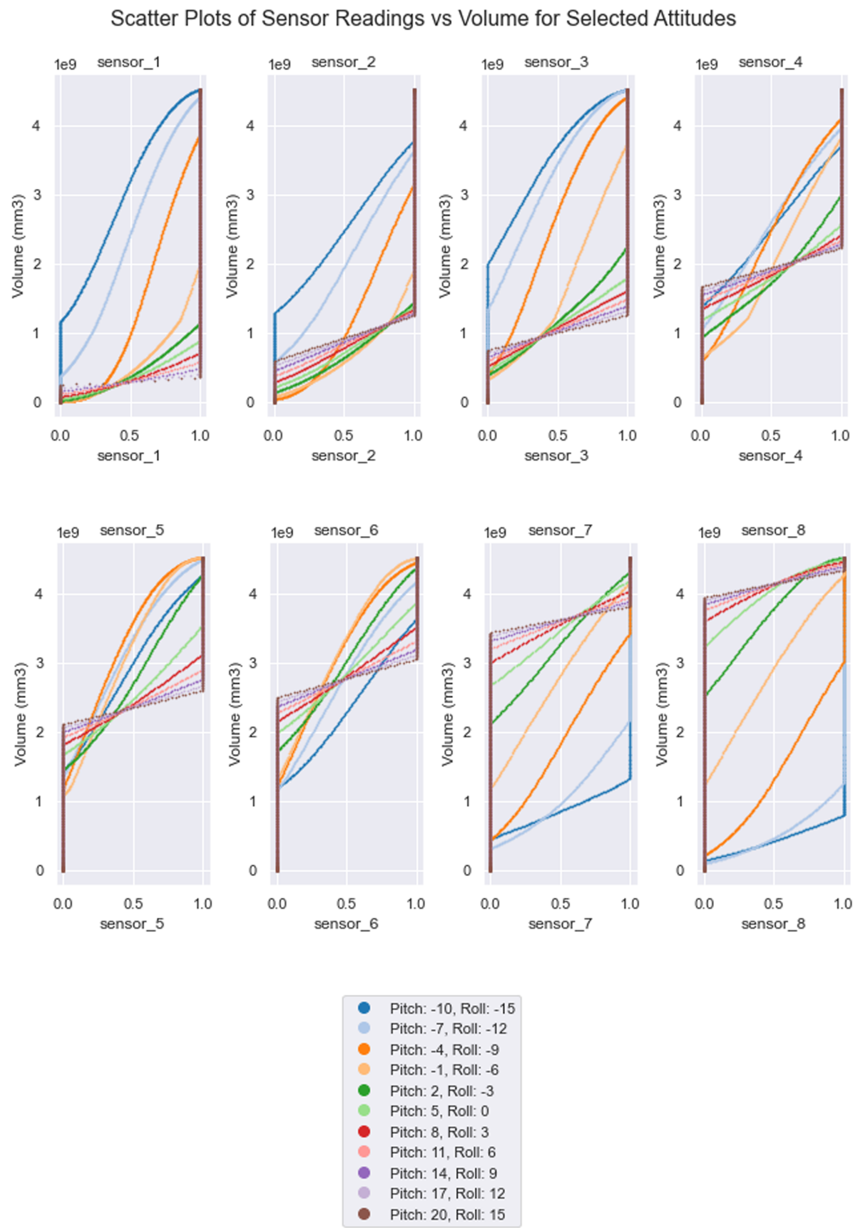


Figure 3 – Scatter plot visualizing the relationships between sensor readings and Volume, on the representative attitudes. All sensors plotted.

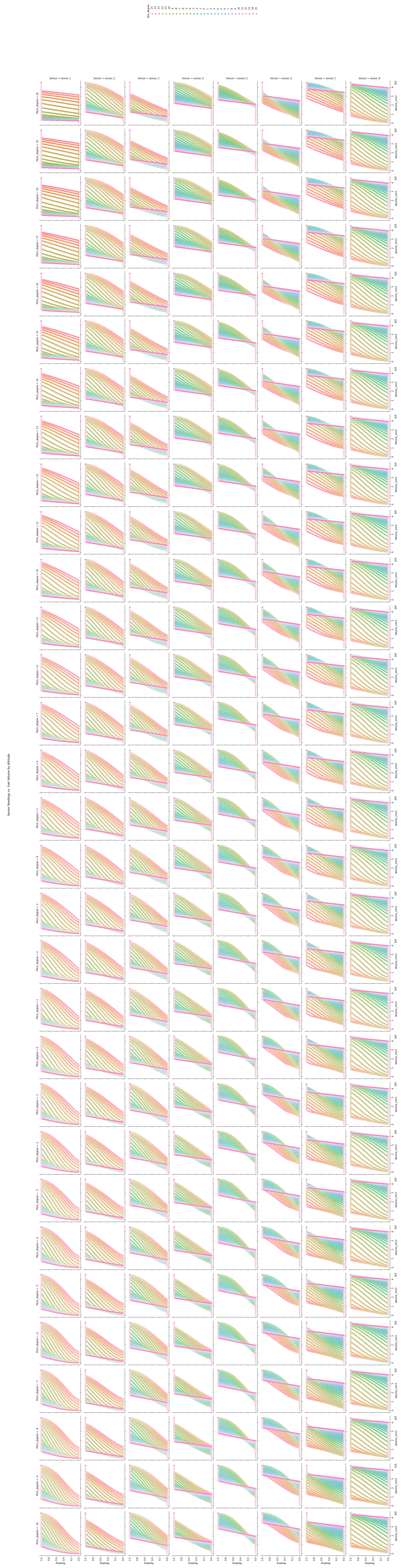
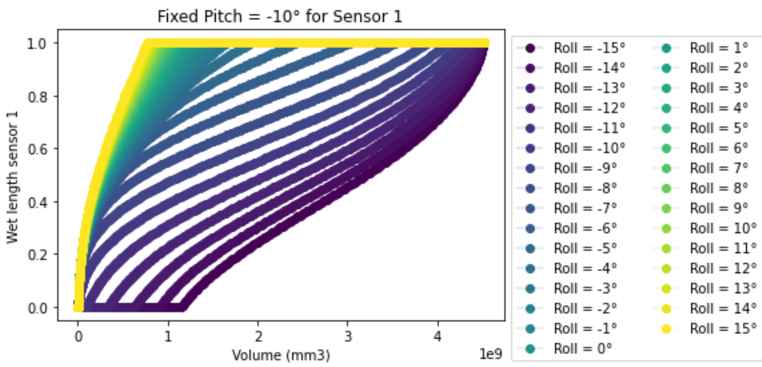
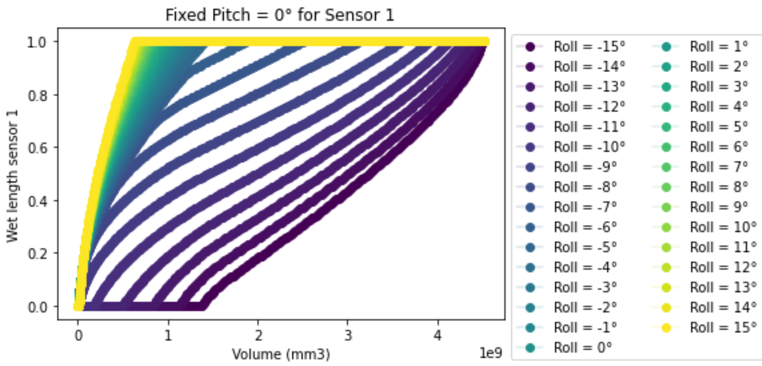


Figure 4 – Scatter plot visualizing the relationships between sensor readings and Volume, on the 961 attitudes.

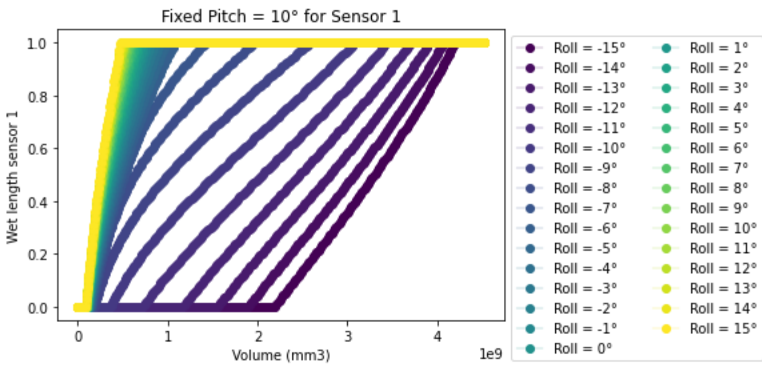
To further enhance the understanding and visualization of this relationship, two additional groups of graphs were plotted: one for sensor 1 and another for sensor 3 (Figures 5 and 6). Each group includes plots for three different pitch angles (-10, 0, 10) with all corresponding roll angles.



((a)) Fixed Pitch = -10, all rolls. Sensor 1 Wet length against volume.

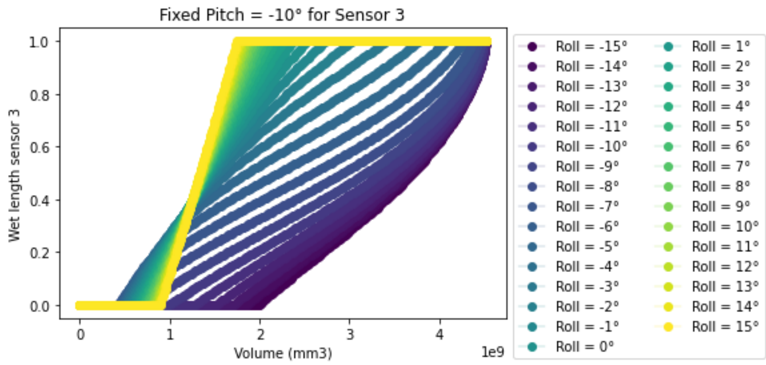


((b)) Fixed Pitch = 0, all rolls. Sensor 1 Wet length against volume.

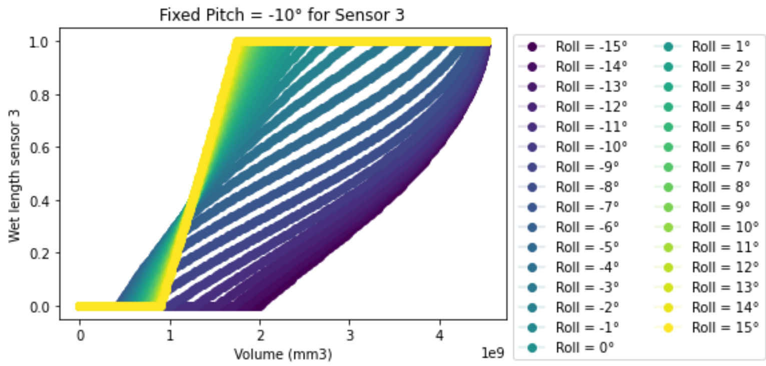


((c)) Fixed Pitch = 10, all rolls. Sensor 1 Wet length against volume.

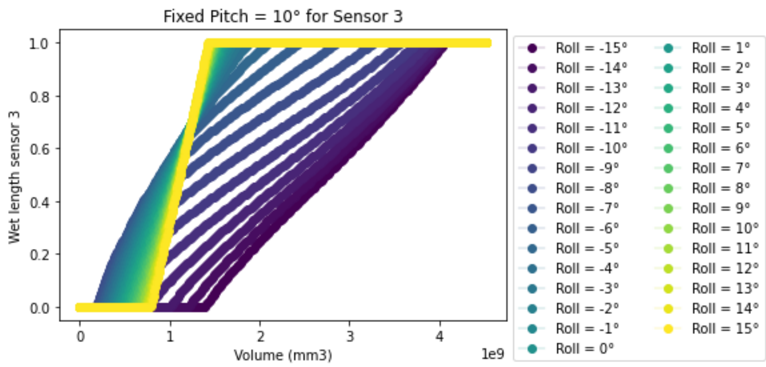
Figure 5 – Sensor1, Fixing Pitch at -10, 0 and 10, with all related rolls.



((a)) Fixed Pitch = -10, all rolls. Sensor 3 Wet length against volume.



((b)) Fixed Pitch = 0, all rolls. Sensor 3 Wet length against volume.



((c)) Fixed Pitch = 10, all rolls. Sensor 3 Wet length against volume.

Figure 6 – Sensor1, Fixing Pitch at -10, 0, and 10, with all related rolls.

5. Correlation Matrix

Statistical analyses are important tools in data science and research, enabling scholars to discern patterns, relationships, and underlying structures within datasets. Among these analyses, correlation matrices play a crucial role by providing a comprehensive view of the relationships between pairs of variables. This section delves into the PhiK correlation, elucidating its applications, and the rationale for its selection in this study.

5.1 PhiK Correlation

PhiK correlation is a novel approach developed to address the limitations of traditional correlation measures like Pearson, especially when dealing with non-linear relationships and categorical data. Proposed by K. W. Meyer et al., PhiK provides a robust measure of association that works uniformly across different data types, including continuous, discrete, and categorical variables [5, 6].
The PhiK correlation coefficient ranges from 0 to 1, where 0 indicates no association and 1 represents a perfect association. Unlike Pearson correlation, PhiK does not assume linearity or a specific distri-

bution, making it highly versatile and suitable for a broader range of data types and structures. PhiK is computed by first constructing a contingency table and then applying a normalization procedure that adjusts for the expected values of the chi-squared statistic under the assumption of independence. This process ensures that PhiK accounts for both the strength and the significance of the association between variables, providing a more comprehensive measure of correlation in complex datasets [6].
In the context of this study, the relationship between sensor readings and fuel volume has been observed to be highly non-linear. The simulated results from the computational tool specifically developed to study aircraft fuel systems have highlighted this non-linearity, independent of the aircraft's attitude (pitch, roll, yaw). Given this non-linearity, traditional linear correlation measures like Pearson are inadequate for accurately capturing the relationships between these variables.
The Pearson correlation coefficient is limited by its assumption of linearity and sensitivity to outliers, which can lead to misleading conclusions when applied to non-linear data. Additionally, ensuring that the data meets the assumption of multivariate normality is critical for the validity of many statistical techniques, including traditional correlation measures [7]. To ensure robust model validation, cross-validation techniques can be employed, providing a more accurate estimation of model performance [8].
Therefore, to accurately quantify the complex, non-linear interactions between sensor readings, fuel volume, and aircraft attitudes, PhiK correlation is employed. PhiK's ability to handle non-linear relationships and its flexibility in dealing with different data types, make it a robust tool for this analysis [5, 6].
This approach has the potential to facilitate better decision-making in sensor placement and system design, ultimately contributing to more reliable aircraft fuel measurement systems. Figure 7 shows the resultant Phik matrix.

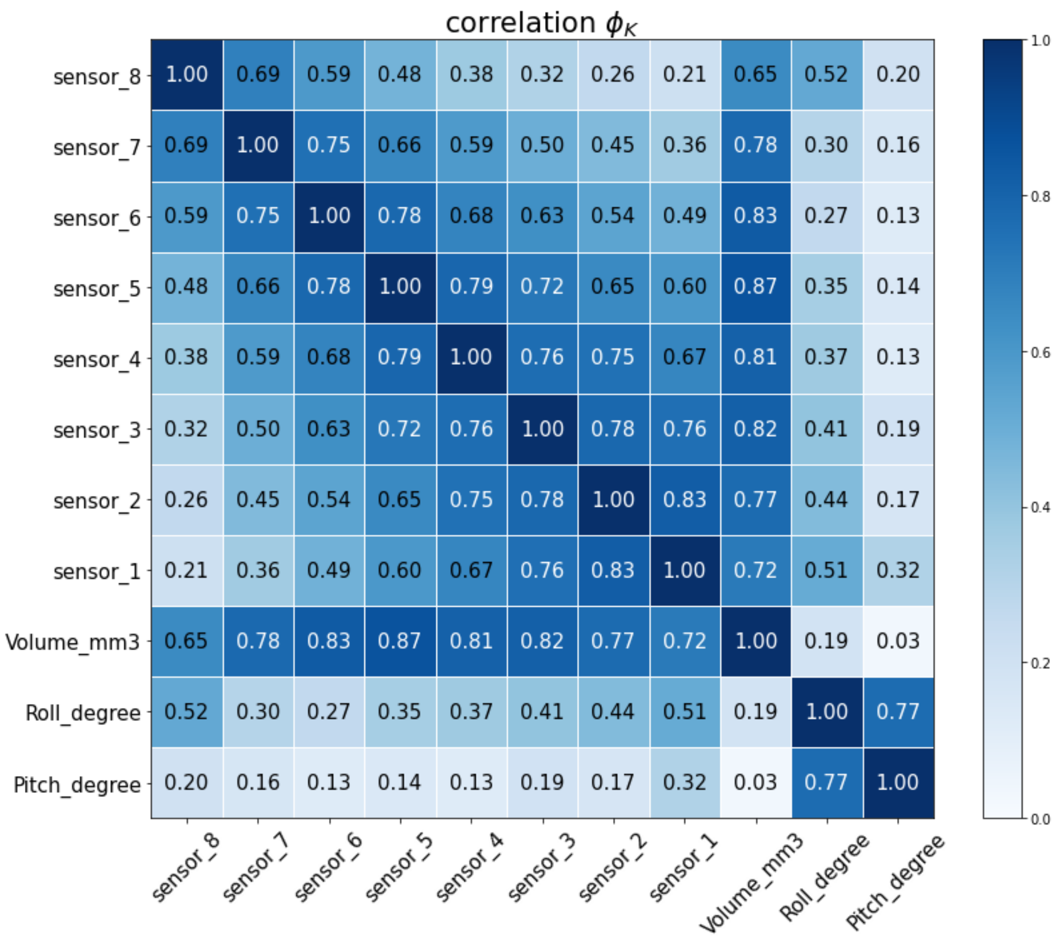


Figure 7 – Phik Correlation Matrix

Some interpretations can be made based on the provided matrix:

5.2 Sensors in Proximity

Sensors located near each other within the fuel tank exhibit high correlations, indicating that their readings are closely related. For instance, sensor pairs such as *sensor_1* and *sensor_2* ($\phi_K = 0.83$), *sensor_4* and *sensor_5* ($\phi_K = 0.79$), *sensor_5* and *sensor_6* ($\phi_K = 0.78$), and *sensor_6* and *sensor_7* ($\phi_K = 0.75$) show strong correlations. This suggests that these sensors are likely to detect similar changes in fuel volume and distribution due to their proximity.

5.3 Sensors and Fuel Volume

The volume of fuel (*Volume_mm3*) shows strong positive correlations with most sensors, particularly with *sensors 5* ($\phi_K = 0.87$), but also *sensors 3* ($\phi_K = 0.82$), *4* ($\phi_K = 0.81$), and *6* ($\phi_K = 0.83$), all above 80%. This indicates that the fuel volume is highly influenced by the readings from these sensors, with sensors in the central region of the tank being particularly important on explaining the fuel volume. This relationship is critical for accurately estimating fuel volume based on sensor readings.

5.4 Roll Angle and Sensor Readings

The roll angle (*Roll_degree*) has notable correlations with *sensor 1* ($\phi_K = 0.51$) and *sensor 8* ($\phi_K = 0.52$). This suggests that the roll angle significantly impacts the readings from these sensors, which are positioned at the edges of the fuel tank. The roll-induced movement of fuel could cause these sensors to experience more significant changes in readings compared to others.

5.5 Pitch Angle

The pitch angle (*Pitch_degree*) shows generally low correlations with all sensors (ϕ_K values ranging from 0.03 to 0.32). This indicates that changes in the pitch angle have a minimal linear relationship with sensor readings. This observation might be due to the shape of the tank and the orientation of the sensors, which are less sensitive to pitch-induced fuel movements. However, there is a high correlation between pitch and roll ($\phi_K = 0.77$)

5.6 Additional Insights

The strong correlations among *sensors 3, 4, 5, and 6* (ϕ_K values between 0.76 and 0.79) suggest a cluster of sensors that provide redundant information regarding the fuel volume. This redundancy can be useful for cross-verification of readings, improving the reliability of fuel volume estimations.
The lower correlation between *Volume_mm3* and the outermost sensors (*sensor 1* and *sensor 8*) compared to the central sensors indicates that the outer sensors are more affected by aircraft movements (roll and pitch).

6. PhiK Significance Matrix

A significance matrix provides a statistical measure of the reliability of the correlations identified in a correlation matrix. Specifically, it assesses the probability that an observed correlation could have occurred by random chance. This probability is expressed as a p-value, with lower p-values indicating stronger evidence against the null hypothesis (which posits that there is no actual correlation between the variables) [9, 10].

6.1 PhiK Significance Matrix

In the context of the PhiK correlation, the significance matrix helps to identify which correlations between sensor readings, fuel volume, and aircraft attitudes are statistically significant. Given the high non-linearity and potential complexity of these relationships, the significance matrix is crucial for distinguishing genuine correlations from those that might be spurious or due to random variation [11, 12].
Figure 8 shows the Phik significance matrix. The numbers in the cells represent the significance level of the correlation between the respective pairs of variables. Positive values indicate significant correlations, while negative values indicate less significant or potentially inverse correlations. The color intensity on the heatmap reflects the significance level. Darker green indicates higher significance, while lighter colors indicate lower significance or potential inverse relationships.

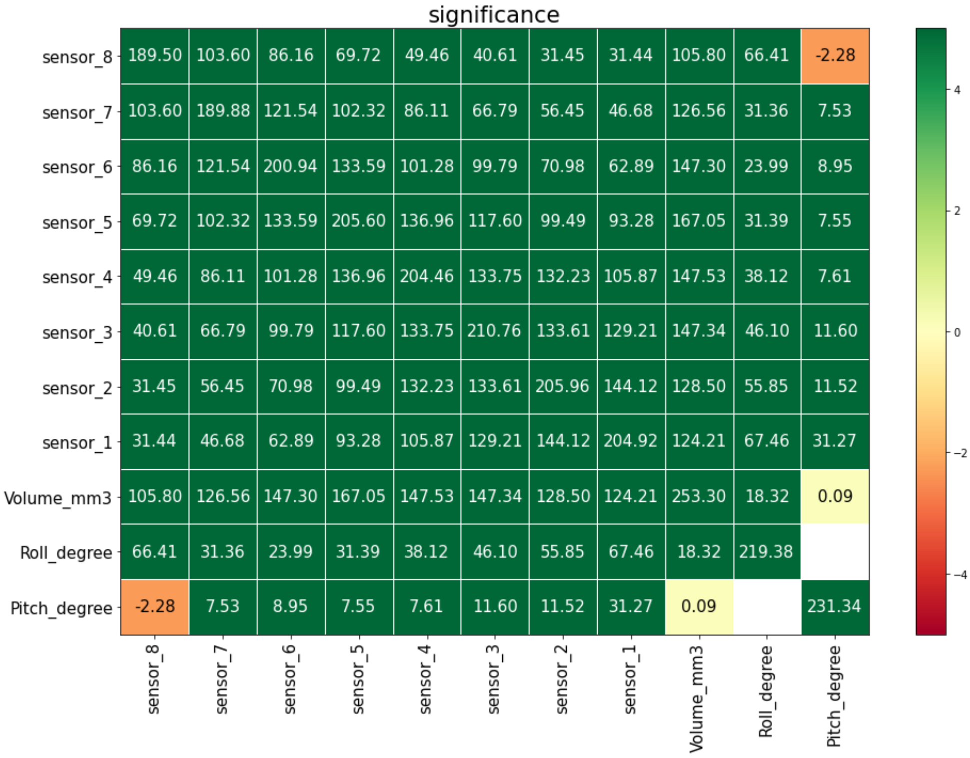


Figure 8 – Phik Significance Matrix

In the matrix, the value of **231.34** indicates a very high level of significance, suggesting a very strong correlation between *Pitch_Degree* and *Roll_Degree*. The value of **-2.28** indicates a potentially inverse correlation between *Pitch_Degree* and *Sensor_8*. *Volume_mm3* has a significance value of **167.05** with *Sensor_5*, indicating a highly significant correlation. On the other hand, the lower significance value of **0.09** with *Pitch_Degree* indicates a weak correlation.

7. Range Sensor Readings

In this section, following the intention of understanding the sensors' behavior under different aircraft attitudes, the Figure 9 illustrates the points at which each sensor starts registering non-zero readings across various pitch and roll angles. Each subplot represents a fixed pitch angle with varying roll angles, and each point within the subplots denotes the volume at which a sensor starts registering non-zero readings for a given roll angle.

Opposite Effects and Correlation

- It can be seen a similar pattern on the sensor readings behavior occurring when pitches go from negative values up to neutral ones (around 0). However, when the pitch starts to become more positive, the reading patterns start to change, and lower sensors have more difficulties registering lower volumes on too-negative rolls.
- Sensors at the extremes (such as Sensors 1 and 8) are more sensitive to attitude variations. Middle sensors (4, 5, 6) have a more stable response to attitude variation.
- On high positive pitches and high negative rolls, the extreme sensors invert functions, where the higher sensors start to register lower volumes and lower sensors mark higher volumes.

Stability and Modeling Implications

Sensor 5 (Most Stable Sensor)

- Sensor 5 consistently provides stable readings across different attitudes, making it a reliable indicator for modeling fuel volume.

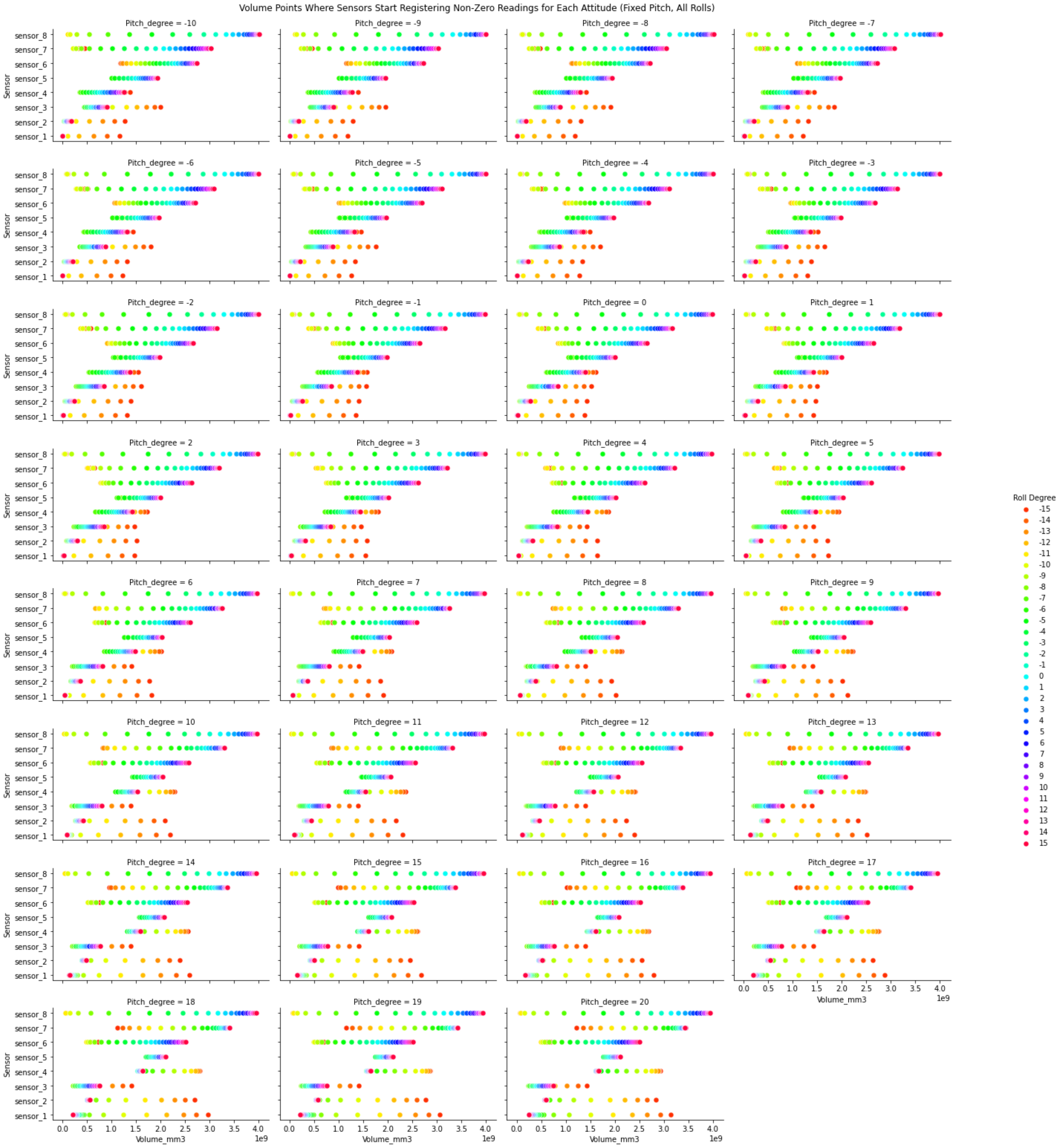


Figure 9 – Figure capturing where each sensor starts to measure in all attitudes.

- Its central positioning likely contributes to its stability, as it is less affected by pitch and roll compared to sensors at the tank's extremes, indicating that the central area of the tank is a strategic location to place sensors.

Modeling Volume Estimation

- **Position and Attitude Correlation:** To accurately model fuel volume, it's essential to account for the relative positions of the sensors and the aircraft's attitude.
- **Algorithm Calibration:** Calibration algorithms should incorporate the observed patterns, par-

ticularly the stable readings from central sensors like Sensor 5, and adjust for the variability in sensors at the extremes (Sensor 1 and Sensor 8).

- **Predictive Accuracy:** Using data from stable sensors in conjunction with sensors at varying positions allows for a more accurate prediction of fuel volume under different flight conditions.

In other words, this plot shows again the importance of sensor 5 on the readings, and that sensors at the extremes (both ends of the tank) are more sensitive to changes in pitch and roll, while central sensors provide more stable readings. Besides, it shows that for accurate fuel measurement, sensors should be strategically placed to account for these shifts in fuel distribution.

8. Application of Normalized Sensitivity Matrix

Now, the next step towards the understanding of the behavior analyses, was to apply a Normalized Sensitivity Matrix (NSM), which is an analytical tool designed to quantify the influence of system inputs on outputs [13]. In the context of the studied aircraft fuel measurement analysis, we aim to utilize NSM to understand the intricate relationships between the aircraft's orientations (pitch, roll), the height at which sensor readings are captured, and the resulting sensor readings.
In applying NSM to our dataset, we keep gaining nuanced insights into how each sensor's reading is affected by changes in the aircraft's orientation and the measurement height.

Structure and Computation of NSM

Each element a_{ij} of the NSM corresponds to the sensitivity of the i^{th} output (sensor reading) to the j^{th} input (aircraft orientation or height), mathematically expressed as:

$$a_{ij} = \frac{\partial y_i}{\partial x_j} \frac{x_j}{y_i} \tag{1}$$

where y_i denotes the i^{th} output, x_j denotes the j^{th} input, and $\frac{\partial y_i}{\partial x_j}$ is the partial derivative indicating the rate of change of the output with respect to the input.
The sensitivities are normalized, rendering a standardized measure of influence that enables a direct comparison of the various inputs' effects on the outputs.

Mathematical Approach

A this stage, a Random Forest algorithm was applied, which is an ensemble learning method, known for its flexibility and ability to model non-linear relationships. It builds multiple decision trees during training and outputs the mean prediction of the individual trees for regression problems [14]. The equation governing the Random Forest regression model is given by:

$$\hat{y} = \frac{1}{n} \sum_{i=1}^{n} T_i(x) \tag{2}$$

where:

- $\hat{y}$ is the predicted output,
- n is the total number of decision trees in the forest,
- $T_i(x)$ is the prediction of the i^{th} decision tree for the input vector x.

We trained a separate Random Forest model for each sensor to understand its specific sensitivity to changes in pitch and roll. The feature importances derived from the models quantify the impact of each feature on the sensor readings.

Visual Insights and Findings

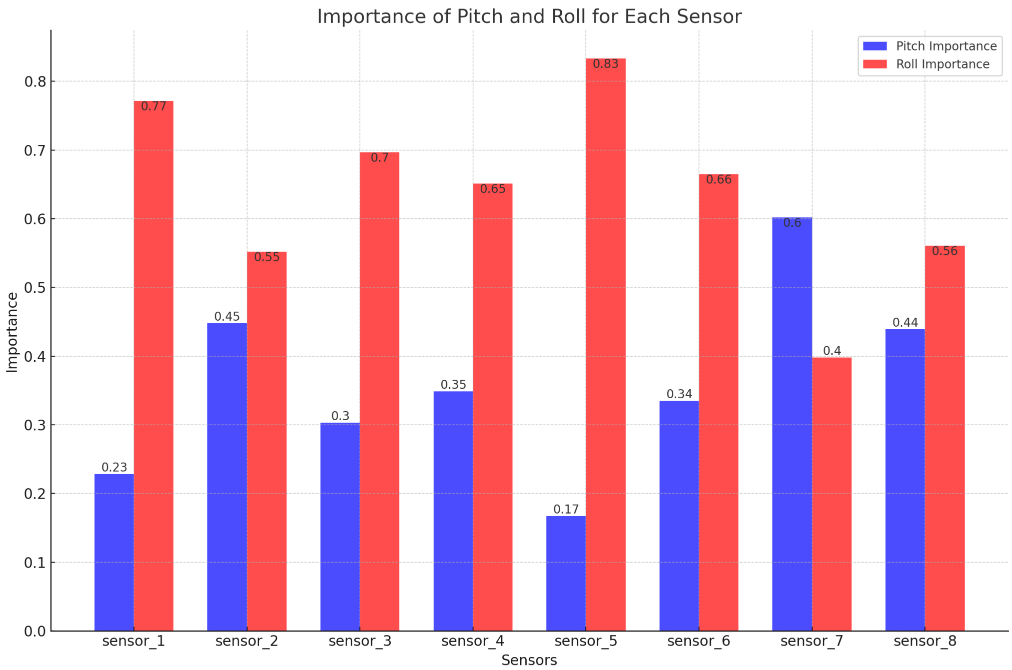


Figure 10 – Importance of Pitch and Roll for Each Sensor

Figure 10 visualizes the importance of pitch and roll for each sensor. The findings suggest a complex interplay between the aircraft's flight dynamics and the sensor readings. Each sensor exhibits distinct sensitivity to pitch and roll, influenced by its specific location within the fuel tank.
Through this method, the plot in Figure 10 provides valuable insights into the significance of pitch and roll orientations for the sensors. For instance, *sensor_1* has a higher roll importance (0.77) compared to pitch (0.23), indicating that this sensor is more sensitive to roll changes. Similarly, *sensor_5* exhibits the highest roll importance (0.83), making it highly responsive to roll orientation variations.
In contrast, some sensors show a more balanced sensitivity between pitch and roll. *sensor_7*, for example, has a pitch importance of 0.6 and a roll importance of 0.4, suggesting that both orientations equally influence it. This balanced sensitivity is also observed in *sensor_4*, which has a relatively close importance for both pitch and roll.
Overall, the dominance of roll importance is noticeable in most sensors, with sensors such as *sensor_1*, *sensor_5*, and *sensor_6* showing a substantial influence of roll over pitch. These insights can guide adjustments in sensor placement or calibration to enhance accuracy in aircraft fuel measurement systems, ensuring that the sensors provide reliable data under varying aircraft orientations.

9. Discussion

The analysis of sensor data from aircraft fuel tank readings reveals the complex and nonlinear interactions between attitude angles and sensor readings. The contour plots illustrate distinct patterns in sensor behavior influenced by pitch and roll angles, with middle sensors (e.g., sensor_4, sensor_5) showing stable readings across various attitudes, while sensors at the tank's ends (e.g., sensor_1, sensor_8) demonstrate significant sensitivity to these changes. This highlights the necessity for careful calibration and potential compensation in fuel management algorithms to ensure accurate fuel level estimations and shows the importance of certain sensor locations within the tank for system modeling.
The PhiK correlation matrix underscores significant relationships between sensors and aircraft attitude parameters, with values such as 0.83 between sensor_6 and Volume_mm3, and 0.77 between Roll_degree and Pitch_degree, indicating strong relationships. These findings imply that changes in aircraft attitude can lead to predictable sensor responses, which are crucial for accurate fuel level monitoring.

Further insights are provided by the PhiK significance matrix, where high values, such as 231.34 for Pitch_Degree vs. Roll_Degree, denote extremely significant correlations, emphasizing the interdependence of these parameters. Similarly, strong correlations between Volume_mm3 and some sensors (e.g., 167.05 with sensor_5) highlight their critical role in fuel volume measurement. Conversely, low significance values, such as 0.09 between Pitch_Degree and Volume_mm3, indicate weak correlations, suggesting a minimal direct relationship.

The Normalized Sensitivity Matrix (NSM) reveals the relative sensitivity of sensors to attitude changes. Sensors with high sensitivity to pitch and roll are suitable for key monitoring positions within the fuel tank.

In summary, this analysis highlights the complex, nonlinear behavior of sensor responses to aircraft attitude changes, underscoring the need for advanced techniques for predicting and modeling fuel estimation and optimizing sensor placement. The strong correlations and high significance values provide valuable insights into sensor placement strategies and lay a foundation for future enhancements in aircraft fuel measurement systems. Future research will focus on developing fuel volume estimation models, applying machine learning techniques, and further using these models to optimize sensor quantity and placement, ensuring more accurate and reliable fuel management.

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11. Copyright Statement

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