



DYNAMIC FREE FLIGHT TESTS WITH A SUB-SCALE AIRPLANE DESIGNED ACCORDING TO THE FROUDE NUMBER

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Abstract

The use of sub-scales to study flight dynamics is an area that can provide excellent results. With the development of electronics, free flight tests to obtain flight dynamics data on sub-scale aircraft have become increasingly attractive. This paper presents the development of a sub-scale aircraft following the Froude number scaling technique used to achieve representativeness in flight dynamics.

Keywords: Sub-scale, free flight tests, Froude Number

1. Introduction

The use of scale models in aviation has been prevalent for several decades, highlighting the effectiveness of these techniques in the development of new aircraft concepts. There are various methods for the design of airplanes, with the most commonly used accounting for a large proportion of studies dealing with the study of aerodynamics [1]. However, the number of papers focusing on dynamic testing is relatively lower. Dynamic wind tunnel tests are more complicated but pose a challenge as most wind tunnels are not capable of performing such tests. In such cases, a viable solution is to use models originally developed for free flight tests [2].

Small free-flight models play a crucial role in studying the dynamic behavior of aircraft at different angles of attack, including stall behavior, configuration adjustments to improve stability and control, all in a cost-effective and time-saving manner. These models facilitate the study of rotational characteristics, recovery techniques, gust mitigation strategies to extend the fatigue life of aircraft structures, and methods to evaluate and improve control systems to advance concepts such as vehicles configured for control [3].

In the recent past, the incorporation of electronic systems into scale models was hampered by their excessive weight and cost, making such projects unfeasible. However, the advent of miniature autopilots has revolutionized this scenario and enabled the development of low-cost and lightweight data acquisition systems. This breakthrough has opened up the possibility of conducting free flight tests with sub-scale aircraft, making the exploration of dynamic test planning techniques a promising avenue for the further development of new aircraft concepts.

The tradition of employing scaling techniques in the development of aviation systems has a rich history. In a technical paper [3] provided a comprehensive review of similarity requirements applicable to a broad spectrum of test conditions, encompassing low-velocity incompressible flows to high-velocity supersonic flows. The paper also highlights limitations in test techniques, with particular attention to free-flight models. It showcases sizing procedures for free-flight models in both incompressible and compressible flows. In the context of incompressible flows, kinematic properties are preserved through velocities scaled based on Froude number similarity requirements (Froude scaling). Meanwhile, for compressible flows, compressibility effects are maintained by scaling velocities according to Mach number requirements (Mach scaling).

Once the desired physical effects are defined, selecting the appropriate technique for observation becomes crucial. Similarity requirements play a key role in establishing the relationship between the nondimensional aerodynamic features of the scale model and the original model. The requirements ensure that the characteristics depend on the same factors involved in the flight test for both the original and scaled models, aligning with the intended purpose of the scale model. Essentially, these requirements are derived from the forces, moments, geometry, kinematics, and aerodynamics of the aircraft.

The objective of this paper is to present the development of a sub-scale aircraft designed using the Froude number method to conduct free flight tests. The experimental surveys used to determine the aircraft's inertias will be presented. Also present the results of these flight tests as well as the design of the maneuvers used in the flight test to excite the aircraft's flight modes. This approach provides a comprehensive system for gathering data from a sub-scale platform, specifically designed for the study of flight dynamics.

2. Development

A real airplane was chosen as a reference for the design of the undersized aircraft. The selected aircraft was a Cessna Cardinal 177B, as shown in Figure 1. This aircraft was subjected to flight tests and the results obtained, together with a description of the flight conditions, are presented in [5].



Figure 1 – Cessna Cardinal 177B [4]

In this project, the Froude number scaling method was developed, which is used so that the scaled object presents similarities in flight dynamics.

The Froude number is computed by:

$$F_r = f\left(\frac{\text{Inertial force}}{\text{Gravitational force}}\right) = \frac{V^2}{lg}, \quad (1)$$

where V is the velocity, l is the characteristic dimension, and g is the acceleration due to gravity.

Project points were selected in the following steps: Initially, the flight test points conducted using the Cessna Cardinal 177B, as outlined in the study, were scrutinized. After evaluating all the points from the conducted tests, the flight conditions detailed in Table 1 were selected.

Table 1 – Flight conditions of the Cessna Cardinal 177B used in flight tests.

Variable	Value	Unit
Airspeed, V_A	58.33	m/s
Altitude, h_A	2286	m
Weight, m_A	1134	kg
mass density of fluid, ρ_A	0.978	kg/m ³

For the selected scaling factor (n) of 22.2%, the wingspan of a full-size aircraft of 10.82 meters, the sub-scale wingspan was set to 2.4 meters. This model is used in wind tunnel tests, as the scale factor is selected so that the model corresponds to the dimensions of the wind tunnel. The model was built from composites such as fiberglass and carbon fiber, as well as balsa wood and marine plywood. Figure. 2 shows the prototype built and installed in the wind tunnel for testing.



Figure 2 – Cessna 177 subscale.

The flight test conditions for the sub-scale aircraft were chosen so that the flight tests could be conducted in the city of São José dos Campos (SJC) with a height AGL(Above Ground Level) of about 40 meters. Tab. 2 shows the desired flight conditions for conducting the test.

Table 2 – Intended flight conditions for Cessna 177B subscale flight test.

Variable	Value	Unit
Altitude from SJC, LFE (Landing Field Elevation)	600	m
Test Flight Altitude, h_m	640	m
mass density of fluid, ρ_m	1.1516	kg/m ³

With the data in Tables 1 and 2, it is possible to calculate the mass and weight of the scale model using equations (2) and (3), as described in [3]. The results show that the mass of the model is 14.60kg and the test speed is 27.48 m/s. For this project, the Fr calculated by Eq. (2) is $Fr = 32.05$:

$$m_m = m_A \left(\frac{\rho_m}{\rho_A} \right) n^3 \Rightarrow m_m = 1134 \left(\frac{1.1516}{0.978} \right) 0.222^3 \Rightarrow m_m = 14.60 \text{ kg} \quad (2)$$

$$V_m = V_A \sqrt{n} \Rightarrow V_m = 58.33 \sqrt{0.222} \Rightarrow V_m = 27.48 \text{ m/s} \quad (3)$$

The proportions of the geometrical dimensions of the airplane resulting from the direct relation of the scale factor n , and the values of both the airplane and the projected scale model are shown in Table. 3

The data acquisition system consists of a controller, in this case the Pixhawk Cube Orange, which contains several sensors, including accelerometers and gyroscopes that can calculate Euler angles, vibrations and velocities. It also contains a GPS, an airspeed sensor, telemetry for monitoring data during flight tests, a system for communication between the radio control and the Pixhawk, measurement of engine RPM, and a power supply and control system for the servomotors. The software used is ArduPilot, which is open-source and has been adapted to record variables of interest at a rate of 100 Hz.

A data acquisition system has been implemented to collect the angles of attack and sideslip using an air data boom. A vane that aligns with the airflow is connected to a potentiometer. The voltage across the potentiometer is acquired by an Arduino Nano, which records the information onto a memory card at a constant sampling rate of 100 Hz.

Figure 3 shows the system installed in the Cessna 177 B sub-scale aircraft,

Table 3 – Geometric dimensions of the aircraft and projected model of the Cessna 177B.

Variable	Wings		Horizontal Stabiliser		Vertical Stabiliser		Unit
	Aircraft	Model	Aircraft	Model	Aircraft	Model	
Length	10820	2402	3607	801	1523	338	mm
Area	16.26	0.80	3.25	0.16	1.74	0.09	m ²
Root chord	1768	392	902	200	1453	323	mm
Tip chord	1237	275	902	200	832	185	mm
Dihedral Angle	1.5	1.5	-	-	-	-	deg
Fuselagem	-	-	-	-	-	-	
Length	7720	1714	-	-	-	-	mm
Maximum width	1200	266	-	-	-	-	mm
Maximum height	1304	289	-	-	-	-	mm
Control Surfaces	Flap		Aileron		Rudder		
Length	2920	648	1692	376	1453	323	mm
Total Area	2.74	0.067	1.80	0.089	1.19	0.059	m ²
Root chord	512	114	564	125	516	115	mm
Tip chord	425	94	500	111	304	67	mm
Deflection	30	30	+20 to -15	+20 to -15	± 24	± 24	deg
There is no elevator because the vertical stabilizer is all moving.							
Deflection	+20 to -5	+20 to -15	-	-	-	-	deg



Figure 3 – Electronic system installed on the Cessna 177B subscale aircraft.

3. Measurement of aircraft inertia in sub-scale.

To gain a comprehensive understanding of the aircraft under study, measuring its inertia is crucial. Initially, the position of the aircraft's Center of Gravity (CG) was precisely calibrated according to the desired weight and location in the design, implying a mass of 14.6 kg and a CG location relative to the aerodynamic mid-chord of 19

The technique employed to determine the CG location is outlined in the work of [6], where two scales are used, and the aircraft is gradually tilted to obtain an inclination graph, allowing for the calculation of the exact CG position in Z, as shown in Figure 4.

Na figura 5 é apresentado o método de medição utilizado para obter os valores de R_M e R_N os quais somados são o valor do peso W.

The data collected for this measurement are detailed in Table 4, considering that the axis orientations are defined with the x-axis pointing towards the front of the aircraft, the y-axis pointing towards the right wing of the aircraft, and the z-axis completing a right-handed coordinate system.

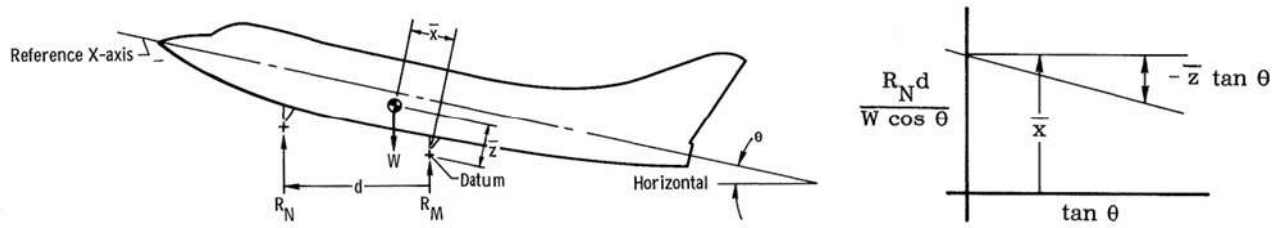


Figure 4 – Weight and vertical and horizontal position of center of gravity.[6]

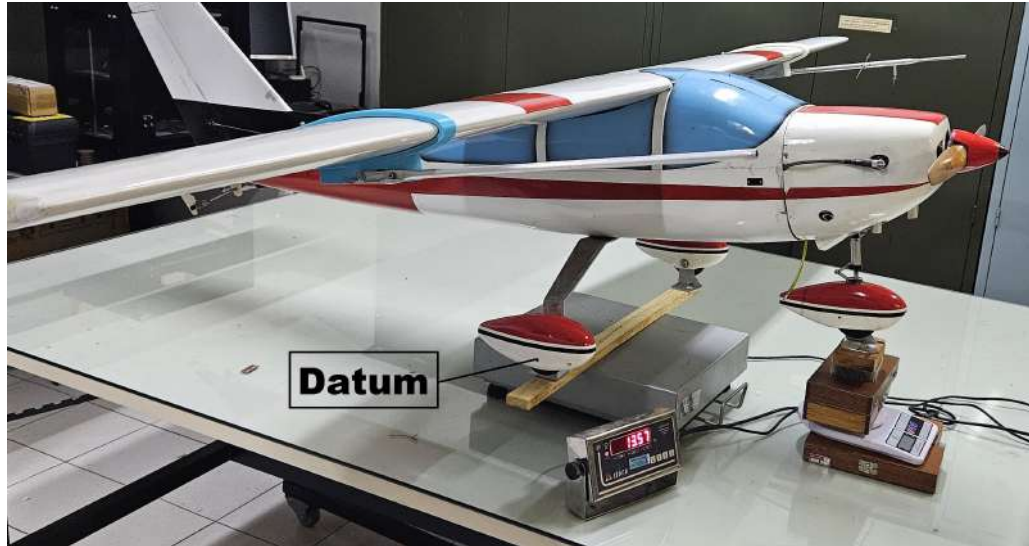


Figure 5 – Measurement of the CG of the Cessna 177 in sub-scale.

Table 4 – Data acquired for determining the Z-position of the CG.

θ (deg)	R_M (kg)	R_N (kg)	W (kg)	d (mm)
-5.5	11.42	3.10	14.52	415
0	12.00	2.52	14.53	417
5.4	12.53	1.98	14.52	415
7.8	12.80	1.71	14.51	413
10.4	13.15	1.37	14.52	410
12.6	13.31	1.20	14.52	406
15.3	13.57	0.94	14.51	402

Based on the collected data, it was possible to construct the proposed graph in Figure 4, shown in Figure 6. The CG position in Z was determined to be 170.05 mm relative to the Datum, and the horizontal CG position was found to be 72.48 mm relative to the Datum, thus achieving the desired design conditions. There was a discrepancy of only 9 grams between the determined aircraft mass and the projected value.

To measure the aircraft inertias, we employed the compound pendulum method for the I_{xx} and I_{yy} inertias. This method involves positioning the aircraft in an arrangement that allows oscillations around the axes of interest, as shown in Figure 7. Inertia is calculated using Eq. 4, where W is the aircraft mass, T is the oscillation period, L is the distance from the center of oscillation to the aircraft's CG and g is gravity. [8], [7],[9]

In this equation, we did not consider the weight of the arrangement used to suspend the aircraft, as materials such as nylon lines, carbon fiber tubes, and low-density steel cables were employed. This ensured that the suspension system had a much lower mass compared to the system being measured, thereby preserving the measurement accuracy by disregarding the support system's mass.

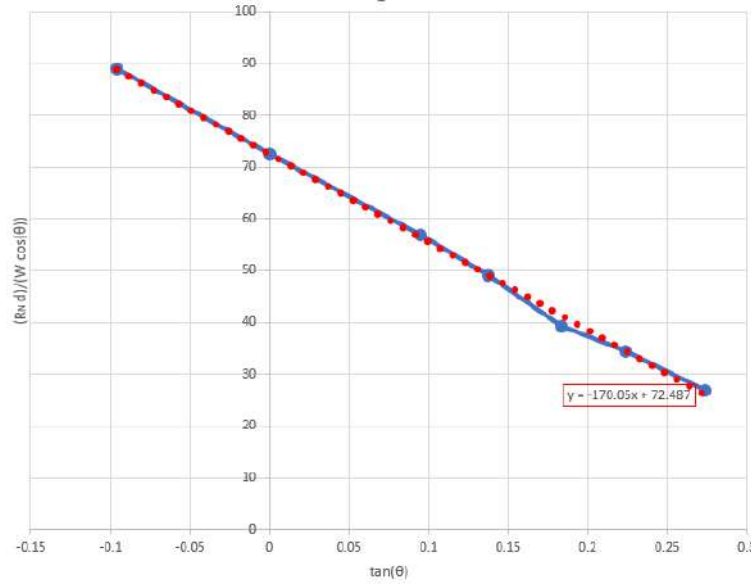
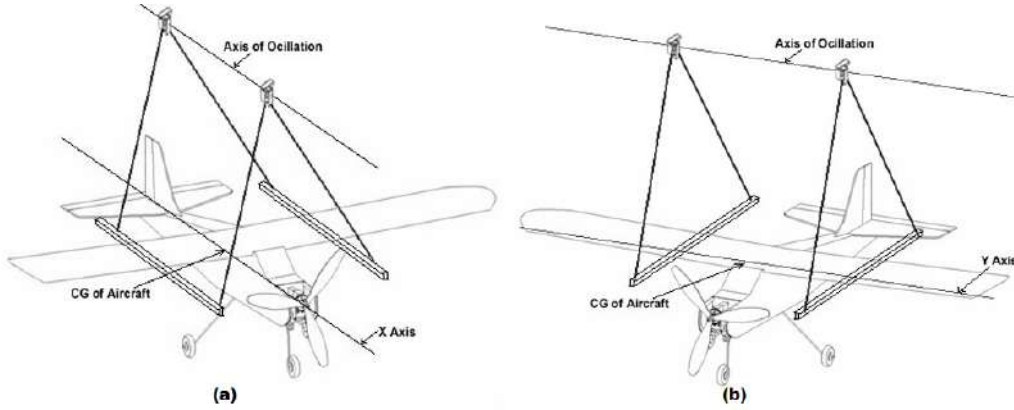


Figure 6 – Horizontal and vertical position of center of gravity

$$I_{xx} = \frac{WT^2Lg}{4\pi^2} - WL^2 \quad (4)$$


 Figure 7 – Set up for (a) I_{xx} and (b) I_{yy} inertia measure [7]

To validate the inertia measurement system, we initially calibrated it with a known mass iron cylinder. Subsequently, we conducted inertia measurement tests for I_{xx} and I_{yy} , where the oscillation period was acquired by the inertial data collection system of the onboard controller.

In Figure 8, the image and frequency oscillation results are presented for the inertia test using the compound pendulum to determine I_{xx} , with $T = 2.7031$ s, $L = 1.7920$ m, and $W = 14.24$ kg. The mass value is lower than projected because the aircraft's fuel was drained to prevent interference with the inertia measurement. With this data, the calculated result for I_{xx} is 0.8379 kgm^2 .

In Figure 9, the image and frequency oscillation results are presented, with $T = 2.9682$ s, $L = 2.15$ m, and $W = 14.24$ kg. So the calculated result for I_{yy} is 1.1581 kgm^2 .

To measure the moment of inertia I_{zz} , we used the structure of a compound pendulum as shown in Figure 11. The inertia in I_{zz} is calculated by Eq. 5, and the results of the experiment are presented in Figure 11.

The obtained values were $T = 2.4738$ s, $L = 0.915$ m, $A = 0.81$ m, and $W = 14.24$ kg. The calculated result for the inertia I_{zz} is 3.8804 kgm^2 .

$$I_{zz} = \frac{WT^2A^2g}{16\pi^2L} \quad (5)$$

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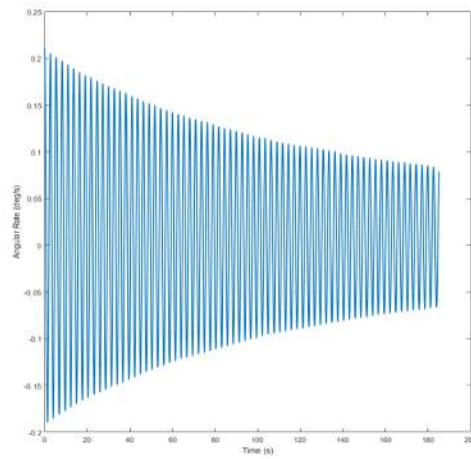


Figure 8 – Test conducted for the measurement of inertia in I_{xx} .

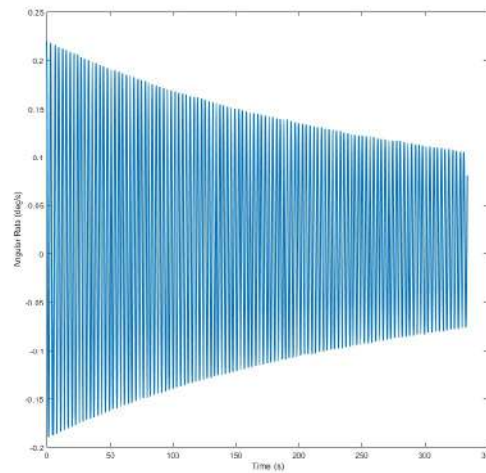


Figure 9 – Test conducted for the measurement of inertia in I_{yy} .

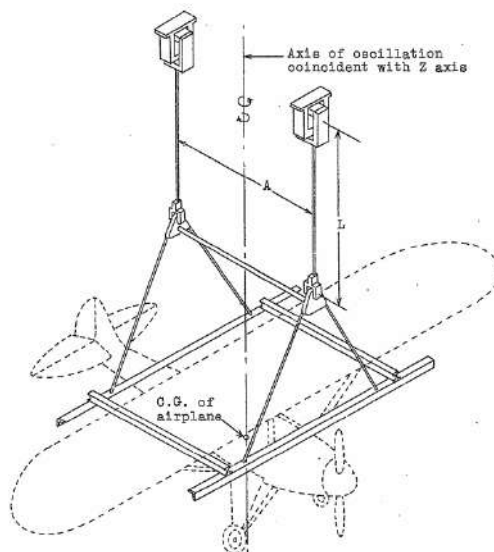


Figure 10 – Set up for I_{zz} inertia measure [8]

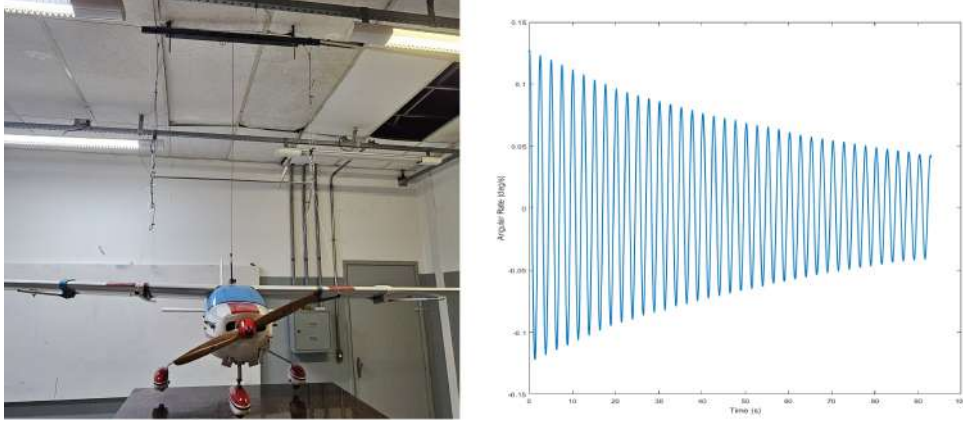


Figure 11 – Test conducted for the measurement of inertia in I_{zz} .

4. Results

After characterizing and specifying all aircraft construction variables, we proceeded with free flight tests. In Figure 12, the first flight conducted with the subscale model is presented. Following this flight, new tests were planned and prioritizing the need to excite the short period, phugoid and roll flight modes. Choosing the right maneuvers is crucial to ensure that the obtained results are truly representative. [10] For the short period mode, a doublet maneuver was designed to excite the



Figure 12 – Cessna 177 B on its first test flight

frequency around 8.5 rad/s, using Equation 6 described by Ravindra [11]. Therefore, the value of the doublet interval $\Delta t_{doublet}$ is 0.2706 s.

$$\Delta t_{doublet} = \frac{2.3}{\omega_n} \quad (6)$$

In Figure 13, the results obtained from a doublet maneuver applied to the Elevator are presented. The magnitude of the maneuver applied to the elevator is 2 degrees. It is observed that the short period mode was successfully excited. Upon graphical analysis, the achieved frequency for the short period is approximately 6.79 rad/s, close to the intended frequency for the maneuver. For the phugoid mode, it was not possible to obtain frequency data graphically due to the limited maneuver execution time, determined by the subscale model's flight range. The intended frequency, as determined by the actual aircraft, is approximately 0.33 rad/s, resulting in an oscillation period of about 19 seconds [5]. The subscale flight was maintained straight and level for a maximum of 8 seconds.

To excite the dutch roll mode, the same strategy was employed, designing a maneuver to excite the mode around 4.33 rad/s, resulting in a $\Delta t_{doublet}$ of 0.53 seconds and a surface deflection magnitude of 5 degrees. In Figure 14, the results obtained from a doublet maneuver applied to the rudder of the aircraft are presented. From the collected data points on the graph, the oscillation frequency of the dutch roll mode is calculated to be 4.52 rad/s, very close to the desired value.

The result of the "bank to bank" maneuver is shown in Figure 15. It can be observed that the maneuver was executed with approximately 10 degrees of deflection to one side and -5 degrees to the other,

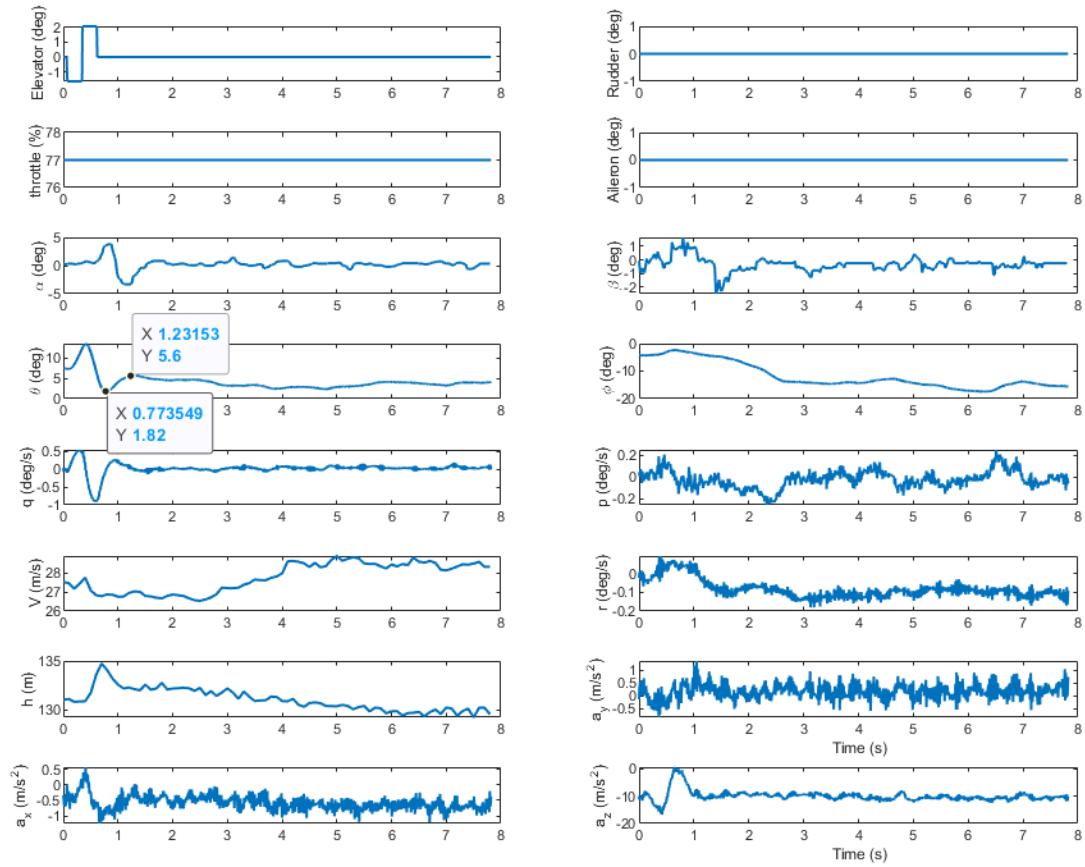


Figure 13 – Control variables and states for a doublet maneuver on the elevator

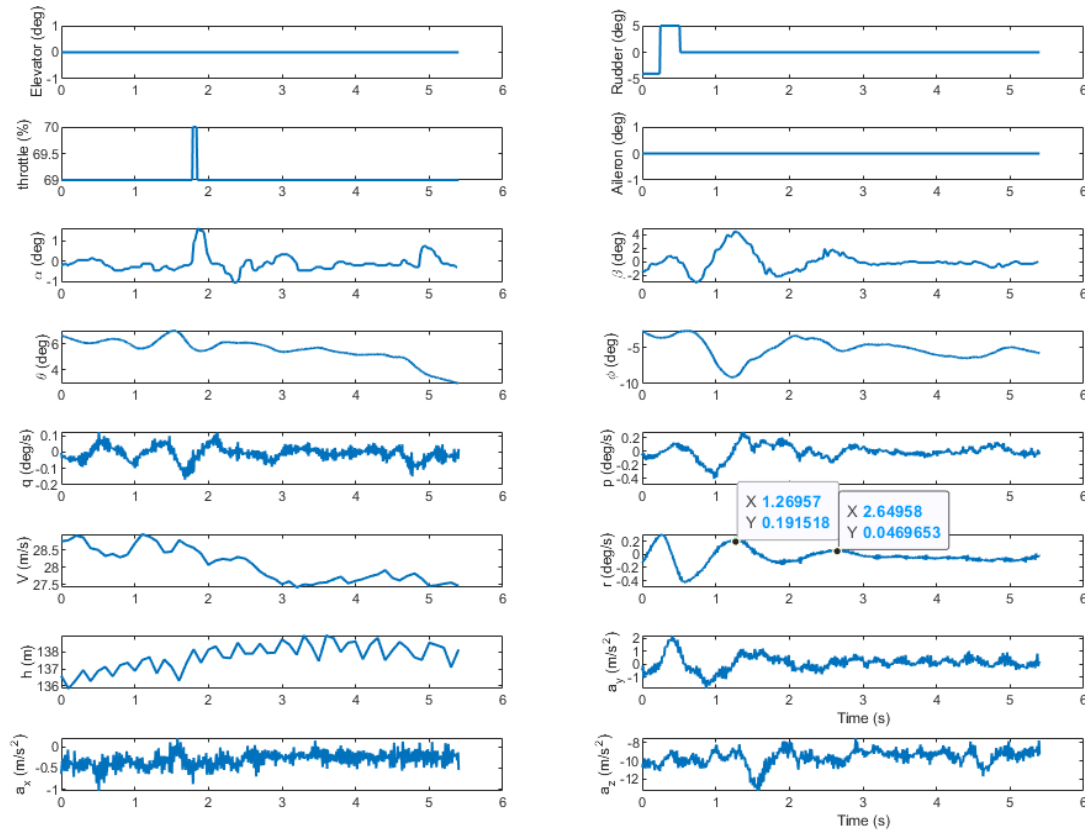


Figure 14 – Control variables and states for a doublet maneuver on the Rudder

due to the test conditions and visibility that provided sufficient safety to perform these maneuvers with full aileron deflection.

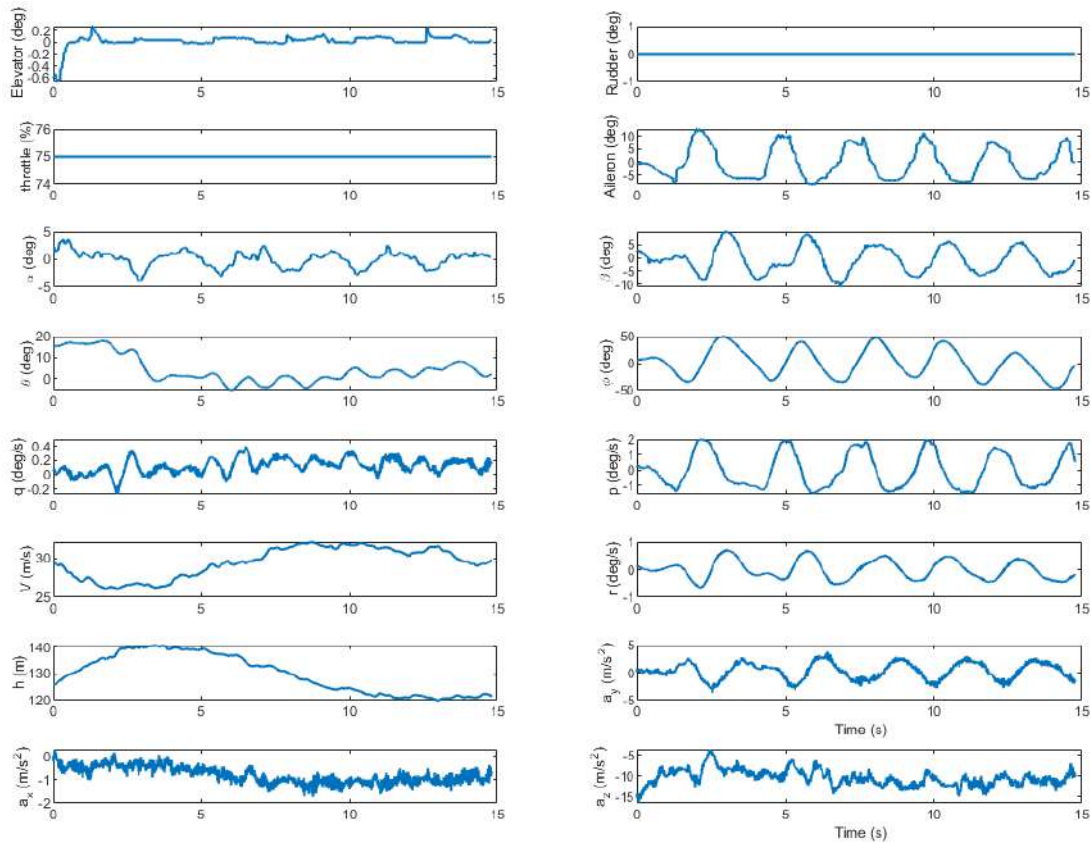


Figure 15 – Bank-to-Bank applied to Aileron

5. Conclusion

This paper presented the development of a sub-scale aircraft for flight dynamics testing, as well as the data acquisition system and preliminary results.

The design methodology for the development of the Cessna 177B subscale aircraft using the Froude number technique is practicable as all scaling criteria have been followed, resulting in flights with excellent stability. Part of the results obtained show that the data acquisition system is efficient and capable of collecting the data of interest for this work.

The inertia data of the real aircraft are not available in the literature, which creates uncertainty regarding the dynamic scaling, as the inertia scale is an important criterion to consider.

The results obtained from the experimental determination of the CG and inertia of the sub-scale aircraft were successfully achieved, demonstrating that reliable results can be obtained using techniques described in the literature and with low investment for the development of experimental work. Flight tests were successfully completed, generating results that can be used to identify parameters and compare the proximity of dutch roll values between the real aircraft and the sub-scale model. However, the evaluation of the short period in comparing the two aircraft could not be performed because this data is not available. Additionally, it was not possible to obtain this data with the sub-scale model for the phugoid due to the limited range to perform such long maneuvers.

Suggestions for future work include conducting 3-2-1-1 maneuvers to more effectively excite the short period mode, as well as implementing a camera system so that the pilot can stabilize the aircraft before starting maneuvers. Planning maneuvers to excite the pure rolling mode with greater efficiency could also be considered.

Overall, this work demonstrates that the use of small-scale aircraft with reduced dimensions can be applied as a cost-effective approach for testing designs and flight conditions that may not be achievable using simulation software or wind tunnels.

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