

# REDUCED ORDER MODELLING OF UNSTEADY AERODYNAMICS FOR GROUND FLUTTER TEST SYSTEM

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#### **Abstract**

Ground Flutter Test (GFT) technique is a recently developed flutter test technique for carrying out flutter test on full scale aircraft configures based on GVT devices and an efficient Reduce Order Model (ROM) of unsteady aerodynamics. The key techniques of GFT system are the loading of equivalent unsteady aerodynamic force by using electromagnetic shakers and the convergence of continuously distributed aerodynamic force into equivalent concentrated forces. In this paper, an equivalent ROM of unsteady aerodynamic force on several concentrated points are developed based frequency domain unsteady aerodynamic theory and surface spline interpolation technique. The genetic optimization algorithm is used to search the optimized location of concentrated points for the best fits of equivalent unsteady aerodynamic forces. Then, the minimum state method in rational function fitting is used to transform the frequency domain ROM of aerodynamic force into time domain. A real-time simulation of GFT system is carried out based the presented ROM model and Simulink units of other physical devices used in GFT system. Flutter phenomenon appears in flutter test using GFT system. The frequency domain and time domain validation of equivalent unsteady aerodynamics are carried out using equivalent unsteady aerodynamics for flutter calculation. The results show that frequency domain error does not exceed 5% and the time domain error does not exceed 8%. Which validates correctness of the present method.

**Keywords:** Aeroelasticity, Flutter, Ground flutter test, Ground Flutter Test

### 1. Introduction

Ground Flutter Test (GFT) system is a semi-virtual and semi-physical flutter test technique to study the flutter characteristics of aircrafts on the ground by simulating the continuously distributed aerodynamic forces using concentrated forces. In the implement of GFT system, vibration signals (displacements, velocities and accelerations) are acquired by sensors, the aerodynamic forces feedback to the structures are calculated by a time-domain aerodynamic solver. Then the calculated concentrated forces are fulfilled by several robustly controlled electro-magnetic shakers

The theory of GFT is firstly proposed by Kearns in 1962s. In the 21th century, the theory of GFT was put into practice based on the development of control and sensor techniques. In 2021, TsAGI [1] proposed the electromechanical modeling technique of aerodynamics. In 2011, ZONA technique Inc. [2] proposed the concept of dry wing tunnel, and successfully fulfilled this concept by using configurations of aircrafts.

Chinese scholars have also carried out relevant research on issues related to ground flutter test. Xu [3] established unsteady aerodynamic model based on subsonic DLM method and piston theory, the reduced aerodynamic force in time domain is obtained by surface spline interpolation and RFA fitting. Gao [4] analyzed the fitting accuracy of aerodynamic rational function. Zhang [5] studied the influence of dynamic characteristics of loading system in ground flutter simulation test.

On the whole, domestic scholars has carried out much research on GFT technique, however there are many areas needs in-depth study, including ROM modelling of unsteady aerodynamics, and robust controller design for concentrated ROM model of aerodynamics. In this paper, an equivalent ROM of unsteady aerodynamic force on several concentrated points are developed based frequency domain unsteady aerodynamic theory and surface spline interpolation technique. A real-time simulation of GFT system is carried out based the presented ROM model and Simulink units of other physical devices used in GFT system. Flutter phenomenon appears in flutter test using GFT system.

# 2. Reduce Order Model (ROM) of unsteady aerodynamics

# 2.1 Linearized small disturbance velocity potential equation

The linearized small disturbance velocity potential equation is as follows,

$$(1 - M_{\infty}^{2})\phi_{Axx} + \phi_{Ayy} + \phi_{Azz} - 2\frac{M_{\infty}}{a_{\infty}}\phi_{Axt} - \frac{1}{a_{\infty}^{2}}\phi_{Att} = 0$$
 (1)

In the appearance of flutter, the structure of an aircraft is doing harmonic vibration, the unsteady velocity potential can be written as,

$$\phi_1 = \phi e^{i\omega t} \tag{2}$$

Using Green's function method to solve equation (1), the solution ca be written as:

$$\phi(x_o, y_o, z_o) = \frac{1}{4\pi} \iint \Delta \phi(x, y, z) e^{i\lambda M_{\infty} \xi} \frac{\partial}{\partial n} K dS$$
(3)

In which,  $x_0, y_0, z_0$  is the field point, and x, y, z is source point,  $\Delta \phi$  represent the distribution of dipole, n is the direction vector, K is Green's function. Take the spatial derivatives of Eq. (3), the expression of velocity can be written as,

$$(u_i, v_i, w_i) = \sum_{i=1}^{N} (u_{W_{ij}}, v_{W_{ij}}, w_{W_{ij}}) \Delta C_{p_j}$$
(4)

Kutta condition,

$$\vec{\nabla} \phi \cdot \vec{n} = F_{w} \tag{5}$$

where,

$$F_{w} = (\frac{\partial}{\partial x} + \frac{i}{L}k)z \tag{6}$$

in which, z the modal displacement in z coordinate,  $F_{w}$  is the downwash function. Applying the Kutta function on the DLM boxes, the following equation can be derived,

$$[NIC]\Delta C_n = F_w \tag{7}$$

$$[NIC] = [n_x][UIC] + [n_y][VIC] + [n_z][WIC]$$
 (8)

in which, [NIC] is the influence coefficient matrix of normal velocity. By using Eq. (7), the pressure difference coefficient can be derived, and the unsteady aerodynamic force ca be derived,

$$F = q_{\infty} A[NIC]^{-1} \left(\frac{\partial}{\partial x} + \frac{i}{L}k\right) z \tag{9}$$

$$q_{\infty} = \frac{1}{2} \rho V^2 \tag{10}$$

In which, F is the force,  $q_{\infty}$  is the dynamic pressure of inflow,  $\rho$  is the density of inflow, V is the velocity of inflow, V is the integration matrix of aerodynamic boxes. In general, Eq. (9) can be written as

$$F = q_{\infty}[AIC]z \tag{11}$$

where.

$$[AIC] = A[NIC]^{-1} \left(\frac{\partial}{\partial x} + \frac{i}{L}k\right) \tag{12}$$

is the aerodynamic influence coefficient matrix. The papers should be prepared, if possible, using the format like this document.

## 2.2 Reducing order of unsteady aerodynamic model

Eq. (11) gives the transfer relationship between unsteady aerodynamic force F and displacement z, which can be simulated by hard devices including shakers and sensors, however this relationship cannot be directly used in GFT, because the numbers of excitation points and collection points are too many to be controlled. Therefore, the order of Eq. (11) must be reduced.

By using the spline interpolation technique, the transfer relationship of the aerodynamic force from the excitation point to the control point is

$$\{F_{shak}\}_{NA\times 1} = [G_f]_{NA\times NP} \{F\}_{NP\times 1}$$
 (13)

And the transfer relationship from the collection point to the control point is

$$\{z\}_{NP\times 1} = [G_{s1}]_{NP\times NS} \{z_{sen}\}_{NS\times 1}$$
(14)

$$\frac{\partial \{z\}_{NP\times 1}}{\partial x} = [G_{s2}]_{NP\times NS} \{z_{sen}\}_{NS\times 1}$$
(15)

Where,  $G_f$ ,  $G_{s1}$ ,  $G_{s2}$  are the spline interpolation matrices of forces and displacements. By substituting Eq. (13)-(15) into Eq.(9), the following equation can be derived,

$$F_{shak} = q_{o}[\overline{AIC}]z_{son} \tag{16}$$

Where,

$$[\overline{AIC}] = G_f A[NIC]^{-1} (G_{s2} + G_{s1} \frac{i}{L} k)$$
(17)

in which,  $[\overline{AIC}]$  is the order reduced aerodynamic influence coefficient matrix. Eq. (16) is order reduced frequency-domain aerodynamic forces. By using the rational function fitting method, the frequency-domain equation can be transferred into Laplace domain, and then into time domain.

$$F = q_{\infty}[A_0 + A_1 p + A_2 p^2 + D(pI - R)^{-1} p]z$$
(18)

where,

$$A_0 = \text{Re}[[\overline{AIC}](0)] \tag{19}$$

$$A_{1} = \frac{1}{k_{\text{max}}} \text{Im}[[\overline{AIC}](ik_{\text{max}})] + D(k_{\text{max}}^{2}I + R^{2})^{-1}E$$
 (20)

$$A_{2} = \frac{1}{k_{\text{max}}} \text{Re}[[\overline{AIC}](0)] - \text{Re}[[\overline{AIC}](ik_{\text{max}})] + D(k_{\text{max}}^{2}I + R^{2})^{-1}E$$
 (21)

# 3. Physically implement of ROM of unsteady aerodynamics

By using Eq.(18), the vibration information denoted by z can be measured by sensors, and based on Eq.(18) the aerodynamic forces F can be calculated with a time-domain solver running on NI instrument. Then, the calculated forces can be physically implemented by electromagnetic shaker as shown in Fig. 1.

Since the electromagnetic shaker and amplifier are complicated electromechanical system, a robust controller should be designed for exactly applied the calculated air forces.

Firstly, the system identification method should be used to identified a MIMO state-space model of the coupled shaker and amplifier system. Four chirp signals of voltage are input into the amplifiers, then the output signals are acquired by using loadcells, as shown in Fig. 2. The inputs and outputs signal are detrended and remove means, the SID identification method is used to identified the state-space model.

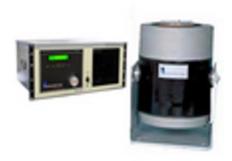


Figure 1 – Electromagnetic shaker and amplifier.

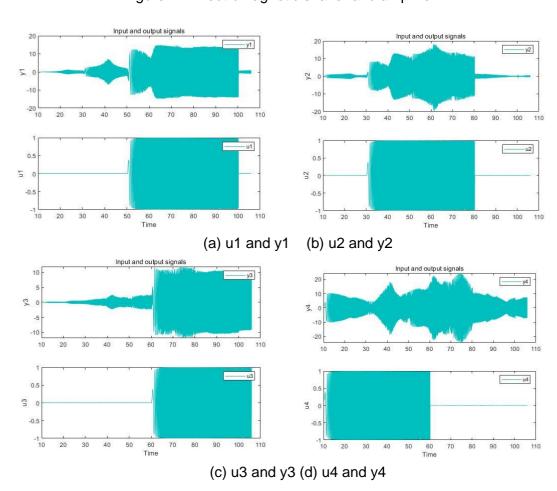


Figure 2- Input voltage and output force signals.

In order to validate the identified state-space model of shaker and amplifier, a Simulink model is built, as shown in Fig. 3. The input chirp signals are input into the identified model, and the output signals are compared with the acquired signals. Fig. 4 shows the comparison of simulated signals and acquire signals. By using the 30-order state-space model, the fitting values of four channels are all above 95%, which validate effectiveness of the identified state-space model of shakers and amplifiers.

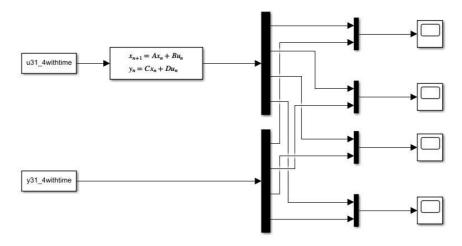


Figure 3 - Time-domain Simulink of identified model.

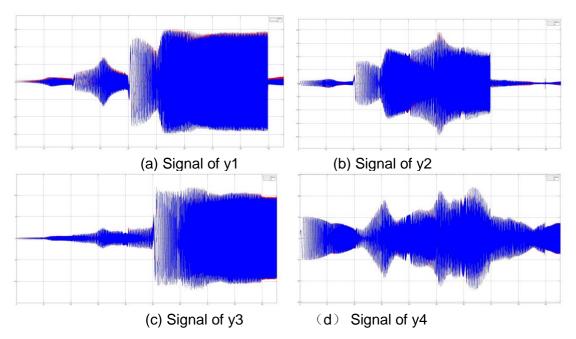


Figure 4- Comparison of simulated signals and acquire signals.

In order to realize multi-channel decoupling and precise loading of air forces, a hybrid sensitivity robust control method is used to design a MIMO robust controller for the identified shaker and amplifier system.

In the implement of W1 is the performance weighting function matrix of the system, which represents the frequency characteristics of interference signals, It can reflect the requirements of the system on the sensitivity function S, sensitivity function S decreases with the increase of W1. According to the previous simulation results, the model's flutter frequency is about 15Hz. Considering the working frequency band, W1 is selected as a fourth-order diagonal array, the diagonal element is a 6-order Butterworth bandpass filter, and the bandpass frequency is 12-18Hz.

W2 is the control weighting function matrix, which is used to control the output voltage amplitude of the controller to ensure that the control signal u will not be too large and cause supersaturation. The specific value is adjusted according to the safety voltage of the actuator. The larger the value, the smaller the KS, and KS should maintain a small value in the full frequency band. KS is selected as the fourth-order constant diagonal matrix.

W3 is the robustness weighting function matrix of the system, reflecting the uncertainty of the model, which is determined by the non-structural uncertainty of the model, that is, the high frequency unmodeled dynamic characteristics and the uncertainty of the model parameters. W3 must contain the multiplicative uncertainty of the controlled object, and generally take the approximate upper bound of the relative error of the identification model.

After the weighted function matrix is determined, the controller K is solved based on the commands of MATLAB robust control toolbox, and a closed-loop feedback control system including the controller is established in Simulink to verify the decoupling effect of the closed-loop system and the tracking effect of the input signal under the action of the controller, see Fig.5.

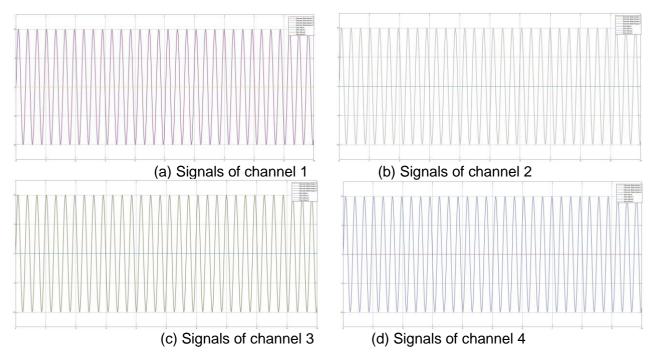


Figure 5 - Decoupling and tracking effects of signals under robust controller

It can be seen that when one channel is given the input signal, the output force of the other channels is almost zero, the four channels are decoupled, and the output force of each channel has a better tracking effect on the target signal.

## 4. Validation of GFT system by time-domain simulation

With the ROM of unsteady aerodynamics and aerodynamic force solver in hand, and the robust controller developed, the GFT system can be established as shown in Fig.6.

In this paper, an aluminium alloy plate is manufactured, and the flutter characteristic is analyzed by PK method, as shown in Fig. 8. The flutter speed is 163.365m/s and the flutter frequency is 15.0 Hz.

Before doing the ground flutter test by GFT system, a Simulink block diagram is constructed by using the time-domain order reduced aerodynamic force solver and the robust controller, and a time-domain simulation is conducted by using the Simulink model as shown in Figs. 7. From Figs. 9-11,

we can see that the time domain simulation results coincide well with the v-g and v-f digram, which validate correct of the GFT system for predicting the flutter speed of aircraft configurations.



Figure 6 - GFT system.

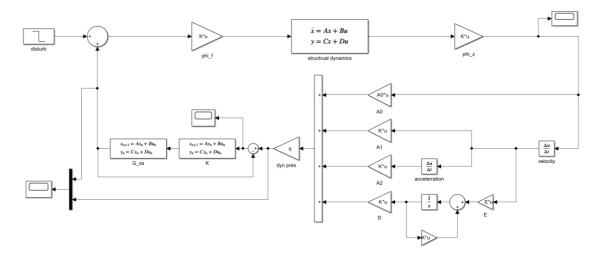


Figure 7- Simulink model of the GFT system.

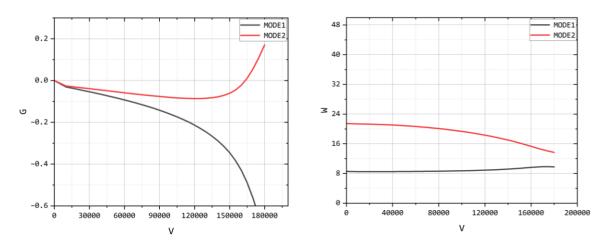


Figure 8 - V-g and v-f digram of a plate.

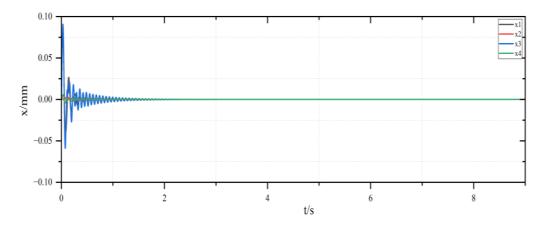


Figure 9 - Time history of displacements with the inflow velocity of 156m/s.

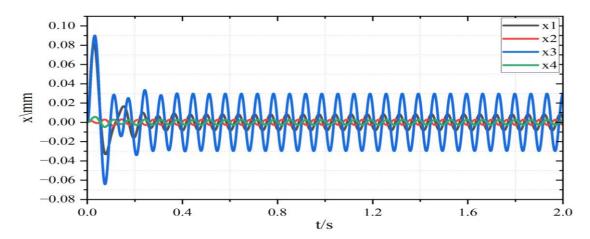


Figure 10 - Time history of displacements with the inflow velocity of 166.54m/s.

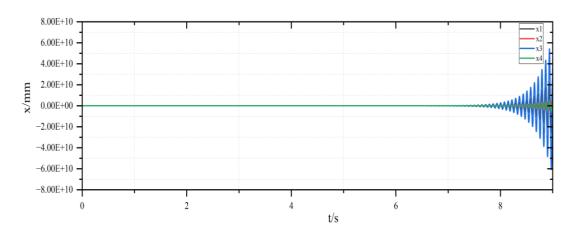


Figure 11- Time history of displacements with the inflow velocity of 176m/s.

## 5. Conclusion

In this paper the ROM of aerodynamic force was developed and a robust controller is designed based on an identified state-space model of shaker and amplifier. Finally, the GFT system is established and a Simulink model of the GFT system is built. From the simulation results, we can see the GFT system is effective and precise in predicting the flutter speed of aircrafts.

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