Fatigue Crack Life Prediction Based on Bayesian Dynamic Update

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Abstract

In this paper, the damage monitoring data (crack length) obtained by different damage monitoring sensors such as intelligent coating, eddy current array and piezoelectric guided wave sensor are used to construct the crack propagation model considering uncertainty by Bayesian dynamic updating method, and the uncertainty is considered by randomizing the relevant parameters of the crack growth model. The random characteristics of the parameters are obtained from the corresponding material test data. The distribution of crack growth life and crack length is obtained by using the distributed parameter sampling and replacing the crack growth model, and the result of crack distribution is used to predict the crack growth result and the prediction accuracy is more accurate, which provides a feasible method for prediction of crack growth life under structural health monitoring.

Keywords: crack growth, uncertainty, Bayesian update, distributed parameter

1. Introduction

As one of the important components of PHM(Prognostics and Health Management, PHM), aircraft Structural Health Monitoring (SHM) can play an important role in the design, flight and maintenance of aircraft. The information of structural response, operation and service environment can be obtained through the built-in sensor network in the aircraft structure. The diagnosis results based on the sensor data can be used for prediction of structural health status and decision-making of auxiliary maintenance and maintenance.

In recent years, many studies have focused on more technical research on the health monitoring of aircraft structures, but the research on how to predict and guide the repair of structures from the damage data of structural monitoring still needs further research, and research on the design of health monitoring system of aircraft structures and the life prediction based on the monitoring data is not supportive in engineering practice.

In this paper, the damage monitoring data (crack length) obtained by different damage monitoring sensors such as intelligent coating, eddy current array and piezoelectric guided wave sensors are used to construct the crack propagation model considering uncertainty by Bayesian dynamic updating method, and the uncertainty is considered by randomizing the relevant parameters of the crack growth model. The random characteristics of the parameters are obtained from the corresponding material test data. The distribution of crack growth life and crack length is obtained by using the distributed parameter sampling and replacing the crack growth model, and the result of crack distribution is used to predict the crack growth result and the prediction accuracy is more accurate, which provides a feasible method for prediction of crack growth life under structural health monitoring.

2. Methodologies

2.1 Dynamic updating steps of crack growth life prediction

The distribution of crack growth life and crack length was obtained by sampling the distributed parameters and replacing the crack growth model, and the results of crack growth were predicted by using the results of crack distribution. The steps of crack propagation analysis based on monitoring data are as follows.

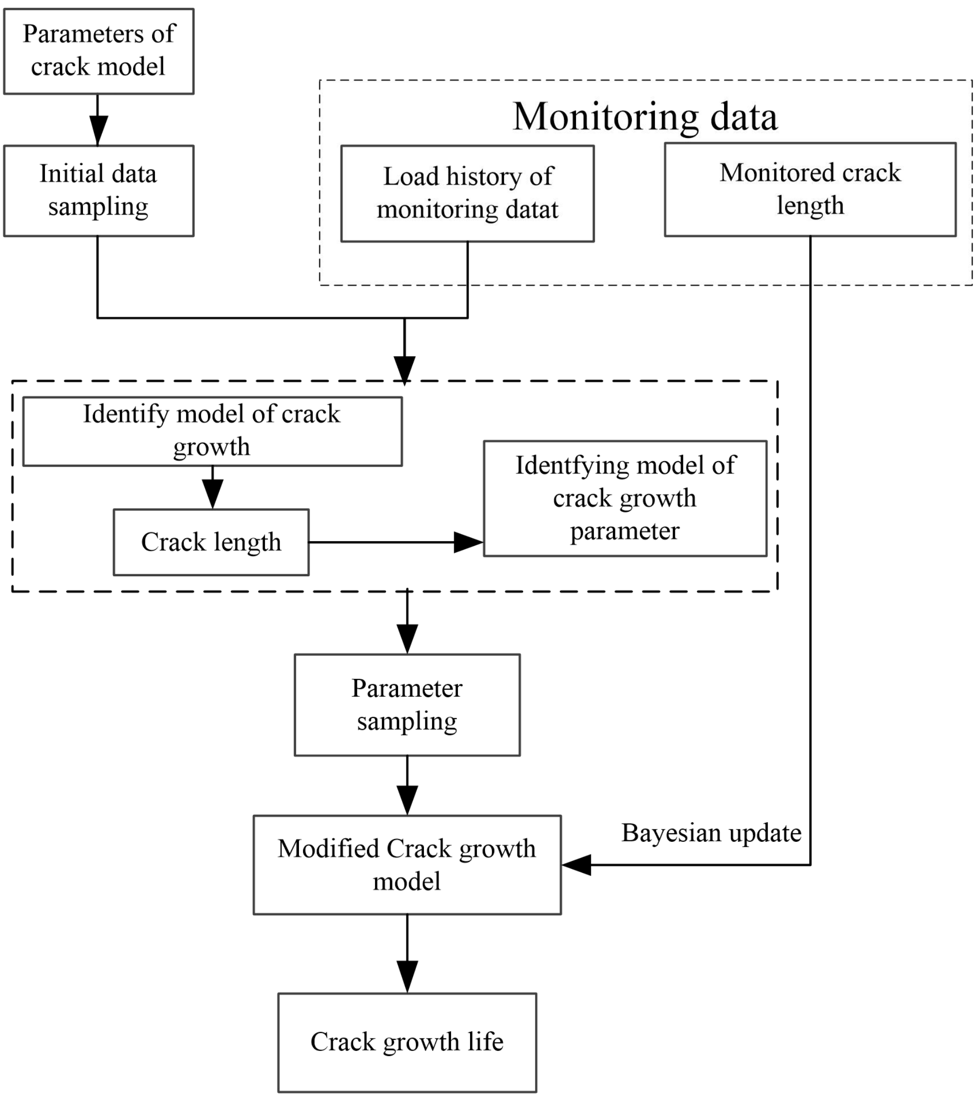


Fig.1 Dynamic updating steps of crack growth life prediction

2.2 Crack growth model

A crack growth model based on probability distribution is established by using the suitable crack growth model. The usual crack growth model can be adopted, taking the classical Paris growth model as an example.

$$\frac{da}{dN} = C(\Delta K)^n \quad (1)$$

Where a represents the length of crack, N represents the number of cycles, da and dN represents the growth of crack under unit cycle, K represents the amplitude of stress intensity factor, C and n represent the growth coefficient of damage.

The method of determination for ΔK is as follows.

$$\Delta K = \Delta\sigma \cdot \beta \cdot \sqrt{\pi \cdot a_i} \quad (2)$$

When dN is assumed to be a unit cycle, we have

$$\Delta a_i = C(\Delta K_i)^n \quad (3)$$

And the unit crack growth is as follows.

$$a_i = a_{i-1} + \Delta a_i \quad (4)$$

In the traditional method, *C, m* and *n* are set to definite values, so the length of crack is also determined. The length of crack cannot be predicted very well, because not all the same structural parameters *C*, *m* and *n* are identical with different structure parameters. This difference is due to the divergence of process and material properties and the difference of structure size, so it should be taken into account when predicting crack growth.

In this study, the parameter distribution is introduced into Walker's formula, the crack propagation distribution is obtained by multiple sampling, and the crack propagation results are predicted by using the crack distribution results.

2.3 Sampling method of parameters

Crack propagation has great dispersion, which mainly includes, (1) inherent dispersion of material, i. e. non-uniformity of material micro-structure. (2) External dispersion, such as uncertainty of external load, specimen geometry, working environment, etc.

In order to improve the precision of life prediction, a crack growth model considering uncertainty should be constructed, and the uncertainty can be considered by randomizing the parameters of the crack growth model, and the random characteristics of the parameters should be obtained from the corresponding material test data.

The distribution of crack growth life and crack length can be obtained through multiple sampling, and the crack growth results can be predicted by using the results of crack distribution.

Generally, when the parameter distribution is normal or uniform, the direct sampling method is used for sampling. While the parameter distribution is complex, the Markov chain Monte Carlo (MCMC) sampling method is recommended.

a) Direct sampling.

Assuming that the Probability Density Function (PDF) of the sample population is known, the Cumulative Distribution Function (CDF) of the sample population is required. Assuming that the PDF to be sampled is $f(x)$, then the CDF to be sampled is $\int f(x)\,dx$. A value a ~ U (0,1) is extracted from the 0-1 uniform distribution and mapped to the CDF axis, and its corresponding x-axis value is the sample value extracted.

b) Accept-reject sampling

Due to the fact that many of the PDF to be sampled are difficult to solve, or the integral of the PDF to be sampled is difficult to obtain, the accept-reject sampling is proposed on this basis, as shown in Figure 2.

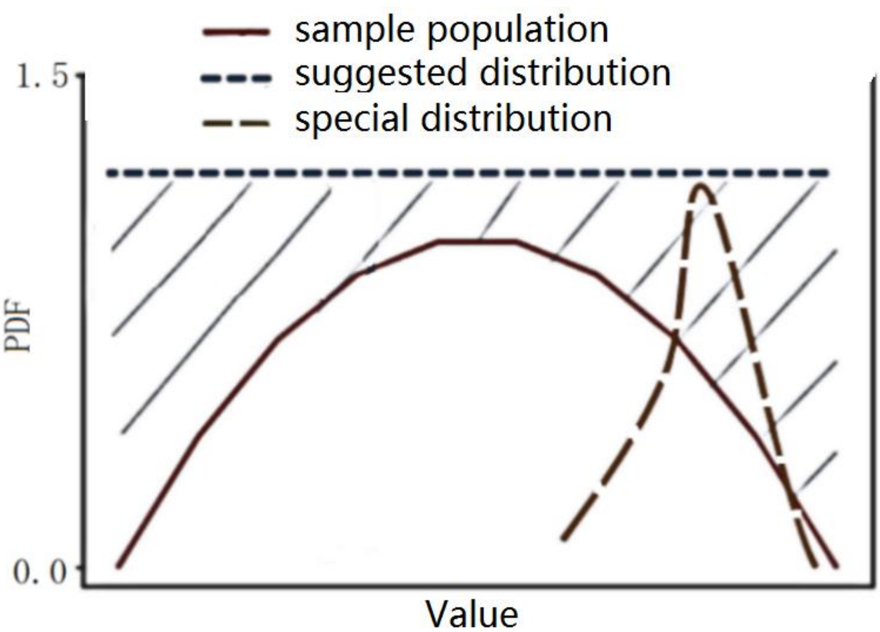


Fig.2 Accept-reject Sampling

Assuming that the PDF to be sampled is *f (x)*, look for a simple distribution of the recommended distribution *G*: *g (x)* and constant *c*, so that all values of the *c* times of recommended distribution are higher than those of the sample to be sampled, that is:

$$c * g(x) \geq f(x) \tag{5}$$

It is suggested that the direct sampling of the distribution is b, and the proportion of the sample to b sampled is *f (x)/[c * g (x)]*, and the value is between 0 and 1.

Uniform distribution takes the random number *u ~ U (0,1)*, if

$$u \leq f(x) \,/\, [c * g(x)] \tag{6}$$

Then accept *b*, or reject *b* as a sample. In order to increase the acceptance ratio and increase the computing speed, we expect that the smaller the *c* the better, and the best value is 1. The commonly used suggested distributions are uniform distribution and Gaussian distribution.

c) Markov Chain Monte Carlo (MCMC) sampling.

Assuming that the sampling time conforms to the Markov chain condition, the corresponding Markov chain transfer matrix *P* can be found when the probability distribution *π (i)* is sampled, so that the probability distribution *π (i)* satisfies the fine stationary condition, which is

$$\pi(i)P(i,j) = \pi(j)P(j,i) \tag{7}$$

Where *P (i, j)* represents the probability of transition from state *i* to state *j*.

It is difficult to get the *P* matrix directly in mathematics, so the transition matrix *Q* is chosen randomly and *α* is introduced to make the probability distribution *π (i)* satisfy the fine stationary condition.

$$\pi(i)Q(i,j)\alpha(i,j) = \pi(j)Q(j,i)\alpha(j,i) \tag{8}$$

Where

$$\alpha(i,j) = \min\left\{\frac{\pi(j)Q(j,i)}{\pi(i)Q(i,j)}, 1\right\} \tag{9}$$

If the selected transition matrix Q has symmetry, that is, Q *(i, j)* = Q *(j, i)*, it can be further reduced as

$$\alpha(i,j) = \min\left\{\frac{\pi(j)}{\pi(i)}, 1\right\} \tag{10}$$

The parameters of *C*, *m* and *n* in this study are distributed according to the actual measurement, and the Gaussian distribution is judged: if satisfied, direct sampling is adopted; if unsatisfied, M-H sampling is adopted.

2.4 Bayesian update fusion

In order to improve the precision of life prediction, the crack length predicted by crack growth model should be fused with the crack length data obtained by the damage monitoring sensor to improve the accuracy of the prediction results.

Generally, Bayesian method is used to predict crack growth analysis based on monitoring data:

$$P(X|Y) = \frac{P(Y|X)}{P(Y)}P(X) \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \tag{11}$$

Among them, *P (X)* represents the probability of *X* occurrence, *P (X |Y)* represents the probability of *X* occurrence when the *Y* occurrence has been known. In this study, *X* represents the length of crack propagation, and *Y* represents the length of crack measured by the sensor. *P (Y |X)* indicates that the probability of *Y* occurrence is the likelihood probability, i. e. the actual measured value of the crack, in the case of known *X* occurrence.

Assuming that the prior information conforms to Gaussian distribution $x \sim N(\mu_1, \sigma_1^2)$, the likelihood probability distribution conforms to Gaussian distribution $y|x \sim N(\mu_2, \sigma_2^2)$.

$$\int_X (x) = \frac{1}{\sqrt{2\pi\sigma_1^2}} \exp\left(-\frac{(x-\mu_1)^2}{2\sigma_1^2}\right) \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \tag{12}$$

$$\int_{y|x}(y\mid x)=\frac{1}{\sqrt{2\pi\sigma_2^2}}\exp(-\frac{(x-\mu_2)^2}{2\sigma_2^2}) \quad (13)$$

Because *P (y)* is irrelevant with *x*, so:

$$P(X|Y) \propto P(Y|X) * P(X) \quad (14)$$

Then, we can obtain distribution as follows.

$$f_{x|y}(x|y) \sim N\left(\frac{\sigma_1^2}{\sigma_1^2+\sigma_2^2} * \mu_2 + \frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2} * \mu_1, \frac{\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2}\right) \quad (15)$$

The corresponding σ_1 and σ_2 are the variance or precision of the two methods respectively, and μ_1 and μ_2 are the expectation of the two methods respectively.

2.5 Crack growth life prediction

The prediction of crack growth life is carried out in the following steps.

a) According to the crack growth curve at the analysis site, the damage and crack growth curves corresponding to the crack growth under the reference load are calculated.

b) Based on the growth curve under the reference load and the actual crack size under the load predicted by the monitoring data, the actual crack size was determined by the Bayesian update fusion method.

c) According to the actual crack size, the crack propagation parameters are sampled and the distribution law is determined.

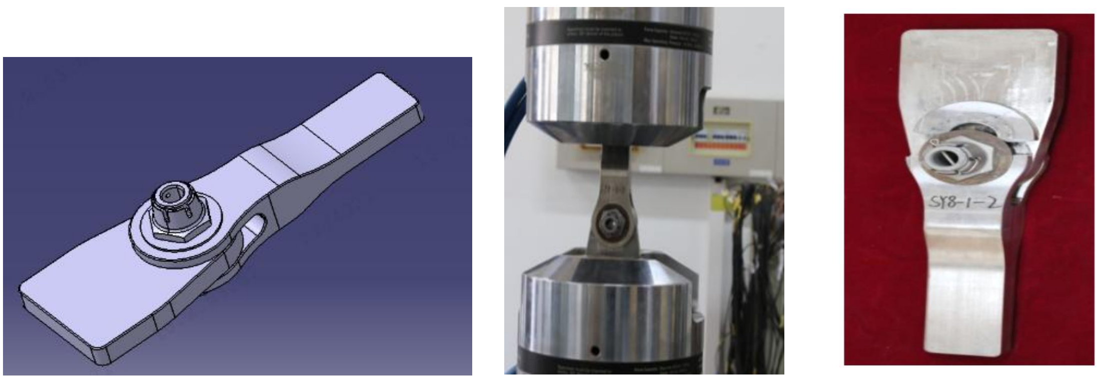
d) Based on the actual crack size of b) and the crack growth model parameters of c), the crack growth life under different parameters distribution is calculated, and the crack growth curve distribution is given.

e) According to the distribution of crack growth curve, the life span of initial crack length to critical crack length is determined by damage tolerance analysis, and the check interval is determined by means of checking.

3. Examples

3.1 Prediction of crack growth life of attachment lugs

Take the real structure at attachment lugs of a certain aircraft as a calculation example. The size of the test piece is 468 mm × 120 mm × 4 mm. The equipment used in the test is 500kN electro-hydraulic servo fatigue testing machine MTS810 with load accuracy of 1%. The test diagram is shown in Figure 3 (a), and the test loading and test piece failure diagram is shown in Figure 3 (b) and Figure 3 (c).



(a) Lugs connection schematic (b) Test loading schematic (c) Test piece failure photograph

Fig.3 Test diagram of attachment lugs

3.2 Prediction of crack growth life of attachment lugs

The fatigue test of a typical lugs piece connector is taken as an example for analysis. According to the literature, the fatigue expansion parameters of 2024-T351 alloy are: *C* = 1.74297 × 10^{-10} and *n*

= 3.6404, respectively.

$$\frac{da}{dN} = CK_i^n \tag{16}$$

$$\ln \frac{da}{dN} = \ln C + n \ln K_i \tag{17}$$

In order to ensure the rate of sampling and to consider the process of parameter solving, the parameters C and n are regarded as random variables, obeying the normal distribution. And then assume that $\ln C \sim N(-22.4645, 1)$, $n \sim N(3.6404, 0.5)$.

Because the crack growth rate is too fast during the final fracture stage, considering the actual situation, the maximum block number is set as 4000 blocks. And a block is defined as a cycle, and less than a cycle part will be rounded.

3.3 Calculation of crack growth

A set of parameters are extracted from the samples, and the procedure is used to record the crack length and the number of cycles, and to remove the samples which are too fast and seriously unfit for practice. *a*-*N* curves of 1000 sets of data are drawn as shown in Figure 4.

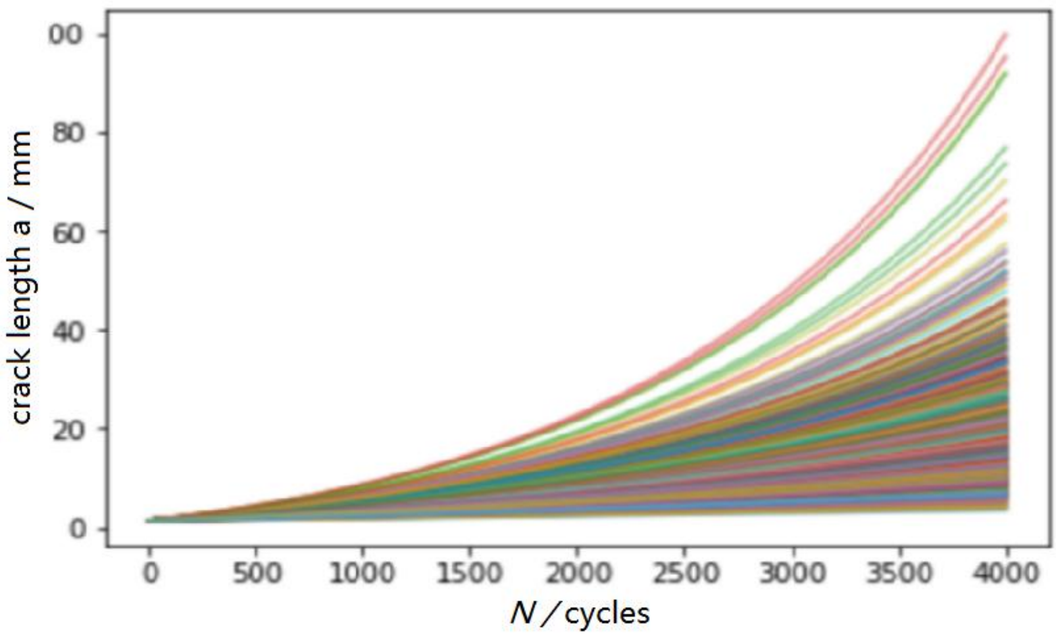
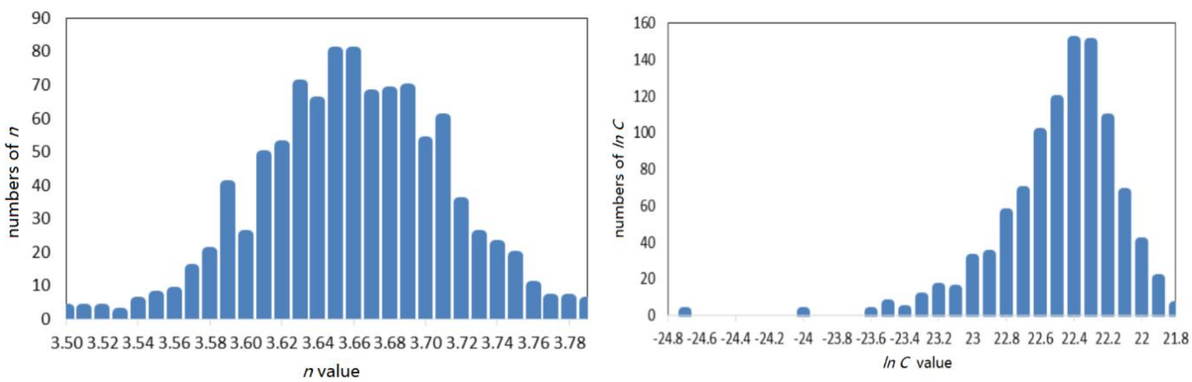


Fig.4 Prediction of crack propagation and cyclic relation curve of parametric samples of lugs

Extract the sample parameters and divide the parameters into 30 equal parts, record the number of intervals, and draw the distribution histogram of the parameters as shown in Figure 5(a) and 5 (b).



(a) Distribution histogram of parameter *n* (b) Logarithmic distribution histogram of parameter *C*

Fig.5 Histograms of parameter distribution of attachment lugs

As can be seen from the graph, the parameters roughly meet the normal distribution, take 90% confidence interval, and get the parameter distribution as shown in Table 1. From the *N-a* curve, the confidence 5%,50% and 95% curves are selected respectively and shown in Figure 6.

Table 1 Value of confidence interval for the parameters of attachment lugs

Parameter	5%	50%	95%
C	9.6891×10^{-10}	1.7598×10^{-10}	2.6112×10^{-10}
n	3.582	3.6411	3.6968

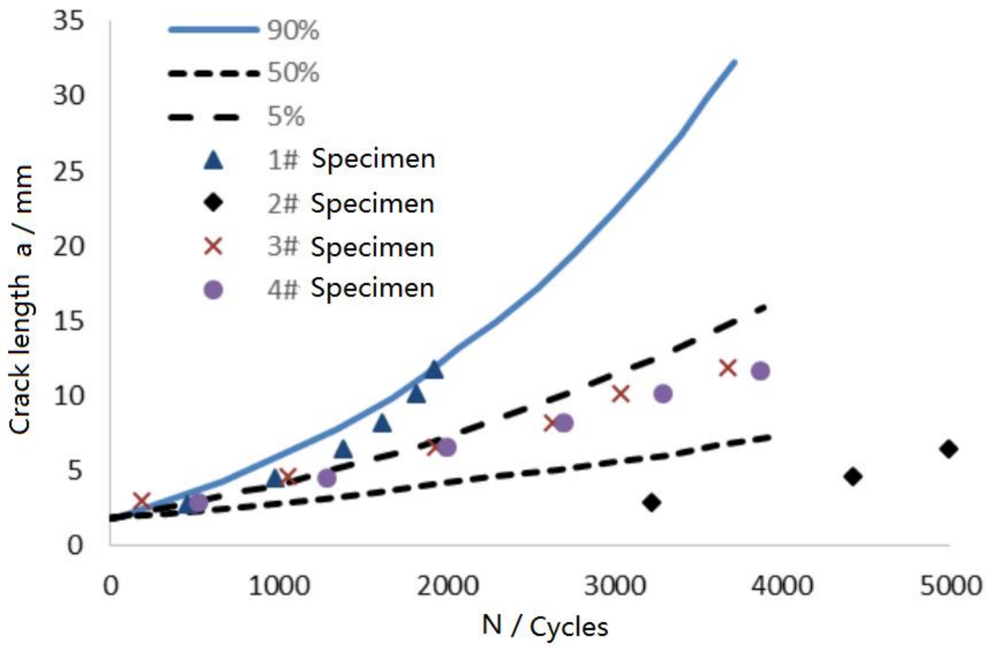


Fig.6 Prediction and test results of crack propagation in 90% confidence interval of attachment lugs

3.4 Analysis

An example is given based on the fatigue test of the joint structure of the ear piece. In this paper,3 # test pieces and 4 # test pieces were used to update the crack information on the mean value of crack length 8.6mm. At this time, the cycle number N=2713 and crack length a = 8.6mm. Assuming the crack measurement accuracy is 1.0mm, it is considered that the crack length distribution conforms to the normal distribution $N(\mu_2, \sigma_2^2)$, in which μ_2 = 8.6mm, σ_2^2= 1.

In the case of N = 2713, the upper and lower limits of normal distribution are represented by the mean line, and the upper and lower limits of normal distribution are represented by the range of 5% and 95%. The approximate normal distribution $N(\mu_1, \sigma_1^2)$ is obtained, and the upper and lower ranges are 3.32 times variance. According to Fig.6, we can obtain the values of $a_{2713}^{5\%}$、$a_{2713}^{50\%}$、$a_{2713}^{95\%}$ when N = 2713 are 5.002、10.092、19.232,respectively.Then, it can be obtained that $\mu_1 = 2a_{2713}^{50\%} = 10.004, \sigma_1 = (a_{2713}^{95\%} - a_{2713}^{5\%})/3.32 = 4.286$.

Using the fusion method given in Section 2.4, the result is N (8.8483, 0.8230). Since the predicted prior distribution is not completely consistent with the normal distribution, the updated posterior distribution should be scaled with equal prior distribution proportion, which makes the prior distribution deviate from the posterior distribution approximately, so $a_1/a_2= b_1/b_2$, the schematic diagram is shown in Figure 7. The updated crack prediction results are shown in Figure 8.

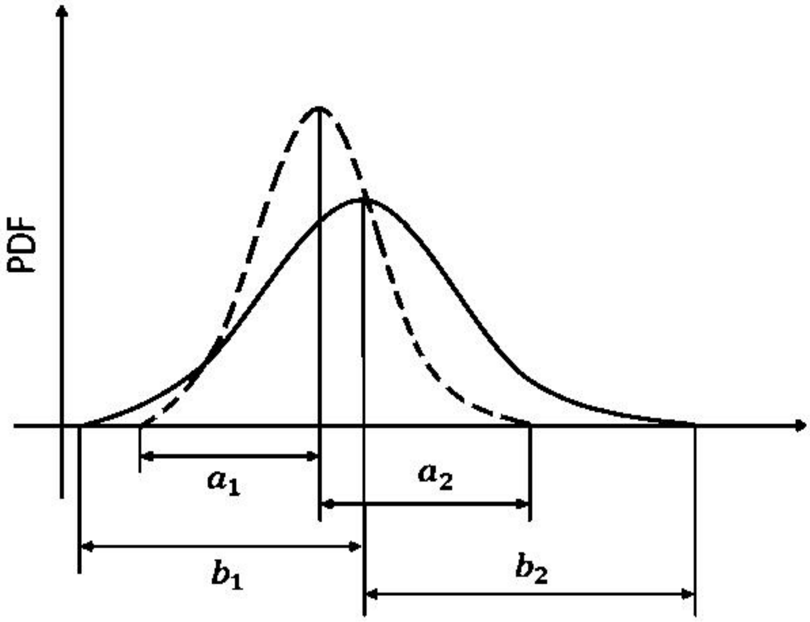


Fig.7 Posterior distribution approximate deviation graph

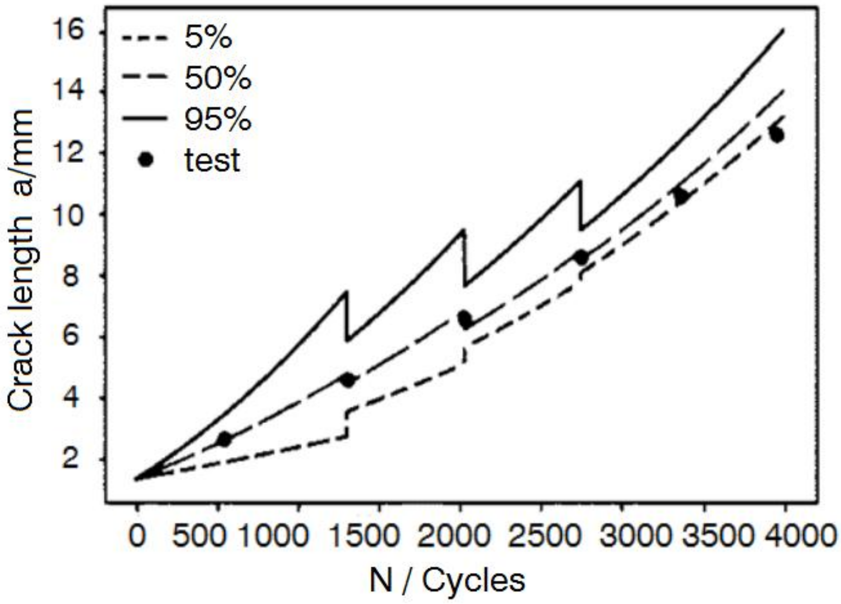


Fig.8 Updating Results of Crack Propagation in 90% Confidence Interval of Ear Plate Samples

4. Conclusions

In this paper, a method for predicting crack growth life based on Bayesian dynamic updating is presented, and an example is given with the attachment lugs. The conclusions are as follows.

(1) Based on the dynamic Bayesian network application to fatigue crack growth, the prediction accuracy of fatigue crack growth life is improved by continuously collecting the crack length data and inputting them into the model, gradually reducing the influence of uncertainty factors on the prediction accuracy.

(2) With the improvement of the performance of crack monitoring sensor and the development of artificial intelligence technology, the method presented in this paper will play a more important role in the design and application of aircraft structure health monitoring.

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6. Copyright Statement

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