A SENSITIVITY ANALYSIS ON THE ENDURANCE EQUATION FOR HYBRID-ELECTRIC AIRCRAFT

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Abstract

The paper performs a sensitivity analysis on the proposed endurance equation [13] for a hybrid-electric aircraft model. The endurance is a key aircraft performance parameter which gives the time taken by an aircraft to fly from an airport A to airport B on a certain amount of fuel. In this research, a subsequent analysis is carried out on the case study of a regional turboprop ATR 72 aircraft [14]. This paper performs the sensitivity analysis based on the partial differentials and subsequently using the plots. The effects of parameters such as coefficient of lift and drag, payload and empty weight, energy consumption, efficiencies, battery energy density, degree of hybridization, etc. are observed on the aircraft endurance and the deductions were carried out. The analysis also shows the impact of the important parameters according to an aircraft designer's perspective using the forecast for the year 2050 on the case study aircraft model.

Keywords: hybrid-electric aircraft, aircraft endurance, aircraft range, battery energy density, degree of hybridization

1. Introduction

During the time when the effects of climate change continue to be felt around the world, there is a pressing need to reduce emissions [1]. Transport is one of the major contributors to climate change. Aviation industry contributes to about 3% of global greenhouse emissions. Additionally, only 10% of the global population use air travel at present. As the world economy grows and more countries are pulled out of poverty, the citizens of these countries too would use air travel, which would increase the emissions further. The COVID-19 pandemic played a significant role in disrupting air traffic, the aviation is expected to recover quickly to its pre-COVID rates. In fact, the ICAO post-COVID-19 forecasts suggest that the aviation industry would recover at an average growth of 3.6 % per annum. In the European Union, programs such as Flightpath 2050 [2] and the Green Deal [3] have goals of reducing in-flight CO_2 emissions by 70%, NOx emissions by 90% and significant reduction in noise levels. To achieve these goals, state of the art solutions such as electrified aircraft [4], hydrogen-based aircraft [5] and Sustainable Aviation Fuel (SAF) [6] based aircraft are being proposed.

The electric propulsion systems have advantages such as reduction of environmental effects of aviation. However, battery energy densities higher than 800 Wh/kg is required to sustain an Airbus A320 / Boeing 737 sized aircraft with around 1,111 km aircraft range [7]. The existing state of the art of battery technology (2024) provides an energy density of approximately 200-250 Wh/kg. This value is expected to double every 23 years [8] and thus it will take around five decades to achieve a power density greater than 800Wh/kg. Other environmentally friendly options such as hydrogen still face significant challenges related to aircraft design, the development of fuel infrastructure, and industry to provide green hydrogen. Likewise, Sustainable Aviation Fuel (SAF) also has its challenges. It is costlier than the traditional jet fuel and they only make up a small portion of the current market fuel and significant commercialization is needed in this case for them to be widely available.

There is a possibility that as a short to medium term solution, hybrid-electric propulsion systems [9] may fill this existing gap. The Hybrid-electric propulsion system combines conventional fueled gas turbines with electric drives which are powered by battery packs. Although the hybrid-electric aircraft would still produce some emissions during the flight mission, these were shown to offer financial and environmental benefits [10]. Such designs could be effective in reducing emissions in sensitive phases such as during takeoff and landing (i.e. near airports) or when flying through saturated parts of the atmosphere so as to reduce the formation of contrails entirely. To achieve these goals, important aircraft design parameters have to be established adequately. In our previous work [11], we proposed a new range equation for the hybrid-electric aircraft using a new definition of the degree of hybridization (φ) which includes efficiencies of the components in the electric or the fuel powered drivetrain. A sensitivity analysis of the hybrid aircraft parameters on range was investigated in [12]. However, aircraft range is not the only important flight performance property that is of interest. Endurance, or the amount of time an aircraft stays in air with a load of fuel, is also important. We have recently developed a theory and an endurance equation for the hybrid-electric aircraft [13]. The equation is applicable over the entire range of the degree of hybridization (from fully-fueled to fully-electric).

In this paper, we carry out a sensitivity analysis on the proposed hybrid-electric endurance equation, which shows the effect of important parameters such as coefficient of lift, coefficient of drag, payload weight, empty weight, planform area, energy consumption, efficiencies, battery energy density, and degree of hybridization, on the resulting aircraft endurance. The effects of these parameters are investigated as some of these values are expected to continue to improve in the coming decades, thereby showing the feasibility for operating a hybrid electric aircraft at any given degree of hybridization. The analysis also shows the implications that changes in such parameters would have on the design of the aircraft itself. The paper also highlights how range and endurance may be more sensitive to some parameters than others.

This paper is planned as follows: Section 2 provides a brief overview of the hybrid-electric aircraft configurations, thereby outlining clearly the concept of operation for a hybrid electric aircraft. The endurance equation for such aircraft is also listed. Section 3 introduces a case study using the ATR72 turboprop aircraft [14] as an example. Section 4 describes the sensitivity analysis using partial differentials and identifies the parameters impacting the endurance of a hybrid-electric aircraft. Based on this, the paper puts attempts to predict how key technological components are expected to progress by 2050, and therefore puts forward an analysis of a potential hybrid-electric ATR72. In the end Section 6 provides a discussion on benefits and conclusion, stressing on the results and outcomes of the carried-out sensitivity analysis.

2. Hybrid-electric aircraft configurations and governing equations

The hybrid-electric aircraft propulsion system [11] can be broadly divided into series and parallel configurations. The series hybrid configuration, as the name suggest, involves components arranged back-to-back and the propeller or the output is driven by an electric motor. The energy in this case can be supplied by a battery or by a generator driven by a fuel powered gas turbine. This type of configuration is depicted in Figure 1. Conversely, a parallel hybrid configuration consists of two parallel paths – electrical and mechanical. The electric path is composed of an electric motor powered by a battery. On the other hand, the mechanical path is a fuel powered gas turbine. This configuration is described in Figure 2. The above configurations can also be shown in a generalized form as shown in Figure 3.

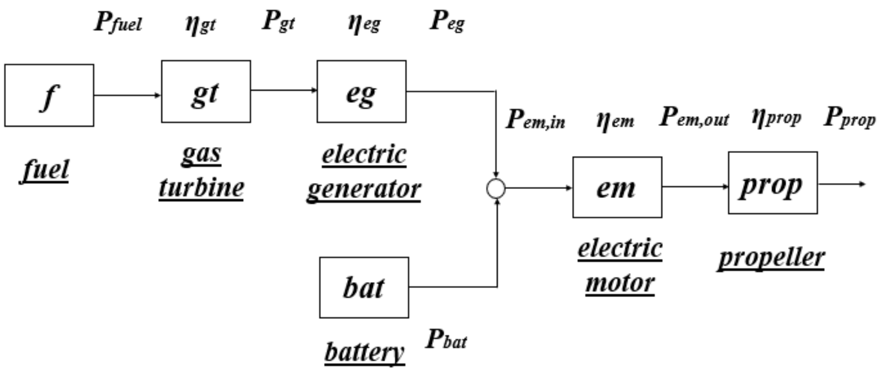

Figure 1. Schematic of the series hybrid-electric powerplant [11].

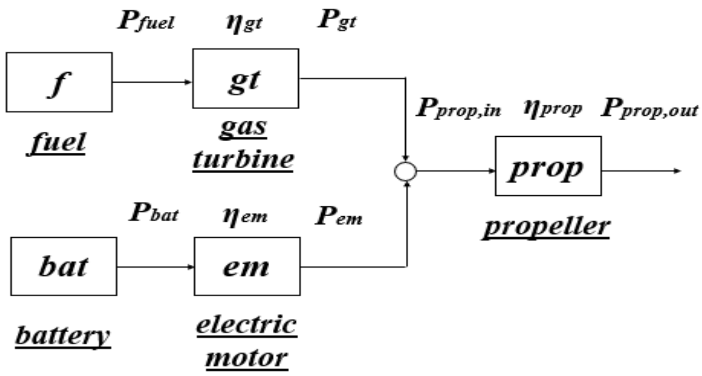

Figure 2. Schematic of the parallel hybrid-electric powerplant [11].

The symbols shown in the above figures are as follows: *gt* is the gas turbine, *eg* is the electric generator, *bat* is the battery, *em* is the electric motor, *prop* is the aircraft propeller, *f* stands for the fuel, *P* denotes power and *η* is the efficiency. The general efficiencies definitions of the series and parallel hybrid-electric configurations are given in Table 1. The schematic for a generalized representation is shown in figure 3.

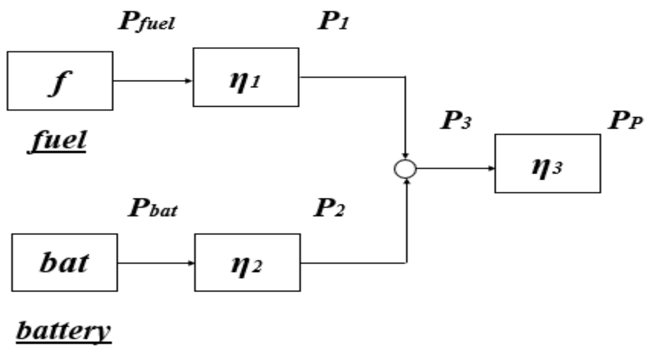

Figure 3. Schematic of a simplified hybrid-electric powerplant representation [11].

Simplified representation	Mechanical-node architecture (or parallel configuration)	Electrical-node architecture (or series configuration)
η_1	η_{gt}	$\eta_{gt}\eta_{eg}$
η_2	η_{em}	1
η_3	$\eta_{gb}\eta_p$	$\eta_{em}\eta_{gb}\eta_p$

Table 1. Efficiencies definitions for series and parallel hybrid-electric configurations [11].

An endurance equation for a hybrid electric aircraft has been proposed in [13] and is replicated below:

$$E = \frac{2^{\frac{1}{2}} \eta_1 \eta_3 e_f C_L^{\frac{3}{2}} S^{\frac{1}{2}} \rho^{\frac{1}{2}}}{(1-\varphi) C_D g} \left[\left(W_{OE} + W_{PL} + \frac{g}{e_{bat}} \frac{\varphi * E_{o,tot}}{\eta_2} \right)^{-\frac{1}{2}} - \left(W_{OE} + W_{PL} + \frac{g}{e_{bat}} \frac{\varphi * E_{o,tot}}{\eta_2} + \frac{g}{e_f} \left(\frac{1-\varphi}{\eta_1} \right) E_{0,tot} \right)^{-\frac{1}{2}} \right] \quad (1)$$

Where:

E is the aircraft endurance
W_{OE} is the empty weight of the aircraft
W_{PL} is the payload weight of the aircraft
S is the wing planform area
ρ is the air density at that altitude
e_f is the energy density of the fuel
Φ is the degree of hybridization
e_{bat} is the energy density of the battery
c_L is the coefficient of lift
c_D is the coefficient of drag
g is the acceleration due to gravity
$E_{o,tot}$ is the total energy contained in fuel and batteries

In earlier work [15], it was shown that the parallel hybrid-electric configuration performs better than the series hybrid-electric configuration as it has less weight and higher overall efficiency. Therefore, in this study only the parallel configuration is adopted for carrying out the sensitivity analysis. Using the efficiency definitions from Table 1, the endurance for a hybrid-electric aircraft in parallel drive train configuration can be written as:

$$E_{parallel} = \frac{2^{\frac{1}{2}} \eta_p \eta_{gt} e_f C_L^{\frac{3}{2}} S^{\frac{1}{2}} \rho^{\frac{1}{2}}}{(1-\varphi) C_D g} \left[\left(W_{OE} + W_{PL} + \frac{g}{e_{bat}} \frac{\varphi E_{o,tot}}{em} \right)^{-\frac{1}{2}} - \left(W_{OE} + W_{PL} + \frac{g}{e_{bat}} \frac{\varphi E_{o,tot}}{em} + \frac{g}{e_f} \left(\frac{1-\varphi}{\eta_{gt}} \right) E_{0,tot} \right)^{-\frac{1}{2}} \right] \quad (2)$$

The following section gives an introduction of the conducted case study based on a regional turboprop ATR 72 aircraft.

3. The ATR 72 aircraft (case study)

The ATR 72 is a popular regional turboprop aircraft that flew for the first time in the year 1988. Such regional aircraft perform around 40% of all the flights worldwide. It is forecasted that around 3020 more of these aircraft types would be needed in the near future [15]. Therefore, such aircraft designs provide an opportunity to implement small improvements which may collectively account to a significant impact on the environment. Figure 4 gives a schematic view of the ATR 72 aircraft and Table 2 details its specifications.

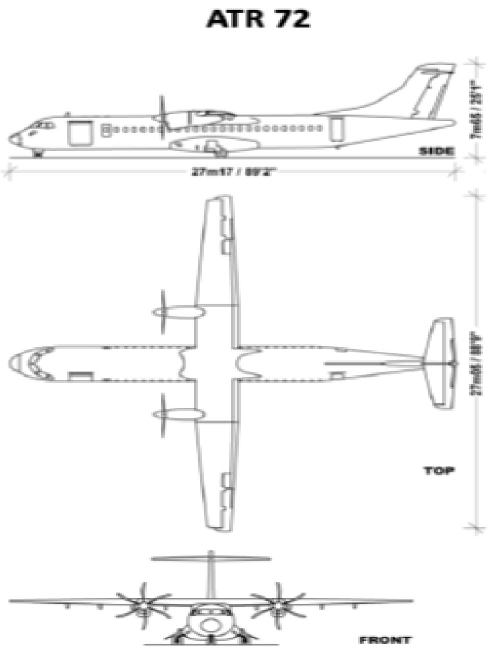

Figure 4. Schematic of the ATR 72 aircraft [16].

Parameter	ATR 72-600	Units
Maximum Passengers	72	
Length	27.17	m
Wingspan	27.05	m
Height	7.65	m
Wing Area	61	m2
Empty weight	13500	kg
Maximum Take-Off Weight (MTOW)	23000	kg
Max. Payload	7500	kg
Engine Power	1846	kW
Cruise Speed	510	km/h

Table 2. Specifications of the ATR 72 aircraft [14].

Having described the aircraft which will be used as a test case, the next section describes the sensitivity analysis of endurance equation (1) carried out by a partial differential method. This analysis shortlists parameters which have the highest impact on the aircraft endurance.

4. Sensitivity analysis

4.1 Sensitivity analysis by partial differential method

The sensitivity analysis is a method to linearly extrapolate the change in output of a mathematical model due to the variation in different existing input parameters [17]. The first method to carry out the sensitivity analysis used here is the partial differentials method. The hybrid-electric aircraft endurance equation (1) was differentiated with respect to dedicated performance parameter, one at a time. This was then multiplied with a certain relative change in the parameter value. The output of the considered equation gives then the change in the expected aircraft endurance. This is described in equation (3). An example is also given in equation (4) which describes the procedure explained in this paragraph.

$$\Delta E = \left(\frac{dE}{d(x)}\right)\Delta(x) \tag{3}$$

$$\Delta E(Change\ in\ endurance) = (104.48)(differentiation\ of\ E\ w.r.t\ x) * (0.20\ (\%\ change\ in\ parameter) * 0.8\ (\%\ initial\ value\ of\ parameter)) = 16.72\ min \tag{4}$$

This analysis was carried out for various parameters such as propulsive efficiency, energy density of battery, payload weight, energy density of fuel and gas turbine efficiency. The corresponding change in the aircraft endurance for various parameters is given in Table 3. It was observed that the propulsive efficiency and energy density of the battery have the most impact on the aircraft endurance as compared to the other parameters. Hence the aircraft endurance is most sensitive to these parameters. Thus, these parameters were shortlisted and further analysis was carried out based on their variation.

Table 3. Change in the aircraft endurance due to the first method of performing sensitivity analysis

Parameter	Partial differential value at initial conditions	% change	Initial value	Change in endurance (min) (dE)	% change in endurance
Propulsive efficiency	104.48	20	0.8	16.7168	20
Energy density battery (Wh/kg)	0.24103	20	228	10.99	13.15
Payload weight (kg)	-0.00035	20	7180	0.5026	0.601
Energy density fuel (Wh/kg)	0.000000033	20	11944.45	0.000079	0.000095
Gas turbine efficiency	4.03	20	0.35	0.2821	0.34

Having identified the parameters which offer the most impact on endurance, the following section focuses on such parameters with the aim to assess how a hybrid-electric aircraft performance improves as the performance of the components also improves in the future until 2050.

4.2 Extrapolating important parameters into the future.

4.2.1 Variation of propulsive efficiency

To project the advances of the improvements in propulsive efficiency over the coming decades, the historical data available in [18], [19] was used and extrapolated. This is presented in the plot of Figure 5. Figure 6 gives the resulting aircraft endurance obtained using the endurance equation 2 by varying the propulsive efficiency with different degrees of hybridization. From the figure, it can be seen that the first change is from original model (point O) to point A. It can be noted that as the hybridization value increases, the endurance time decreases. In this study, a hybridization factor of up to 0.5 is considered. It can be seen that a change in φ from 0 to 0.5 will half the endurance time from 190 minutes to under 85 minutes. An improvement in propulsive efficiency to 0.87 by 2050 will have a minor improvement in endurance to over 90 minutes (point B).

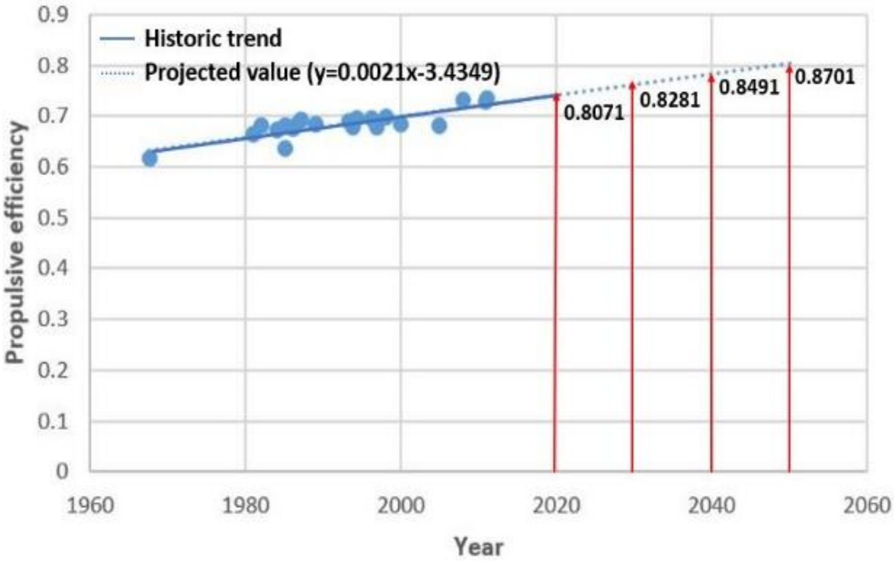

Figure 5. Propulsive efficiency prediction over the upcoming decades [18].

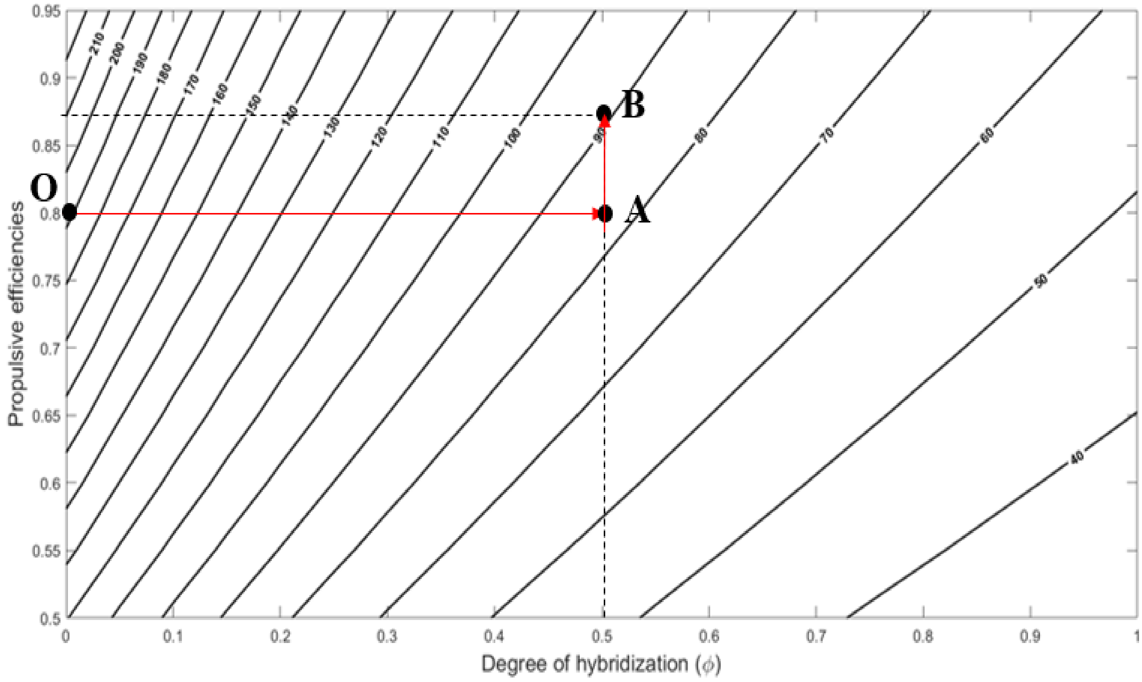

Figure 6. Sensitivity analysis using carpet plots for propulsive efficiency.

The change in the aircraft design from point A to B is also shown schematically in Figure 7. This schematic shows the impact on the aircraft endurance due to the change in propulsive efficiency.

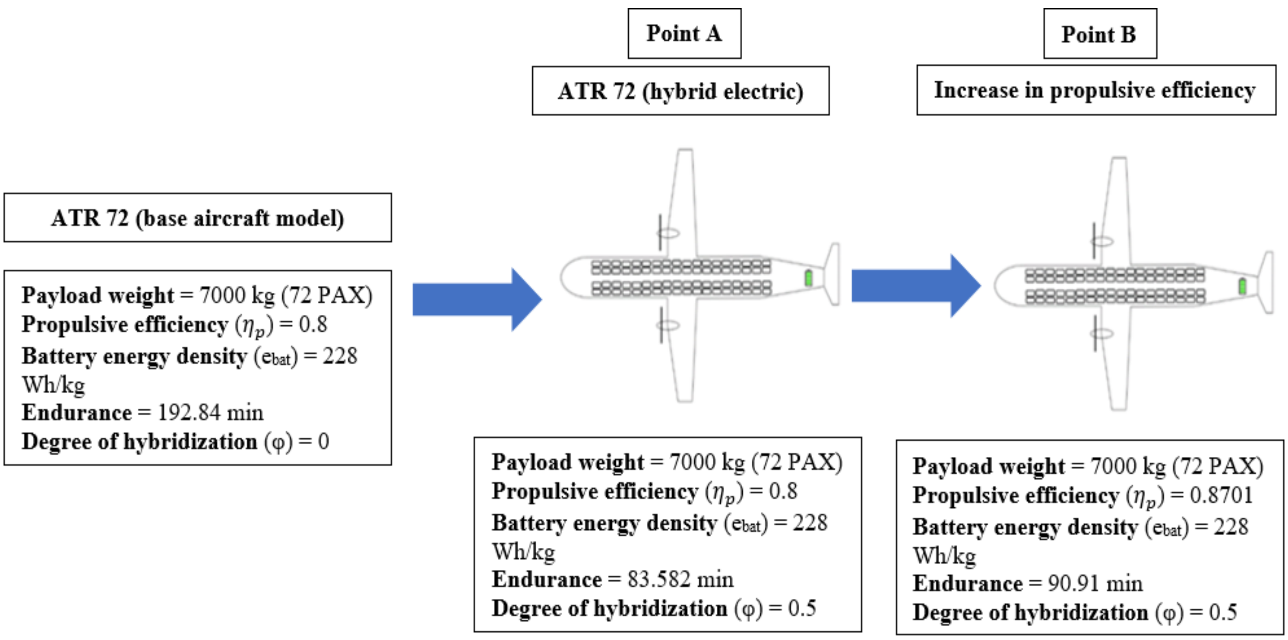

Figure 7. The proposed hybrid-electric ATR 72 aircraft based on the prediction of propulsive efficiency for the year 2050.

4.2.1 Variation of battery energy density

Similar to the previous section, historical battery energy density data from [20] was used to project battery energy density values for the upcoming decades. This is shown Figure 8. Figure 9 gives the resulting aircraft endurance obtained using endurance equation 2 as the battery energy density improves over time, while maintaining a fixed degree of hybridization at 0.5. It can be shown that as the battery energy density improves from current levels (228Wh/kg) to predicted levels of 570 Wh/kg, the aircraft endurance time improves from 90.91 minutes to 143.68 minutes.

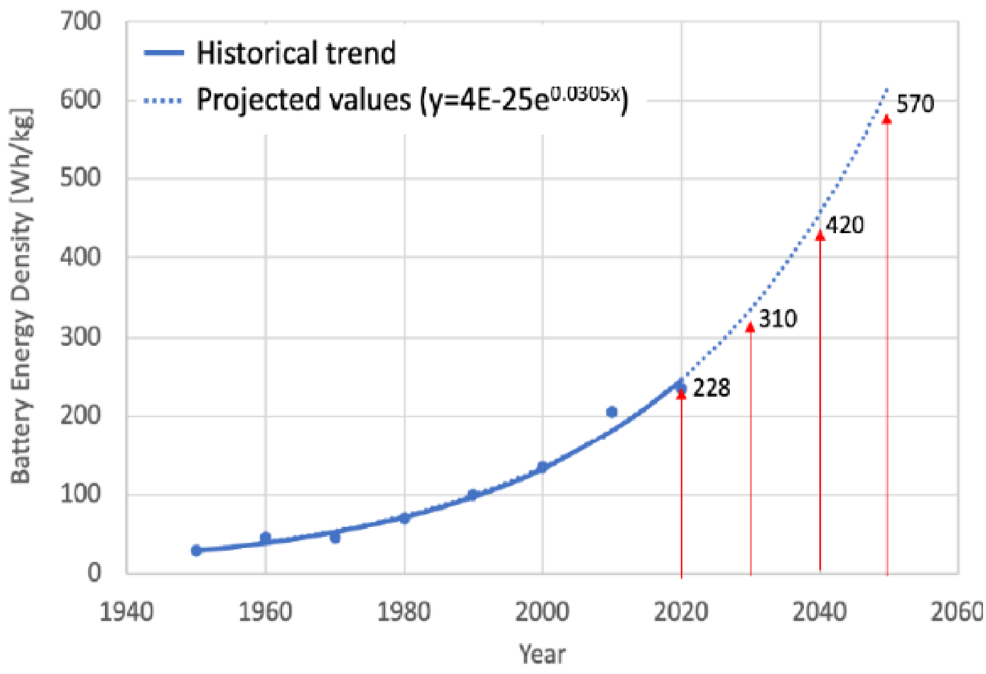

Figure 8. Battery energy density prediction over the upcoming decades [20].

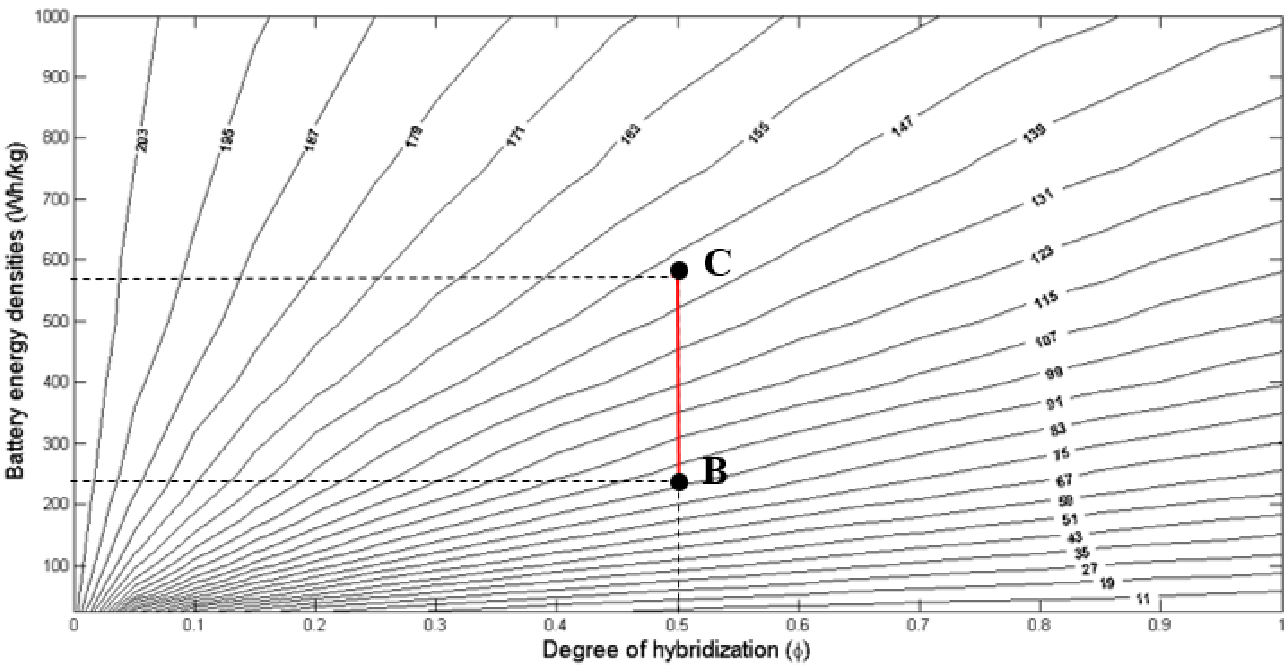

Figure 9. Sensitivity analysis using carpet plots for battery energy density.

The change in the configuration from the last point B (from previous sub-section) to next point C is also depicted schematically in Figure 10. The aircraft endurance changes by 52.77 minutes due to the increase of the battery density from 228 Wh/kg to 570 Wh/kg predicted for the year 2050. The change in aircraft layout would require a battery pack that is physically smaller due to the higher energy density. Alternatively, for the same geometrical dimensions, a larger energy storage would be required. The results are reflected in Figure 10 (where green battery packs depict more batteries).

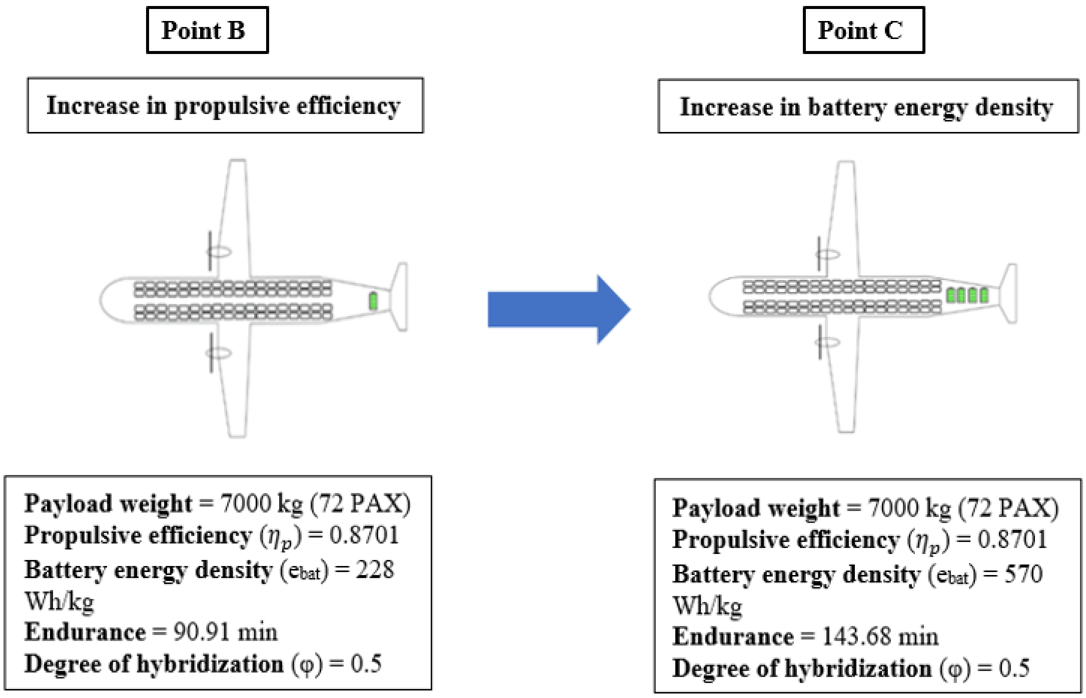

Figure 10. The proposed hybrid-electric ATR 72 aircraft based on the prediction of battery energy density for the year 2050.

4.2.3 Variation of payload

Figure 11 showcases the potential impact on the aircraft performance due to reducing payload weight. Despite that this is not the most desired option as it limits the aircraft carrying capacity, it is also an option for increasing aircraft endurance. It can be shown that as the payload weight is reduced from 7000 kg (72 PAX) to 3500 kg (36 PAX) the aircraft endurance is increased from approximately 143 minutes to 178 minutes. The modification in the configuration from the last point C (from previous sub-section) to next point D is also depicted schematically in Figure 12. Here, a reduction in the aircraft layout is shown so as to depict a decrease in number of total passengers. The total increase in endurance due to this change is about 34.36 minutes.

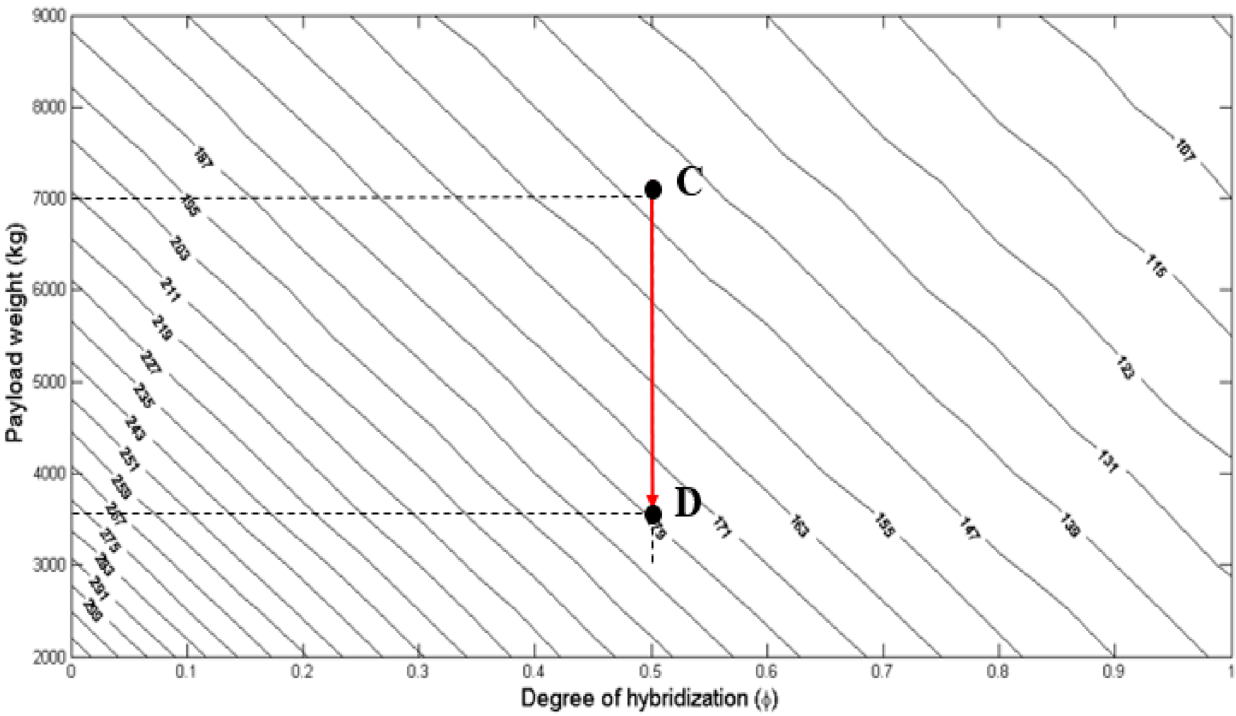

Figure 11: Sensitivity analysis using carpet plots for payload weight.

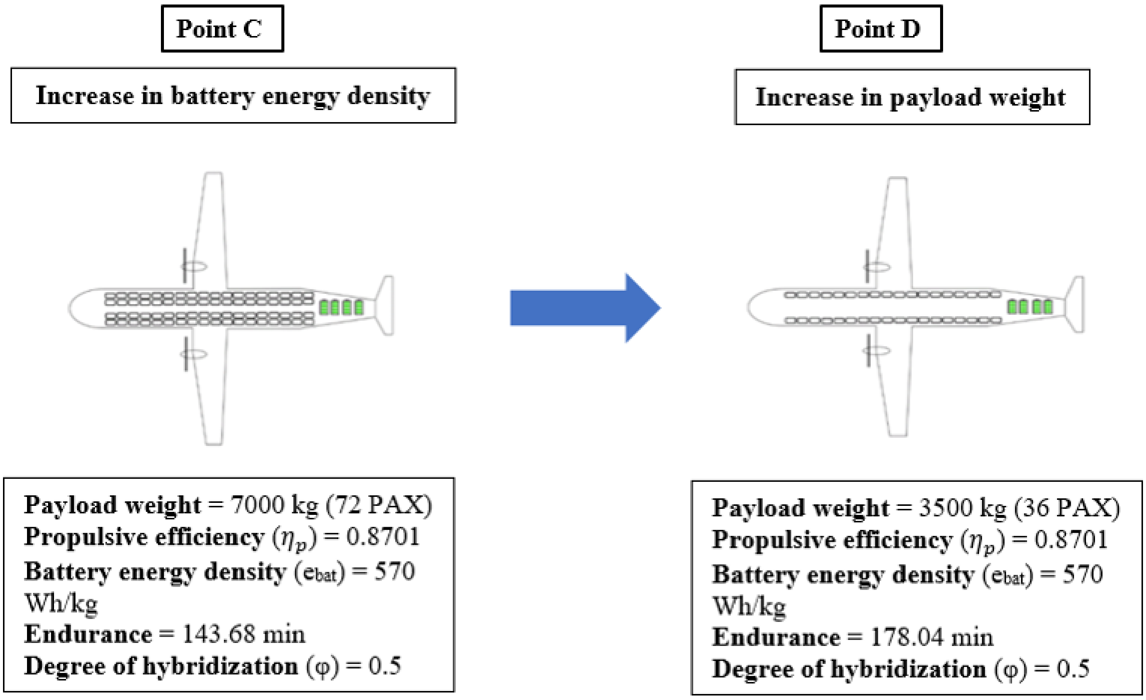

Figure 12: Proposed hybrid-electric ATR 72 aircraft based on the change in payload weight.

The first initial thought is always that the aircraft performance would be strictly depended on the battery energy density and improving that is the only way to enhance the performance. This is true while on the other hand, it was shown in this research that other parameters such as propulsive efficiency, payload weight and other efficiencies can also be used in enhancing the endurance performance of the hybrid-electric aircraft (ATR 72 in this case). In the previous work by the authors [22], it was seen that the aircraft performance (range in that case) was also dependent on the lift to drag ratio. while in this research the aircraft performance (endurance values) is only dependent on the coefficients of lift and drag separately and they are present individually not as a ratio.

The figure 13 shows the points A, B, C and D of the hybridization process in the previous section in a single plot. This figure also depicts the improvement in the endurance values from the first point (A) to the final point (D). The characteristics of that point are also mentioned above each point.

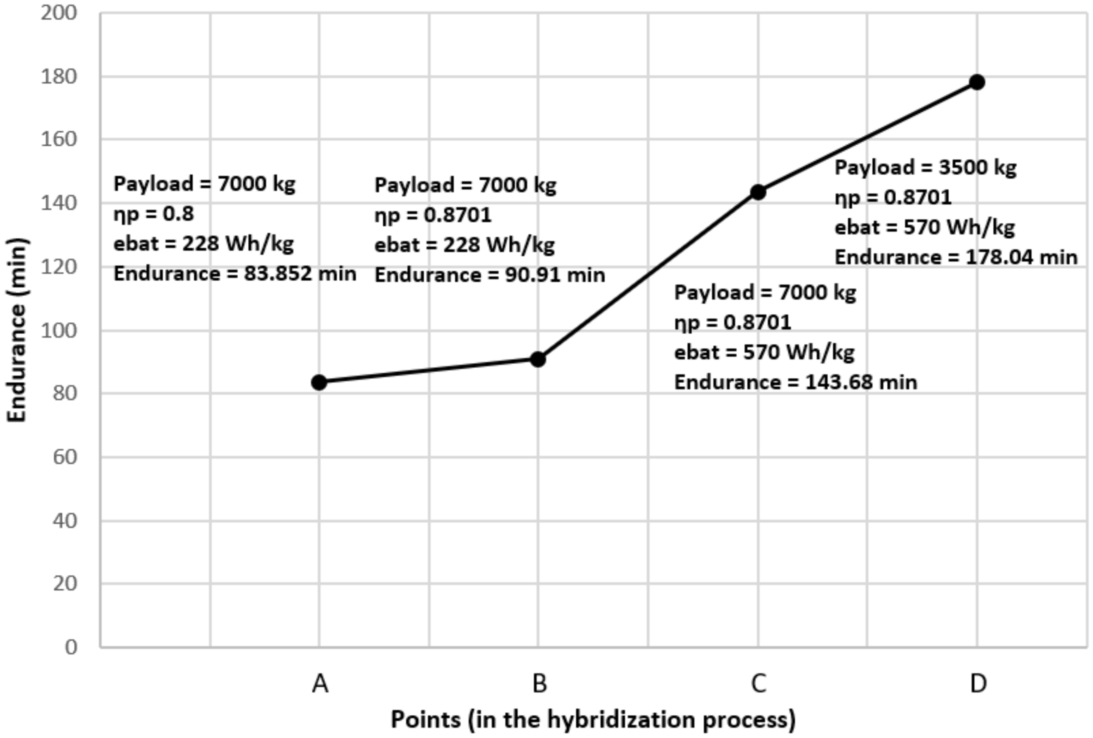

Figure 4: This figure depicts endurance (min) vs the points (in the hybridization process).

5 Discussion and Conclusion

This paper, carries out a sensitivity analysis on the hybrid-electric endurance equation. When considering battery or hybrid-electric aircraft, the initial thought is that the aircraft performance is strictly limited by the battery energy density. Despite that this is true, it does not show the full picture. This paper studied the impact of battery energy density and reviewed other parameters such as propulsive efficiency and payload weight which also offer a potential to enhance the endurance of the hybrid-electric aircraft.

It is also interesting to note that while in [11] it was also shown that the aircraft range was heavily dependent on the lift to drag ratio, the endurance is not strictly dependent on this ratio, as the lift coefficient is to the power of 3/2. The sensitivity analysis on the endurance equation was done using a regional turboprop ATR 72 at 50 % hybridization. The hybrid-electric aircraft at a very low degree of hybridization (for example 5%) is a feasible solution with current technology and would still return significant fuel savings and reduction in emissions [10]. However, as the technology achieves incremental improvements over the coming decades, the degree of hybridization can be increased leading to an aircraft design with similar performance to today's fully fuel design.

References

[1] Intergovernmental Panel on Climate Change (IPCC) 2022 report Available online: https://www.ipcc.ch/report/ar6/wg2/ , accessed on April 24, 2024.

[2] European Comission, "Flightpath 2050 CARE2050, http:// Flightpath 2050, Europe's Vision for Aviation." Available online: https://ec.europa.eu/transport/sites/transport/files/modes/air/doc/flightpath2050.pdf, accessed on April 25, 2024.

[3] European Comission, "European Green Deal" Available online: https://ec.europa.eu/commission/presscorner/detail/en/ip_23_2389, accessed on March 22, 2024.

[4] Schwab A, Thomas A, Bennett J, Robertson E and Cary S. "Electrification of Aircraft: Challenges, Barriers, and Potential Impacts," *National Renewable Energy Laboratory (NREL)*, Oct. 2021, NREL/TP-6A20-80220.

[5] Hoelzen J, Silberhorn D, Zill T, Bensmann B, and Rauschenbach R. H. ''Hydrogen-powered aviation and its reliance on green hydrogen infrastructure – Review and research gaps,'', *Elsevier*, Vol. 47, No. 5, 2022, pp. 3108-3130, https://doi.org/10.1016/j.ijhydene.2021.10.239.

[6] Zhang L, Butler, T. L., & Yang B. (2020). Recent trends, opportunities and challenges of sustainable aviation fuel. *Green Energy to Sustainability: Strategies for Global Industries*, 85-110.

[7] Schäfer A.W., Barrett S.R.H., Doyme K. *et al.* Technological, economic and environmental prospects of all-electric aircraft. *Nature Energy* 4, 160–166 (2019). https://doi.org/10.1038/s41560-018-0294-x.

[8] V. Viswanathan, A. H. Epstein, Y. Chiang, E. Takeuchi, M. Bradley, J. Langford and M. Winter. 'The challenges and opportunities of battery-powered flight,' Nature Perspectives, Vol. 601, Jan. 22, pp.519-525, https://doi.org/10.1038/s41586-021-04139-1

[9] Rossi N, Trainelli L, Riboldi C E D and Rolando A. 'Conceptual Design of Hybrid-Electric Aircraft' *Politechnico Di Milano,* 2016-17.

[10] Batra, A., Raute, R. and Camilleri, R., Hybrid-electric aircraft propulsion systems: Economic and Environmental Prospects, AIAA Science and Technology Forum and Exposition 2023 (SciTech), 23-27 Jan. 2023, National Harbour, MD, USA.

[11] Batra A, Raute R and Camilleri R. 'On the range equation for a hybrid-electric aircraft', *MDPI Aerospace 10*(8) p 687; https://doi.org/10.3390/aerospace10080687

[12] Batra, A., Raute, R. and Camilleri, R., Sensitivity analysis of the range equation for a hybrid-electric aircraft, 13th EASN Int. Conf. Sept. 5-8, 2023, Salerno, Italy.

[13] Batra A, Raute R and Camilleri R. 'An endurance equation for a hybrid-electric aircraft', *MDPI Aerospace* (under review).

[14] The ATR-72 aircraft Available online: https://www.atr-aircraft.com/aircraft-services/aircraft-family/atr-72-600/, accessed on April 25, 2024.

[15] Batra, A., Raute, R. and Camilleri R., Series or parallel hybrid-electric propulsion systems? Case studies of the ATR42 and ATR72, 33rd Congress of the Int. Council of the Aeronautical Sciences (ICAS2022), 4-9 Sept 2022, Stockholm, Sweden.

[15] Travel News Asia Available online: https://www.asiatraveltips.com/news18/37 ATRMarketForecast.shtml, accessed March 23, 2024.

[16] ATR 72 aircraft Available online: https://en.wikipedia.org/wiki/ATR_72, accessed March 22 2024.

[17] Sensitivity Analysis Available online: https://en.wikipedia.org/wiki/Sensitivity_analysis, accessed March 23 2024.

[18] National Academies of Sciences, Engineering and Medicine 2016 *Commercial Aircraft Propulsion and Energy Systems Research: Reducing Global Carbon Emissions Washington, DC: The National Academies Press*. https://doi.org/10.17226/23490.
[19] Soares C 2008 *Gas Turbines A Handbook of Air, Land and Sea Applications* p 679-719. https://doi.org/10.1016/B978-075067969-5.50024-7
[20] Energy density Available online: https://en.wikipedia.org/wiki/Energy_density#In_batteries, accessed March 23, 2024.