REVIEW ON NONLINEAR AEROELASTIC ANALYSIS WITH FREEPLAY

Yukai Sun[1, *], Zheng Han[1], He Cao[1], Wenyuan Cheng[1], Zhigang Wu[2] & Chao Yang[2]

[1] Aviation Industry Development Research Center of China, Beijing 100029, China.
[2] School of Aeronautic Science and Engineering, Beihang University, Beijing 100191, China.

Abstract

This article summarizes the progress and challenges in the field of freeplay nonlinearity in aeroelastic analysis. The impact of structural freeplay on aircraft flight and ground tests are described. The main models and modeling methods of freeplay aeroelastic system are summarized. The theoretical and experimental research of nonlinear aeroelastic analysis with freeplay. We also look forward to the future work.

Keywords: aeroelasicity; freeplay nonlinearity; review; aeroelastic analysis; nonlinear aeroelasticity

Aeroelasticity is the study of the interactions among inertial, aerodynamic, and elastic forces and their interactions on aircraft design[1]. It explores the mechanical behavior of elastic structures in the airflow[2]. Nowadays, the theory of linear aeroelasticity based on linear system assumption has matured significantly, which was well-documented in numerous esteemed publications worldwide[1]-[10]. Nevertheless, with the escalating demands for enhanced aircraft performance and the continuous advancements in aircraft design technologies, the linear aeroelasticity theory is increasingly inadequate for the requirements of the aircraft design and performance analysis correntlly[11].

Structural freeplay, a common form of concentrated structural nonlinearity in aircraft, typically occurs between different components capable of relative motion, such as control surface hinges, bearings, and external stores. These freeplay are predominantly found in the control surface actuation systems, the roots of all-moving control surfaces, and folding wing hinges. Nonlinear aeroelastic issues induced by freeplay nonlinearity may introduce potential risks to the aircraft.

In this paper, the advancements and challenges in the field of aeroelastic analysis of freeplay nonlinearity were summarized. Section 1 describes the impact of structural freeplay on aircraft flight and ground testing, then the considerations and analyses for freeplay in industry were reviewed. Section 2 reviews the primary models and methods for nonlinear aeroelastic modeling of freeplay. Section 3 summarizes the research on methods for nonlinear aeroelastic analysis of freeplay. Section 4 compiles the recent wind tunnel test studies and validations of nonlinear aeroelasticity of freeplay. Finally, Section 5 assesses the current state of nonlinear aeroelastic analysis of freeplay. Moreover, the insights into future development prospects are introduced.

1. Nonlinear Aeroelasticities Induced by Freeplay in Aircraft Design

The aeroelastic design, analysis, and validation of aircraft are critical for preventing flutter and other aeroelastic issues within the design envelope. In engineering applications, the aeroelastic analysis process during the development phase of the aircraft is illustrated in Figure 1.

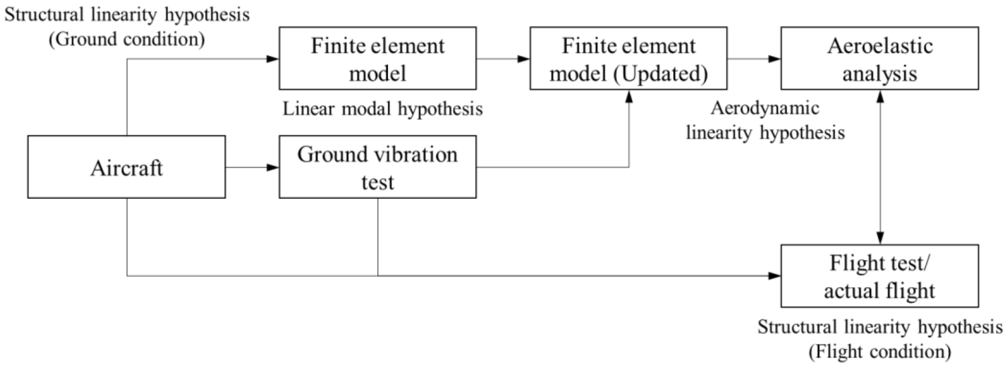
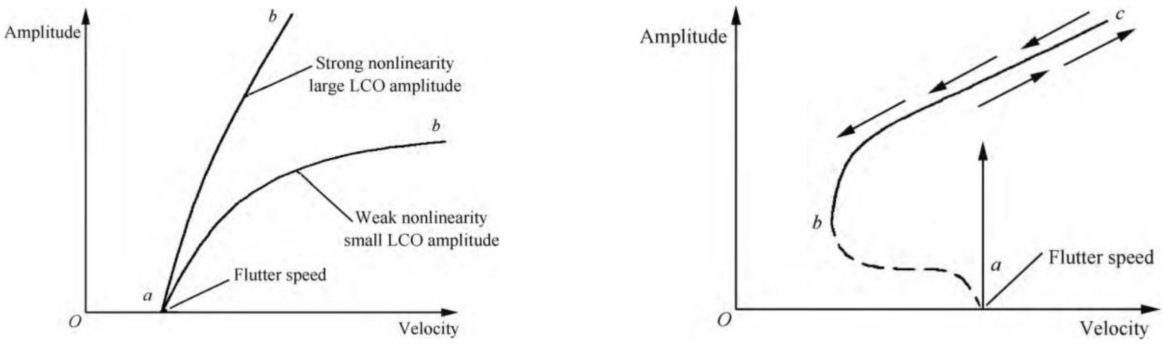


Figure 1 General process of aircraft aeroelastic analysis in engineering application

In aeroelastic analysis based on linear assumptions, conservative design is typically employed to address nonlinear aeroelastic issues induced by freeplay. This approach may involve sacrificing some aircraft performance and increasing structural weight to ensure the flight safety. International military standards[12] and civil airworthiness regulations[13] impose strict requirements on the permissible gaps of various components. Similarly, China's military standard GJB67.7A-2008[14] specifies the permissible gaps between control surfaces and trim tabs. Within these permissible gaps, the system can be considered linear, and conservative design can be performed using linear methods. In addition to increased weight, however, potential dangers such as nonlinear flutter may still exist even with conservative design.

Extensive researches around the world have demonstrated that nonlinear aeroelastic phenomena induced by freeplay primarily involve limit cycle oscillations (LCO), bifurcations, and chaotic motions, which alter the system's original flutter characteristics (the underlying linear system's flutter characteristics). The most direct influences of these changes are supercritical flutter and subcritical flutter, as illustrated in Figure 2(a) and Figure 2(b), respectively. Supercritical flutter can elevate the original flutter speed, meaning that when wind speeds exceed the linear flutter speed, the system transitions from divergent flutter to LCO. On the other hand, subcritical flutter may reduce the original flutter speed, making the system susceptible to large amplitude LCO or even divergence below the original flutter speed.



(a) supercritical flutter (b) subcritical flutter

Figure 2 Influence of freeplay nonlinearity on linear flutter characteristics of an aeroelastic system[5]

1.1 Freeplay in Real Flight

If the originally tightly connected components of the aircraft become loose due to factors such as vibration impact, friction wear, or vibration loosening during flight, the gap may appear or expand into beyond the permissible range. Then the overall aeroelastic characteristics of the aircraft may be significantly compromised, which might render the conservatively designed safe flight state unsafe. Excessive freeplay may induce larger amplitude LCO, significantly reducing the structural fatigue life. Moreover, the nonlinear LCO may even trigger flutter, causing direct structural failure.

On March 24, 1968, an Aer Lingus Vickers Viscount aircraft crashed 3.1km northeast of Tuskar Rock, Ireland, resulting in the deaths of all 61 occupants, including 4 crew members. The accident investigation report indicated that the probable cause was excessive freeplay in the elevator control mechanism, leading to fatigue failure of the components under cycle oscillations and subsequently causing the elevator flutter[15], [16].

During the flight telemetry of a specific missile model in the 1970s, the UK discovered that some missiles exhibited oscillations at a frequency of about 70Hz under high dynamic pressure. Figure 3 shows a comparison between the flutter boundary predicted by linear methods and the actual flight test results. As dynamic pressure increased, approximately 50% of the missiles experienced oscillations. With further increases, all missiles oscillated. Moreover, the continued increases in dynamic pressure led to divergent oscillations. Although linear design and analysis indicated that such phenomena would not occur within the flutter boundary, the presence of nonlinear freeplay in the servo system led to the LCO below the flutter boundary[17].

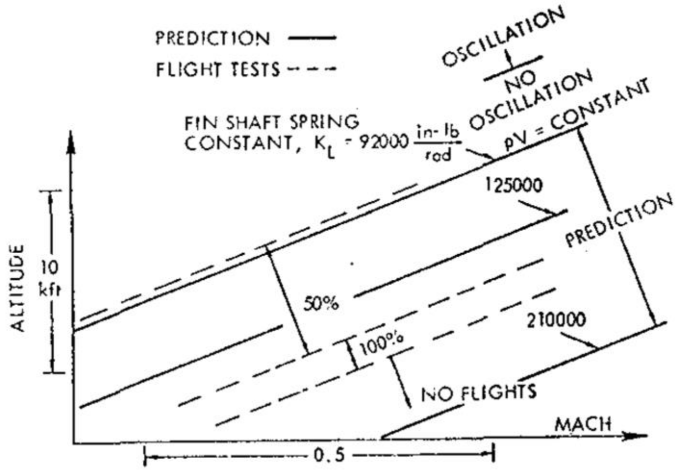
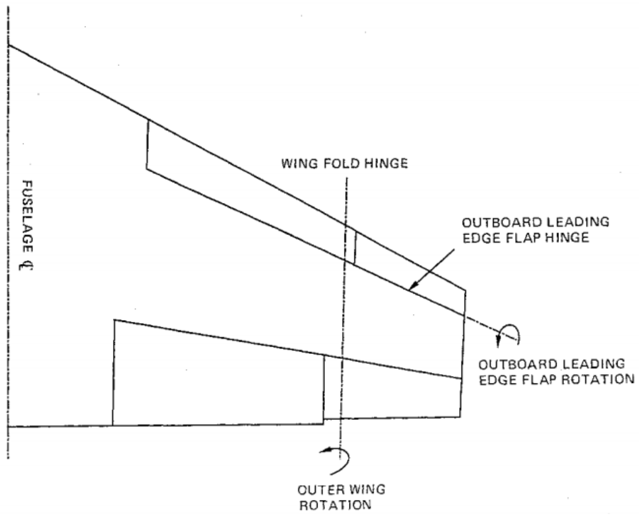
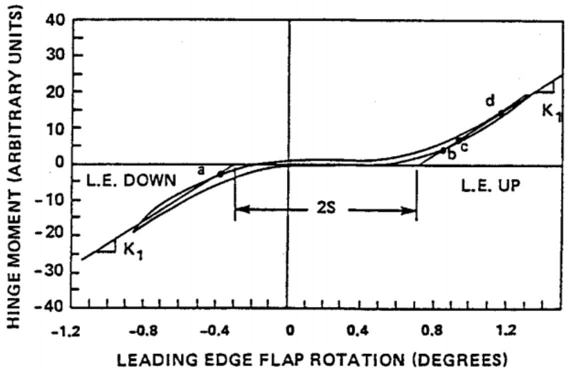


Figure 3 Comparison between the linear prediction and the actual test results of missile flutter boundary[17]

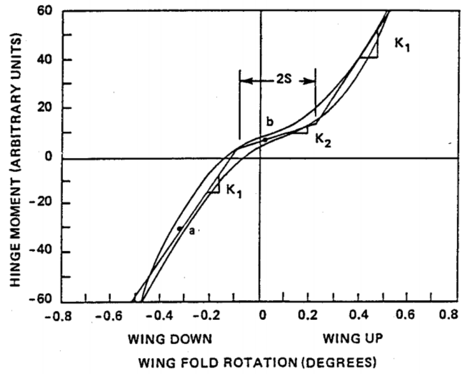
In 1985, the Canadian Air Force observed that the CF-18 fighter frequently exhibited severe and sustained periodic oscillations in the wing structure within the normal flight envelope. Lee B.H.K.[18] conducted research on the CF-18 and found notable wear and looseness at the outer leading-edge flap hinge and the wing fold hinge. Displacement-torque test results showed that some CF-18 fighter, after a period of service, displayed nonlinear structural characteristics such as freeplay, hysteresis, and cubic stiffness at both the flap hinge and fold hinge, as depicted in Figure 4.



(a) the layout of CF-18 fighter control surface and folding wing[19]



(b) Nonlinear test results at outboard leading edge flap



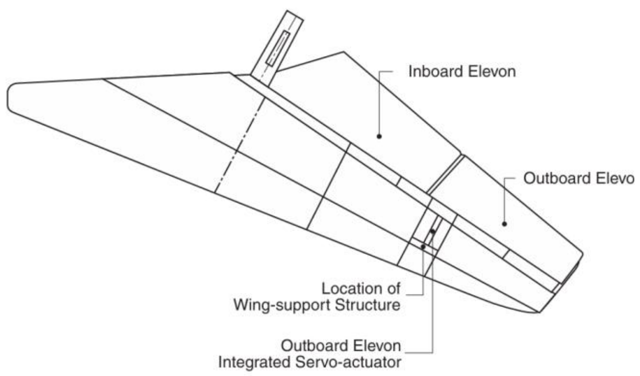
(c) Nonlinear test results at the folding axis of the wing

Figure 4 Two nonlinear structures of CF-18 aircraft wing

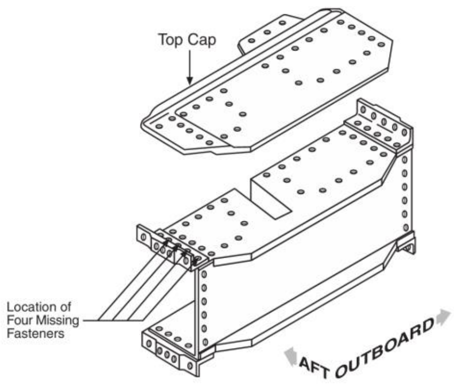
The Twin Commander 690 series aircraft were involved in three flight accidents in the late 20th century. The report on the August 1993 accident involving a Twin Commander 690A[20] explains that the pilot suddenly encountered an uncontrollable right roll, with control only being regained after a 360° roll. The investigation revealed that an excessive freeplay of 0.584cm in the servo actuator on the left aileron led to an elevator malfunction. Evidence of impact vibrations was found on the elevator pushrod, and the inner hinges of both ailerons contacted the aileron surfaces, indicating that the ailerons had exceeded their deflection range. As a result, the National Transportation Safety Board (NTSB) mandated that the Federal Aviation Administration (FAA) reassess the control system of this aircraft model.

During an air show near Baltimore on September 14, 1997, a Lockheed Martin F-117A stealth bomber was ascending when the pilot felt an unusual vibration. The aircraft abruptly rolled to the left, leading to the left wing detaching. The pilot ejected, and the aircraft crashed shortly thereafter.

According to the investigation findings[21], the accident was caused by the absence of four fasteners, which compromised the rigidity of the connection point between the inner and outer sections of the left wing (as depicted in Figure 5). This reduction in rigidity created a gap, resulting in the loosening of the outer wing aileron servo system, which should have been securely fixed. This looseness induced nonlinear oscillations in the outer wing aileron, leading to flutter and ultimately causing the left wing to break off.



(a) layout of the wing



(b) structure

Figure 5 F-117A accident investigation during air show in 1997[21]

On October 4, 2016, a Boeing 737-4Q8 cargo aircraft operated by ASL Airlines Belgium experienced shimmy in its landing gear during landing at Belfast International Airport, UK. The right main landing gear's upper and lower torsion links fractured. The accident investigation report suggested that an excessive freeplay between the landing gear shimmy damper and the torsion links caused the shimmy during high-speed landing, leading to the fracture of the torsion links[22].

1.2 Freeplay in Ground Vibration Tests

In contrast to linear systems, nonlinear systems inherently exhibit a frequency-amplitude dependency, whereas linear theory predicts structural natural frequencies that are independent of amplitude[23]. Structural freeplay caused by control systems, complex connections, manufacturing errors, and tolerance requirements can influence the results of aircraft ground vibration tests (GVTs), thereby impacting finite element model (FEM) updates and subsequent aircraft design analyses. Figure 7 shows the frequency response function (FRF) results in GVTs for the folding wing under sweep frequency excitation, as illustrated in Figure 6. Due to the frequency-amplitude dependency of nonlinear systems, the peak of the structure's FRF curve shifts with increasing excitation levels. Figure 8 provides a more intuitive representation of this phenomenon.

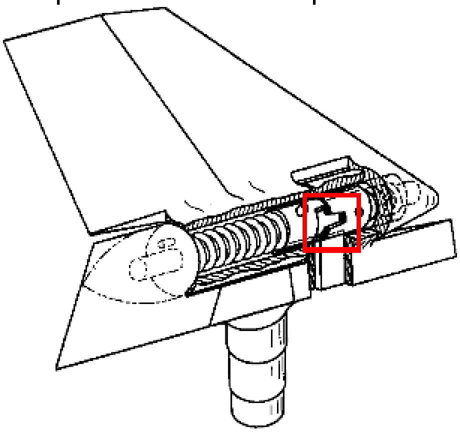
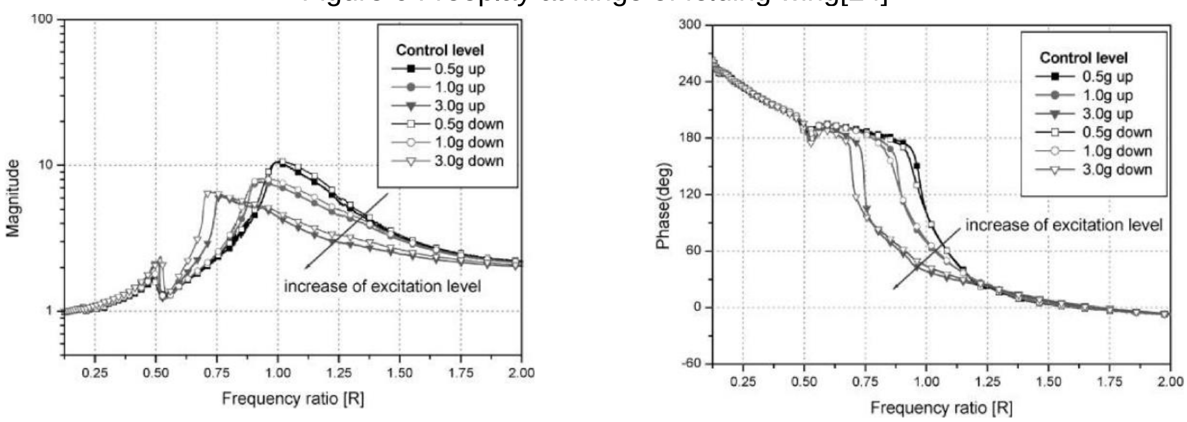


Figure 6 Freeplay at hinge of folding wing[24]



(a) amplitude (b) phase

Figure 7 Results of the FRFs of folding wing GVTs under sweep-sine excitation[25]

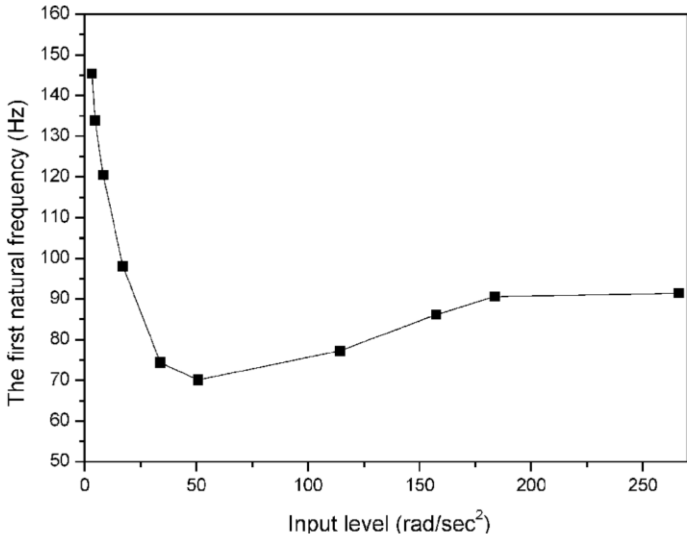
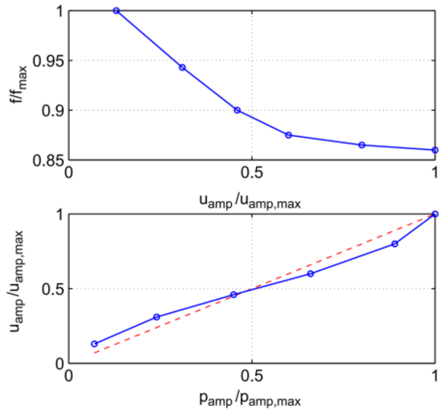


Figure 8 Variation of the first-order natural frequency with the excitation level of the folding wing during the GVTs under sweep-sine excitation[25]

During the ground vibration tests of the A380, the influence of nonlinearity on the frequency response function was also observed. The linearity plots method proposed by Dennis[26] allows for a more intuitive visualization of the amplitude-frequency dependency phenomenon, as shown in Figure 9. Dennis[27], [28] systematically described and studied this phenomenon, discovering amplitude-frequency dependency in tests of various aircraft components such as engines, ailerons, and elevators, which indicates the presence of structural nonlinearity. Similar nonlinear results have also been observed in recent ground vibration tests of the A350 XWB[29]. The shift in the frequency response curve with varying excitation levels causes the modal parameters obtained by traditional modal identification methods based on the frequency response function to also shift, thus presenting significant challenges to finite element model updates based on linear assumptions.

(a) GVT set up



(b) Linearity plots

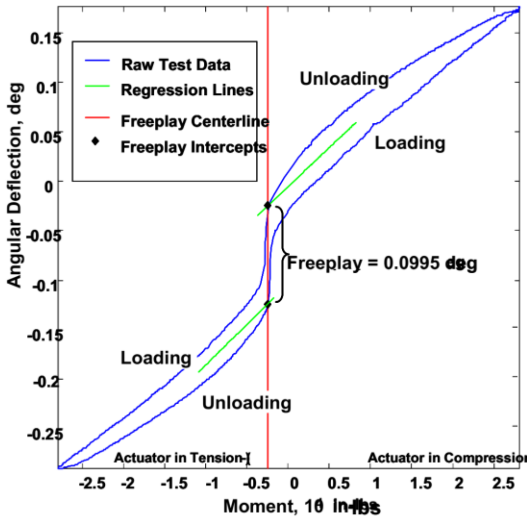
Figure 9 Nonlinear phenomena in A380's GVTs[26]

1.3 Research Towards the Aircrafts

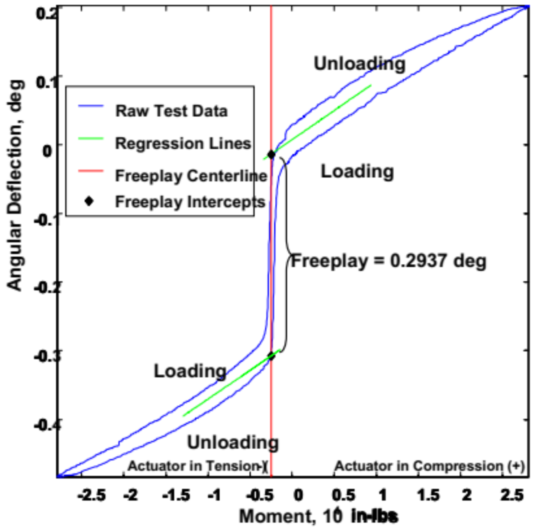
To address the challenges posed by nonlinearity, international scholars have begun to conduct research during the development and refinement stages of aircraft models such as the A380[26], A350 XWB[29], F-16[30], CF-18[18], F-22[31]-[33], F-35, and B-2. These efforts aim to understand the impact of concentrated nonlinearity on aeroelastic characteristics and ground vibration tests. Recently, researchers have utilized nonlinear methods to carry out a series of ground vibration tests on the F-16[34]-[36]. Aerial photography and video footage of the CF-18 carrier-based aircraft during flight tests at the Cold Lake Test Center of the Canadian Army base showed that the nonlinearity of the folding wing mechanism resulted in lightly damped low-frequency oscillations, suspected to be limit cycle oscillations due to nonlinearity. Consequently, Lee et al.[18] conducted extensive research on the effects of two nonlinear hinges on the CF-18 wing's aeroelastic characteristics, revealing the underlying mechanisms.

During the design phase of the F-22, the impact of freeplay nonlinearity on the aircraft's structural response and control systems was taken into account[33]. Control surface freeplay and LCO simulations were incorporated into the design, leveraging multidisciplinary design optimization (MDO) techniques to achieve tasks such as aeroelastic tailoring[31]. This approach thoroughly explored the F-22's design potential under linear assumptions. However, during the validation of the classic flutter scale model, unexpected LCOs appeared before the scale model encountered flutter failure, as depicted in Figure 10, due to the influence of freeplay nonlinearity. As a result, static freeplay measurement tests were performed on the F-22 prototype, uncovering the presence of freeplay nonlinearity in the control surface transmission mechanism (refer to Figure 11), and a series of analyses on its nonlinear flutter characteristics were conducted[32]. Furthermore, the control surface freeplay was factored into the F-22 control system design[33], and after extensive testing, an acceptable freeplay in control surface deflection was allowed.

Figure 10 Destruction of control surface of scaled F-22 in classic flutter test[33]



(a) Nominal freeplay



(b) Maximum freeplay

Figure 11 Ground static loading test results of F-22's left flaperon[32]

The United States Air Force Research Laboratory (AFRL) conducted wind tunnel tests on a scaled model of the F-35 featuring horizontal tail surface freeplay[37], as illustrated in Figure 12. The study revealed that without preload on the horizontal tail control surfaces, even minor freeplay could trigger LCOs. However, applying a small preload was sufficient to eliminate these oscillations at small deflection angles. As a result, researchers developed strategies to prevent LCOs and suggested that the design requirements specified by the U.S. military standard MIL-A-8870[12] might be excessively conservative.

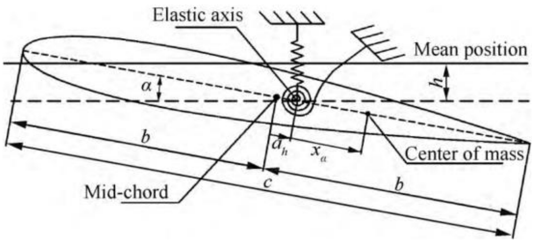
Figure 12 Scaled F-35 flutter model wind tunnel test[37]

2. Modeling of Nonlinear Aeroelasticity of Freeplay

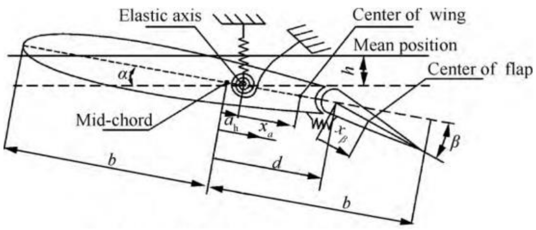
In nonlinear aeroelastic research, the subjects are primarily categorized into two main types: two-dimensional (2D) wing-section model and three-dimensional (3D) model. This classification is based on the research objectives and the complexity of the structural and aerodynamic forces involved.

2.1 Airfoil Model

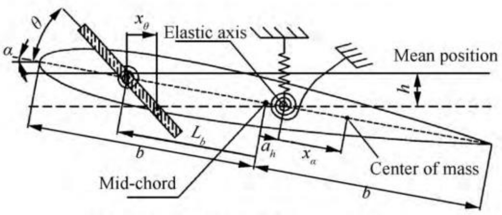
The frequently employed airfoil models in research[38] are: the plunge-pitch model (Figure 13 (a)), the plunge-pitch-control model (Figure 13 (b)), and the plunge-pitch-store model (Figure 13 (c)). Additionally, the plunge-pitch-double control model (Figure 13 (d)) as a special configuration, has also been explored[39], [40].



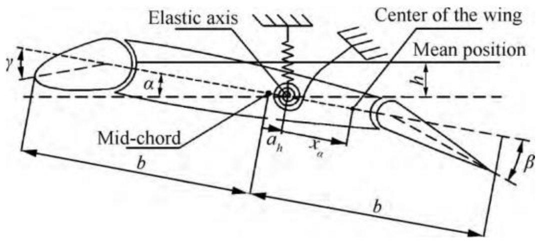
(a) plunge-pitch model



(b) plunge-pitch-control model



(c) plunge-pitch-store model



(d) plunge-pitch-double control surface model

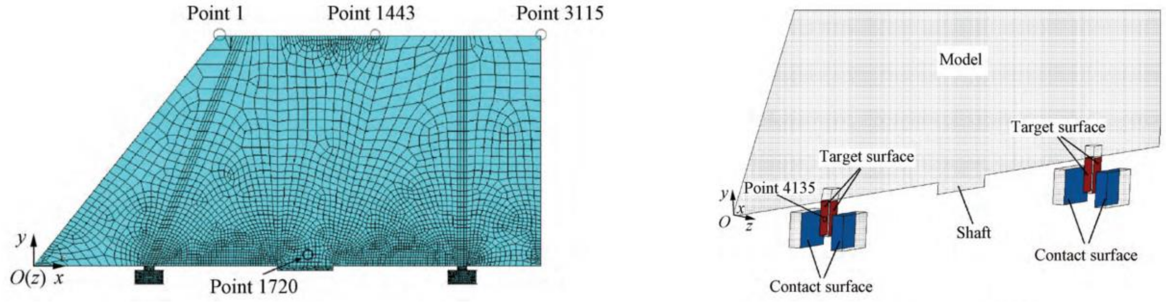
Figure 13 Airfoil model in nonlinear aeroelasticity research[38]

The primary advantage of airfoil model is it fewer degrees of freedom (DOF) and the ability to use 2D aerodynamic calculations, eliminating the need to consider spanwise aerodynamic effects. The analytical contributions from Theodorsen[41] and Lee[42], along with the approximate analytical solutions by Li[43], have made airfoil model a popular choice for studying new methods and technologies. Extensive and thorough research on these models has been carried out by numerous researchers worldwide[44].

2.2 3D Structural Model

The investigation of 3D aeroelastic model is notably more complex, primarily due to the difficulties in fomulating and calculating the 3D aerodynamic forces. For rapid aerodynamic calculations in 3D models, subsonic aerodynamics generally use the doublet-lattice method (DLM), while supersonic aerodynamics typically employ the piston theory. These methods are suitable for engineering analysis but only provide linear unsteady aerodynamic forces. In contrast, CFD techniques based on Euler or Navier-Stokes equations involve fewer assumptions and can simulate more complex characteristics of the aerodynamics, but these require considerable computational resources. Yang et al.[38] have systematically summarized the various types and applications of unsteady aerodynamic models in aeroelastic analysis. It should be emphasized that when analyzing aeroelastic systems with structural nonlinearity, linear unsteady aerodynamic forces are usually sufficient. However, the large angle-of-attack (AOA) separation flows and the transonic situations are not involved.

Alternatively, various approaches are available for representing freeplay nonlinearity in 3D structures. The most direct approach, as shown in Figure 14, involves meticulously defining the gap boundary using contact boundary elements in finite element (FE) software. Then the CFD/CSD simulation using numerical methods is performed. However, it makes parametric analysis, such as adjusting the gap size or changing the connection stiffness, particularly challenging.



(a) FE model (b) freeplay modeling

Figure 14 FE modeling of a hypersonic nonlinear wind tunnel test model with freeplay[45]

In the modeling process, when dealing with nonlinear connections, the freeplay nonlinearity at the connection points can be represented as a nonlinear restoring force fomulation (see Figure 15). For certain simple connection models, after obtaining the FE model of the structure in physical coordinates, the corresponding DOFs can be directly altered from a linear connection to a freeplay nonlinear connection. Then the nonlinear FE model described in physical coordinates is conducted As shown in Figure 16 (a) and Figure 16 (b), this nonlinear hinge connects only a few nodes between different components, with little modification to the established linear FE model.

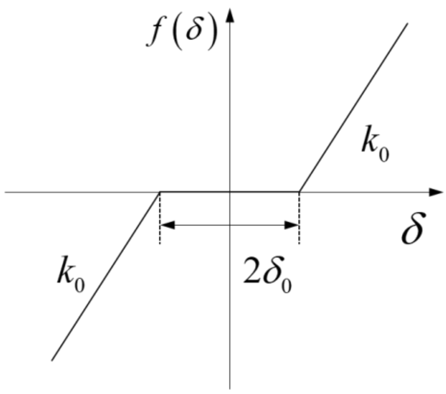
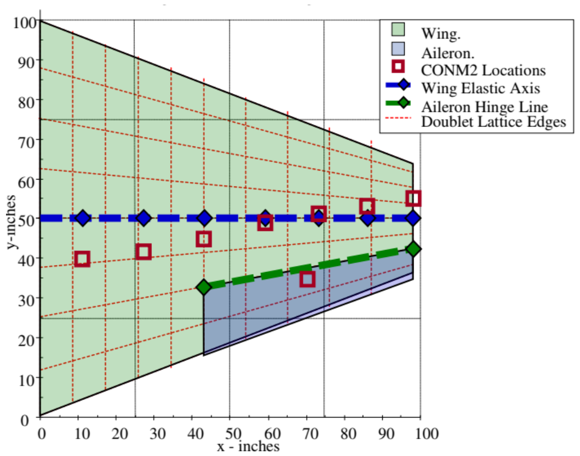
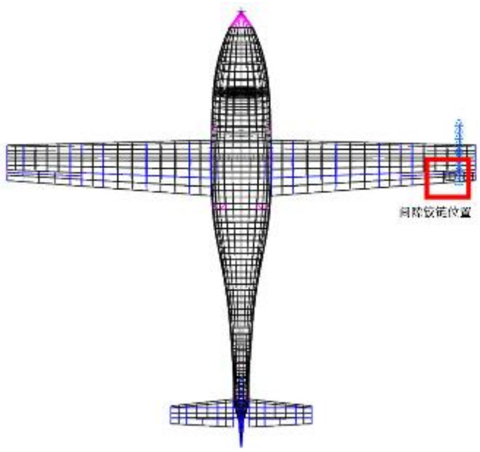


Figure 15 Nonlinear restoring force fomulation for freeplay



(a) a general model of the aileron[46]



(b) HA145J model[47]

Figure 16 Modeling of nonlinear 3D structures with freeplay in physical coordinates

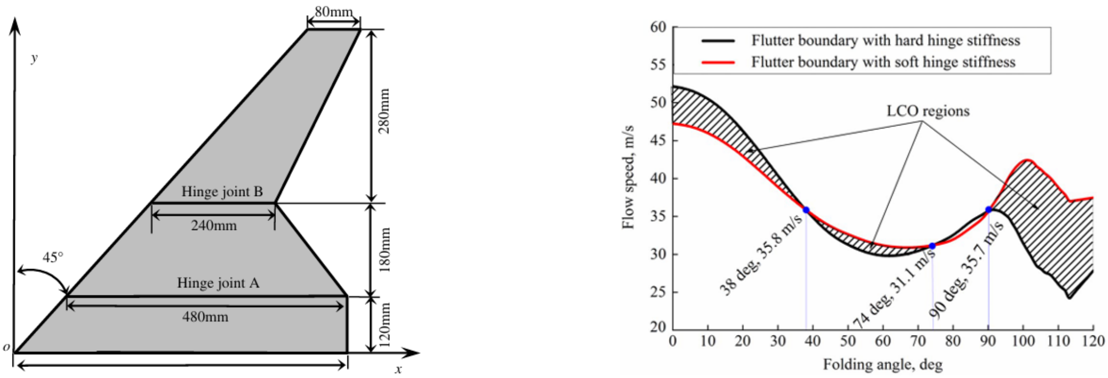
Although it is feasible to establish a nonlinear FE model using the aforementioned approach, the order of the structural dynamic equations remains unchanged. For FE models conducted by solid elements, the number of DOF of which exceeding tens of thousands, renders rapid analysis in physical coordinate descriptions exceedingly challenging. Conversely, adopting a simplified "fishbone" model fails to considerably reduce the model order when confronted with complex structures. In well-established linear aeroelastic analysis methods, the structural model is typically described using generalized modal coordinates that effectively reducing the model order by retaining low-order modes. However, due to the violation of the linear modal assumption, generalized modal coordinates cannot be directly employed to describe nonlinear components.

Consequently, addressing the dimensionality reduction issue in nonlinear structural models necessitates an approach that can consider the application of generalized modal coordinate descriptions for linear structures while simultaneously preserving the physical coordinate descriptions for nonlinear components. To address these issues, two predominant methodologies are employed within the field: the fictitious-mass (FM) method and the nonlinear component mode synthesis (CMS) method.

2.2.1 Fictitious-mass Method

The FM method was originally proposed by Israeli scholar Karpel[48]-[50]. This method revolves around maintaining the overall structure with freeplay unaltered while introducing a concentrated fictitious-mass element of significant magnitude exclusively at the connection points of the freeplay. Subsequently, modal analysis is performed on this modified structure, resulting in the determination of fictitious-mass modes[49], [50]. By exploiting the discrepancies between the fictitious-mass modes and the original linear orthogonal modes, then the novel modal bases are constructed[19]. Since its inception, the fictitious-mass method has showcased extensive applicability in diverse nonlinear aeroelastic analyses. However, during the practical implementation of the fictitious-mass method, determining the appropriate magnitude of the fictitious-mass for different structures primarily relies on the user's experience.

In recent years, Karpel has led a team in developing the commercial software Dynresp and providing solution consulting[51]. Currently, Dynresp has been utilized in some projects by foreign companies, including Airbus. Huang utilized the virtual mass method to analyze a folding wing with segmented linear hinges (see Figure 17)[52]. The analysis examined the LCOs under different folding angles and illustrated the boundaries of these LCOs.



(a) Geometry and dimensions of folding wings (b) LCO boundary of the folding wing

Figure 17 LCO analysis of a folding wing with a piecewise linear hinge[52]

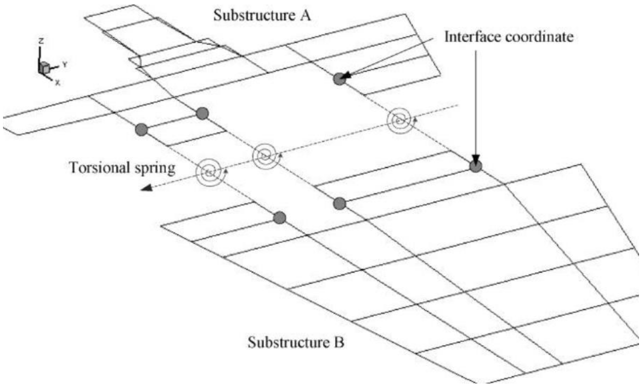
2.2.2 Nonlinear Component Mode Synthesis Method

Another method involves the extension of the mature linear CMS technique to nonlinear systems. In China, this method is also known as the dynamic substructure method, while MSC.Nastran employs a similar definition called super-elements. Developed between the 1950s and 1980s, this approach utilizes modal analysis to reduce the complexity of structures and is fundamentally based on the Rayleigh-Ritz method. The method entails partitioning the overall structure into several substructures, obtaining the reduced-order model (ROM) of each substructure, then subsequently reassembling these ROM models into the overall ROM model[53].

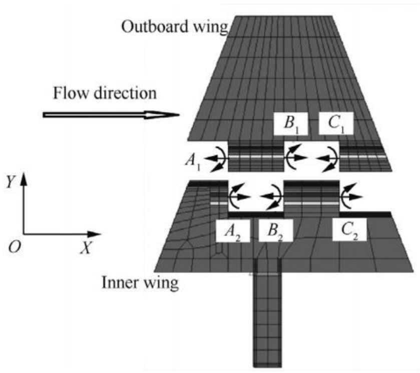
The CMS methods are classified based on the form of the interface, which includes fixed-interface CMS method, free-interface CMS method, and hybrid-interface CMS method[53]. The most classic methods are the fixed-interface CMS method utilizing the substructure principal modes and constraint modes as the modal basis, and the free-interface CMS method utilizing substructure principal modes and released modes as the modal basis. Generally, the fixed-interface CMS method is commonly employed for theoretical analysis due to its high precision and simplicity in the interface of the substructures. However, its practical usage in experimental scenarios is highly challenging.

The fixed-interface CMS method was initially proposed by Hurty[54], [55] and later widely employed in engineering following the advancements by Craig and Bampton[56]. Conversely, the free interface modal synthesis method was introduced by Hou[57] and further expanded by Rubin[58] and Craig[59]. Currently, the most commonly utilized methods include the Hou free-interface method, Gladwell branch modal method, Benfield modal substitution method, Craig-Bampton fixed-interface method, dual compatible free-interface CMS method, Hurty assumed modes method, Leung superelement method, interface-less modal synthesis method, et al. Among these, the dual compatible free-interface CMS method proposed by Wang[60], has exhibited outstanding performance in terms of synthesis efficiency and experimental applications. Over several decades of development, researchers have conducted extensive systematic reviews of CMS methods[53], [61]-[63].

Kim, Yang, and other researchers have made outstanding contributions in the study of modal synthesis modeling methods for nonlinear clearances. Kim[25] improved the Craig-Bampton fixed-interface CMS method and established the nonlinear aeroelastic equation of a missile's folding control surface, as shown in Figure 18 (a). By comparing the results with the modal analysis of the FE model, the effectiveness of the proposed method was validated. Furthermore, Kim conducted corresponding GVTs on the actual structure of the folding control surface using the free-interface CMS procedure, thereby providing experimental validation of this method (refer to Figure 8). Yang[64] conducted theoretical and experimental research on the nonlinear freeplay within the folding hinge of the rudder by extending the dual compatible free-interface CMS method. As illustrated in Figure 18 (b), the utilized model in conjunction with linear unsteady aerodynamic theory yielded excellent results in nonlinear aeroelastic analysis. This extended CMS method was also applied in the nonlinear system identification (NSI)[65], [66].



(a) nonlinear fixed-interface CMS method[25]



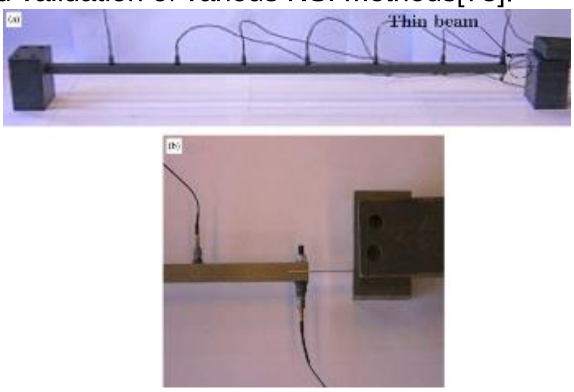
(b) nonlinear dual compatible free-interface CMS method[64]

Figure 18 Modeling of a folding rudder

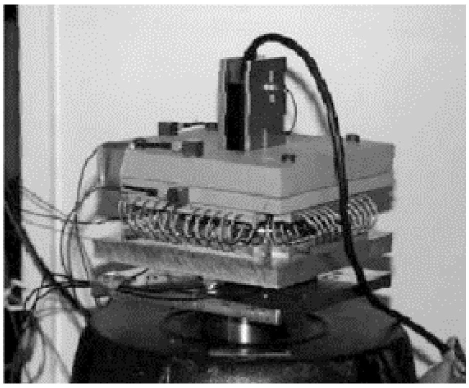
2.3 Nonlinear System Identification of Freeplay

Although it is possible to construct suitable nonlinear structural models using nonlinear modeling techniques for systems with freeplay, two issues still need to be considered. Firstly, the presence of nonlinearity may hinder the acquisition of accurate structural modal parameters through GVTs conducted on the actual structure. Secondly, without knowledge of the true nonlinear parameters of the freeplay, it is not feasible to establish an exact nonlinear structural model. Typically, these two issues are resolved by employing NSI methods.

Since the 1960s to 1970s, there has been a growing interest among scholars in the field of NSI[67]-[69]. Over the past few decades, the field has witnessed rapid development. In the 21st century, NSI has evolved from its initial academic exploration of simple dynamic structures to the analysis of complex nonlinear dynamic systems with practical industrial applications. Noteworthy comprehensive reviews on nonlinear system identification have been conducted by Worden[70], Kerschen[23], Noël[71], et al. Worden and Tomlinson[72] have also compiled the first dedicated book on the NSI. Between 1997 and 2001, European technology cooperation organizations released two benchmark models for nonlinear systems with freeplay, as shown in Figure 19, to facilitate research and validation of various NSI methods[73].



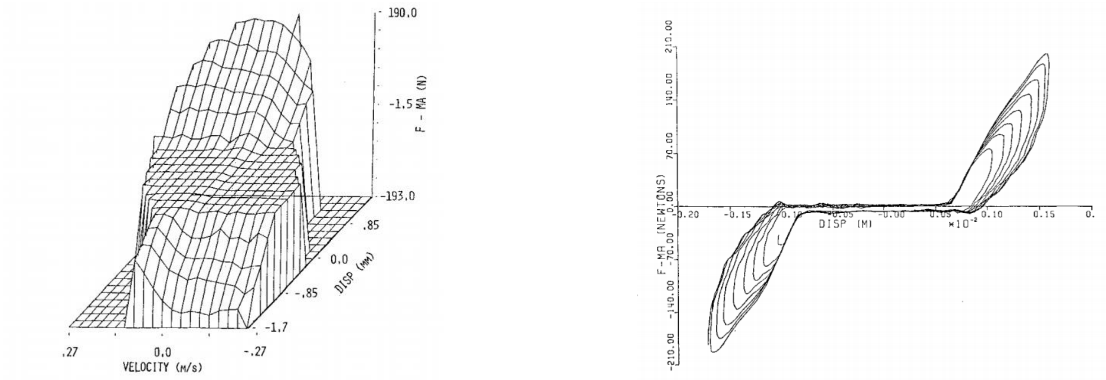
(a) ECL benchmark model



(b) VTT benchmark model

Figure 19 Benchmark model released by European COST F3 program[73]

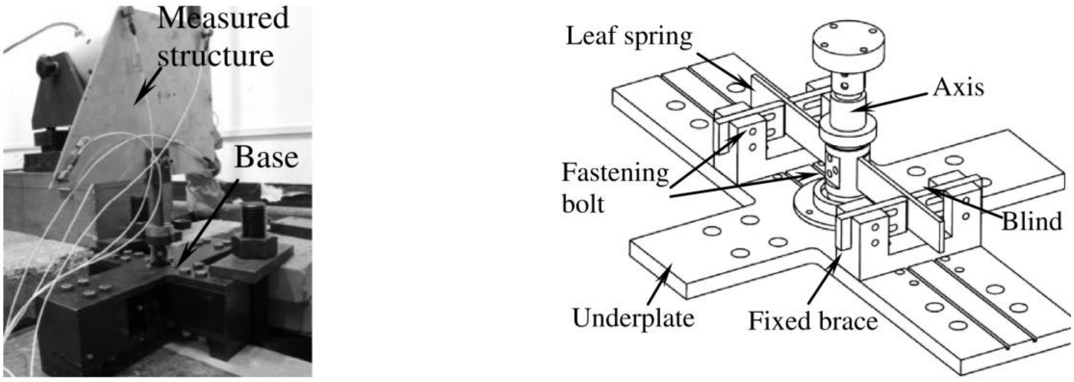
Freeplay nonlinearity, known for its non-smooth characteristics, has not been thoroughly reviewed in contrast to the extensive reviews on general NSI by many researchers. The extended restoring force surface method, which identifies the type of nonlinearity by depicting the restoring force surface, was among the earliest methods applied to freeplay nonlinearity. Crawley and Aubert[74] applied this technique to a single-DOF (SDOF) freeplay system, and the freeplay nonlinearity was effectively illustrated by the restoring force surface as shown in Figure 20. Liu et.al[75]-[77] enhanced this method to make it applicable to multi-DOF (MDOF) nonlinear systems, and they experimentally validated it using the Balachandran cantilever beam model. Noël et al.[78] applied this approach to a SmallSat satellite structure to identify the support clearance between its inertial navigation component and the satellite.



(a) restoring force surface (b) projection of the restoring force surface

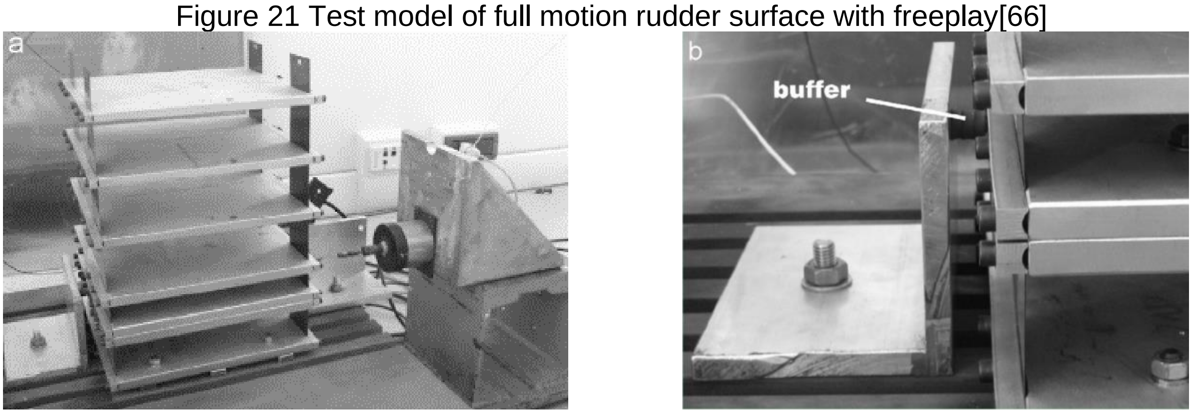
Figure 20 Nonlinear results of freeplay identification using restoring force surface method[74]

The Conditional Reverse Path (CRP) method and the Nonlinear Subspace System Identification (NSSI) method have seen widespread application, and have proven effective in identifying the freeplay. Wu employed the CRP method[66] to identify the rotational freeplay of an all-moving control surface model (see Figure 21) and validated the results experimentally. Despite using a smooth function to approximate the freeplay, the identification results were satisfactory. The time-domain NSSI method has also been applied to a SDOF freeplay system[79] and a five-DOF structure with freeplay (see Figure 22)[80], demonstrating excellent performance in identifying the freeplay in large complex structures (see Figure 23)[81]. Both the freeplay piecewise-linear function model proposed by Marchesiello[79] and the freeplay identification error criterion indicator proposed by Kerschen[81] can directly determine the clearance boundaries. Although the frequency-domain NSSI method's clearance pointer function achieves highly accurate clearance boundaries, it is ineffective for non-central freeplay. To address this, Sun et al.[82] extended the time-domain NSSI method to identify non-central freeplay.



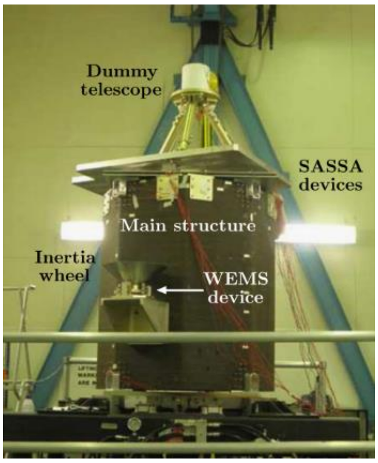
(a) the rudder (b) freeplay device

Figure 21 Test model of full motion rudder surface with freeplay[66]

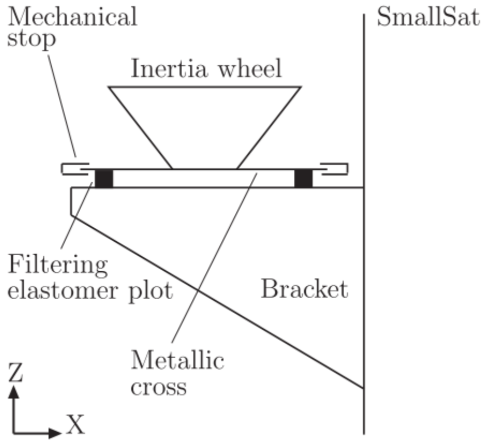


(a) the whole structure (b) freeplay device

Figure 22 5-DOF structure with single side freeplay[80]



(a) SmallSat satellite



(b) freeplay device

Figure 23 SmallSat satellite structure and inertial assembly with freeplay[81]

Freeplay nonlinearity can also be identified through several general mathematical transforms. Feldman[83], [84] proposed the Hilbert identification method and studied a SDOF system with freeplay. Wu[65] extended this method to a MDOF system with freeplay. The Volterra series[85] has also been modified to identify freeplay nonlinearity. The Hammerstein model[86], which is commonly used in control systems, has been introduced into the field of aeroelastic clearance identification. Recently, Han et al.[87] have used neural network methods to identify the nonlinear aeroelastic system with freeplay.

3. Nonlinear Aeroelastic Analysis of Freeplay Nonlinearity

In general, numerous researchers have performed extensive reviews on the structural nonlinear aeroelasticity analysis[38], [88]-[96]. The influence of structural nonlinearity has also been a subject of the special discussion in recent high-quality aeroelasticity textbooks globally[2], [5], [7]-[8]. In 2017, Dimitriadis authored the first foundational book specifically addressing nonlinear aeroelasticity[97].

Lee[90] investigated the nonlinear airfoil and provided a comprehensive review of various methods, emphasizing the phenomena of the Hopf bifurcation and the LCOs caused by cubic stiffness, freeplay, and hysteresis nonlinearity at subsonic speeds. Dowell[91] mainly reviewed structural nonlinear aeroelastic problems in four aeroelastic models: an airfoil with control surface freeplay, a delta wing model, a high-aspect-ratio wing model, and a transonic wing model. An[92] focused on summarizing and forecasting the advancements in CFD/CSD technology. Dowell[93] not only summarized the research on LCOs induced by freeplay, but also systematically reviewed the progress in the reduce order unsteady aerodynamic theory, the transonic flutter, and the lift surface LCOs. Chen[94] concentrated on summarizing semi-analytical and semi-numerical methods in the analysis of nonlinear aeroelastic characteristics of the airfoil. Xiang[95] reviewed the latest research advancements in the nonlinear aeroelastic problems of the airfoil, the high-aspect-ratio wings, and the entire aircraft. Yang[38] described the modeling methods, analysis methods, and the principles of nonlinear aeroelastic stability and the response behavior of the wing with concentrated nonlinearity. Generally, the study of structural nonlinear aeroelastic phenomena can be chiefly approached through methods including the harmonic-balance (HB) method, the numerical integration method, and the numerical continuation techniques, et. al.

3.1 Harmonic-balance Method

The initial studies on structural nonlinear aeroelasticity were conducted by Woolston[98], [99] and Shen[100]. They investigated the plunge-pitch airfoil model (see Figure 13 (a)) and plunge-pitch-control airfoil model (see Figure 13 (b)). In Shen’s work, the unsteady aerodynamic forces were calculated using Theodorsen’s theory[41], which allows for the precise solutions of harmonic motion of the airfoil. Since Theodorsen’s theory operates in the frequency domain, Shen was the first to apply the describing function method, proposed by Kryloff and Bogoliuboff[101], to the nonlinear aeroelastic research. Based on the assumption of harmonic motion[100], this method subsequently became a primary tool for analyzing periodic oscillations in nonlinear aeroelastic systems.

According to the classification by some researchers in China[38], [94], the describing function method is categorized as a semi-analytical and semi-numerical approach. Other methods in this category include the equivalent linearization method and the harmonic-balance method. The equivalent linearization method, proposed by Liu[102] and later extended by Guo[103] for analyzing nonlinear flutter induced by freeplay, falls into this classification. The HB method is based on the truncated Fourier series expansion of time-domain signals. When the Fourier series retains only the first-order primary harmonic, its mathematical expression matches that of the describing function method[90].

Over the decades, the harmonic-balance method has evolved into various forms, such as higher-order-HB (HOHB) method[104], [105], higher-dimensional HB method[106], and the incremental HB method[107],[108]. Recently, Wang et al.[109] proposed the dual quasi-HB method, which optimizes computational speed and enhances the understanding of the LCO. Moulin[110] developed a time-spectral method based on the harmonic balance principles for solving the strong nonlinear flutter, validating its effectiveness through comparison with the Theodorsen's theory.

3.2 Numerical Integration Method

With the advancements in computer performance, the numerical integration method has become increasingly effective tool for nonlinear dynamic analysis[38]. In nonlinear aeroelastic analysis, the most widely used numerical integration methods include the Runge-Kutta algorithm, the Newmark algorithm, the Wilson-θ algorithm, and the recently developed precise integration method[111]-[113]. For addressing the freeplay nonlinearity, which is characterized by its non-smooth nature, three different approaches are typically employed to ensure adequate solution accuracy: the smooth approximation[114], [115], the event-driven algorithms (such as the fourth-order Runge-Kutta algorithm combined with the Hénon method[116] and the extended Hénon method[117]), and the fixed-step algorithms. The Smooth approximation changes the nature of the freeplay, leading to the inaccuracy on reflecting the true nonlinear vibration phenomena. Event-driven algorithms, while capable of high-precision calculations by iteratively approaching the gap boundary, involve extensive computational effort. By choosing an appropriate step size, the fixed-step algorithm offers the faster computation speed but is less accurate compared to the event-driven algorithm.

3.3 Numerical Continuation Technique

The aforementioned methods have enabled the simulation of many intriguing phenomena in the nonlinear aeroelastic systems with freeplay. Many of these phenomena can be explained using nonlinear bifurcation theory, including Hopf bifurcation and fold bifurcation. However, for more complex phenomena such as quasi-periodic responses, non-periodic responses, and chaotic behaviors, the primary difficulty lies in the inability to solve these using periodic methods. Furthermore as the nonlinear parameters vary, it is even more challenging to investigate the characteristics of the nonlinear system by the numerical integration method.

Recently, researchers have introduced numerical methods for solving periodic solutions in nonlinear dynamics into the study of nonlinear aeroelastic mechanics. Alighanbari and Price[118] were among the first to apply numerical continuation methods to the nonlinear aeroelastic system with freeplay. Using the AUTO[119] software and polynomial approximations, they were able to rapidly and accurately calculate LCOs and their bifurcations in an airfoil. Dimitriadis further advanced the application in aeroelastic analysis, conducting a series of comprehensive studies from the airfoil[120]-[122] to the full 3D aircraft[123]-[125]. The branch tracking capability of numerical continuation provides broad prospects for addressing the nonlinear aeroelastic system. Once the periodic solution is obtained, this method can track the evolution of LCOs as system parameters, such as airspeed or dynamic pressure, change.

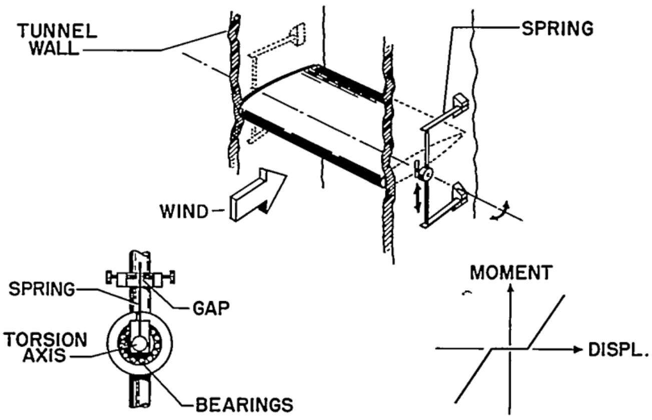
In recent years, the numerical continuation method has seen widespread attention and application. Currently, there are many well-developed commercial software packages based on these methods, such as AUTO[119], MatCont[126], and CoCo. Wang et al.[127] utilized the concepts of numerical continuation to propose a state-space iterative scheme based on time-domain integration, and performed the numerical validation on an airfoil model with freeplay nonlinearity. This provided further mathematical insights into the freeplay-induced LCOs. Yu et al.[128] introduced a variable step-size numerical continuation technique, using the describing function method to address the freeplay and calculating the aerodynamic influence coefficients via the filtered pulse function method. They conducted numerical analysis on an plunge-pitch-control airfoil model with freeplay, significantly improving the speed of analyzing nonlinear phenomena such as LCOs. The results were compared with the classical Theodorsen's theory, confirming their effectiveness. Edward[129] proposed a nonlinear numerical continuation method and applied it to a 51-DOF twin-engine aircraft model with nonlienarity, revealing the potential LCOs.

4. Nonlinear Aeroelastic Wind Tunnel Tests of Freeplay Nonlinearity

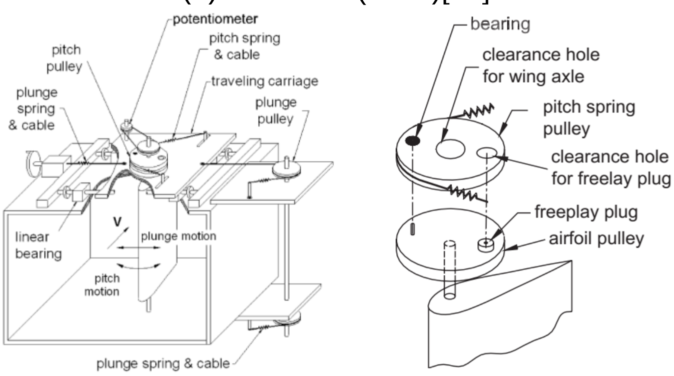
Wind tunnel test is a crucial procedure for identifying freeplay-induced nonlinear aeroelastic phenomena and validating the nonlinear aeroelastic analysis techniques. Researchers have extensively studied the wind tunnel tests of the freeplay nonlinear aeroelastic systems. These studies primarily focus on two types of models: the airfoil test models and the 3D test models. Moreover, the 3D models are further classified based on the freeplay location into the nonlinear test models with the root freeplay (e.g., full-moving control surfaces or T-tails) and the others with the off-root freeplay(e.g., aircraft aileron models).

4.1 2D Airfoil Model

As early as 1955, Woolston[98] (see Figure 24 (a)) designed a 2-DOF airfoil model with pitch freeplay for the wind tunnel tests. This design was both simple and ingenious, and it has been referenced by many later experimental models[130]. However, the model has certain limitations, such as the inability to change the angle of attack (AOA) and the exclusion of control surfaces. In 2005, Marsden[131] (see Figure 24 (b)) used a rope structure to create a more complex 2-DOF airfoil model. This model allows for easy adjustment of the plunge and pitch stiffness.



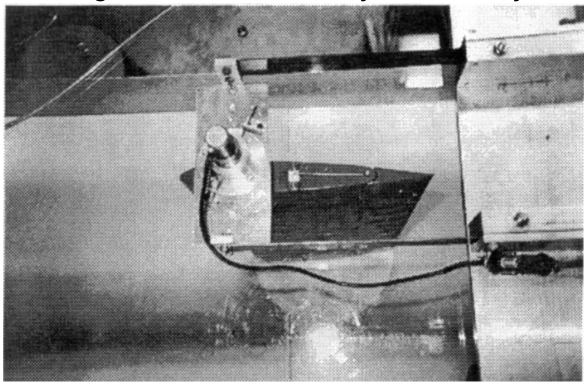
(a) Woolston (1955)[98]



(b) Marsden (2005)[131]

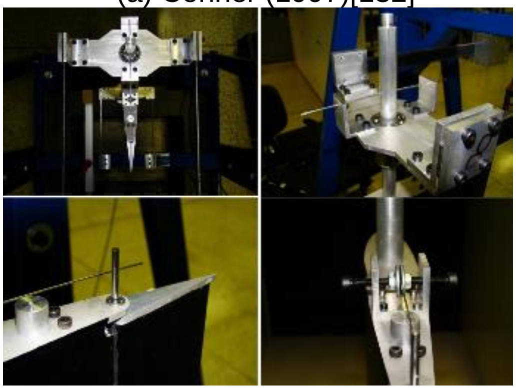
Figure 24 2-DOF airfoil wind tunnel test models

The 3DOF airfoil with the control surface freeplay is the most frequently employed model in wind tunnel tests, as it more accurately represents the cross-sectional profiles of ailerons, elevators, and rudders. The nonlinear characteristics caused by the freeplay in the plunge DOF are less pronounced, while the pitch and control surface DOF are more susceptible to the various LCO phenomena[97]. Conner's model[132] in 1997 (see Figure 25 (a)) has been extensively used to validate theoretical results. Similarly, Kholodar's model[133] in 2014 (see Figure 25 (b)) has been used for a comprehensive study of different methods for predicting LCOs. In 2012, Vasconcellos[134] redesigned the experimental model based on Conner's work (see Figure 25 (c)). During the same period, Tang[135] designed a 3-DOF airfoil model in 2013 for exploring the effects of 4 different sizes control surface freeplay at the larger angles of attack. Later in 2016, Vasconcellos[136] improved his model (see Figure 25 (d)), investigating the impact of the combination of hardening stiffness and freeplay on the aeroelastic properties of the airfoil. Wind tunnel experimental studies of the airfoil have observed a rich variety of LCO phenomena, as well as the different forms of LCO occurring at different wind speed ranges. Additionally, factors, such as the AOA and the freeplay preload, that influencing LCOs have been systematically studied using these models.

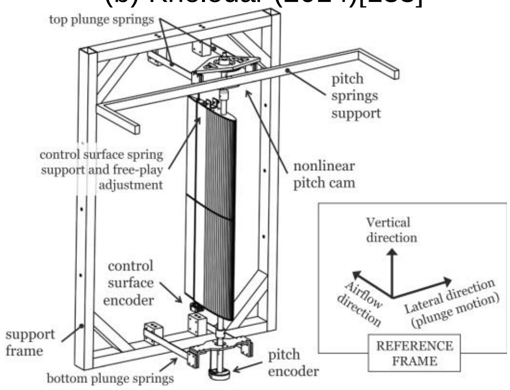


(a) Conner (1997)[132]

(b) Kholodar (2014)[133]



(c) Vasconcellos (2012)[134]



(d) Vasconcellos (2016)[136]

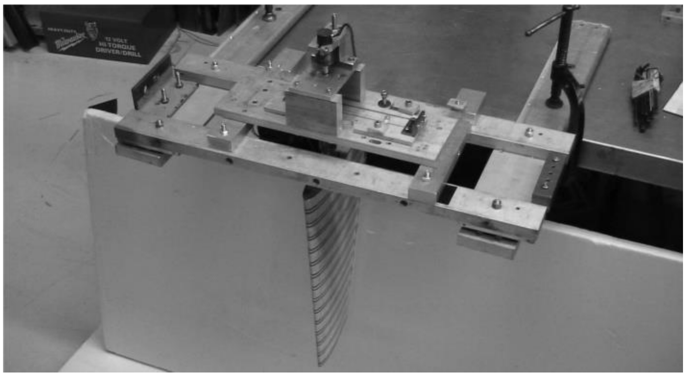
Figure 25 3-DOF airfoil wind tunnel test models

4.2 3D Model with Root Freeplay

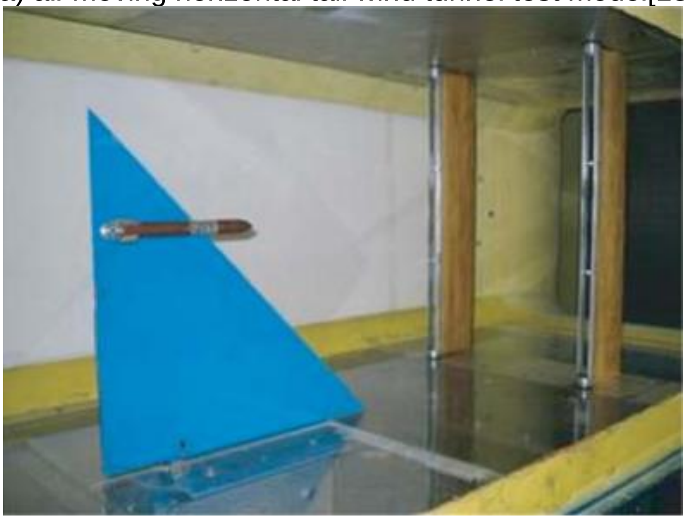
Characterized by fewer DOF and the sufficient structural rigidity, the airfoil wind tunnel test model allows for relatively straightforward experimental implementation. In contrast, designing a 3D model with freeplay nonlinearity is considerably more complex. Firstly, the overall stiffness distribution of the 3D test model must be meticulously planned. Secondly, if the freeplay is not located at the root but needs to be integrated within the model, the internal space constraints make the design of the freeplay device particularly difficult.

A widely studied type of 3D models with freeplay nonlinearity are the all-moving control surface or all-moving horizontal tail model with rotating freeplay. On one hand, new generation military aircraft such as the F22 and F35 use all-moving horizontal tails to enhance maneuverability. On the other hand, for wind tunnel tests, the freeplay device of the all-moving control surface model is located at the root, where the freeplay device can be designed outside the wind tunnel. Additionally, the freeplay implementation mechanism can reference the design of the airfoil wind tunnel test model. Tang and Dowell[137] designed an all-moving horizontal tail wind tunnel test model with freeplay as shown in Figure 26 (a). This model was redesigned based on the all-moving horizontal tail model from the 1954 USAF test report[138] and was manufactured using additive manufacturing technology. Tang and Dowell studied the LCO characteristics of this model, discussed the effects of AOA and gusts. They also focused on analyzing the correlation between theoretical calculations and experiments, achieving excellent results[139]. Chen and Li also designed similar all-moving control surface models. Chen[37] used a traditional beam and wooden wing box structure to design a scaled wind tunnel test model for studying the F-35 horizontal tail freepaly (see Figure 12). Li[140] designed an all-moving control surface model and designed a connecting device that decouples the plunge and pitch stiffness of the control surface.

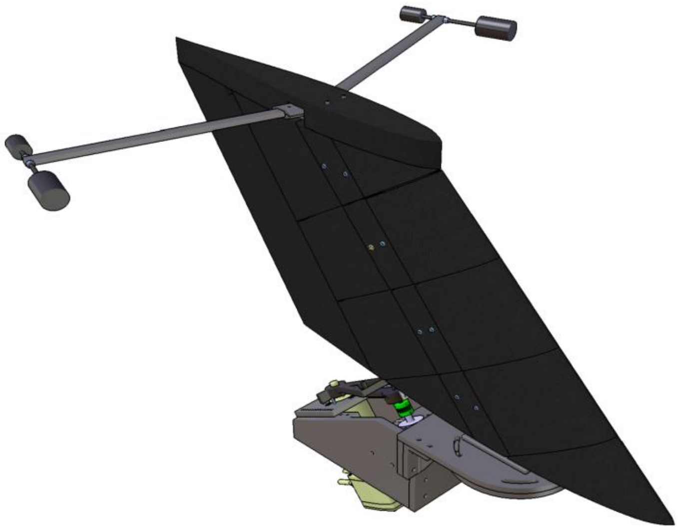
Tang and Dowell[141] performed wind tunnel tests on a delta wing with external stores (see Figure 26 (b)), where they observed LCOs induced by freeplay between the store and the wing pylon. Utilizing the von Karman plate theory and treating the store as a rigid body, they created an aeroelastic model of the overall structure using the CMS method. Fichera[142], [143]developed a T-tail model with adjustable freeplay (see Figure 26 (c)), analyzing the LCOs using numerical integration and the HOHB method. Tang and Dowell[144] introduced a three-segment folding wing wind tunnel test model and conducted tests to explore the flutter characteristics at different folding angles. Yang et al.[64], [145](see Figure 18(b)) also conducted wind tunnel tests on a folding rudder, focusing on its aeroelastic characteristics induced by freeplay. They concluded that the presence of hinge freeplay in the folding hinge increases the flutter boundary. In recent years, with the rise in experimental research on new aeroelastic theories, Tang and Dowell et al.[146] have provided a comprehensive review of wind tunnel test models in nonlinear aeroelastic research.



(a) all-moving horizontal tail wind tunnel test model[139]



(b) delta wing[141]



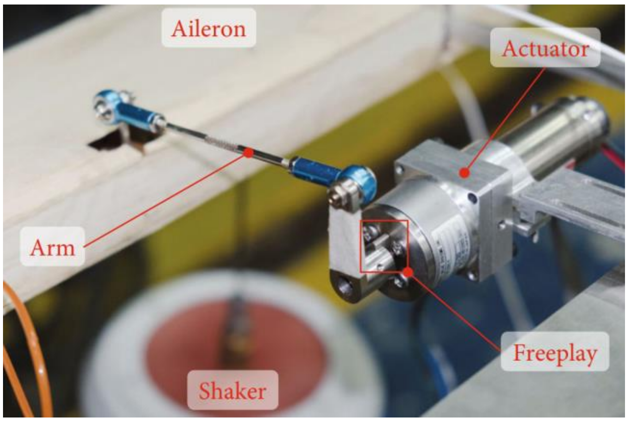
(c) T-tail model[142]

Figure 26 3D nonlinear aeroelastic wind tunnel test model with freeplay

4.3 3D Model with Off-root Freeplay

Wu and collaborators have undertaken comprehensive research on the nonlinear aeroelasticity of aileron rotating freeplay. Wu et al.[11], [109], [147]designed a nonlinear aeroelastic wind tunnel test model, a rectangular wing with aileron rotating freeplay as shown in Figure 27. In wind tunnel tests, they observed LCOs induced by the aileron rotating freeplay. These LCOs occurred at wind speeds notably lower than the linear flutter boundary, gradually approaching the linear flutter boundary and eventually diverging as wind speed increased. Furthermore, Wu et al.[148] developed a nonlinear aeroelastic model of a wing aileron rotating freeplay with the engine, where they observed nonlinear LCO phenomena induced by freeplay during testing.

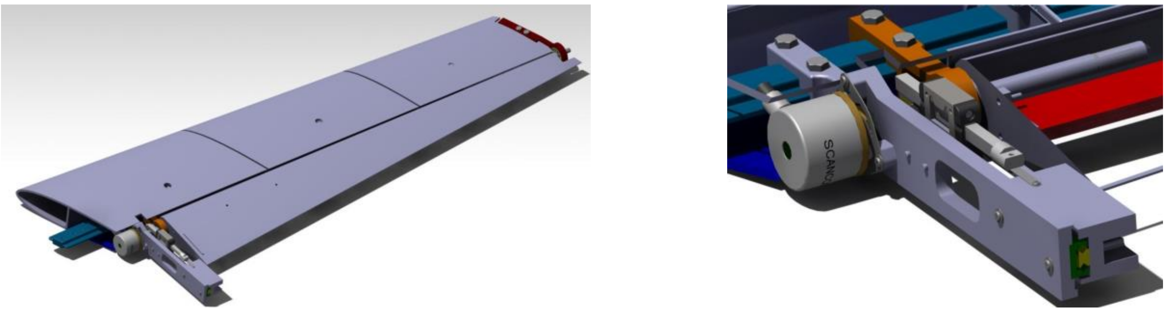
(a) test model



(b) freeplay device

Figure 27 Nonlinear aeroelastic rectangular wing-aileron wind tunnel test model with freeplay[147]

Fonzi et al.[149] conducted comprehensive numerical and experimental research on a T-tail horizontal stabilizer featuring a nonlinear control surface with an adjustable freeplay. They redesigned the T-tail horizontal stabilizer and its control surface based on the X-DIA wind tunnel model at Politecnico di Milano, as depicted in Figure 28, with an adjustable gap size ranging from 0 to 5°. Numerical analysis was performed on the test model utilizing the describing function method and numerical integration, and the results were cross-validated with experimental data. The findings demonstrated that the onset of LCOs is not dependent on the gap size, and increasing the rotational stiffness between the horizontal stabilizer and the control surface does not consistently provide advantages.



(a) CAD model of the horizontal tail plane (b) CAD model of the nonlinearity mechanism

Figure 28 T-tail wind tunnel test model[149]

5. Conclusion

Over several decades of rapid development, scholars worldwide have made substantial progress in the field of nonlinear aeroelasticity analysis of freeplay nonlinearity. Through in-depth studies of the airfoil, numerous mature methods were proposed including the describing function method, the HB method, and the time-domain integration method. Additionally, many new techniques from nonlinear dynamics, such as the HOHB method and numerical continuation method, have been introduced into aeroelastic research. Studies on the nonlinear 2D airfoil have elucidated the mechanisms of various nonlinear dynamic responses, such as periodic LCOs, quasi-periodic LCOs, jumping, and chaotic motion. Researchers like Tang, Dowell, Chen, Cooper, and Dimitriadis have conducted extensive and innovative studies, while other researchers like Wu, Yang, Li, and Han have also shown keen interest in this field. Furthermore, researchers such as Lee and Karpel have performed practical nonlinear aeroelastic analyses on complex 3D wings and full aircraft models. The newly developed analysis methods for airfoils are quickly being applied to engineering practices, and research on the nonlinear aeroelastic analysis of complex structures has also achieved significant results.

For a considerable period in the future, nonlinear aeroelasticity analysis will remain a prominent area of research. Currently, the development of nonlinear aeroelasticity is mainly confined to the analytical realm. While there is abundant research on gap nonlinearity, studies on other nonlinear aspects such as friction and hysteresis are quite limited, and there are few general analytical methods available. Additionally, many techniques from fluid mechanics are still challenging to incorporate into aeroelasticity analysis, and the efficiency of CFD/CSD coupling methods for complex structures is not promising. Therefore, there is still a considerable distance to cover before nonlinear aeroelasticity-based design methods can become powerful tools in aircraft design.

6. Contact Author Email Address

mailto: rrts@163.com

7. Copyright Statement

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