



TRAJECTORY OPTIMIZATION FOR LONG-RANGE AIR-LAUNCHED GLIDING VEHICLES USING CONVEX PROGRAMMING

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Abstract

In this paper, trajectory optimization for long-range air-launched gliding vehicles using convex programming is proposed. The nonlinear dynamics is simplified to compact form by utilizing a novel reformulation technique. The flight-path angle and angle of-attack are expressed as functions of altitude and derivative of altitude. This results in a logarithmic velocity dynamics and linear dynamics of altitude and altitude rate where control input is the second derivative of altitude. Then, the optimal control problem is reformulated into second-order cone programming(SOCP) problem by introducing new input variables. An optimal guidance command is generated by successively solving SOCP problem. The effectiveness and performance of the proposed method are demonstrated by numerical simulations.

Keywords: trajectory optimization, convex programming, gliding vehicle, lossless relaxation

1. Introduction

Proportional navigation(PN) guidance and its variants have been successfully applied to many missile guidance problems due to their simplicity and effectiveness. In addition to reaching the targets, PN guidance with bias command can satisfy additional constraints such as impact angle and impact time[1-4]. The closed-form solutions are derived based on the simplified vehicle dynamics. The velocity is assumed to be constant by neglecting the aerodynamic drag, and the dynamics are linearized using small angle approximations. Then, the analytical solution is obtained from Lyapunov function or optimal control theory with simplified dynamic models[5-7]. Many studies have demonstrated that these closed-form guidance laws derived from simplified dynamics can be effectively applied in realistic short-range scenarios where aerodynamic force and gravity are present. However, these simplified models are not reasonable when the vehicles fly relatively long distances. The velocity drastically varies during flight because of aerodynamic drag. The variable velocity and aerodynamic forces result in highly nonlinear dynamics. The aforementioned PN- or Lyapunov-based guidance laws result in substantial kinematic energy loss when applied to long-range scenarios and the guidance performance can be significantly degraded. In worst cases, PN guidance may fail to reach the targets.

It is important to secure sufficient velocity (or kinetic energy) at the terminal homing phase to cope with the target maneuver or increase the effectiveness of the warhead. In order to achieve high velocity at the end, the vehicle should fly on an efficient flight trajectory while minimizing the aerodynamic drag. This problem of maximizing the terminal velocity with nonlinear dynamics can be formulated as an optimal control problem. To handle the highly nonlinear dynamics of long-range engagement scenarios, new guidance methodologies should be developed. As the computational power of embedded systems has increased, trajectory optimization(or direct method) has been studied as one of the computational methods[8]. The original optimal control problems are discretized and converted to the parameter optimization problems. These converted problems are efficiently solved through optimization solvers or algorithms developed in the field of mathematical programming. The calculated trajectory solutions are used as a guidance command. Trajectory optimization can take into account

many realistic constraints as well as nonlinear dynamics without any approximation. The guidance command is given in numerical form rather than in closed or analytic form. Therefore, trajectory optimization should be performed iteratively for each guidance cycle to compensate for disturbances occurring during flight. However, the computation time of general nonlinear programming(NLP) algorithms is not bounded and unpredictable. Furthermore, NLP cannot guarantee convergence in bounded time, which is the main obstacle to the real-time application of trajectory optimization.

As an alternative to general NLP, convex programming can solve convex problems using polynomial-time algorithms. Since convex programming has predictable and bounded calculation time, it can be reliably implemented in real time. Recently, many studies have been conducted to solve aerospace control and guidance problems using convex programming. Convex programming has been applied to planetary entry[9-10], multiagent path planning[11-12], formation flying[13], powered descent landing[14-15], missile guidance[16-18] and so on. In this paper, trajectory optimization for long-range air-launched gliding vehicles using convex programming is proposed. The nonlinear dynamics is simplified to compact form by utilizing a novel reformulation method. The flight-path angle and angle-of-attack are expressed as functions of altitude and derivative of altitude, and substituted in velocity dynamics. This results in compact logarithmic velocity dynamics and linear dynamics of altitude and altitude rate where control input is the second derivative of altitude. By introducing new input variables, the dynamics are converted to input affine form. Then, the original optimal control problem is reformulated into second-order cone programming(SOCP) problem through partial linearization and solved sequentially until convergence. The main contributions of this paper are twofold. First, the highly nonlinear dynamics are converted to compact form using a novel reformulation method. This compact dynamics requires only minimal partial linearization to be transformed into SOCP problem. The main obstacle of sequential convex programming(SCP) is the inconsistency of the linearized dynamics. Excessive linearization causes oscillations in the iterated solution and slow convergence. However, stable and fast convergence of SCP can be achieved using the proposed novel reformulation method. Secondly, robust convergence is accomplished with crude initial guess using the adaptive trust-region method. To alleviate the inconsistency caused by linearized dynamics, the trust region limits the undue variation of state variables from the previous solutions. Instead of utilizing the conventional fixed trust region, the norm of state change is bounded by slack variables. These slack variables are augmented to the objective function with weighting parameters. The adaptive trust region can assist the stable convergence of SCP even with a crude initial guess. Numerical simulation demonstrates the effectiveness and robustness of the proposed method.

The remainder of the paper is organized as follows. In Section 2, vehicle dynamics and constraints are provided to formulate optimal control problem. In section 3, the original optimal control problem is converted to a convex problem by utilizing the reformulation of dynamics and relaxation. Numerical simulations are conducted to demonstrate the performance of the proposed method in section 4. Finally, conclusions are given in Section 5.

2. Problem Formulation

In this section, the optimal control problem for an air-launched gliding vehicle is formulated. First, the dynamics and constraints of glide vehicles are provided. Then, the optimal control problem is formulated by introducing the performance index of velocity maximization.

2.1 Vehicle Dynamics and Constraints

Consider a two-dimensional engagement geometry in the vertical plane as shown in Fig.1, where the gliding vehicle and target are denoted by M and T . Downrange, altitude, velocity, and flight-path angle are denoted by x , h , V , and γ , respectively. Gravitational acceleration and mass are denoted by g and m . Angle-of-attack, which is the control input, is denoted as α . Aerodynamic lift and drag are L and D . The vehicle dynamics are given as

$$\dot{x} = V \cos \gamma \quad (1)$$

$$\dot{h} = V \sin \gamma \quad (2)$$

$$\dot{V} = -\frac{D}{m} - g \sin \gamma \quad (3)$$

$$\dot{\gamma} = \frac{L}{mV} - \frac{g \cos \gamma}{V} \quad (4)$$

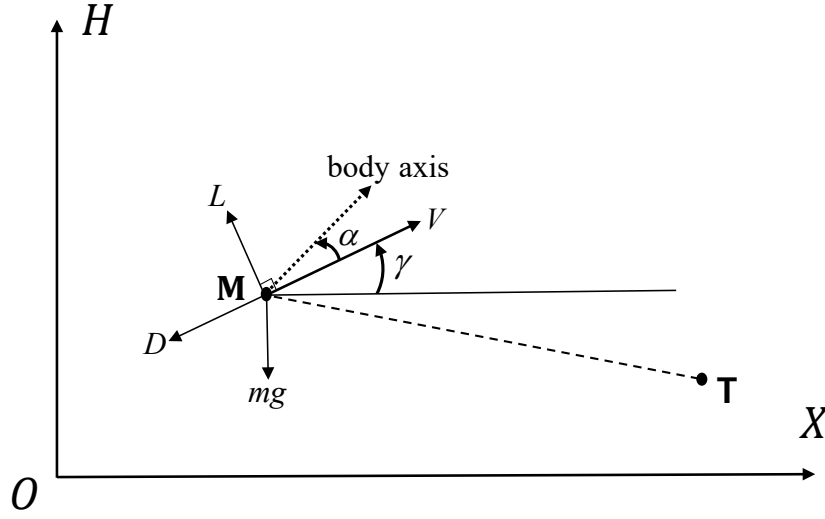


Figure 1 – Engagement geometry in vertical plane

Aerodynamic lift and drag are modeled using drag polar as Eqs. (5) and (6).

$$L = qS_{\text{ref}}C_{L\alpha}\alpha \quad (5)$$

$$D = qS_{\text{ref}}(C_{D0} + KC_{L\alpha}^2\alpha^2) \quad (6)$$

$$\text{where } q = 1/2\rho V^2$$

where $q(= 1/2\rho V^2)$ is dynamic pressure and ρ is density. Density is modeled as an exponential function of altitude as

$$\rho(h) = \rho_0 \exp\left(-\frac{h}{h_s}\right) \quad (7)$$

where h_s is a density parameter that is determined by the standard atmospheric model. Then, initial and terminal conditions are given as follows.

$$x(t_0) = x_0, h(t_0) = h_0, \gamma(t_0) = \gamma_0, V(t_0) = V_0 \quad (8)$$

$$x(t_f) = x_f, h(t_f) = h_f, \gamma(t_f) = \gamma_f \quad (9)$$

The target is assumed to be located at terminal position (x_f, h_f) . The vehicle's terminal flight-path angle is constrained to satisfy the desired impact angle γ_f to enhance the effectiveness of the warhead. The terminal velocity is not constrained since it is used as a performance index.

2.2 Optimal Control Problem

To ensure warhead effectiveness and sufficient kinetic energy, the terminal velocity should be maximized. The performance index for the optimal control problem is set to be

$$J = -V(t_f) \quad (10)$$

By aggregating the vehicle dynamics, constraints, and performance index, the following optimal control problem is formulated.

$$\begin{aligned} \text{Problem A : } & \text{minimize } J = -V(t_f) \\ & \text{subject to } \text{Eqs. (1) – (4), (8), (9)} \end{aligned} \quad (11)$$

It is difficult to directly solve Problem A by indirect methods such as the shooting method since Problem A has highly nonlinear dynamics. By utilizing the reformulation of dynamics and convex relaxation, this problem can be handled in successive convex programming frameworks. The detailed process is explained in the next section.

3. Convex Programming Approach

In this section, the original optimal control problem is converted to a convex programming problem. First, the dynamics is reformulated into compact form using a novel technique. Then, the convex programming problem is derived by utilizing input relaxation and the trust-region method.

3.1 Reformulation of Dynamics

To reformulate the vehicle dynamics, the independent variable is changed to the downrange variable x . If Eqs. (2)- (4) are divided by Eq. (1), the following dynamics is obtained.

$$\frac{dh}{dx} = \tan \gamma \quad (12)$$

$$\frac{dV}{dx} = -\frac{\rho V S_{\text{ref}} (C_{D0} + K C_{L\alpha}^2 \alpha^2)}{2m \cos \gamma} - \frac{g \tan \gamma}{V} \quad (13)$$

$$\frac{d\gamma}{dx} = \frac{\rho S_{\text{ref}} C_{L\alpha} \alpha}{2m \cos \gamma} - \frac{g}{V^2} \quad (14)$$

From Eq. (12), the following relations can be obtained.

$$\frac{d^2 h}{dx^2} = \sec^2 \gamma \frac{d\gamma}{dx} \quad (15)$$

$$\cos \gamma = \frac{1}{\sqrt{\left(\frac{dh}{dx}\right)^2 + 1}} \quad (16)$$

Eq. (7) can be rearranged with respect to angle-of-attack α as follows.

$$\alpha = \frac{2m \cos \gamma}{\rho S_{\text{ref}} C_{L\alpha}} \left(\frac{g}{V^2} + \frac{d^2 h}{dx^2} \cos^2 \gamma \right) \quad (17)$$

By substituting Eqs. (16)- (17) into Eq. (13), the following velocity dynamics is obtained. The derived velocity dynamics is a function of altitude and its derivatives.

$$\begin{aligned} \frac{dV}{dx} = & -k_1 \rho V \sqrt{1 + (h')^2} - k_2 \frac{V}{\rho \sqrt{1 + (h')^2}} \left(\frac{g}{V^2} + \frac{h''}{1 + (h')^2} \right)^2 - \frac{gh'}{V} \\ \text{where } & k_1 = \frac{S_{\text{ref}} C_{D0}}{2m}, k_2 = \frac{2Km}{S_{\text{ref}}} \end{aligned} \quad (18)$$

In Eq. (18), $()'$ denotes the derivative with respect to x . Since the gliding vehicle is boosted after being air-launched, the velocity is significantly high. From this observation, Eq. (18) is approximated by neglecting the gravity effect as

$$\frac{dV}{dx} = -k_1 \rho(h) V \sqrt{1 + (h')^2} - k_2 \frac{V (h'')^2}{\rho(h) (1 + (h')^2)^{5/2}} \quad (19)$$

By dividing both sides of Eq. (19) by velocity V ,

$$\frac{1}{V} \frac{dV}{dx} = \frac{d(\log V)}{dx} = -k_1 \rho(h) \sqrt{1 + (h')^2} - k_2 \frac{(h'')^2}{\rho(h) (1 + (h')^2)^{5/2}} \quad (20)$$

The velocity dynamics can be simplified by introducing the new state variable $P(= \log V)$ and $\omega(= h')$. The vehicle dynamics is reformulated using new states $[P, h, \omega]^T$ as follows.

$$\frac{dP}{dx} = -k_1 \rho(h) \sqrt{1 + \omega^2} - k_2 \frac{(h'')^2}{\rho(h) (1 + \omega^2)^{5/2}} \quad (21)$$

$$\frac{dh}{dx} = \omega \quad (22)$$

$$\frac{d\omega}{dx} = h'' \quad (23)$$

where the second derivative of altitude h'' is regarded as a control input.

3.2 Lossless Relaxation and Linearization

To convert the dynamics of Eqs. (21)- (23) into control affine form, the following new control inputs of u_1 and u_2 are utilized.

$$u_1 = (h'')^2, \quad u_2 = h'' \quad (24)$$

Since u_1 and u_2 are not independent variables, the following additional constraint should be imposed.

$$u_1 = u_2^2 \quad (25)$$

Then, this additional constraint of Eq. (25) is relaxed to convex inequality as

$$u_1 \geq u_2^2 \quad (26)$$

which is a quadratic constraint. This constraint can be handled in quadratically constrained quadratic programming(QCQP) or second-order cone programming(SOCP). Relaxation of Eq. (26) results in a wider feasible solution space than the original constraint of Eq. (25). However, it can be mathematically proven that this relaxation leads to the same optimal solution as the original problem[16-17], which is lossless relaxation. The detailed proof is omitted in this paper. Then, the dynamics of Eqs. (21)- (23) is converted to the following control affine form.

$$\begin{aligned} \frac{d\mathbf{z}}{dx} &= \mathbf{f}(\mathbf{z}) + \mathbf{b}(\mathbf{z})\mathbf{u} \\ \text{where } \mathbf{z} &= [P \ h \ \omega]^T, \ \mathbf{u} = [u_1 \ u_2]^T \\ \mathbf{f}(\mathbf{z}) &= \begin{bmatrix} -k_1 \rho(h) \sqrt{1 + \omega^2} \\ \omega \\ 0 \end{bmatrix}, \ \mathbf{b}(\mathbf{z}) = \begin{bmatrix} \frac{-k_2}{\rho(h)(1 + \omega^2)^{5/2}} & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned} \quad (27)$$

The dynamics of Eq. (27) should be linearized to be applied to a convex programming framework. The linearized dynamics are given as

$$\frac{dz}{dx} = A(\mathbf{z}^k)\mathbf{z} + B(\mathbf{z}^k)\mathbf{u} + C(\mathbf{z}^k)$$

where $A(\mathbf{z}) = \begin{bmatrix} 0 & -k_1 \frac{\partial \rho(h)}{\partial h} \sqrt{1+\omega^2} & -k_1 \frac{\rho(h)\omega}{\sqrt{1+\omega^2}} \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $B(\mathbf{z}) = \begin{bmatrix} \frac{-k_2}{(1+\omega^2)^{5/2}} & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$, $C(\mathbf{z}) = \mathbf{f}(\mathbf{z}) - A(\mathbf{z})\mathbf{z}$ (28)

In Eq. (28), $\mathbf{z}^k$ represents a specific linearization point, which is constant. In successive convex programming framework, $\mathbf{z}^k$ is the solution obtained at the previous iteration. Then, the dynamics in the continuous time domain is transcribed to a finite-dimensional domain. In this study, the pseudo-spectral(PS) method is utilized for transcription of dynamics[20,21]. PS method is one of the direct collocation methods that uses a global interpolation function to approximate states and control. PS method does not exhibit Runge phenomenon problem and shows a quasi-exponential convergence rate. There are three types of collocation points in PS method: Legendre-Gauss-Lobatto(LGL), Legendre-Gauss(LG), and GaussRadau(LGR) point. LGR points are used in this study. A detailed explanation and description can be found in many literatures[22-24]. Only the transcription results for the PS method are presented as follows.

$$\mathbf{D}_i \cdot \mathbf{Z} = \kappa \left[A(\mathbf{z}_i^k)\mathbf{z}_i + B(\mathbf{z}_i^k)\mathbf{u}_i + C(\mathbf{z}_i^k) \right], \quad (i = 1, \dots, N)$$

where $\mathbf{Z} = [\mathbf{z}_0^T \mathbf{z}_1^T \dots \mathbf{z}_N^T]^T$, $\kappa = \frac{x_f - x_0}{2}$ (29)

where $\mathbf{D}_i$ is the differentiation matrix or differential operator derived via Lagrange polynomial and LGR collocation points[25]. N is the number of collocation points. Note that the number of discretized states is $N + 1$ since LGR collocation points include initial points.

The linearized dynamics can cause solution divergence or oscillation in the solution profile because of the inconsistency of dynamics. To handle the inconsistency of dynamics, the following modified trust-region method is utilized.

$$\left[\mathbf{z}_i - \mathbf{z}_i^k \right]^T \left[\mathbf{z}_i - \mathbf{z}_i^k \right] \leq s_i, \quad (i = 1, \dots, N)$$
 (30)

Generally, trust-region with constant bound is used in some studies[9,16]. However, it is not trivial to select the appropriate bound. The problem with improper bound may become infeasible. Instead, the modified trust-region of Eq. (30) utilizes slack variables s_i to prevent excessive change in the current solution from the previous one. The slack variables $s_i (\geq 0)$ are augmented to performance index as

$$\bar{J} = -V(x_f) + w_{\text{trust}} \sum_{i=1}^N s_i$$
 (31)

The weighting parameter w_{trust} should be chosen to be small enough not to affect the original performance index $-V(x_f)$. As the solution converges, slack variables s_i converge to zero. The initial and terminal conditions are redefined by the independent variable of the downrange x and new states.

$$P(x_0) = \log V_0, \quad h(x_0) = h_0, \quad \omega(x_0) = \tan(\gamma_0)$$
 (32)

$$h(x_f) = h_f, \quad \omega(x_f) = \tan(\gamma_f)$$
 (33)

Finally, the second-order cone programming problem can be formulated as follows.

$$\begin{aligned} \text{Problem B : } & \text{minimize} \quad \bar{J} = -V(x_f) + w_{\text{trust}} \sum_{i=1}^N s_i \\ & \text{subject to} \quad \text{Eqs. (29) - (30), (32) - (33)} \end{aligned}$$
 (34)

Problem B is a convex problem that can be efficiently and rapidly solved by an interior-point algorithm[26].

3.3 Successive Convex Programming

An optimal solution to Problem A(the original problem) can be obtained by solving Problem B successively until the solution converges. The termination condition of the successive process is given as

$$\max_i |\mathbf{z}_i^k - \mathbf{z}_i^{k-1}| < \varepsilon_z \text{ or } \frac{|\bar{J}^k - \bar{J}^{k-1}|}{|\bar{J}^{k-1}|} < \varepsilon_J \quad (35)$$

where $\varepsilon_z = [\varepsilon_P \ \varepsilon_h \ \varepsilon_\omega]$

The successive process terminates when all state variables or performance index have not changed significantly from the previous iteration. The tolerance parameters ε_P , ε_h , and ε_ω can be easily selected by observing the physical scale of each state variable. Parameter ε_J is selected as 0.01% in this study. The solution process for successive convex programming can be summarized as follows.

Successive Convex Programming

1. Set initial guess of states $\mathbf{z}_i^0$ ($i = 0, 1, \dots, N$) at every collocation point. And set $k = 1$.
2. For $k \geq 1$, obtain a new solution $\mathbf{z}_i^k, \mathbf{u}_i^k$ ($i = 0, 1, \dots, N$) by solving Problem B with the previous solution $\mathbf{z}_i^{k-1}$ ($i = 0, 1, \dots, N$).
3. Check the termination condition of Eq. (35). If Eq. (35) is satisfied, complete the iterative process and set the optimal solution to $\mathbf{z}_i^* = \mathbf{z}_i^k, \mathbf{u}_i^* = \mathbf{u}_i^k$ ($i = 0, 1, \dots, N$). Otherwise, set $k = k + 1$ and go back to step 2.

States should be initialized to start the successive convex programming process. We used a simple way to initialize the states. P is initialized by a line connecting $\log V_0$ and $(\log V_0)/2$. h is initialized by a line connecting the initial and terminal conditions. ω is set to be zero.

To apply the proposed method as a guidance law, an angle-of-attack command should be generated from the solution of successive convex programming. From Eq. (17), the angle-of-attack command can be generated at each guidance cycle as follows.

$$\alpha_{\text{cmd}} = \alpha_{\text{cvx}} + \alpha_{\text{grav}}$$

where $\alpha_{\text{cvx}} = \eta (u_2^*)_0 \cos^2 \gamma$, $\alpha_{\text{grav}} = \eta \frac{g}{V^2}$

$$\eta = \frac{2m \cos \gamma}{\rho S_{\text{ref}} C_{L\alpha}}$$
(36)

where $(u_2^*)_0$ denotes the optimal solution of u_2 at first collocation point. Since the proposed method ignores the gravitational acceleration when approximating the dynamics in Eq. (19), the bias command α_{grav} is augmented to compensate gravity effect.

4. Numerical Simulations

In this section, the proposed method is applied to the guidance of the long-range air-launched gliding vehicle. The following parameters in Table 1 are used in the simulations. The optimality and effectiveness of the proposed method are demonstrated by the two representative simulations. First, the convergence profile of the proposed method is presented. Second, the simulation results of the proposed method are compared to the solution of a nonlinear trajectory optimization.

Table 1 – Simulation parameters

Parameter	Value
Surface density, ρ_0	1.225 kg/m ³
Density parameter, h_s	8420
Zero-lift drag coefficient, C_{D0}	0.25
Induced drag factor, K	0.06
Lift coefficient slope, $C_{L\alpha}$	15
Reference area, S_{ref}	0.025
Mass, m	100 kg
Weighting factor of trust region, w_{trust}	1/100
Number of collocation points, N	30
Tolerance parameters, $(\epsilon_P, \epsilon_h, \epsilon_\omega, \epsilon_J)$	$(10^{-3} \log(\text{m/s}), 0.3 \text{ m}, 10^{-3} \text{ m/s}, 0.01 \%)$

First, the convergence characteristics of the proposed method are shown. Figure 2 shows the convergence of the solution. Figure 2(a) demonstrates that the trajectory rapidly converges to the optimal solution. Although the initial trajectory is roughly guessed as a line connecting the initial and terminal position, the solution converges quickly to the optimal one in 8 iterations. It takes only 6 to 10 msec to solve Problem B at each iteration using Intel Core i5-12500 at 3GHz. The solution can converge in less than 0.1 sec total. To reduce computation time, the iteration can be reduced by increasing

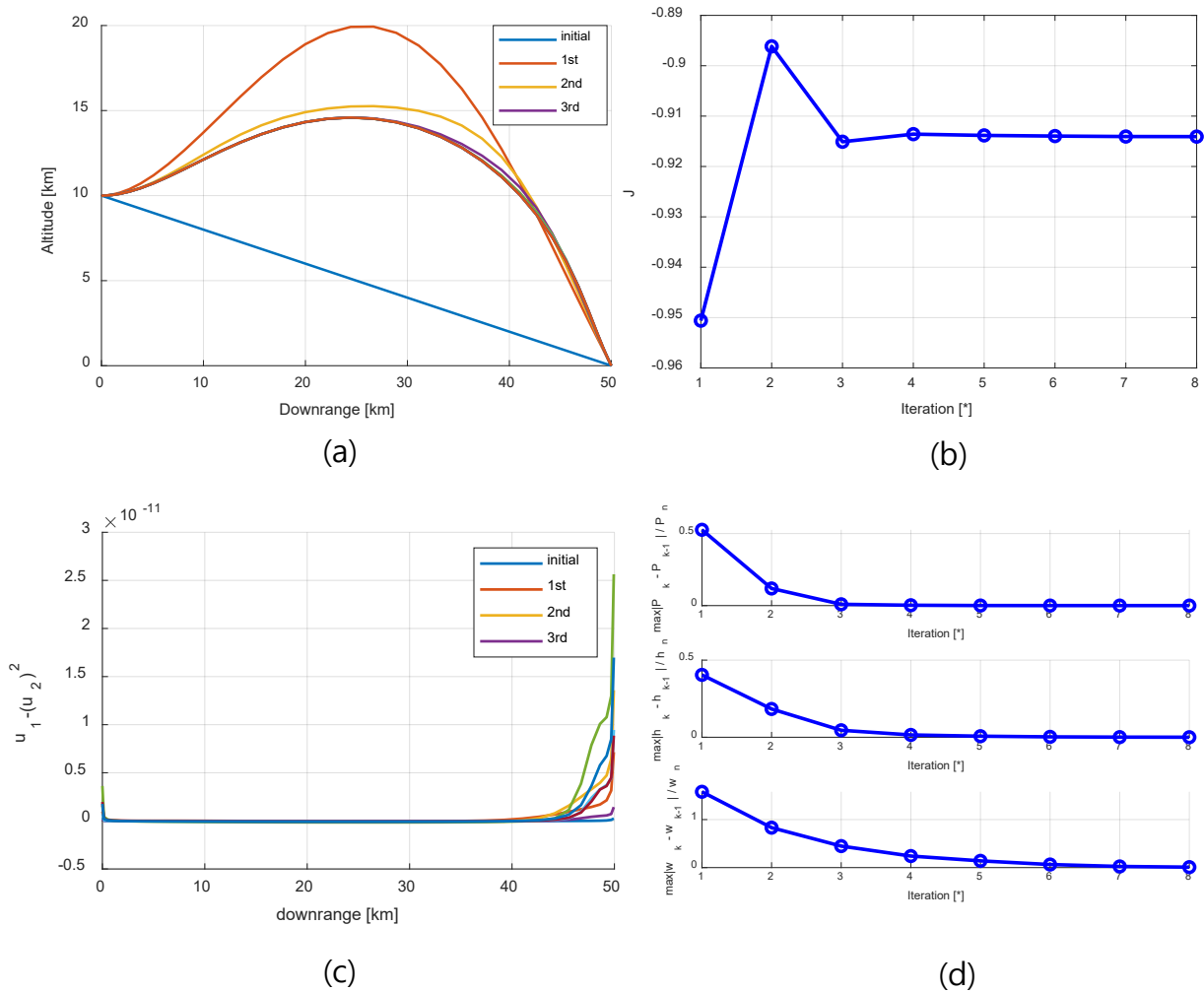


Figure 2 – Convergence characteristics of the proposed method : (a)Trajectories, (b)Performance index, (c)Lossless relaxation, (d)Change of states

the tolerance parameter at the expense of some optimality. If the proposed method is applied to the guidance algorithm, the previous converged solution can be used as a new initial guess, which means a warm start for successive convex programming. Using a warm initial guess, 2 or 3 iterations are enough for convergence. Figure 2(b) shows the performance index. It can be seen that the performance index stably converges to the optimal value. Furthermore, it can be validated that convex relaxation of Eq. (26) is lossless, i.e. Eq. (26) is active, from Figure 2(c). This figure denotes $u_1 - u_2^2$ value which is almost zero. The maximum change in state variable from the previous solution is presented in Figure 2(d), which shows stable convergence.

Secondly, the simulation results of the proposed method are compared to the solution of a nonlinear trajectory optimization. The simulation results of the proposed method are obtained via applying α_{cmd} of Eq. (36) as a guidance law. The optimal solution is obtained from general-purpose optimal control software(GPOPS-II), and is used as a reference for optimality validation of the proposed method. Figure 3 compares the simulation results of the proposed method and the optimal solution of GPOPS-II with the same parameters and initial/terminal conditions. It can be seen that the proposed method guides the vehicle to intercept the terminal position with a desired impact angle of -50 deg. Furthermore, the proposed method shows similar flight trajectories and flight-path angle profiles. Even though there is some deviation in the angle-of-attack profile, the velocity profile is nearly identical. The terminal velocity is 951.5 m/s for the proposed method, and 960.1 m/s for GPOPS-II, respectively. It can be seen that the terminal velocity difference is only 0.9 %, which denotes the optimality of the proposed method.

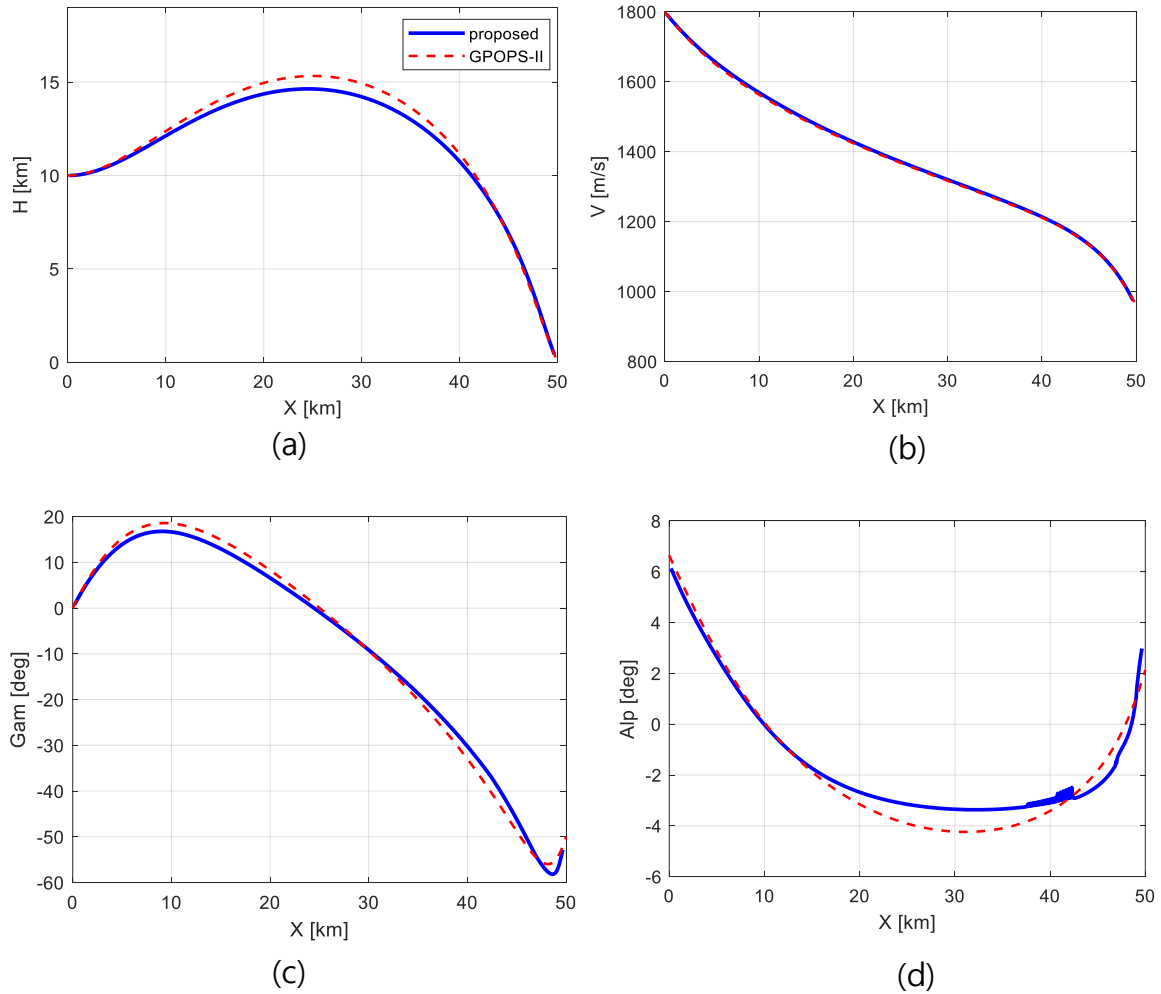


Figure 3 – Results of simulation 1 : (a)Trajectories, (b)Velocity, (c)Flight-path angle, (d)Angle-of-attack

An additional simulation is conducted with the same initial/terminal condition except for the terminal

downrange. Similar to the previous simulation, the proposed method produces an analogous trajectory to the optimal solution of GPOPS-II. The terminal velocity is 917.2 m/s for the proposed method and 927.6 m/s for GPOPS-II, respectively. The terminal velocity difference is only 1.12 % in this simulation. When the proposed method is applied as a guidance law, it successfully achieves terminal conditions and optimality of velocity maximization. Through numerical simulations, it is demonstrated that the proposed method can achieve optimality with real-time application capability.

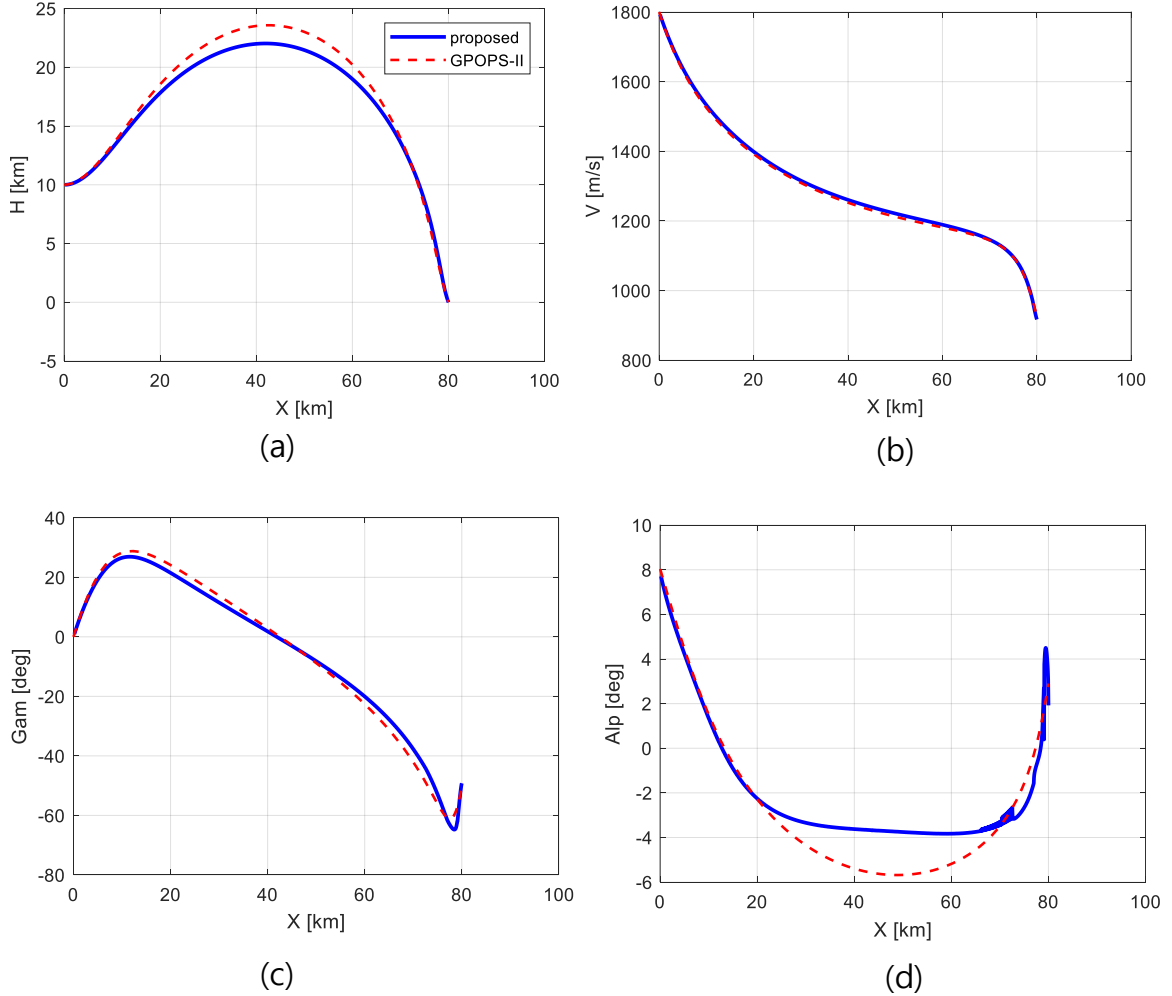


Figure 4 – Results of simulation 2 : (a)Trajectories, (b)Velocity, (c)Flight-path angle, (d)Angle-of-attack

5. Conclusions

In this paper, trajectory optimization for long-range air-launched gliding vehicles using convex programming is proposed. The original nonlinear dynamics is simplified to compact form by utilizing a novel reformulation technique. The flight-path angle and angle-of-attack are expressed as functions of altitude and altitude rate, and substituted in velocity dynamics. This technique results in the logarithmic velocity dynamics and linear dynamics of altitude and altitude rate where control input is the second derivative of altitude. Then, the optimal control problem is converted to second-order cone programming(SOCP) problem by introducing new control variables and conducting partial linearization. Optimal guidance command is generated by successively solving the SOCP problem. The effectiveness and performance of the proposed method are demonstrated by numerical simulations.

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References

- [1] Ryoo, C.K., Cho, H., and Tahk, M.J., "Optimal guidance laws with terminal impact angle constraint," *Journal of Guidance, Control, and Dynamics*, Vol. 28, No. 4, 2005, pp.724-732.
- [2] Ryoo, C.K., Cho, H., and Tahk, M.J., "Time-to-go weighted optimal guidance with impact angle constraints," *IEEE Transactions on control systems technology*, Vol. 14, No. 3, 2006, pp.483-492.
- [3] Lee, J.I., Jeon, I.S., and Tahk, M.J., "Guidance law to control impact time and angle," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 43, No. 1, 2007, pp.301-310.
- [4] Jeon, I.S., Lee, J.I., and Tahk, M.J., "Impact-time-control guidance law for anti-ship missiles," *IEEE Transactions on control systems technology*, Vol. 14, No. 2, 2006, pp.260-266.
- [5] Ates, U.H., "Lyapunov based nonlinear impact angle guidance law for stationary targets," In *AIAA guidance, navigation, and control conference*, 2015, p.863.
- [6] Saleem, A., and Ashwini, R., "Lyapunov-based guidance law for impact time control and simultaneous arrival," *Journal of Guidance, Control, and Dynamics*, Vol. 39, No. 1, 2016, pp.164-173.
- [7] Bryson, A.E., and Ashwini, R., *Applied optimal control: optimization, estimation and control*, Routledge, 2018.
- [8] Rao, A.V., "Trajectory optimization: a survey," *Optimization and optimal control in automotive systems*, 2014, pp.3-21.
- [9] Wang, Z., and Michael, J.G., "Constrained trajectory optimization for planetary entry via sequential convex programming," *Journal of Guidance, Control, and Dynamics*, Vol. 40, No. 10, 2017, pp.2603-2615.
- [10] Wang, Z., and Ye, L., "Improved sequential convex programming algorithms for entry trajectory optimization," *Journal of Spacecraft and Rockets*, Vol. 57, No. 6, 2020, pp.1373-1386.
- [11] Chen, Y., Mark, C., and Jonathan, P.H., "Decoupled multiagent path planning via incremental sequential convex programming," In *2015 IEEE International Conference on Robotics and Automation*, 2015, pp.5954-5961.
- [12] Bento, J., Derbinsky, N., Mathy, C., and Yedidia, J., "Proximal operators for multi-agent path planning," In *Proceedings of the AAAI Conference on Artificial Intelligence*, Vol. 29, No. 1, 2015.
- [13] Wu, Y.H., Cao, X.B., Xing, Y.J., Zheng, P.F., and Zhang, S.J., "Relative motion coupled control for formation flying spacecraft via convex optimization," *Aerospace Science and Technology*, Vol. 14, No. 6, 2010, pp.415-428.
- [14] Liu, X., "Fuel-optimal rocket landing with aerodynamic controls," *Journal of Guidance, Control, and Dynamics*, Vol. 42, No. 1, 2019, pp.65-77.
- [15] Acikmese, B., and Ploen, S.R., "Convex programming approach to powered descent guidance for mars landing," *Journal of Guidance, Control, and Dynamics*, Vol. 30, No. 5, 2007, pp.1353-1366.
- [16] Liu, X., Shen, Z., and Lu, P., "Exact convex relaxation for optimal flight of aerodynamically controlled missiles," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 52, No. 4, 2016, pp.1881-1892.
- [17] Jung, C.G., Kim, B., Jung, K.W., and Lee, C.H., "Thrust Integrated Trajectory Optimization for Multipulse Rocket Missiles Using Convex Programming," *Journal of Spacecraft and Rockets*, Vol. 60, No. 3, 2023, pp.957-971.
- [18] Roh, H., Oh, Y.J., Tahk, M.J., and Kwon, K.J., and Kwon, H.H., "L1 penalized sequential convex programming for fast trajectory optimization: With application to optimal missile guidance," *International Journal of Aeronautical and Space Sciences*, Vol. 21, No. 2, 2020, pp.493-503.
- [19] Kim, C.J., and Sung, S., "A comparative study of transcription techniques for nonlinear optimal control problems using a pseudo-spectral method," *International Journal of Aeronautical and Space Sciences*, Vol. 16, No. 2, 2015, pp.264-277.

- [20] Garg, D., Patterson, M., Hager, W., Rao, A., Benson, D.R., and Huntington, G.T., "An overview of three pseudospectral methods for the numerical solution of optimal control problems," 2017.
- [21] Sagliano, M., "Pseudospectral convex optimization for powered descent and landing," *Journal of guidance, control, and Dynamics*, vol. 41, no. 2, 2018, pp.320–334.
- [22] Benson, D., "A Gauss pseudospectral transcription for optimal control," PhD Thesis, Massachusetts Institute of Technology, 2005.
- [23] Darby, C.L., Hager, W.W., and Rao, A.V., "An hp-adaptive pseudospectral method for solving optimal control problems," *Optimal Control Applications and Methods*, vol. 32, no. 4, 2011, pp.476-502.
- [24] Sagliano, M., Theil, S., Bergsma, M., D'Onofrio, V., Whittle, L., and Viavattene, G., "On the Radau pseudospectral method: theoretical and implementation advances," *CEAS Space Journal*, vol. 9, 2017, pp.313-331.
- [25] Jung, C.G., Lee, C.H., and Tahk, M.J., "Legendre Pseudo-Spectral Method for Missile Trajectory Optimization with Free Final Time," In *Asia-Pacific International Symposium on Aerospace Technology*, 2021, pp.569-581.
- [26] Wächter, A., Biegler, L.T., "On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming," *Mathematical programming*, Vol. 106, 2006, pp.25-57.