



PURSUIT-BASED LONG-RANGE AIR-TO-AIR MISSILE MIDCOURSE GUIDANCE ROBUST TO CHANGES IN THE PREDICTED IMPACT POINT

Minjae Shin¹, Min-Jea Tahk², Boseok Kim³ & Chang-Hun Lee⁴

¹Aerospace Engineering, KAIST, 34141 Daejeon, Korea, shinmj@kaist.ac.kr

²Aerospace Engineering, KAIST, 34141 Daejeon, Korea, mjtahk@kaist.ac.kr

³Aerospace Engineering, KAIST, 34141 Daejeon, Korea, kbobo6@kaist.ac.kr

⁴Aerospace Engineering, KAIST, 34141 Daejeon, Korea, lckdgns@kaist.ac.kr

Abstract

This paper proposes a midcourse guidance method for long-range air-to-air missiles that is robust to changes in the predicted impact point (PIP) during flight. The guidance method consists of a reference trajectory with an altitude constraint and a pursuit-based guidance that follows the reference trajectory. Simulation comparisons with a suboptimal trajectory that maximizes terminal speed show that the proposed guidance method is significantly robust to changes in the PIP. The intercept performance for various peak altitudes and PIP change magnitudes is also analyzed.

Keywords: midcourse guidance, predicted impact point, augmented pursuit guidance

1. Introduction

To increase the probability of target interception for a long-range air-to-air missile, it is important to reach the predicted impact point (PIP) with a high terminal speed through proper midcourse guidance. Although the concept of PIP has been used in various midcourse guidance methods [1, 2], and trajectory modification methods have been proposed to deal with changing PIPs [3], the robustness of the midcourse trajectories for PIP changes has not been investigated thoroughly in existing studies.

For a given PIP, the optimal trajectory that maximizes terminal speed typically takes the form of initial climb to a high altitude to minimize drag, followed by a pull-down maneuver to reduce altitude to reach the PIP. However, as the air density decreases at high altitudes and the total angle of attack increases due to the pull-down maneuver, there is less room left for additional maneuvers. Therefore, if the PIP changes are due to evasive maneuvers of the target during midcourse guidance, the total angle of attack can be easily saturated when the guidance command is modified to reach the new PIP, resulting in a course correction failure.

One possible solution to this problem is to optimize the trajectory considering the uncertainty in the PIP due to the target's evasive maneuvers. However, this approach has the disadvantage that its real-time application requires a considerable increase in computation time. Therefore, we suggest an alternative solution, which is to apply a constraint on the peak altitude of the missile's trajectory. An appropriate altitude limit can be calculated off-line to avoid an excessive onboard computation time. In order to derive an appropriate altitude constraint, it is necessary to conduct extensive simulation studies with various altitude constraints and PIP changes, considering the characteristics of the target's evasive maneuvers.

In this paper, Augmented Pursuit Guidance (APG) proposed in [4, 5] is adopted to impose the altitude constraint on the trajectory. Then, numerical simulations are conducted to examine the robustness of the trajectory to PIP change for various altitude constraints.

In Section 2, the formulation of the midcourse guidance problem is presented. Section 3 briefly describes the guidance law. Section 4 presents the simulation results showing the missile's response to the PIP changes. Concluding remarks are given in Section 5.

2. Problem Formulation

In this paper, the 3-DOF dynamics of a missile with varying speed is considered. The equations of motion of the missile are as follows:

$$\begin{aligned}
 \dot{x} &= V \cos \gamma \cos \psi \\
 \dot{y} &= V \cos \gamma \sin \psi \\
 \dot{z} &= -V \sin \gamma \\
 \dot{V} &= \frac{T \cos \alpha_T - D}{m} - g \sin \gamma \\
 \dot{\gamma} &= \frac{1}{V} \left(\frac{T \sin \alpha_T + L_T}{m} \cos \phi - g \cos \gamma \right) \\
 \dot{\psi} &= \frac{T \sin \alpha_T + L_T}{mV \cos \gamma} \sin \phi
 \end{aligned} \tag{1}$$

where x represents the downrange distance traveled, y is the crossrange, z is the negative value of the altitude, V is the velocity, γ is the flight-path angle, and ψ is the heading angle. The profiles of thrust T and mass m are given as functions of time. The total angle of attack α_T and the aerodynamic roll angle ϕ are the control inputs. The total lift L_T and drag D are expressed as

$$L_T = \bar{q} S C_{L_\alpha} \alpha_T \tag{2}$$

$$D = \bar{q} S (C_{D_0} + k(C_{L_\alpha} \alpha_T)^2) \tag{3}$$

where $\bar{q}$ is the dynamic pressure and S is the reference area of the missile.

The PIP is assumed to be stationary until it is modified any time during the engagement. The objective of the missile is to reach the modified PIP with as high velocity as possible.

3. Guidance Law

The guidance law considered in this study consists of the pursuit guidance (PG) for lateral guidance and the augmented pursuit guidance (APG) proposed in [4, 5] for longitudinal guidance.

3.1 Pursuit Guidance

PG is a guidance law which directs the velocity vector of the missile toward the target. There are two types of pursuit guidance: velocity pursuit using feedback of the lead angle and attitude pursuit using feedback of the look angle. Only velocity pursuit is considered in this study. Velocity pursuit is expressed as

$$a_{com} = -NV\lambda \tag{4}$$

where a_{com} is the normal acceleration command, N is the guidance gain, V is the missile's speed, and λ is the lead angle.

3.2 Augmented Pursuit Guidance

APG is a simple modification to PG, expressed as

$$a_{com} = NV(\lambda_{com} - \lambda) \tag{5}$$

where λ_{com} is the lead-angle command.

APG can be used to track a reference trajectory if the altitude of the reference trajectory is given as a polynomial of the range to go. Let ξ and h denote the range to go and the altitude, respectively. If h is given by

$$h(\xi) = a_0 + a_1\xi + a_2\xi^2 + \cdots + a_n\xi^n, \tag{6}$$

then the lead-angle command λ_{com} of APG to follow the trajectory is approximated as follows [4]:

$$\lambda_{\text{com}} = -[a_2\xi + 2a_3\xi^2 + \dots + (n-1)a_n\xi^{n-1}]. \quad (7)$$

As shown in [5], APG with an appropriately chosen quadratic altitude function provides a suboptimal trajectory that closely matches the optimal trajectory maximizing the terminal speed. However, such a trajectory is vulnerable to PIP changes, due to its high peak altitude which limits the maneuverability of the missile.

Therefore, we propose a reference trajectory with a limited peak altitude to provide robustness to changes in the PIP. The altitude profile of the trajectory is given by a 4th-order polynomial of range to go, and the coefficients of the polynomial are determined by specifying the initial lead-angle bias and the desired altitudes at two intermediate points.

4. Numerical Simulations

In this section, numerical simulations are conducted to investigate the performance and robustness of various reference trajectories.

4.1 Reference Trajectories

Three trajectories with different peak altitudes are considered in the simulation studies. The three trajectories share the same engagement scenario, where the missile is launched at the altitude of 10 km, and the initial coordinates of the PIP are given by [100, 0, -10] km. The details of the three reference trajectories are summarized in Table 1. Trajectory 1 has the form of a quadratic polynomial, and the lead-angle bias of APG is given as a linear function of the range to go. The coefficients are chosen so that the terminal speed of the trajectory is maximized. This trajectory has the highest peak altitude among the three. Trajectories 2 and 3 are in the form of 4th-order polynomials, with the peak altitudes of 30 km and 25 km, respectively. The shapes of the reference trajectories are shown in Fig. 1.

Table 1 – Reference trajectories.

Trajectory No.	Reference trajectory shape	Peak altitude
1	Quadratic polynomial	33 km
2	4th-order polynomial	30 km
3	4th-order polynomial	25 km

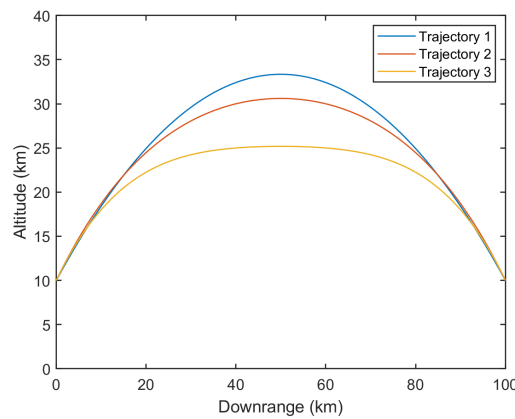


Figure 1 – Shapes of the reference trajectories.

4.2 Nominal Engagement with No PIP Change

Simulations are conducted to analyze the nominal performance of each reference trajectory when there is no PIP change. The initial flight-path angle is set to zero, and APG with the gain of 2 is used for longitudinal guidance. The total angle of attack of the missile is limited to 20 degrees. Fig.

2 illustrates the simulation results. The terminal speed of Trajectory 1 is the fastest with 1045.8 m/s, followed by Trajectory 2 with 1015.2 m/s, and Trajectory 3 with 918.5 m/s. Therefore, Trajectory 1 provides the best chance for intercepting the target if the PIP does not change. However, the angle of attack for Trajectory 1 is saturated for a large portion of flight, starting at the downrange of around 30 km and ending only at 95 km. On the other hand, Trajectories 2 and 3 use much smaller angles of attack overall compared to Trajectory 1, and angle of attack saturation occurs only in the short initial portion of flight.

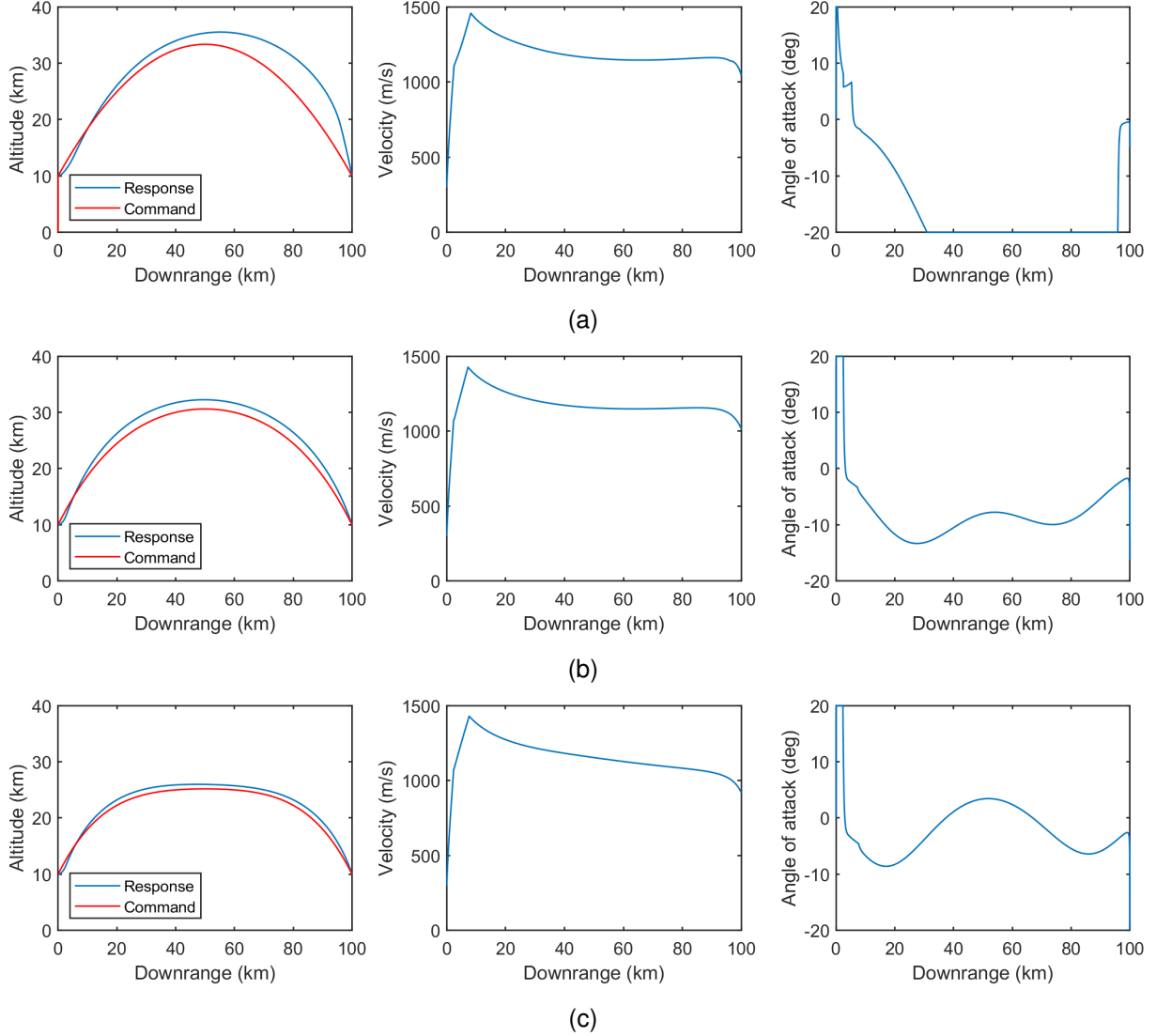


Figure 2 – Simulation results for each reference trajectory with no PIP change.
(a) Trajectory 1. (b) Trajectory 2. (c) Trajectory 3.

4.3 Robustness Analysis for PIP Changes

In this subsection, robustness of the three reference trajectories for various PIP change scenarios is analyzed. The initial conditions for the simulations are the same as in the previous subsection, but now the y-coordinate of the PIP is modified once during the engagement. In addition to APG for longitudinal guidance, PG with the gain of 1 is applied for lateral guidance. The missile downrange at the time of PIP change denoted as X_{CHG} and the magnitude of PIP change denoted as ΔY_{PIP} are variable parameters in the simulations.

Fig. 3 shows the effect of X_{CHG} on the terminal speed for ΔY_{PIP} values of 10 and 20 km. The cases where the terminal position error is larger than 1 km are considered as intercept failures and not shown in the figure. Trajectory 1 fails to intercept the target for the X_{CHG} range of 22–45 km

when $\Delta Y_{PIP} = 10$ km, and 18–89 km when $\Delta Y_{PIP} = 20$ km. On the other hand, Trajectories 2 and 3 successfully reaches the PIP for all cases. This clearly shows the robustness of the 4th-order trajectories over the quadratic trajectory. For all cases, Trajectory 2 has a larger terminal speed than Trajectory 3. This is because Trajectory 3 has a lower peak altitude, and therefore the missile should travel through the higher-density part of the atmosphere. Considering these factors, it is evident that Trajectory 2 is the best among the three trajectories in terms of performance and robustness for the given engagement scenarios. One interesting thing to note is that as the value of X_{CHG} increases, the course correction becomes more and more costly in terms of terminal speed.

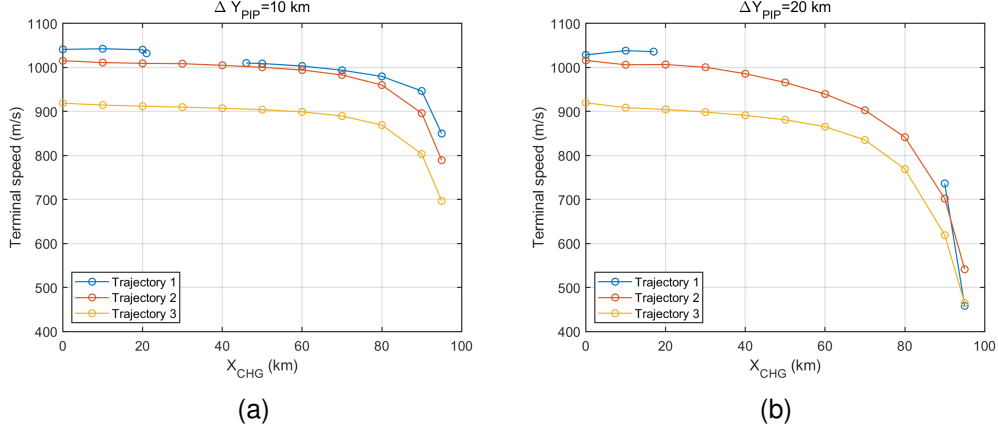


Figure 3 – Terminal speed for various X_{CHG} values. (a) $\Delta Y_{PIP} = 10$ km. (b) $\Delta Y_{PIP} = 20$ km.

Fig. 4 illustrates terminal altitude and crossrange errors of Trajectory 1 for various values of X_{CHG} . It can be seen that a larger value of ΔY_{PIP} causes not only a wider area of intercept failure but also larger terminal position errors. At an early stage of flight, the position error due to the PIP change is predominantly in the altitude, whereas a late PIP change causes error mostly in the crossrange direction.

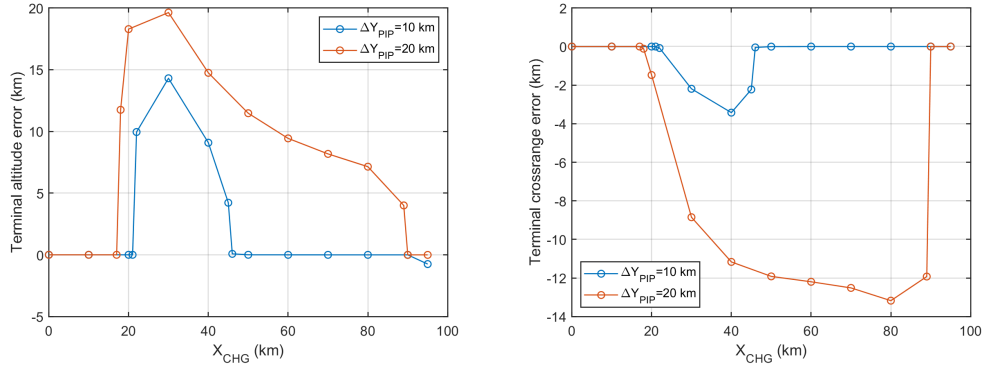
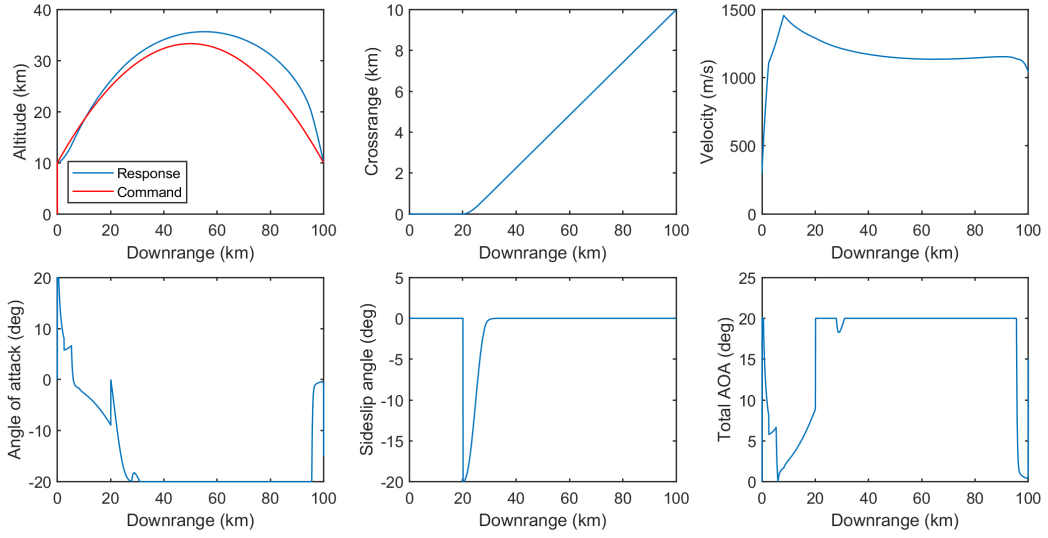
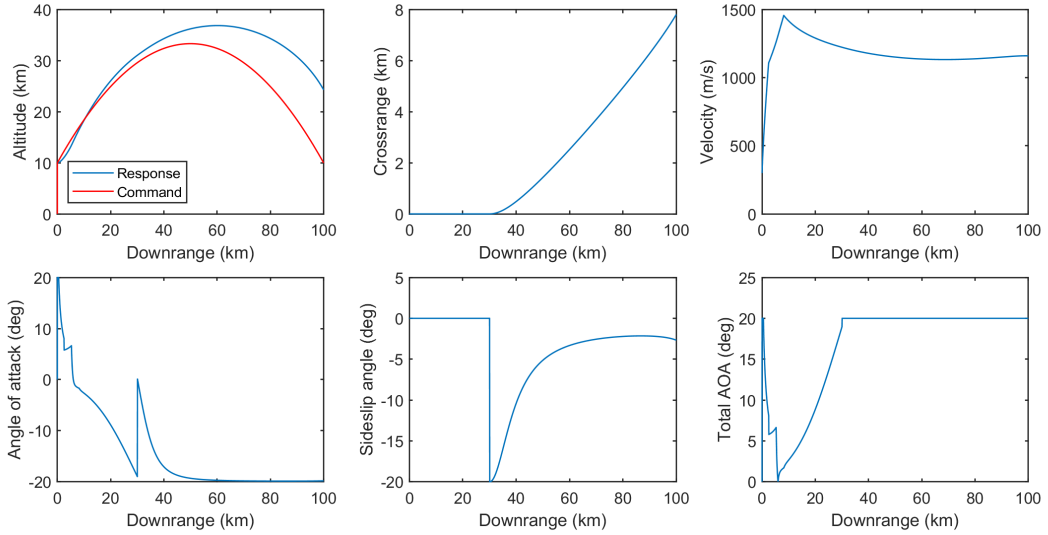


Figure 4 – Terminal altitude and crossrange errors of Trajectory 1.

To analyze the cause of intercept failure of Trajectory 1, the simulation results for $X_{CHG} = 20$ km, $\Delta Y_{PIP} = 10$ km and $X_{CHG} = 30$ km, $\Delta Y_{PIP} = 10$ km are compared. As shown in Fig. 5, if $X_{CHG} = 20$ km, the missile is able to change the heading angle toward the new PIP in the early stage and spend the control energy for longitudinal maneuver. The missile escapes from the angle of attack saturation near the end of the flight and reach the PIP. However, for the case of $X_{CHG} = 30$ km shown in Fig. 6, the missile starts the lateral maneuver at an altitude of 30 km, where the atmospheric density is low, so that heading correction is not complete and altitude control is not successful, either. Note that the total angle of attack is saturated until the end of the engagement for $X_{CHG} = 30$ km. These two scenarios reveal that the intercept failures are due to insufficient maneuverability at high altitudes.

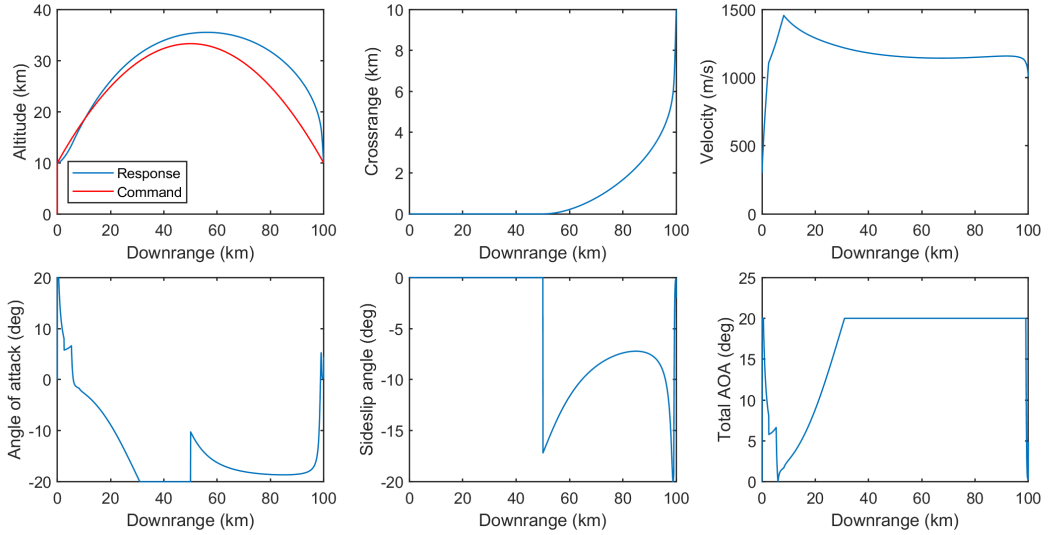
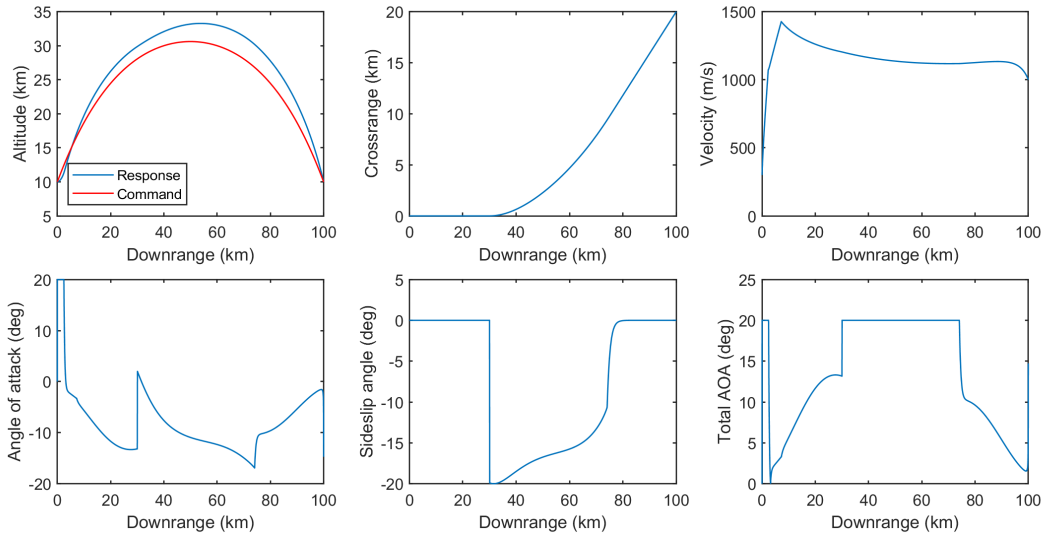
It is interesting that a successful intercept is possible if X_{CHG} is delayed to 50 km or later. As shown in Fig. 7, the missile is focused on altitude control until the downrange of 50 km and the angle


 Figure 5 – Simulation results of Trajectory 1 with $\Delta Y_{PIP} = 10\text{km}$ and $X_{CHG} = 20\text{ km}$.

 Figure 6 – Simulation results of Trajectory 1 with $\Delta Y_{PIP} = 10\text{km}$ and $X_{CHG} = 30\text{ km}$.

of attack rapidly decreases after the downrange of 95 km. Hence, the missile is able to produce large side forces to correct the heading angle in the last minutes. In this case the terminal heading angle increases to 79.2 deg while it is only 7.4 deg for $X_{CHG} = 20\text{ km}$. The simulation results for larger ΔY_{PIP} values show the same characteristics that we observe for $\Delta Y_{PIP} = 10\text{km}$, but it is more difficult to intercept the PIP successfully.

The merit of an 4th-order trajectory is that the midcourse guidance performance is insensitive to X_{CHG} . Since its peak altitude is constrained, the missile can conduct necessary maneuvers for longitudinal direction and lateral direction simultaneously. The PIP is successfully intercepted for all X_{CHG} values when Trajectory 2 or 3 is used. Fig. 8 shows that the angle of attack is not saturated along Trajectory 2. Hence, sufficient heading correction for PIP changes can be executed any time during the flight, and a much larger magnitude of PIP changes can be handled.

The results of the robustness analysis show that the altitude of the missile's trajectory is the main factor that affects the robustness of the trajectory to PIP changes. However, the average altitude is also highly correlated with the terminal speed of the missile. Therefore, the altitude constraint should be determined so that the missile can respond to PIP changes at any point on the trajectory, while minimizing the terminal speed loss due to drag. Therefore, a 4th-order trajectory is an excellent option for midcourse guidance of long-range air-to-air missiles in terms of the robustness to PIP changes


 Figure 7 – Simulation results of Trajectory 1 with $\Delta Y_{PIP} = 10\text{km}$ and $X_{CHG} = 50\text{ km}$.

 Figure 8 – Simulation results of Trajectory 2 with $\Delta Y_{PIP} = 20\text{km}$ and $X_{CHG} = 30\text{ km}$.

and the terminal speed.

5. Concluding Remarks

In this paper, a midcourse guidance law robust to Predicted Intercept Point (PIP) variations based on Augmented Proportional Guidance (APG) is proposed, and the differences in the robustness to PIP variations with respect to the flight trajectory profile are analyzed. According to the robustness analysis results, the optimal trajectory that maximizes the terminal speed exhibits a high risk of angle-of-attack saturation due to low atmospheric density when PIP changes in the middle of flight. This saturation leads to the inability to modify the trajectory, increasing the likelihood of intercept failure. On the other hand, the 4th-order reference trajectory, which limits the peak altitude, demonstrates robustness to PIP variations by ensuring sufficient atmospheric density. However, excessively restricting the peak altitude increases drag, resulting in a significant loss of terminal speed. Therefore, a trade-off between terminal speed and robustness to PIP variations must be made when choosing the flight altitude constraint. For the long-range engagement scenario considered in this paper, there should be an upper limit on the peak altitude to ensure successful interception since the timing of PIP changes is unknown. Future research should extend the robustness analysis to various engagement scenarios, considering not only lateral PIP variations but also variations in altitude and downrange.

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7. Contact Author Phone Number

+82-10-3404-3718

8. Contact Author Email Address

mjtahk@kaist.ac.kr

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