PAIRWISE SWAPPING SEQUENCE OPTIMIZATION BY METROPOLIS-HASTING ALGORITHM WITH QUANTUM ANNEALING FOR AIR TRAFFIC CONTROL

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Abstract

The sequence optimization method with quantum annealing sampler-based Metropolis-Hasting algorithm is investigated. Quantum annealing is power full tools to find the optimal solution of combinatorial optimization problem like aircraft sequencing problems. However in actual large airport, the number of aircraft to be considered is too large to compute with the quantum annealer, and it leads constraints violating solutions. To avoid this issue, pairwise swapping sequence optimization on Markov Chain Monte Carlo algorithm-based evolutionaly computation is proposed. This method can solve the sequencing problem with more than 100 aircraft and multiple runway assignment problem in 10 sec. This paper shows that proposed quantum algorithm show 5 times faster than classical algorithm.

Keywords: air traffic management, optimization, quantum annealing

1. Introduction

The air traffic demand is expected to grow. Expanding the capacity of airport is a crucial challenges in the field of air transport management (ATM) system. The optimization of the aircraft sequencing to improve the runway throughput is one of the efficient way to increase the airport capacity. Several methods has been proposed [1, 2]. However actual implementation to the airport of the large cities is difficult because of the computation cost.

Traditional approach to this optimization problem have typically involved formulations as Mixed Integer Programs (MIP). But as the number of targeted aircraft increases, the computation time required for these algorithms escalates exponentially. This has led researchers to explore alternative strategies, such as evolutionary computation methods. Genetic Algorithms (GA) [3, 4], and Simulated Annealing (SA) [5] are some of the evolutionary computational techniques that have been proposed. However, these approaches still face the challenge of inefficient random sampling due to the large solution spaces and a tendency to fall into local minimum with local search strategies.

In these days the Quantum Annealing (QA) has attracted much attention as high speed and efficient solver for combinatorial optimization problems. Studies in various traffic control and management systems have indicated potential benefits of QA, e.g. Automated Guided Vehicle (AGV) navigation [6], road signal control [7], and air traffic control [8]. However effective formulation to obtain available solution remains a challenge. Constraints are added to the cost function as penalty term and it leads that the contribution of the cost function for the sample probability get small. Furthermore constraint violating solution can be yield and such solution is infeasible, which are unacceptable in the ATM system. A mechanism that consistently outputs feasible solutions is thus imperative.

In this study, the quantum-classical hybrid algorithm for aircraft sequencing problem is investigated. This method is based on the Metropolis-Hastings algorithm, which is one of the Markov Chain Monte Carlo (MCMC) algorithm. The candidate solutions in MH is sampled with QA sampler to improve the efficiency of exploration. For QA sampling the sequence optimization problem is redefined from total sequence optimization to pairwise swapping problem to improve the feasibility of sampled solution.

This reformulation focuses on localized changes within the sequence, which is expected to enhance the computational efficiency and feasibility of finding optimal solutions within the vast solution space of the runway assignment problem.

2. Preliminary

2.1 Aircraft Sequencing Problem

In this study, the air traffic management to optimize runway throughput is considered. This problem is formulated as the aircraft sequencing problem (ASP) around the airport. The ASP is formulated in a number of studies. Here the mixed integer programming (MIP) formulation called the runway capacity management (RCM) is shown as the followings:

$$\text{Minimize} \sum_{f \in F} t_f - \underline{t}_f \tag{1a}$$

subject to

$$\sum_{f \in F} z_{rf} = 1, \quad \forall r \in R, \tag{1b}$$

$$\underline{t}_f \leq t_f \leq \bar{t}_f, \quad \forall f \in F, \tag{1c}$$

$$t_{f_2} \geq t_{f_1} + S_{f_1 f_2} - M(1 - y_{f_1 f_2}), \quad \forall f_1 \neq f_2 \in F, \tag{1d}$$

$$y_{f_1 f_2} + y_{f_2 f_1} \geq z_{rf_1} + z_{rf_2} - 1, \quad \forall r \in R, \forall f_1 < f_2 \in F, \tag{1e}$$

$$y_{f_1 f_2}, z_{rf} \in \mathbb{B}, \tag{1f}$$

where F is the flight set to be assigned runways, R is the available runway set, t_f is the decision variables indicating the assigned time of aircraft f, and $\underline{t}_f$ and $\bar{t}_f$ are the ready time and the due time for flight f which is a time window to be assigned, $y_{f_1 f_2} \in \mathbb{B}$ is a flag to represent flight f_1 leads flight f_2 then $y_{f_1 f_2} = 1$, and $z_{rf} \in \mathbb{B}$ is a flag to indicate that the flight f is assigned at the runway r. Eq.(1a) is the objective function representing the sum of delays to be minimized, Eq.(1b) is constraint to ensure that each aircraft is assigned to the only one runway at only once between its time window Eq.(1c) with the minimum separation from leading aircraft Eq.(1d), and 1e and 1f the flags to distinguish the aircraft pair is assigned into same runway or not.

2.2 Metropolis-Hastings method

Metropolis-Hastings (MH) method is a one of the Markov Chain Monte Carlo (MCMC) method used for sampling from complex and high-dimensional probability distribution. In the MH mehod used to propose the new candidate from the probability distribution characterized with the cost function of optimization problem as the following:

$$Q(x'|x) = \exp\left(-\beta E_s(x', x)\right) / Z(\beta), \tag{2}$$

where the $Q(x'|x)$ is a proposal function as a sample probability of x' when the current candidate is x, $\beta \in > 0$ is thermodynamic beta which is one of hyper parameter to tune the sensitivity for cost value, E_s is a cost function of transition from x to x', and $Z(\beta) = \sum_x$ is the partition function. The new sampled candidate are evaluate whether it follows the desired probability density $P(x)$. This probability function is described by using the cost function $E(x)$ of optimization problem as:

$$P(x) = \exp(-\beta E(x)) \tag{3}$$

And in the MH algorithm, the new sampled candidate is accepted with the following ratio:

$$A(x', x) = \min\left(1, \frac{P(x')}{P(x)} \frac{Q(x|x')}{Q(x'|x)}\right). \tag{4}$$

Therefore the partition function is not required to compute explicitly because it cancels out. As the propose distribution Following this manner, the lowest candidate have largest probability to be solution.

2.3 Quantum Annealing

Quantum Annealing (QA) is an innovative approach for solving optimization problems, leveraging quantum mechanics principles. It encodes the problem into a quantum system, where the optimal solution is the system's lowest energy state. Unlike traditional methods, QA exploits quantum tunneling to escape local minima, potentially outperforming classical algorithms. By gradually adjusting the system's parameters, QA guides the quantum bits (qubits) towards this optimal state. This makes QA a promising tool for complex optimization problems that are challenging for classical algorithms, offering a new frontier in optimization techniques.

3. Methods

The candidate sampling algorithm of aircraft sequencing problem with the quantum annealing (QA) is shown. The random sampling is quite inefficient in aspect of considering the combinatorial nature of problem. The local search algorithm proposes the candidate which is randomly altering the current solution. This candidate is sampled from neighborhood of current solution. Therefore, it allows for gradual improvements through the probabilistic acceptance of the MH algorithm. However, because of sampling from neighborhood of current solution, the local search algorithm is prone to getting trapped in local minimum. A method that samples from a region that is neither random nor too close to the neighborhood is required to optimize efficiently. Therefore, the annealing formulation based on ASP with the current solution to sample non-random and non-too-close candidate is proposed.

3.1 Converting into pair swapping problem

Total queue optimization with QA is proposed in [9]. There are several efforts to calculate rapidly with QA, but it still takes high computation cost to adopt real time operation. Here the total sequence optimization problem is converted into the pair swapping problem. In this pair swapping problem, all aircraft is paired, and the decision is that the paired aircraft is swapped or not. The pairs are assigned between adjacent aircraft in the current solution. Because significant changes of the sequence require the large operation cost in ASP. Therefore, swapping neighborhood is not overly severe relaxation. The method focuses solely on determining the sequence, subsequently calculating the actual assignment times heuristically based on the sequence. In the implementation, the sum of separation time is adopted as the cost function of the annealing as followings:

$$
\begin{aligned}
E_{qa}(x) &= \sum_{k\in K} E_{in}(k) + \sum_{j\in K}\sum_{k\in K^0/\{j\}} E_{ex}(j,k) \qquad (5)\\
E_{in}(k) &= S_{k_1k_2}(1-x_k) + S_{k_2k_1}x_k\\
E_{ex}(j,k) &= S_{j_2k_1}(1-x_j)(1-x_k) + S_{j_2k_2}(1-x_j)x_k + S_{j_1k_1}x_j(1-x_k) + S_{j_1k_2}x_jx_k
\end{aligned}
$$

where K is the set of pairs, aircraft k_1 and $k_2 \in J$ belong to the pair k, K^0 is the subset of pairs exclude the first pair of the sequence, variable x_k is decision for the aircraft pair k is swapped, $x_k = 1$ then swap, $x_k = 0$ then keep the current position.
Using this cost function, the candidates are sampled by the quantum annealer or simulated annealer. And also the MIP solver can be used to get sample.

3.2 Separating the runway assignment decision

In this study, the decision of the runway assignment is sampled through the local search independently of the QA sampling. The runway assignment sampling and the sequence sampling are probabilistically selected based on hyperparameter ρ.

3.3 Removing the time window constraint

The time window constraint is necessary to be considered in the realistic problem setting. To consider the time window, the variables for representing assign time is required. In the QA formulation, variables are binary. Therefore, number of variables are consumed to represent the integer variable for the assign time. Increasing number of variables causes also increasing the number of consumed qubits and it is unsuitable for the rapid iterative operation due to increasing the embedding time. Therefore, the time window constraints are omitted to formulate the cost function of the QA sampling.

And penalties for the violation of the time window constraint are applied to the objective function of cost function of original problem via penalty coefficient, described as bellow:

$$E_p(x) = \sum_{j \in J} v_j, \tag{6}$$

$$v_j = \begin{cases} 1 & \text{if } \underline{t}_j \leq t_j \leq \bar{t}_j \\ 0 & \text{otherwise} \end{cases}, \tag{7}$$

where $v_j \in \mathbb{B}$ is the binary flag indicates assign time of aircraft j violates the time window.
Because of converting problem, the solution represents the sequence of aircraft, not concrete assign times. The assign time is required to calculate from the sequence to evaluate the solution in time space, and described as the following:

$$\begin{cases} t_{f_1} = \underline{t}_{f_1}, & f_1 \text{ is head ofqueue} \\ t_{f_2} = \max\left[\underline{t}_{f_2}, t_{f_1} + S_{f_1 f_2}\right] & \text{otherwise.} \end{cases} \tag{8}$$

The all aircraft assignment time can be calculated recursively.
Then the cost function for the target distribution of MH method is described as:

$$E(x) = \sum_{f \in F} \left(t_f - \underline{t}_f\right) + cE_p(x), \tag{9}$$

where c is a penalty coefficient. Then total MH algorithm with QA for ASP is shown as the follows:

Algorithm 1 MH Algorithm for ASP with QA

Input: N_{iter}: Number of iteration, ρ: switch probability for sequence/runway

$x \leftarrow Initialstate$
for $i = 0$ until N_{iter} **do**
  $u, v \leftarrow$ Generate random uniform $\sim U(0,1)$
  if $u < (1-\rho)$ **then**
    $x' \leftarrow$ Sample candidate via sequence swap $\sim Q(x'|x)$
  else
    $x' \leftarrow$ Sample candidate via runway swap
  end if
  $e' \leftarrow$ Evaluate $E(x')$
  $r \leftarrow$ Calculate acceptance probability $A(x'|x)$
  if $v < r$ **then**
    $x \leftarrow x'$, $e \leftarrow e'$
  end if
end for

4. Numerical Experiments

To show the effecit of proposed algorithm, the numerical experiments are performed with following conditions: the wake turbulence categories of RECAT-1 standard, the airport with 4 runways, and 100 taking-off/landing requests whose interval is Poisson arrival with interval $\lambda = 60$ sec. Cost value decays of Metropolis-Hastings method (blue) and Quantum Metropolis-Hastings (orange) for ASP are shown in Fig.1. Each experiment is performed with 5 seeds. The Solid line corresponds the average, and shaded region is minimum and maximum values, respectively.
The proposed method solved 5 times faster than classical MH algorithm in aspect of calculation time until converging. The 4 final solution of 5 experiment of the quantum MH algorithm is same, and another has 1 pair difference from others. Therefore the proposed algorithm can be said that faster and better convergence. This can be considered from the high efficiency of the quantum annealing search of the combinatorial optimization problem.

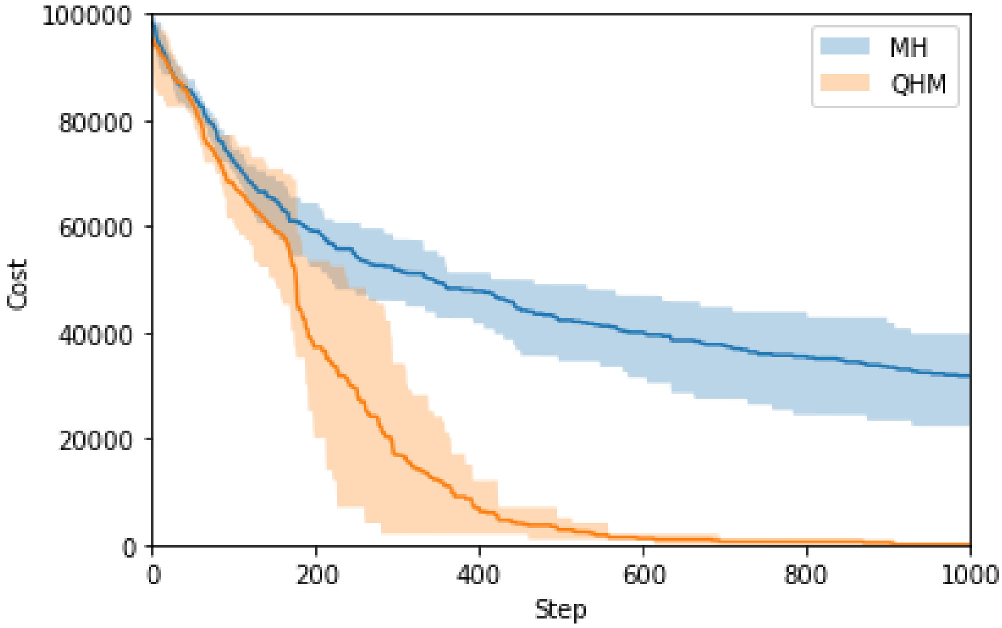

Figure 1 – cost decay

The Computation time of quantum MH algorithm for each number of aircraft is shown in Fig. 2. The computation time is measured with 5 different scenario which has different number of flight from 10 to 100 and 3 seeds for same scenario. The computation time is increased as the number of flight. However scenario with 100 aircraft can be solved in 10 sec.

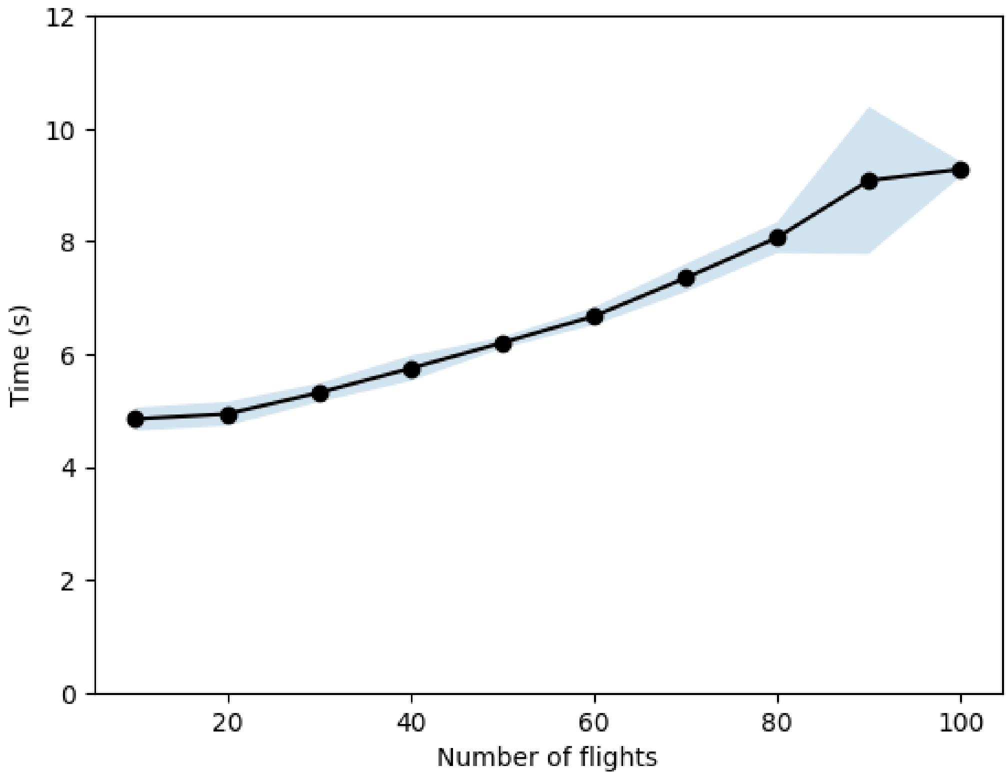

Figure 2 – Computation time

5. Conclusion

In this paper, the quantum annealing-based sampling enhanced Metropolis Hasting algorithm for the aircraft sequencing problem of AMAN/DMAN. This algorithm solved the ASP problem 5 times faster than classical MH algorith, and scenario with 100 aircraft can be solved within 10 sec. The compution time is increased as the number of flight, however it is enought to solve ASP problem at actual airport such like Tokyo Haneda airport.

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7. Copyright Statement

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