EFFECTIVENESS EVALUATION METHOD FOR AVIATION EMERGENCY RESCUE SYSTEM OF SYSTEMS BASED ON EFFECT CHAIN

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Abstract

Aiming at the problem that the effectiveness of aviation emergency rescue system of systems is difficult to evaluate due to complex functional interactions, the interdependent network theory is introduced to solve it. Firstly, a two-layer heterogeneous interdependent network model is constructed, which includes task network, information support network and the asymmetric dependent network. Secondly, the concept of effect chain is proposed. Taking the comprehensive impact of various heterogeneous factors on effect chains as the entry point, an improved Ullmann algorithm and a network state sampling method based on Monte Carlo are proposed to realize the effectiveness evaluation. Finally, these methods are used to an aviation emergency rescue system of systems for sea and land, and the results validate the rationality of the modeling method and effectiveness evaluation method.

Keywords: effectiveness evaluation, aviation emergency rescue (AER), system of systems (SoS), effect chain, interdependent network

1. Introduction

Aviation emergency rescue (AER) plays an irreplaceable role in dealing with emergencies due to its advantages of fast, flexible and efficient [1-3]. As the material basis of AER, aviation emergency rescue system of systems (AERSoS) is a typical complex giant system, which covers different types of functional equipment (such as helicopters and unmanned aerial vehicles) and information interactions (such as target reconnaissance, command and control). Considering the above factors, how to evaluate the effectiveness of AERSoS scientifically is a basic work, which can not only lead the development of equipment and technology, but also guide the overall construction of AERSoS.

Modeling is the basis of effectiveness evaluation. For complex giant system and system of systems (SoS), the modeling based on complex network is an important method. At present, this method can be divided into two categories. One is not to distinguish the types of nodes and edges and establish the model by homogeneous network, such as power grids [4-6] and traffic networks [7-9]. The other is to use heterogeneous networks to model the function and structure according to the types of nodes and edges, which is common in combat networks [10-12]. In the process of application, the former is suitable for the statistical analysis of large-scale networks, but it is difficult to reflect the complex functional interactions. The latter is more suitable for the actual situation of AER, but it is not perfect in the reflection of functional dependence and the description of information interaction process.

For the network effectiveness evaluation of SoS, there are also two main methods. One is to improve the network efficiency, average path length, clustering coefficient and other complex network characteristic parameters to make them suitable for the effectiveness evaluation [13-14]. These methods are relatively simple in calculation and implementation, but they don't reflect the influence of specific elements and are only suitable for macro evaluation of SoS. So some researchers choose to describe the effectiveness of SoS through the number of functional structures formed in the network, comprehensive ability and other indicators [15-16]. These methods take the influence of some practical factors into account, but do not reflect the impact of heterogeneous factors such as the success rate of different equipment, the importance of tasks and different functional dependencies on the effectiveness of the SoS, which have certain defects.

In this paper, a two-layer heterogeneous interdependent network model is established to reflect the complex functional interactions of AERSoS. The concept of effect chain is introduced, and the effect chains are obtained by the improved Ullmann algorithm and Monte Carlo method. Based on the influence of various factors on the effect chains, the effectiveness evaluation for AERSoS is realized.

2. The Interdependent Network Model for AERSoS

2.1 Analysis of AER Based on OODA

AER can be considered as a paramilitary operation, which includes disaster detection, information transmission, decision-making and operations. Therefore, the OODA loop theory [17], which is commonly used in the military field, is introduced to analyze the AER process and the function of equipment. A complete OODA loop consists of four stages (as shown in Figure 1): observe, orient, decide and act.

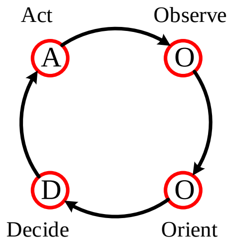

Figure 1 – The four stages of the OODA loop

Correspondingly, AER can also be divided into these four stages: detect the disaster situation, make situation judgments, make decision and implement rescue operations. According to the above analysis framework and the functional characteristics of the equipment in AERSoS, the equipment nodes are mapped into five categories.

1) Sensor nodes (S), the main function is to conduct reconnaissance on the disaster situation, obtain disaster information and transmit the information to other nodes.

2) Decision nodes (D), the main function is to make decisions based on the disaster situation and transmit the decisions to other nodes.

3) Action nodes (A), the main function is to execute disaster relief tasks according to decision instructions.

4) Communication nodes (C), the main function is to provide communication and information transmission support for all stages of AER.

5) Target nodes (T), it's an abstraction of disaster event and disaster relief task.

It should be noted that each node in the AERSoS network can only be mapped from one equipment entity. For some multi-function equipment in AERSoS such as multi-purpose helicopter and unmanned aerial vehicle (UAV), it may have multiple functions. So it should be mapped into multiple functional nodes according to the actual functional characteristics of the equipment entity.

According to the actual operation law of the AERSoS, a two-layer interdependent network model is established to achieve the description of the AERSoS. The two-layer networks contain task network and information support network, and the functional interaction between the two networks constitutes the asymmetrical dependent network. The modeling process will be explained in detail below.

2.2 Modeling of Task Network

According to the functional types of nodes, it can be seen that S, D, I and T nodes directly participate in the OODA loop. That means they are directly related to the rescue task. Based on this consideration, these four types of nodes are classified as task nodes. The edge is the reflection of the specific task activities of the equipment. Due to the obvious order and dependence between the functions of the task nodes, directed edges are introduced to reflect this characteristic. According to [15], there are 7 kinds of directed edges with high frequency, which are shown in Table 1.

Table 1 Seven types of directed edges in task network

Type	Explanation	Type	Explanation
T→S	Detection and monitoring of node T by node S	S→S	Cooperative detection and disaster information sharing among S nodes
S→D	Node S transmits the disaster information to node D	D→D	Cooperative command and decision information sharing between D nodes
D→A	Node D transmits the decision information to node A	D→S	Node D feeds back the decision information to node S
A→T	Node A performs rescue operations for node T		

During an AER operation, the task activities represented by directed edges have different success rates because of the different capabilities of equipment nodes. To reflect this important factor, weights are assigned to directed edges in task network. Let the *t*-th directed edge in task network be $e_t = \left(v_u^X, v_v^Y\right)$, where v_u^X and v_v^Y (*X* and *Y* represent the four types of task nodes) are the nodes at both ends of edge e_t. The weight of e_t can be expressed in terms of w_t. For A→T and T→S, the weight w_t can be regarded as the probability that edge e_t is connected, which can be obtained by formula (1).

$$w_t = h\left(a_u^X, a_v^Y\right), \quad h(\bullet) \in (0,1) \tag{1}$$

Where: a_u^X and a_v^Y represent the sets of capability indicators of v_u^X and v_v^Y; $h(\bullet)$ is the mapping function between the capability indicators and w_t, which can be obtained by mathematical analysis, expert analysis or simulation.

For other types of edges in task network, the weight w_t is the probability that edge e_t is connected under the condition of effective information transmission, which can be obtained by formula (2).

$$w_t = h\left(a_u^X, a_v^Y \middle| C\right), \quad h(\bullet) \in (0,1) \tag{2}$$

Where: *C* stands for the condition of effective information transmission, and the specific meaning of which will be explained later.

Task nodes and corresponding directed edges together form the task network. In this paper, the task network is represented by $G_{task} = (V^{task}, E_{task}, W)$. $V^{task} = V^S \cup V^D \cup V^A \cup V^T$ is the set of task nodes, where $V^S = \{v_1^S, v_2^S, \ldots, v_{n_S}^S\}$, $V^D = \{v_1^D, v_2^D, \ldots, v_{n_D}^D\}$, $V^A = \{v_1^A, v_2^A, \ldots, v_{n_A}^A\}$ and $V^T = \{v_1^T, v_2^T, \ldots, v_{n_T}^T\}$ are the sets of S, D, A and T nodes. E_{task} is the set of directed edges and *W* is the set of weights. In addition, due to the differences in the urgency of the tasks and the number of people involved, the importance of each task is usually different. Therefore, T nodes should be assigned weights to reflect this characteristic. Let $\mathbf{w}^T = (w_1^T, w_2^T, \ldots, w_{n_T}^T)$ be the importance weight vector of T nodes, and w_l^T represents the weight of the *l*-th T node. At the same time, the sum of all the elements in $\mathbf{w}^T$ is 1.

2.3 Modeling of Information Support Network

For C nodes in AERSoS, they don't participate in the OODA loop directly. But they provide necessary information support for task nodes finishing the OODA loop. Therefore, C nodes are classified as information support nodes. In general, the establishment of communication relationships and the transmission of information in AER operations are bidirectional. So, the edges between two information support nodes are undirected. If there is an edge between two information support nodes, it indicates that they have the ability of information transmission. In addition, information transmission between two C nodes can also be transferred by other C nodes. But due to the impact of time delay and reliability, the more times of transfer, the worse the quality of received information. Usually, the transmission of effective information has the maximum number of limits, which can be expressed by n_h.

Assuming n_h=1, for the connection relationship shown in Figure 2, we carry out an analysis of node C_1. It can be seen that node C_1, node C2 and node C_3 have the condition of effective information transmission. However, since the establishment of the communication relationship between node C_1 and node C_4 requires the transit of two nodes (C_2 and C_3), node C_1 and node C_4 don't have the condition of effective information transmission.

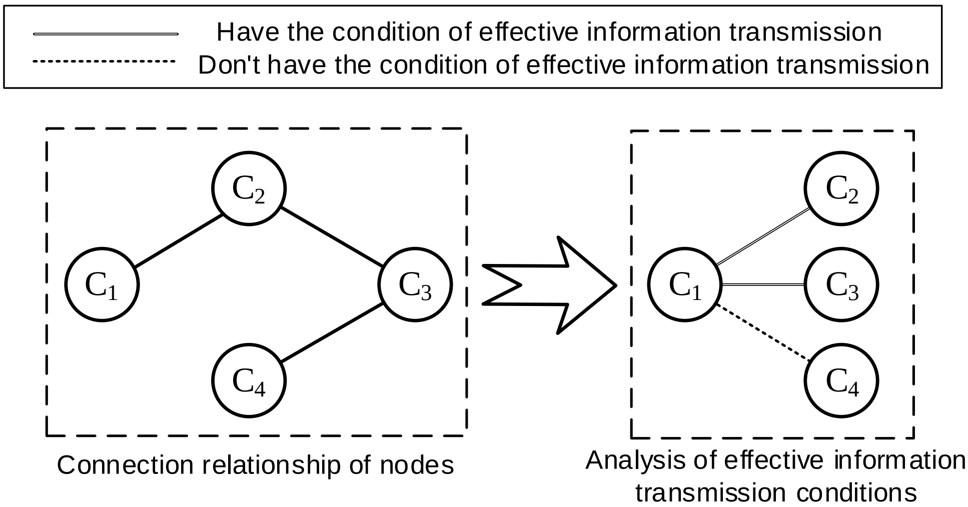

Figure 2 – Description of effective information transmission conditions

On this basis, the information support nodes and corresponding undirected edges together form the information support network, which can be represented by $G_{sup} = (V^C, E_{sup})$. $V^C = \{v_1^C, v_2^C, \ldots, v_{n_c}^C\}$ denotes the set of information support nodes, and E_{sup} represents the set of undirected edges among information support nodes.

2.4 Modeling of Asymmetrical Dependent Network

For task nodes and the information support nodes, there is a unilateral dependence between them in function. That is, the information transmission of task nodes needs to be carried out through the information support nodes. If a certain task node does not have the communication support of the corresponding information support nodes, its function may fail due to the failure of information transmission. On the contrary, the function of information support nodes cannot be affected by task nodes. In this paper, the asymmetrical dependency between task nodes and information support nodes are reflected through the directed edges from information support nodes to task nodes.

On this basis, the task nodes, the information support nodes and the corresponding directed edges form the asymmetrical dependent network together, which can be expressed as $G_{in} = (V, E_{in})$. V is the set of task nodes and information support nodes, and E_{in} is the set of directed edges of asymmetrical dependent network.

Furthermore, a triple $G=(G_{task},G_{sup},G_{in})$ can be used to represent the two-layer heterogeneous interdependent network model for AERSoS, and the construction of AERSoS network model is completed.

3. Effectiveness Evaluation Method Based on Effect Chain

3.1 The Definition of Effect Chain

According to the analysis in Section 2.1, for a successful AER operation, it must mean the completion of the process from detection to action. This process can be regarded as the basic functional unit of AERSoS. Inspired by the kill chain theory [18-19] in the field of air combat, this basic functional unit of AERSoS is called effect chain. Considering the constructed interdependent network model, the definition of effect chain is given as follows: If the closed continuous path "detect-orient-decide-act" is formed in the AERSoS network, the mission is successful and the closed continuous path is called the effect chain.

The standard form of effect chain is shown in Figure 3, that is, under the information support of C nodes, the node S obtains disaster information and transmits it to the node D, the D node processes the information and makes decisions, and the node A relies on the decisions to carry out rescue operations against T nodes. The standard form of effect chain can be represented by T→S→D→I→T. Obviously the effect chain should start and end with the same T node. When the effect chain containing T node is formed, the task represented by T node is completed.

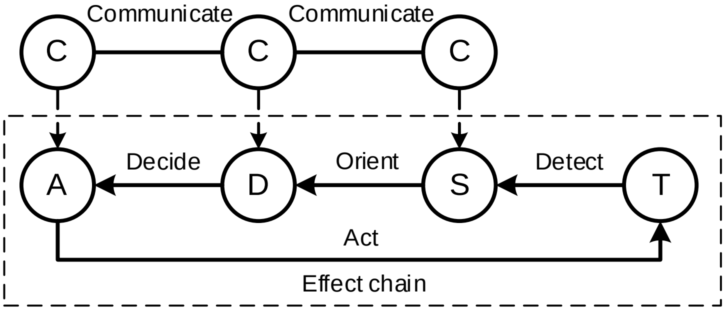

Figure 3 – The standard form of effect chain

For a typical AERSoS network, there are seven main types of effect chains which are ubiquitous, as shown in Table 2.

Table 2 Seven types of effect chains

Type	Effect chain	Explanation
1	T→S→D→A→T	The standard form of effect chain.
2	T→S→S→D→A→T	Effect chain having the ability of cooperative detection.
3	T→S→D→D→A→T	Effect chain having the ability of decision information sharing.
4	T→S→S→D→D→A→T	Effect chain having the ability of cooperative detection and decision information sharing.
5	T→S→D→S→D→A→T	Effect chain having the ability of decision feedback.
6	T→S→S→D→S→D→A→T	Effect chain having the ability of cooperative detection and decision feedback.
7	T→S→D→D→S→D→A→T	Effect chain having the ability of decision information sharing and decision feedback.

The formation of effect chains is the key to the effectiveness of AERSoS, and the influence of various factors on the effectiveness is derived from their influence on the effect chains. Based on this analysis, the next step is to quantitatively characterize the influence of various factors on the effect chains.

3.2 Search Method for Effect Chain

In the interdependent network for AERSoS, the task network G_{task} is directly involved in the formation of effect chains, so it is necessary to search all possible effect chains in G_{task} as the basis for analysis. $G_{task}^{d} = (V^{task}, E_{task})$ is used to represent the network with the initial task network edge weights removed, and its adjacency matrix is $\mathbf{B}^{d} = \left[b_{ij}^{d}\right]_{n_{task} \times n_{task}}$ (n_{task} is the number of task nodes and $n_{task} = n_S + n_D + n_I + n_T$). On this basis, the seven types of effect chains can be regarded as specific subgraph structures in G_{task}^{d}, so the search of effect chains is transformed into the problem of subgraph isomorphism matching in G_{task}^{d}.

Subgraph isomorphism matching mainly studies how to find the mapping set of all nodes with the same structure as query graph from target graph under the premise of given target graph and query graph. The common methods consist of Ullmann [20], VF2 [21] and GraphQL [22]. In this paper, seven types of effect chains are regarded as query graph, and the network G_{task}^{d} is regarded as target graph. According to the characteristics of the problem, Ullmann algorithm is partially improved and the search of effect chains is realized.

Let G_l^n be the *n*-th ($n = 1, 2, \ldots, 7$) type of effect chain containing target node v_l^T ($l = 1, 2, \ldots, n_T$) and its adjacent matrix is $\mathbf{A}_l^n = \left[a_{ij}^n\right]_{p_l^n \times p_l^n}$, where p_l^n is the number of nodes in G_l^n. The initial mapping matrix between effect chain G_l^n and network G_{task}^{d} is defined as $\mathbf{M}_l^0 = \left[m_{ij}^0\right]_{p_l^n \times n_{task}}$, which is a 0-1 matrix. If the element m_{ij}^0 in $\mathbf{M}_l^0$ is 1, it indicates the type of the *i*-th node in G_l^n is the same as that of the *j*-th node in G_{task}^{d}, so they have a mapping relationship. Otherwise, there is no mapping relationship between them. It should be noted that the difference in the weights of T nodes makes each T node different, so the type of T nodes needs to be refined to $\{T_1, T_2, \ldots, T_{n_T}\}$. Only when the refined types of the T nodes in G_l^n and G_{task}^{d} are the same, the two have a mapping relationship.

Next, pruning optimization algorithm is used to reduce the search space. The pruning strategy of the classical Ullmann algorithm don't consider the connection characteristics of nodes, resulting in low pruning efficiency when applied to this problem. According to the connection rules of task nodes, the new pruning rule constructed is shown in formula (3).

$$m_{ij} = Prun\left(m_{ij}^0\right) = \begin{cases} 1 & \text{if: } m_{ij}^0 = 1 \text{ and } type\_pre(i) \in type\_Pre(j) \\ & \quad \text{and } type\_suc(i) \in type\_Suc(j) \\ 0 & \text{others} \end{cases} \tag{3}$$

Where, m_{ij} is the element of the node mapping matrix $\mathbf{M}_l = \left[m_{ij}\right]_{p_l^n \times n_{task}}$ after pruning; $Prun(\bullet)$ represents the pruning optimization function; $type\_pre(\bullet)$ and $type\_suc(\bullet)$ respectively represent the type of the forward node and the backward node of the certain node, and $type\_Pre(\bullet)$ and $type\_Suc(\bullet)$ respectively represent the set of the types of the forward node and the backward node of the certain node. After pruning, the final output is $\mathbf{M}_l$.

Define a 0-1 matrix $\mathbf{M}_l^{'} = \left[m_{ij}^{'}\right]_{p_l^n \times n_{com}}$ to represent a specific mapping relationship between G_l^n and

G_{task}^{d} in $\mathbf{M}_l$, so $\mathbf{M}_l^{'}$ must meet: Each column has at most one 1; Each row has only one 1; When $m_{ij}^{'}$ is 1, m_{ij} must be 1. Moreover, define matrix $\mathbf{C}$ as shown in formula (4).

$$\mathbf{C} = \left[c_{ij}\right]_{p_l^n \times p_l^n} = \mathbf{M}_l^{'}\mathbf{B}^d\mathbf{M}_l^{'\mathrm{T}} \tag{4}$$

Where, T stands for transpose.

According to the isomorphism rule of Ullmann algorithm, if the condition of formula (5) is satisfied, $\mathbf{M}_l^{'}$ is an effect chain of the *n*-th type in G_{task}^{d} that contains v_l^T.

$$(\forall i \forall j)_{1 \le i, j \le p_l^n} \quad a_{ij}^n = 1 \Rightarrow c_{ij} = 1 \tag{5}$$

Finally, the enumeration algorithms such as depth-first search can be used to obtain all possible $\mathbf{M}_l^{'}$ in $\mathbf{M}_l$, and formula (5) is used to filter qualified $\mathbf{M}_l^{'}$ to complete the search of the *n*-th type of effect chain containing v_l^T. Iterating over the values of *l* and *n* yields the set of all possible effect chains in task network, denoted $\boldsymbol{Q}^0$.

3.3 Screening Method of Effective Effect Chain

The effect chains in set $\boldsymbol{Q}^0$ are not all effective. Only the effect chains that can carry out effective communication and information transmission between nodes in set $\boldsymbol{Q}^0$ are effective. And the effective effect chains should meet two conditions: 1) For any type of effect chain, task node (except T node) should have at least one directed edge from the information support nodes to the node in the asymmetrical dependent network G_{in}. 2) The information support nodes relied on by neighboring nodes in the effect chain have the ability of effective information transmission in the information support network G_{sup}. If any of the conditions are not met, the effect chain is a noneffective effect chain.

First of all, traverse each effect chain in $\boldsymbol{Q}^0$ and check whether each node (except T node) in the effect chain has a directed edge from the information support node to the node in the asymmetrical dependent network G_{in}. If there is a node that does not meet the condition, which means that information cannot be effectively transmitted in the effect chain, remove the corresponding effect chain from $\boldsymbol{Q}^0$.

Then, assume that the adjacency matrix of information support network G_{sup} is $\boldsymbol{D} = \left[d_{ij}\right]_{n_c \times n_c}$ and the information support node that two adjacent nodes of an effect chain in $\boldsymbol{Q}^0$ depend on are the *i*-th and *j*-th node in G_{sup}. Let $\boldsymbol{D}^k = \left[d_{ij}^{(k)}\right]_{n_c \times n_c}$ be the *k*-th $(1 \le k \le n_h + 1)$ power matrix of $\boldsymbol{D}$. When *k* is 1, according to the meaning of the adjacency matrix, the element d_{ij} in $\boldsymbol{D}$ represents whether the path of length 1 from node *i* to node *j* in G_{sup} exists. If $d_{ij} = d_{ji} = 1$, it indicates that node *i* and node *j* have edges directly connected in G_{sup} and have the ability of effective information transmission. When $k > 1$, according to the meaning of the power of the adjacency matrix, the element $d_{ij}^{(k)}$ in $\boldsymbol{D}^k$ represents the number of paths of length *k* from node *i* to node *j* in G_{sup}. If $d_{ij}^{(k)} = d_{ji}^{(k)} \ne 0$, it indicates that node *i* can transmit information to node *j* through a relay of other information support nodes. Therefore, if $d_{ij}^{(k)} = d_{ji}^{(k)} \ne 0$ exists in the matrix of the first to n_h-th power of $\boldsymbol{D}$, node *i* and node *j* have effective information transmission capability in G_{sup}.

Finally, traverse all adjacent nodes of each effect chain in $\boldsymbol{Q}^0$ based on the above rules and remove the effect chains that don't have effective information transmission capability. In this way, the set $\boldsymbol{Q}$ of effective effect chains is output.

3.4 Effectiveness Evaluation Method

In the task network, the weights of edges reflect the success rate of the rescue activities of different equipment affecting the formation probability of effect chains in the set ***Q***. At the same time, the weights of the T nodes reflect the importance of different tasks, which makes the effect chains in set ***Q*** play the different roles in the AERSoS effectiveness. After comprehensive consideration of the above factors, the quantitative relationship between effect chains and AERSoS effectiveness can be established by Monte Carlo sampling.

Assuming that the states of edges in task network G_{task} are independent of each other, 0 and 1 are used to represent the state of the edge not connected (equipment not complete the rescue activity) and connected (equipment complete the rescue activity). Then the states of edges in G_{task} can be sampled by generating uniform random numbers between 0 and 1. $\boldsymbol{x}^k = (x_1^k, x_2^k, \ldots, x_{n_e}^k)$ is used to represent the state vector of edges after the k-th sampling, and the calculation method of element value x_t^k is shown in formula (6).

$$x_t^k = \begin{cases} 1, & r_t^k \le w_t \\ 0, & r_t^k > w_t \end{cases} \qquad t = 1,2,\ldots,n_e \quad k = 1,2,\ldots,K \tag{6}$$

Where, K is the total number of Monte Carlo sampling; x_t^k is the state of the t-th edge in the k-th sampling; r_t^k is the random number corresponding to the t-th edge in the k-th sampling.

According to Kolmogorov strong law of large numbers [23], if X_t is the total number of t-th edge connected after K samplings, X_t/K is an unbiased estimate of w_t when K is large. So the statistical results obtained by a large number of Monte Carlo samplings can effectively reflect the influence of different factors.

According to $\boldsymbol{x}^k$, the existence of the effect chains in ***Q*** after the k-th sampling can be judged, that is, when every edge of the effect chain is connected, the effect chain exists in the k-th sampling. At the same time, F_l^k is used to record the existence of effect chain containing v_l^T in ***Q*** after the k-th sampling. $F_l^k = 1$ indicates the existence of the corresponding effect chain and $F_l^k = 0$ indicates the absence of the corresponding effect chain.

Considering the influence of the weights of T nodes, the difference of the weights of edges in G_{task}, the functional dependence of nodes and the effective information transmission ability of the information support nodes, the effectiveness evaluation method of the two-layer heterogeneous interdependent network based on kill chain is given in formula (7).

$$Ef = \frac{1}{K}\sum_{l=1}^{n_T} w_l^T \bullet \sum_{k=1}^{K} F_l^k \tag{7}$$

Where, Ef stands for the AERSoS effectiveness.

4. Example Analysis

Taking an AERSoS for sea and land (as shown in Figure 4) as an example, there are two boats capsized at sea and a fire on land, both of which require aviation emergency rescue. The AERSoS consists of one command aircraft, two helicopters, two reconnaissance UAVs, one communication satellite and one command center. They work together according to the functional relationships shown in Figure 4 to accomplish this task. For this AERSoS, we carry out an example analysis to verify the feasibility of the proposed methods.

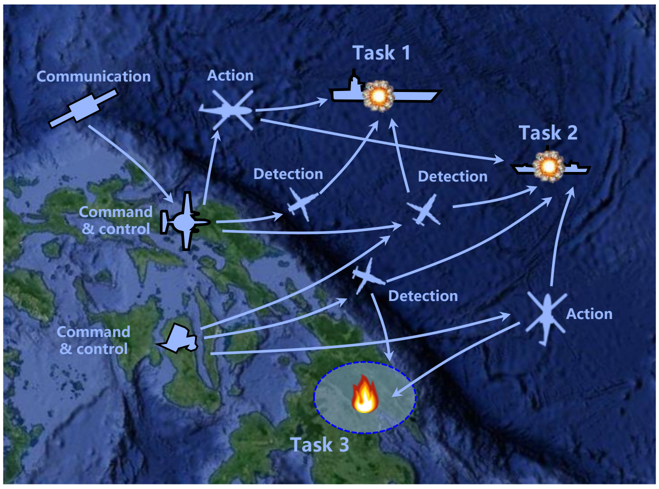

Figure 4 – Schematic diagram of an AER scene for sea and land

4.1 Interdependent Network Model of AERSoS for Sea and Land

Firstly, the equipment entities in the AERSoS can be mapped into five types of functional nodes, as shown in Table 3.

Table 3 The mapping relationship between equipment and nodes

Equipment entity	Nodes	Equipment entity	Nodes
Three tasks	T_1, T_2, T_3	Reconnaissance UAV 1	S_2, C_2
Command aircraft	S_1, D_1, C_1	Reconnaissance UAV 2	S_3, C_4
Helicopter 1	A_1, C_3	Communication satellite	C_5
Helicopter 2	A_2, C_6	Command center	D_2

Obviously, the node set of the task network is $V^{task} = \{S_1, S_2, S_3, D_1, D_2, A_1, A_2, T_1, T_2, T_3\}$. According to the functional interactions among nodes, the set of directed edges of the task network can be obtained, and weights are assigned to the edges by expert experience. The results are shown in Table 4. In addition, the importance weight vector of the T nodes is $\boldsymbol{w}^T = (0.3,\ 0.5,\ 0.2)$.

Table 4 Directed edges and weights in the task network

Edge	Weight	Edge	Weight	Edge	Weight	Edge	Weight
$T_1 \to S_1$	0.70	$S_1 \to D_1$	0.90	$D_2 \to A_2$	0.80	$S_1 \to S_2$	0.95
$T_1 \to S_2$	0.90	$S_2 \to D_2$	0.90	$A_1 \to T_1$	0.85	$D_1 \to S_2$	0.95
$T_2 \to S_2$	0.90	$S_3 \to D_2$	0.90	$A_1 \to T_2$	0.75		
$T_2 \to S_3$	0.70	$D_1 \to A_1$	0.80	$A_2 \to T_2$	0.90		
$T_3 \to S_3$	0.80	$D_2 \to A_1$	0.80	$A_2 \to T_3$	0.90		

Also, the node set of the information support network is $V^C = \{C_1, C_2, C_3, C_4, C_5, C_6\}$. The adjacency matrix $\boldsymbol{D}$ is used to represent the undirected edges among information support nodes, as shown in formula (8).

$$\boldsymbol{D} = \begin{array}{c} \\ \\ \end{array} \begin{matrix} C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \end{matrix} \\ \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{matrix} \tag{8}$$

Then, the node set of the asymmetrical dependent network is the union of the task nodes and the information support nodes, that is, $V = V^{task} \cup V^{C}$. According to the dependence relationship between the task nodes and the information support nodes, the directed edges in the asymmetrical dependent network is shown in Table 5.

Table 5 Directed edges in asymmetrical dependent network

No.	Edge	No.	Edge	No.	Edge	No.	Edge
1	$C_1 \rightarrow S_1$	3	$C_2 \rightarrow S_2$	5	$C_4 \rightarrow S_3$	7	$C_6 \rightarrow A_2$
2	$C_1 \rightarrow D_1$	4	$C_3 \rightarrow A_1$	6	$C_5 \rightarrow D_2$		

Through the above steps, the two-layer heterogeneous interdependent network model of this AERSoS is finally constructed, as shown in Figure 5.

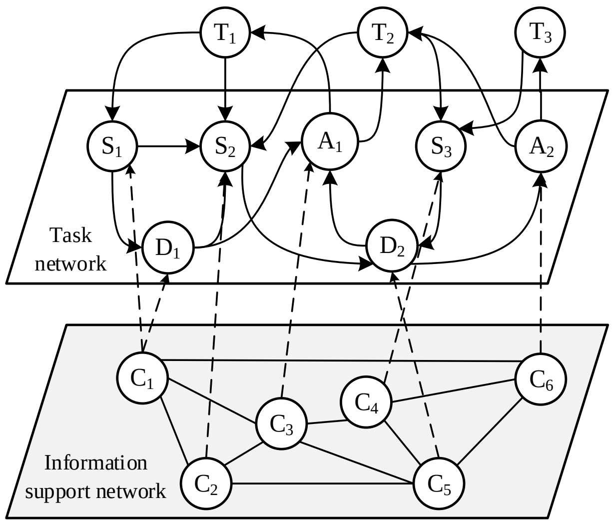

Figure 5 – The interdependent network model of this AERSoS

4.2 Result of Effectiveness Evaluation

Firstly, obtain all possible effect chains in the task network based on the idea of subgraph isomorphic matching. Take the first type of effect chain G_2^1 ($T_2 \rightarrow S \rightarrow D \rightarrow A \rightarrow T_2$) containing T_2 as an example to illustrate the search process. Let the numbers of the task nodes S_1~S_3, D_1~D_2, A_1~A_2 and T_1~T_3 of the AERSoS be 1~3, 4~5, 6~7 and 8~10, respectively. The initial mapping matrix $\boldsymbol{M}_2^0$ between effect chain G_2^1 and the task network with the weights removed G_{com}^d is shown in formula (9).

$$\boldsymbol{M}_2^0 = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \tag{9}$$

According to formula (3), the obtained mapping matrix $\boldsymbol{M}_2$ after pruning is shown in formula (10).

$$\boldsymbol{M}_2 = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \tag{10}$$

Use depth-first search obtain all specific mapping relationships between G_2^1 and G_{com}^d in $\boldsymbol{M}_2$, and one $\boldsymbol{M}_2^{'}$ obtained during this process is shown in formula (11).

$$\boldsymbol{M}_2^{'} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \tag{11}$$

The corresponding matrix $\boldsymbol{C}$ is shown in formula (12).

$$\boldsymbol{C} = \boldsymbol{M}_2^{'} \boldsymbol{B}^d \boldsymbol{M}_2^{'\mathrm{T}} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix} \tag{12}$$

The adjacency matrix of G_2^1 is shown in formula (13).

$$\boldsymbol{A}_2^1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix} \tag{13}$$

According to formula (5), $\boldsymbol{M}_2^{'}$ satisfies the isomorphic rule, so an effect chain of type 1 that containing T_2 in G_{com}^d is found: $T_2 \rightarrow S_2 \rightarrow D_2 \rightarrow A_1 \rightarrow T_2$. Next, traverse all $\boldsymbol{M}_2^{'}$ in $\boldsymbol{M}_2$ to obtain all effect chains of type 1 that contain T_2. Moreover, traverse all T nodes and all types of effect chains, and the set $\boldsymbol{Q}^0$ of all possible effect chains in the task network are obtained, as shown in Table 6.

Table 6 All possible effect chains in the task network

No.	Type of effect chain	Equipment composition	No.	Type of effect chain	Equipment composition
1	1	$T_1 \rightarrow S_1 \rightarrow D_1 \rightarrow A_1 \rightarrow T_1$	6	1	$T_2 \rightarrow S_2 \rightarrow D_2 \rightarrow A_2 \rightarrow T_2$
2	1	$T_1 \rightarrow S_2 \rightarrow D_2 \rightarrow A_1 \rightarrow T_1$	7	3	$T_2 \rightarrow S_3 \rightarrow D_2 \rightarrow A_1 \rightarrow T_2$
3	2	$T_1 \rightarrow S_1 \rightarrow S_2 \rightarrow D_2 \rightarrow A_1 \rightarrow T_1$	8	3	$T_2 \rightarrow S_3 \rightarrow D_2 \rightarrow A_2 \rightarrow T_2$
4	5	$T_1 \rightarrow S_1 \rightarrow D_1 \rightarrow S_2 \rightarrow D_2 \rightarrow A_1 \rightarrow T_1$	9	5	$T_3 \rightarrow S_3 \rightarrow D_2 \rightarrow A_2 \rightarrow T_3$
5	1	$T_2 \rightarrow S_2 \rightarrow D_2 \rightarrow A_1 \rightarrow T_2$			

For the effect chains in $\boldsymbol{Q}^0$, all task nodes have a directed edge from the information support node to this node in the asymmetrical dependent network, so all effect chains meet this requirement. The next step is to determine whether the information support nodes that the adjacent task nodes in the effect chain depend on have the ability of effective information transmission. Taking the effect chain $T_2 \rightarrow S_2 \rightarrow D_2 \rightarrow A_1 \rightarrow T_2$ as an example, the information support nodes that S_2, D_2 and A_1 rely on are C_2, C_5 and C_3, respectively. And the quadratic power matrix $\boldsymbol{D}^2$ of $\boldsymbol{D}$ is shown in formula (14).

$$\boldsymbol{D}^2=\begin{pmatrix}3&1&1&2&3&0\\1&3&2&2&1&2\\1&2&4&1&2&3\\2&2&1&3&2&1\\3&1&2&2&4&1\\0&2&3&1&1&3\end{pmatrix} \tag{14}$$

We can see that $d_{2,5}=d_{5,2}=d_{3,5}=d_{5,3}=1$, $d_{2,5}^{(2)}=d_{5,2}^{(2)}=1\neq 0$ and $d_{3,5}^{(2)}=d_{5,3}^{(2)}=2\neq 0$. Therefore, C_2 and C_5, C_5 and C_3 have effective information transmission capability and the effect chain $T_2 \rightarrow S_2 \rightarrow D_2 \rightarrow A_1 \rightarrow T_2$ is an effective effect chain.

Traversing the effect chains in $\boldsymbol{Q}^0$, we can see that all effect chains are effective effect chains, so $\boldsymbol{Q}=\boldsymbol{Q}^0$.

Let K=100 and repeat the Monte Carlo sampling for the task network for K times. The edge state of the task network obtained by a certain sampling is shown in Figure 6, where the dotted line indicates that the corresponding edges in this sampling are disconnected and the solid line indicates that the corresponding edges are connected. According to Figure 6, effect chains numbered 1, 2, 3 and 5 in $\boldsymbol{Q}$ are formed, so $F_1^k=F_2^k=1$ and $F_3^k=0$ in this sampling.

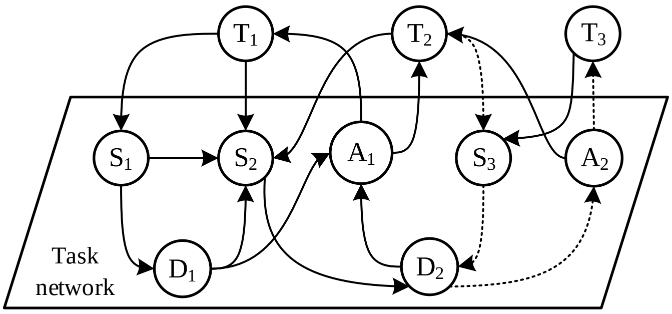

Figure 6 – The edge state of the task network from a certain sample

After K repeated Monte Carlo samplings, $\sum_{k=1}^{K}F_1^k=69$, $\sum_{k=1}^{K}F_2^k=85$ and $\sum_{k=1}^{K}F_3^k=52$ are obtained. According to formula (7), the effectiveness of this AERSoS Ef is 0.736, which validates the rationality of the modeling method and effectiveness evaluation method.

Finally, the contribution of each equipment node to the effectiveness is quantitatively analyzed by using the node deletion method [24], and the result is as shown in Figure 7. It can be seen that D_2 and C_5 have higher contribution than other nodes, because they participate in the formation of more effect chains with higher probability of the formation of more important tasks. This conclusion can also provide support for the design and optimization of the AERSoS.

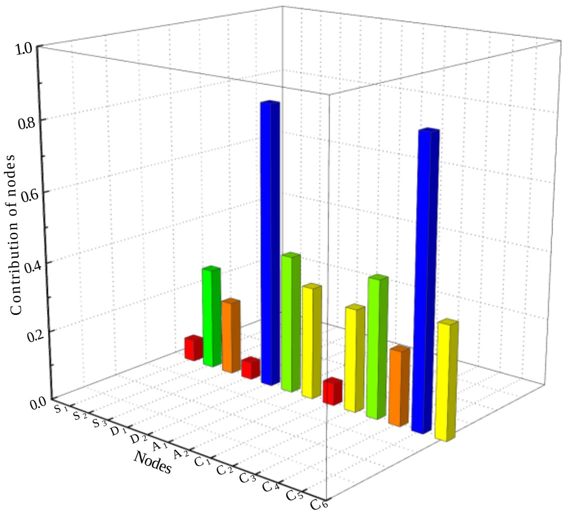

Figure 7 – The contribution of each equipment node to the effectiveness of this AERSoS

5. Conclusion

In order to comprehensively reflect the impact of the success rate of different equipment, the importance of tasks and different functional dependencies on the AERSoS effectiveness, the following studies are mainly carried out.

1) In order to reasonably reflect the influence of various heterogeneous factors, the two-layer heterogeneous interdependent network model of AERSoS is proposed, which includes task network, information support network and the asymmetric dependent network.

2) The smallest functional unit of the AERSoS to complete the task is defined as effect chain. Regarding effect chains as the specific structure in the network, the subgraph isomorphism matching theory and an improved Ullmann algorithm are introduced to realize the search of the possible effect chains. And the existence of the effective information transmission capability is judged through the power operation of the adjacency matrix of the information support network.

3) By using Monte Carlo sampling, the influence of various factors on the AERSoS effectiveness is unified into the impact on the effect chains, and the effectiveness evaluation index is given. The results of example analysis validate the rationality of the proposed methods.

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7. Copyright Statement

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