LARGE-SCALE STRUCTURAL DYNAMIC TOPOLOGY OPTIMIZATION DESIGN FOR AIRCRAFT VIBRATION REDUCTION UNDER RANDOM EXCITING RESPONSE

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Abstract

The aircraft is faced with complex and uncertain aerodynamic load environment when flying at a high speed, the severe vibration caused by which poses a great challenge to the safety and life of the aircraft. Nevertheless, so far, the vibration reduction design of aircraft has always been a difficult problem to be solved. In particular, considering the randomness of the load is an important aspect that cannot be ignored. Therefore, it is of great significance to carry out vibration reduction design of aircraft under random exciting response. As the engineering application of topology optimization extends from component-level to system-level, large-scale topology optimization has become an international frontier research topic. In recent years, significant progress has been made in large-scale static topology optimization, breaking through the billion-mesh scale and achieving a substantial improvement in structural performance. However, due to the enormous computational cost of dynamic response analysis, large-scale dynamic topology optimization design has not yet been reported. In response to the urgent need for vibration reduction design for aerospace vehicles, this work develops a large-scale high-resolution structural dynamic topology optimization method to obtain excellent vibration reduction structural designs. Firstly, the structure topology optimization system under random exciting response is constructed based on pseudo-excitation method and relative motion method so as to realize vibration reduction design. The adaptive second-order Krylov subspace method and the multi-grid method are combined to solve the problem of stochastic dynamics with high-resolution design, and it is successfully implemented in parallel computing. Some numerical designs validate the validation of the proposed method and show it engineering application ability in aircraft vibration reduction.

Keywords: aircraft vibration reduction; random exciting response; pseudo excitation method; second order Krylov subspace method; topology optimization

1. Introduction

High-speed aircraft can speed up the attack efficiency, with long-range precision attack, high mobility and other characteristics, has become an extremely important development direction in the world's aerospace field. However, the dynamic load failure caused by the complex aerodynamic load environment it faces has always been an urgent problem to be solved in the structural bearing and vibration reduction design [1]. How to carry out the optimization design of aircraft vibration reduction under random load has become a research hotspot.

As early as the 1980s, Adami and Seide [2] et.al. used the time domain numerical simulation method and Galerkin method to solve the random response. Witt and Wentz [3] discussed the mean dynamic response of weakly stationary random excitation and wide-band random acoustic excitation by statistical methods, wherein Wentz also considered the influence of geometric nonlinearity. Over the past decades, many scholars have made large research achievements in different fields of random vibration [4, 5]. Among them, the pseudo excitation method [6,7] can efficiently and accurately analyze the stochastic response of linear time-invariant systems under stationary/non-stationary, fully coherent/partially coherent, uniform modulation/non-uniform modulation evolution random excitation.

With the rapid development of computer technology, structure optimization plays an increasingly important role in configuration design. According to the type of design variables, structure optimization is generally divided into three categories, including size optimization, shape optimization and topology optimization [8]. Among them, the topology optimization method combines numerical simulation and optimization algorithm to optimize the material layout in the design domain to meet the actual performance requirements. This method does not need to rely on existing design experience, and can produce unexpected innovative designs, which is favored by researchers and engineering designers Topology optimization methods have been developed in a variety of ways, including homogenization based methods[9], density-based methods[10, 11], level set methods[12, 13], evolutionary methods[14], evolutionary methods evolutionary methods[15, 16], etc. In recent years, the problem of structural topology optimization under random load has been widely concerned by the academic community, and a series of achievements have been made. Rong et al. [17, 18] first carried out a study on the structural lightweight problem constrained by the stationary random response mean square, based on the asymptotic optimization method[19] and the complete quadratic combination method. Based on the variable density method and the complete quadratic combination method, Zhang et al. [20] realized the topology optimization design of the structure under the combined action of static load and random load. In order to realize the topology optimization design of large-scale structures under random loads, Zhang et al. [21] used pseudo excitation method and modal reduction technology to replace the complete quadratic combination method under the topology optimization framework of variable density method, which greatly improved the efficiency of solving the sensitivity of structural random response in optimization problems.

However, although the topology optimization method considering random response has been successfully applied to some extent, it still faces great challenges in the application of practical engineering problems due to the computational limitations. When analyzing the frequency response of a specific frequency interval, it is necessary to carry out fine frequency dispersion of the frequency interval, especially in the case of formant. Since the essence of frequency response analysis is to solve frequency-dependent linear equations, for a large number of discrete frequency points, the linear equations need to be solved repeatedly at each frequency point. Since the equation matrix is not predecomposed because of the frequency correlation, it brings an unbearable calculation scale. In addition, repeated iterations in topology optimization will further lead to an explosion of computational dimensions, and for non-self-adjoint optimization problems, adjoint equations need to be solved.

To solve this problem, establishing a reduced order model with relatively small degrees of freedom and effective retention of the original system dynamics characteristics becomes an effective means for fast frequency response analysis of large complex structures. Generally, it can be divided into two categories, namely, explicit model reduction method and subspace mapping reduction method.

The basic idea of the explicit model reduction method is to assume a direct explicit function of displacement and frequency with finite unknown parameters, solve the unknown parameters by substituting the frequency response equation, and obtain the explicit frequency response function of displacement. Commonly used explicit functions include Taylor series expansion and Pade series expansion[22]. Among them, Pade expansion method has higher calculation precision. However, due to the large number of matrix conditions caused by computer truncation error and the low accuracy of matrix ill-condition, this method can only provide accurate approximate solution of frequency response in a narrow frequency range.

In order to solve the problem of ill-conditioned matrix in the explicit expansion method, the reduced order method based on subspace mapping has attracted much attention. Modal superposition method[23, 24] is one of the most commonly used model order reduction methods, which first solves the eigenvector of the model, and then projects the original problem into the modal space. However, solving all modes of large-scale eigenvalue problems requires a huge amount of computation and low computational efficiency. Ritz vector method is an alternative non-modal expansion technique. Yoon[25] compared the application of Ritz vector method (RV), quasi-static Ritz vector method (QSRV) and standard Modal displacement method (MDM) in topology optimization. The results show that QSRV method can be used as an alternative ROM scheme which can achieve stable optimization process. Similar to QSRV is the Krylov subspace method[26, 27] ,which forms the basis of many reduced order models. For example, these are commonly used to solve large-scale linear systems and eigenvalue problems. For most dynamic problems, the system is second-order, so the second-order Kryloy subspace method is proposed, which has been successfully applied to the frequency space analysis of large-scale structures[28], acoustic systems[28, 29], etc., but has not been applied to topological optimization.

It is of great importance to develop an efficient random vibration response solution strategy for the topology optimization design of large-scale aircraft vibration reduction. In this paper, the pseudo excitation method and relative motion method are applied to the topology optimization design of structural vibration reduction. Based on the second-order Krylov subspace method, a SOAR method with adaptive addition of extension points is proposed to ensure the universality of the reduced order model, and it is successfully applied to the topology optimization design of structures under random excitation in the broadband domain.

2. Random vibration analysis theory

2.1 Pseudo excitation method

Random vibration means that the vibration size at any time can not be determined in advance, and its waveform changes with time without regular vibration, which can not be expressed by a deterministic function. For example, the vibration of aircraft subjected to complex aerodynamic loads is a typical random vibration. The single test results of random vibration have uncertainty and non-repeatability, but the multiple tests under the same conditions have inherent statistical rules. Generally, it should be described by the method of probability statistics.

The schematic diagram of pseudo excitation method is shown as Figure 1. $S_{xx}(\omega)$ is the self-power spectral density of random excitation $x(t)$, and $H(\omega)$ is the structural frequency response function, then the power spectral density of any output response $y(t)$ is shown at the right end. When a linear system is subjected to a unit harmonic excitation $e^{i\omega t}$, the corresponding response is $He^{i\omega t}$.

If the acceleration power spectral density of the input excitation is $S_{\ddot{x}\ddot{x}}(f)$, when the input virtual excitation amplitude is $\sqrt{S_{\ddot{x}\ddot{x}}}$, the square of the output response is the power spectral density of the output displacement:

$$S_{uu} = |H|^2 S_{\ddot{x}\ddot{x}} = |\tilde{u}|^2 = \tilde{u}^{\mathrm{H}}\tilde{u} \tag{1}$$

Then the output acceleration power spectral density is:

$$S_{\ddot{u}\ddot{u}} = \left|\ddot{u}\right|^2 = \omega^4 S_{uu} \tag{2}$$

The mean square value of the output acceleration is:

$$E\left[\left|\ddot{u}\right|^2\right] = \int_{-\infty}^{+\infty} S_{\ddot{u}\ddot{u}}\left(f\right) df = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S_{\ddot{u}\ddot{u}}\left(\omega\right) d\omega \tag{3}$$

Therefore, the random vibration analysis is essentially a frequency response analysis. This is a means to determine the structural response of a system under simple harmonic load, which is often used in the vibration control design of engineering structures. For general dynamics problems, the equation of state can be written in the following matrix form:

$$\mathbf{M}\ddot{\mathbf{u}}\left(t\right)+\mathbf{C}\dot{\mathbf{u}}\left(t\right)+\mathbf{K}\mathbf{u}\left(t\right)=\mathbf{f}\left(t\right) \tag{4}$$

where $\mathbf{K}$, $\mathbf{C}$ and $\mathbf{M}$ denote the standard global stiffness, damping, and mass matrices, respectively, and $\mathbf{u}\left(t\right)$ is the time-dependent displacement. The dot denotes differentiation with respect to time, and hence $\dot{\mathbf{u}}\left(t\right)$ and $\ddot{\mathbf{u}}\left(t\right)$ represent velocity and acceleration vectors, respectively. Assuming the structure is subjected to a time-harmonic external force $\mathbf{f}\left(t\right)=\mathrm{Re}\left(\mathbf{F}e^{i\omega t}\right)$, the equation of motion can be cast in the frequency space by substituting the solution $\mathbf{u}\left(t\right)=\mathrm{Re}\left(\mathbf{U}\left(\omega\right)e^{i\omega t}\right)$ into equation:

$$\left(-\omega^2\mathbf{M}+i\omega\mathbf{C}+\mathbf{K}\right)\mathbf{U}\left(\omega\right)=\mathbf{F} \tag{5}$$

where $\omega\in\left[\omega_L,\omega_R\right]$ is the excitation frequency, and i is the imaginary number satisfying $i^2=-1$. For convenience, we define the frequency dependent system matrix as

$$\mathbf{S}\left(\omega\right)=\mathbf{K}+i\omega\mathbf{C}\text{-}\omega^2\mathbf{M} \tag{6}$$

Then, formula (5) can be simplified as:

$$\mathbf{S}\left(\omega\right)\mathbf{U}\left(\omega\right)=\mathbf{F} \tag{7}$$

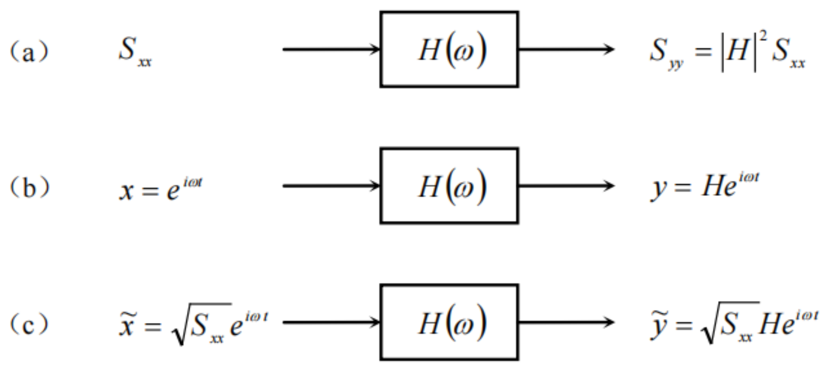


Figure 1 – Schematic diagram of pseudo excitation method.

2.2 Reduced order solution strategy

The reduced order method based on subspace mapping focuses on constructing a set of orthogonal vector basis and expanding into a subspace, and projecting the original model onto the subspace to obtain a reduced order model with greatly reduced number of degrees of freedom, so as to realize fast sweep frequency analysis. Because of the orthogonality of the basis vector of the subspace, the symmetry and positive properties of the original system matrix can be effectively maintained, thus ensuring the accuracy and stability of the reduced order model. In this paper, In this paper, $\mathbf{Q}_n$ is used to represent the reduced subspace, where n represents the dimension of the subspace, which is usually much smaller than the degree of freedom of the original model. Substituting it into the frequency response equation can construct:

$$\left(-\omega^2\mathbf{M}_R + i\omega\mathbf{C}_R + \mathbf{K}_R\right)\mathbf{U}_R\left(\omega\right) = \mathbf{F}_R \tag{8}$$

where

$$\mathbf{K}_R = \mathbf{Q}_n^{\mathrm{H}}\mathbf{K}\mathbf{Q}_n,\ \mathbf{C}_R = \mathbf{Q}_n^{\mathrm{H}}\mathbf{C}\mathbf{Q}_n,\ \mathbf{M}_R = \mathbf{Q}_n^{\mathrm{H}}\mathbf{M}\mathbf{Q}_n,\ \mathbf{F}_R = \mathbf{Q}_n^{\mathrm{H}}\mathbf{F} \tag{9}$$

$$\mathbf{U}\left(\omega\right) = \mathbf{Q}_n^{\mathrm{H}}\mathbf{U}_R\left(\omega\right) \tag{10}$$

The following is the construction of orthogonal basis vectors for Krylov subspace methods, which was originally an iterative method for solving large sparse linear equations and linear eigenvalue problems. It is spanned by a sequence of vectors defined by a linear homogeneous recurrence relation of first order. However, since the equation of the problem under consideration is a second-order system, it needs to be rewritten as a first-order system before applying the Krylov subspace method, i.e. :

$$\left(\begin{bmatrix}\mathbf{C} & \mathbf{K}\\ -\mathbf{I} & \mathbf{0}\end{bmatrix} + i\omega\begin{bmatrix}\mathbf{M} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\right)\begin{bmatrix}i\omega\mathbf{U}\\ \mathbf{U}\end{bmatrix} = \begin{bmatrix}\mathbf{F}\\ \mathbf{0}\end{bmatrix} \tag{11}$$

Where $\mathbf{0}$ and $\mathbf{I}$ represent the zero matrix and the identity matrix respectively. However, this strategy will double the size of the original problem matrix and may destroy good properties of the matrix, such as symmetry and positive character. To solve this problem, a second-order Krylov subspace can be generated at the expansion point:

$$\begin{cases}\mathbf{P}_0\bar{\mathbf{q}}_0 = \mathbf{F}\\ \mathbf{P}_0\bar{\mathbf{q}}_1 = -\mathbf{P}_1\mathbf{q}_0\\ \mathbf{P}_0\bar{\mathbf{q}}_j = -\mathbf{P}_1\mathbf{q}_{j-1} - \mathbf{P}_2\mathbf{q}_{j-2} \quad for\ \ j = 2,...,N-1\end{cases} \tag{12}$$

where

$$\begin{cases}\mathbf{P}_0 = \mathbf{S}(\omega_0) = -\omega_0^2\mathbf{M} + i\omega_0\mathbf{C} + \mathbf{K}\\ \mathbf{P}_1 = \dfrac{\partial\mathbf{S}}{\partial\omega}(\omega_0) = -2\omega_0\mathbf{M} + i\mathbf{C}\\ \mathbf{P}_2 = \dfrac{1}{2}\dfrac{\partial^2\mathbf{S}}{\partial\omega^2}(\omega_0) = -\mathbf{M}\end{cases} \tag{13}$$

Where, $\mathbf{q}_j$ is the basis vector obtained by the orthogonal normalization process of $\bar{\mathbf{q}}_j$. The results show that with the increase of subspace dimension, the generated basis vector is easy to lose orthogonality due to the numerical truncation error brought by the computer, resulting in convergence stagnation. Therefore, it is very important to choose the basis vector orthogonalization method reasonably.

A method to ensure moment matching is to generate orthogonal basis vectors by the Arnoldi strategy proposed by Bai et al. This method is named the second-order Arnoldi method (SOAR), which is expressed in the following pseudo-code format:

Second-order Armoldi method (SOAR)
1. $Solve\ \ \mathbf{P}_0\bar{\mathbf{q}}_0 = \mathbf{F}$
2. $\mathbf{q}_0 = \mathbf{q}_0 / \lVert\mathbf{q}_0\rVert$
3. $\mathbf{p}_0 = 0$
4. $for\ \ j = 1,2,...,N-1, do$
5. $\quad solve\ \ \mathbf{P}_0\mathbf{r} = -\mathbf{P}_1\mathbf{q}_j - \mathbf{P}_2\mathbf{p}_j$
6. $\quad \mathbf{s} = \mathbf{q}_j$

7 $\quad for \;\; i = 0,1,...,j, do$
8. $\quad\quad \mathbf{r} = \mathbf{r} - \langle \mathbf{q}_i, \mathbf{r} \rangle \mathbf{q}_i$
9. $\quad\quad \mathbf{s} = \mathbf{s} - \langle \mathbf{q}_i, \mathbf{r} \rangle \mathbf{p}_i$
10. $\quad end \;\; for$
11. $\quad \mathbf{q}_{j+1} = \mathbf{r} / \|\mathbf{r}\|_2$
12. $\quad \mathbf{p}_{j+1} = \mathbf{s} / \|\mathbf{r}\|_2$
13. $end \;\; for$
14. $Output \;\; \mathbf{Q}_N = span\{\mathbf{q}_0, \mathbf{q}_1, \mathbf{q}_2, ..., \mathbf{q}_{N-1}\}$

where,Lines 1-3 and 5-6 correspond to second-order Krylov processes, and lines 7-10 are for loops that orthogonalize the basis vector.

3. Structural optimization design for random vibration problems

In this paper, the topology optimization design of structural vibration reduction under random excitation is carried out based pseudo excitation method and relative motion method, and the response optimization formula is as follows:

$$\begin{aligned}
&\min_{\rho_e} \quad \max\left(J^i, J^d, J^e\right), \quad J = \sum_k^{N_f} \omega_k^4 \cdot J_k, \quad J_k = \left|\mathbf{U}\left(\omega_k\right)^{\mathrm{H}} \mathbf{L}\mathbf{U}\left(\omega_k\right)\right| \\
&s.t. \quad \sum_{e=1}^{N_e} \bar{\tilde{\rho}}_e^{(d)} v_e / V - V^* \le 0 \\
&\qquad \mathbf{S}\left(\bar{\tilde{\rho}}_e^{(i,d,e)}, \omega_k\right) \mathbf{U}^{(i,d,e)}\left(\omega_k\right) = \mathbf{F}, \quad \left(k = 1,...,N_f\right) \\
&\qquad \mathbf{S}\left(\bar{\tilde{\rho}}_e^{(i,d,e)}, \omega_k\right) = \mathbf{K} + i\omega_k \mathbf{C} - \omega_k{}^2 \mathbf{M} \\
&\qquad 0 \le \rho_e \le 1, \quad \left(e = 1,...,N_e\right) \\
&\qquad \mathrm{c} = \left|\left(\mathbf{F}^s\right)^{\mathrm{T}} \mathbf{U}^s\right| <= 1
\end{aligned} \tag{14}$$

Where, J is the objective function representing the mean square value of the output acceleration. The constraint columns are volume constraint and structural compliance constraint, v_e is the element volume, V^* is the target volume fraction, $\mathbf{F}^s$ and $\mathbf{U}^s$ are the static load and displacement vector respectively. ρ_e is the design variable, N_e represents the number of discrete units, ω is the frequency, and N_f is the number of discrete frequency points.

$$\begin{bmatrix} \mathbf{M}_{ii} & \mathbf{M}_{is} \\ \mathbf{M}_{si} & \mathbf{M}_{ss} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{U}}_i \\ \ddot{\mathbf{U}}_s \end{bmatrix} + \begin{bmatrix} \mathbf{C}_{ii} & \mathbf{C}_{is} \\ \mathbf{C}_{si} & \mathbf{C}_{ss} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{U}}_i \\ \dot{\mathbf{U}}_s \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{ii} & \mathbf{K}_{is} \\ \mathbf{K}_{si} & \mathbf{K}_{ss} \end{bmatrix} \begin{bmatrix} \mathbf{U}_i \\ \mathbf{U}_s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{15}$$

where, $\ddot{\mathbf{U}}_s$, $\dot{\mathbf{U}}_s$ and $\mathbf{U}_s$ are the absolute acceleration, velocity and displacement of the nodes at the supports, $\ddot{\mathbf{U}}_i$, $\dot{\mathbf{U}}_i$ and $\mathbf{U}_i$ represent the absolute acceleration, velocity and displacement of the nodes inside the structure, respectively.

Based on the principle of superposition, the relative motion method divides the absolute displacement $\mathbf{U}_i$ at the non-support position into quasi-static displacement $\mathbf{U}_i^s$ caused by the movement $\mathbf{U}_s^s$ at the support position and dynamic relative displacement $\mathbf{U}_i^d$ caused by the acceleration of the movement $\ddot{\mathbf{U}}_s^s$ at the support position.

$$\begin{bmatrix} \mathbf{U}_i \\ \mathbf{U}_s \end{bmatrix} = \begin{bmatrix} \mathbf{U}_i^d \\ 0 \end{bmatrix} + \begin{bmatrix} \mathbf{U}_i^s \\ \mathbf{U}_s^s \end{bmatrix} \tag{16}$$

where, superscript d and s represent dynamic response and quasi-static response respectively, and subscript i and s represent free node and support node respectively. Support nodes include fixed nodes and nodes that apply excitation.

Ignoring the inertial force and damping force, the quasi-static response satisfies the following formula:

$$\begin{bmatrix} \mathbf{K}_{ii} & \mathbf{K}_{is} \\ \mathbf{K}_{si} & \mathbf{K}_{ss} \end{bmatrix} \begin{bmatrix} \mathbf{U}_i^s \\ \mathbf{U}_s^s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{17}$$

The quasi-static response of the free node can be obtained by solving equation(17):

$$\mathbf{U}_i^s = -\mathbf{K}_{ii} \backslash \mathbf{K}_{is} \mathbf{U}_s^s \tag{18}$$

Substituting formula (16) into formula (15) yields:

$$\begin{aligned} &\begin{bmatrix} \mathbf{M}_{ii} & \mathbf{M}_{is} \\ \mathbf{M}_{si} & \mathbf{M}_{ss} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{U}}_i^s \\ \ddot{\mathbf{U}}_s^s \end{bmatrix} + \begin{bmatrix} \mathbf{C}_{ii} & \mathbf{C}_{is} \\ \mathbf{C}_{si} & \mathbf{C}_{ss} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{U}}_i^s \\ \dot{\mathbf{U}}_s^s \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{ii} & \mathbf{K}_{is} \\ \mathbf{K}_{si} & \mathbf{K}_{ss} \end{bmatrix} \begin{bmatrix} \mathbf{U}_i^s \\ \mathbf{U}_s^s \end{bmatrix} + \\ &\begin{bmatrix} \mathbf{M}_{ii} & \mathbf{M}_{is} \\ \mathbf{M}_{si} & \mathbf{M}_{ss} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{U}}_i^d \\ 0 \end{bmatrix} + \begin{bmatrix} \mathbf{C}_{ii} & \mathbf{C}_{is} \\ \mathbf{C}_{si} & \mathbf{C}_{ss} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{U}}_i^d \\ 0 \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{ii} & \mathbf{K}_{is} \\ \mathbf{K}_{si} & \mathbf{K}_{ss} \end{bmatrix} \begin{bmatrix} \mathbf{U}_i^d \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{aligned} \tag{19}$$

By substituting formula (17) into the above formula, the quasi-static stiffness term is 0, ignoring the quasi-static damping term and focusing on the internal node equation of the structure, the equation can be simplified to:

$$\mathbf{M}_{ii} \ddot{\mathbf{U}}_i^d + \mathbf{C}_{ii} \dot{\mathbf{U}}_i^d + \mathbf{K}_{ii} \mathbf{U}_i^d = -\mathbf{M}_{ii} \ddot{\mathbf{U}}_i^s - \mathbf{M}_{is} \ddot{\mathbf{U}}_s^s \tag{20}$$

By substituting formula (18) into the above formula, the dynamic response satisfies:

$$\begin{cases} \mathbf{S}_{ii} \mathbf{U}_i^d = \mathbf{F}_i \\ \mathbf{F}_i = \left(\mathbf{M}_{ii} \mathbf{K}_{ii} \backslash \mathbf{K}_{is} - \mathbf{M}_{is} \right) \ddot{\mathbf{U}}_s^s \end{cases} \tag{21}$$

Define $\mathbf{U}_i^* = \mathbf{K}_{ii} \backslash \mathbf{K}_{is} \ddot{\mathbf{U}}_s^s$ and substitute it into the above formula to get:

$$\begin{aligned} \mathbf{F} &= \begin{bmatrix} \mathbf{M}_{ii} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{U}_i^* \\ 0 \end{bmatrix} - \begin{bmatrix} 0 & \mathbf{M}_{is} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ \ddot{\mathbf{U}}_s \end{bmatrix} \\ &= \tilde{\mathbf{M}}_{ii} \tilde{\mathbf{U}}_i^* - \tilde{\mathbf{M}}_{is} \tilde{\ddot{\mathbf{U}}}_s^s \end{aligned} \tag{22}$$

The power spectral density of the output displacement can be expressed as:

$$J_k = \left| \mathbf{U}^{\mathrm{H}} \mathbf{L} \mathbf{U} \right| = \left| \left(\mathbf{U}^s + \mathbf{U}^d \right)^{\mathrm{H}} \mathbf{L} \left(\mathbf{U}^s + \mathbf{U}^d \right) \right| \tag{23}$$

Then, the sensitivity of the power spectral density of the output displacement relative to the design variable can be written as:

$$\frac{\partial J_k}{\partial \rho} = 2\left(\mathbf{U}^s + \mathbf{U}^d \right)^{\mathrm{H}} \mathbf{L} \left(\frac{\partial \mathbf{U}^s}{\partial \rho} + \frac{\partial \mathbf{U}^d}{\partial \rho} \right) + 2\boldsymbol{\lambda}_0^{\mathrm{T}} \left(\frac{\partial \mathbf{K}}{\partial \rho} \mathbf{U}^s + \mathbf{K} \frac{\partial \mathbf{U}^s}{\partial \rho} \right) + 2\boldsymbol{\lambda}_1^{\mathrm{T}} \left(\frac{\partial \mathbf{S}}{\partial \rho} \mathbf{U}^d + \mathbf{S} \frac{\partial \mathbf{U}^d}{\partial \rho} - \frac{\partial \mathbf{F}^d}{\partial \rho} \right) \tag{24}$$

where,

$$\lambda_1^{\mathrm{T}} \frac{\partial \mathbf{F}}{\partial \rho} = \lambda_1^{\mathrm{T}} \left(\frac{\partial \tilde{\mathbf{M}}_{ii}}{\partial \rho} \tilde{\mathbf{U}}_i^* + \tilde{\mathbf{M}}_{ii} \frac{\partial \tilde{\mathbf{U}}^*}{\partial \rho} \right) + \lambda_2^{\mathrm{T}} \left(\frac{\partial \tilde{\mathbf{K}}_{ii}}{\partial \rho} \tilde{\mathbf{U}}_i^* + \tilde{\mathbf{K}}_{ii} \frac{\partial \tilde{\mathbf{U}}_i^*}{\partial \rho} \right) \tag{25}$$

Define:

$$\begin{aligned} \boldsymbol{\lambda}_0 &= \mathbf{K} \backslash \left(-\mathbf{L}\bar{\mathbf{U}}\right) \\ \boldsymbol{\lambda}_1 &= \mathbf{S} \backslash \left(-\mathbf{L}\bar{\mathbf{U}}\right) \\ \boldsymbol{\lambda}_2 &= \mathbf{K} \backslash \left(-\tilde{\mathbf{M}}\boldsymbol{\lambda}_1\right) \end{aligned} \tag{26}$$

By substituting it into the formula (24), the power spectral density sensitivity expression of the output displacement can be simplified to:

$$\frac{\partial J_k}{\partial \rho} = 2\,\mathrm{Re}\left(\boldsymbol{\lambda}_0^{\mathrm{T}} \frac{\partial \mathbf{K}}{\partial \rho} \mathbf{U}^s + \boldsymbol{\lambda}_1^{\mathrm{T}} \frac{\partial \mathbf{S}}{\partial \rho} \mathbf{U}^d - \boldsymbol{\lambda}_1^{\mathrm{T}} \frac{\partial \tilde{\mathbf{M}}}{\partial \rho} \tilde{\mathbf{U}}^* - \boldsymbol{\lambda}_2^{\mathrm{T}} \frac{\partial \mathbf{K}}{\partial \rho} \tilde{\mathbf{U}}^* \right) \tag{27}$$

Then the sensitivity of the power spectral density of the output acceleration relative to the design variable is:

$$\frac{\partial J}{\partial \rho} = \sum_{k}^{N_f} \omega_k^4 \cdot \frac{\partial J_k}{\partial \rho} \tag{28}$$

4. Numerical examples

4.1 Damping block design

In order to realize omnidirectional support and vibration reduction, a three-dimensional cube optimization model is designed. The design domain is 0.1m×0.1m×0.1m. The optimized frequency range is 15-2000Hz. Constraints are fixed on the bottom, left side and back of the model, and the input acceleration power spectral density function is shown in Table 1 and Table 2. The optimization objective is to minimize the mean square value of acceleration on the upper surface, right side and front of the structure (optimal vibration damping performance). In order to ensure the bearing performance of the structure, static constraints are applied, and uniform load is applied to the upper surface, right side and front. The resultant force F=500N, and the maximum deformation in x, y and z directions is required not to exceed 10mm. The material properties of dual-material are shown in Table 3.

Table 1 – Acceleration power spectral density function in the X and Y directions.

Frequency (Hz)	15	500	600	900	1000	1200	1400	2000
$S_{\ddot{x}\ddot{x}}(f)$ (g²/Hz)	0.02	0.02	0.05	0.05	0.07	0.07	0.05	0.05

Table 2 – Acceleration power spectral density function in the Z direction.

Frequency (Hz)	15	35	50	300	350	450	500	600
$S_{\ddot{x}\ddot{x}}(f)$ (g²/Hz)	0.05	0.09	0.02	0.02	0.04	0.04	0.1	0.13
Frequency (Hz)	800	850	1200	1300	1600	1700	2000	
$S_{\ddot{x}\ddot{x}}(f)$ (g²/Hz)	0.13	0.06	0.06	0.04	0.04	0.02	0.02	

Table 3 – Material properties.

Materials	Density (kg/m³)	Modulus (MPa)	Poisson's ratio	α	β
Material 1	1120	2000	0.3	1e-6	1e-7
Material 2	1120	1	0.3	1e-5	1e-6

A 24×24×24 grid was used to disperse the design domain, and the volume fraction of material 1 was 0.45. Density filtration is adopted, and the filtering radius is 2.5. Static analysis was carried out to calculate the relationship between the bulk fraction of material 1 and the maximum displacement of the structure under static load and the degree of compliance of the structure. As shown in the following table, in order to ensure that the maximum displacement under static constraint is less than

10mm, the compliance constraint is set from $c \leqslant 1$.

The initial value of the design variable is the total volume fraction. The design variable is updated based on MMA, and the maximum number of iteration steps is 100. The optimization results and control results are shown in the Table 5. Table 6 shows the performance parameters of the 5 groups of results. It can be seen that the optimization results have obvious vibration reduction effect under the bearing conditions, and the output is 20%-30% of the input. The frequency response curves of optimization results and control results are shown in Figure 2.

Table 4 – Material volume fraction and corresponding structural compliance.

Volume fraction of material 1	1	0.2	0.1	0
Maximum displacement (mm)	0.01	0.12	0.26	20
Compliance	0.0036	0.045	0.1	7.2

Table 5 – Optimization results and control results.

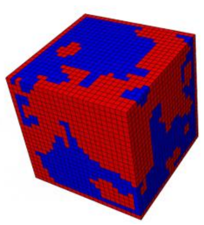
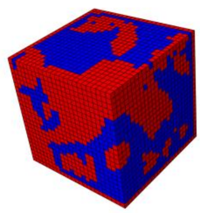
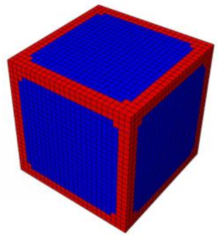
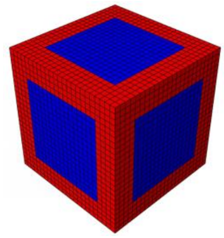
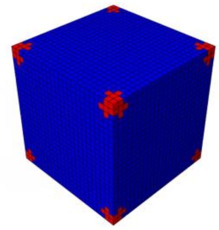
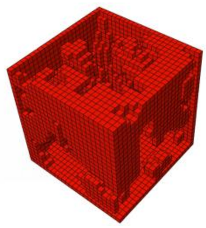
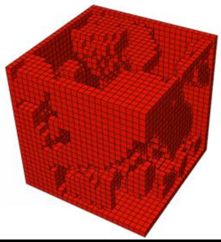
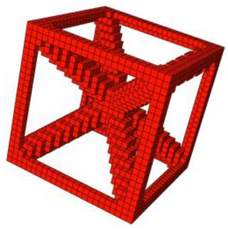
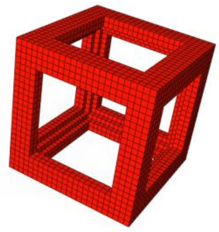
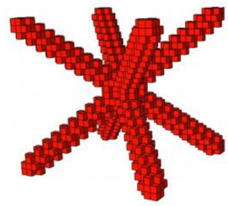
1	2	3	4	5
				
				

Table 6 – The performance parameters of the optimization and comparison results.

Results	1	2	3	4	5
Objective function	1.44	0.974	3.52	4.30	1.85
The proportion of vibration reduction (%)	32.9	22.2	80.2	98.0	42.0
Volume fraction	0.30	0.32	0.25	0.27	0.20
Compliance	0.74	1.82	2.02	1.24	5.96
Maximum displacement (mm)	3.1	9.9	4.3	3.7	18.1

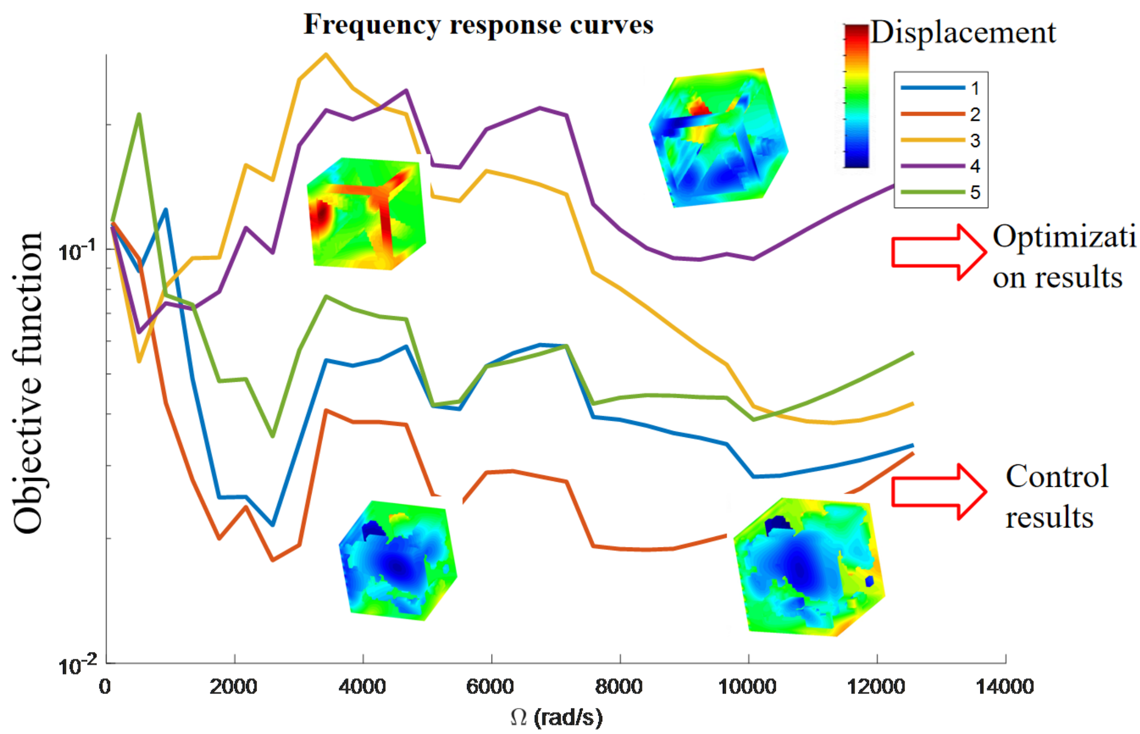


Figure 2 – Frequency response curves of optimization results and control results.

4.2 Reduced order solution strategy

In this work, the topology optimization design for vibration reduction of aircraft wall panels is carried out. The optimized design domain is approximately a rectangular flat plate, as shown in Figure 3. The bottom layer is an undesignable domain, which is fixed and constrained by 8 bolts, and random excitation is applied. The load spectrum value and inflection point frequency are shown in Table 3.

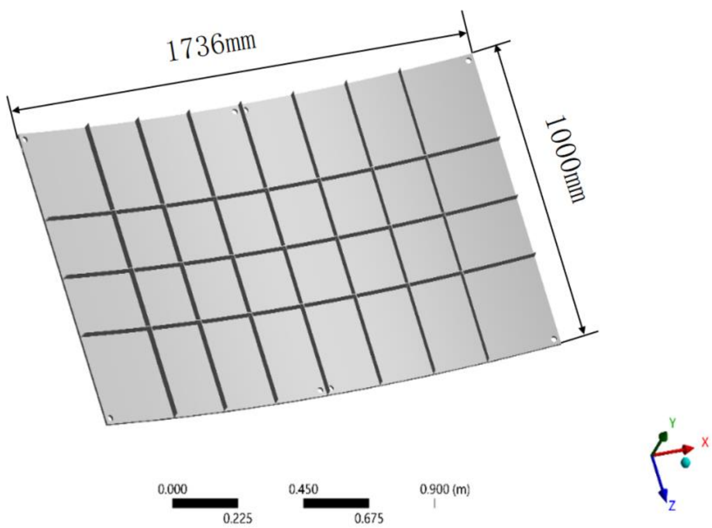


Figure 3 – Aircraft panel model.

Table 7 – Load spectrum value and inflection point frequency.

Inflection point (Hz)	20	89	124	1000	20000
Value (g2/Hz)	0.064	0.064	0.1	0.1	0.025

Aiming at structural stress power spectrum density, this work applies mass and stiffness constraints to carry out random vibration topology optimization design. The optimized topology configuration is post-processed and modeled by software, as shown in Figure 4.

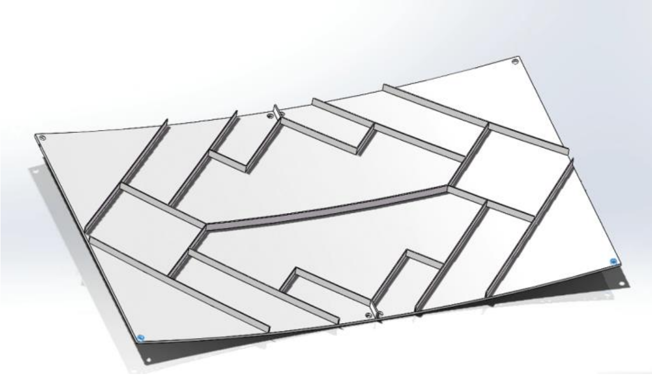


Figure 4 – Topology optimization result.

The vibration reduction performance data of the topology optimization results are shown in Table 4. The peak stress PSD is 6.33e14Pa2 /Hz, which is smaller than 7.63e14Pa2 /Hz of the dimensional optimization solution (Figure 4), and the structural vibration performance is improved by 17%.

Table 8 – Vibration reduction performance.

Max stress （Pa）	Average stress （Pa）	Max strain （m/m）	Stress PSD peak value（Pa2/Hz）
1.12e8	9.11e6	5.13e-4	6.33e14

5. Conclusion

In this paper, a large scale and high resolution structural dynamic topology optimization method is developed for the urgent need of vibration reduction design of aerospace vehicles. (1) Aiming at the random excitation problem, the pseudo excitation method and relative motion method are applied to the topology optimization design of structural vibration reduction. By converting random loads into deterministic virtual excitation, the efficiency of solving random problems is greatly improved, and topology optimization of aircraft vibration reduction design under random loads is successfully realized. (2) In view of the unbearable computational scale brought by the intensive frequency sweeping problem to the optimization design, this paper constructs an adaptive SOAR reduced order model method and successfully applies it to the topology optimization design of structures under random excitation in the broadband domain. This work intends to integrate the solution method established above into the topology optimization software, focusing on the development of reduced order model module and dynamic response analysis module. According to the engineering application background, the optimization objectives and constraint functions will be determined, so as to carry out large-scale dynamic topology optimization design for typical components in aerospace structures.

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7. Copyright Statement

References

[1] Blevins RD, Holehouse I, Wentz KRJJoA. Thermoacoustic loads and fatigue of hypersonic vehicle skin panels. 1993; 30:971-978.

[2] Seide P, Adami CQ. Dynamic stability of beams in a combined thermal-acoustic environment. In; 1983.

[3] Witt M, Sobczyk K. Dynamic response of laminated plates to random loading. International Journal of Solids and Structures 1980; 16(3):231-238.

[4] Lin YKMJM. Probabilistic theory of structural dynamics. 1967.

[5] Roberts JBG, Spanos PD. Random vibration and statistical linearization. In; 1990.

[6] Jiahao L. A fast CQC algorithm of psd matrices for random seismic responses. Computers & Structures 1992; 44(3):683-687.

[7] Lin J, Sun DK, Sun YY, Williams FWJCiNMiE. Structural responses to non-uniformly modulated evolutionary random seismic excitations. 1997; 13:605-616.

[8] Guo X, Cheng G-D. Recent development in structural design and optimization. Acta Mechanica Sinica 2010; 26(6):807-823.

[9] Bendsøe MP, Kikuchi N. Generating optimal topologies in structural design using a homogenization method. Computer Methods in Applied Mechanics and Engineering 1988; 71(2):197-224.

[10]Bendsøe MP. Optimal shape design as a material distribution problem. Structural Optimization 1989; 1(4):193-202.

[11]Zhou M, Rozvany G. The COC algorithm, Part II: Topological, geometrical and generalized shape optimization. Computer Methods in Applied Mechanics and Engineering 1991; 89(1-3):309-336.

[12]Wang MY, Wang X, Guo D. A level set method for structural topology optimization. Computer methods in applied mechanics and engineering 2003; 192(1):227-246.

[13]Allaire G, Jouve F, Toader A-M. Structural optimization using sensitivity analysis and a level-set method. Journal of Computational Physics 2004; 194(1):363-393.

[14]Xie Y, Steven GP. A simple evolutionary procedure for structural optimization. Comput Struct 1993; 49(5):885-896.

[15]Guo X, Zhang W, Zhong W. Doing topology optimization explicitly and geometrically—A new moving morphable components based framework. Journal of Applied Mechanics 2014; 81(8):081009-081012.

[16]Zhou Y, Zhang W, Zhu J, Xu Z. Feature-driven topology optimization method with signed distance function. Computer Methods in Applied Mechanics and Engineering 2016; 310:1-32.

[17]Rong JH, Xie YM, Yang XY, Liang QQ. Topology optimization of structures under dynamic response constraints. Journal of Sound and Vibration 2000; 234(2):177-189.

[18]Rong JH, Tang ZL, Xie YM, Li FY. Topological optimization design of structures under random excitations using SQP method. Engineering Structures 2013; 56:2098-2106.

[19]Xie YM, Steven GP. A simple evolutionary procedure for structural optimization. Computers & Structures 1993; 49(5):885-896.

[20]Qiao Z, Weihong Z, Jihong Z, Tong G. Layout optimization of multi-component structures under static loads and random excitations. Engineering Structures 2012; 43:120-128.

[21]Zhang W, Liu H, Gao T. Topology optimization of large-scale structures subjected to stationary random excitation: An efficient optimization procedure integrating pseudo excitation method and mode acceleration method. Computers & Structures 2015; 158:61-70.

[22]Jensen JS. Topology optimization of dynamics problems with Padé approximants. International Journal for Numerical Methods in Engineering 2007; 72(13):1605-1630.

[23]Ma ZD, Kikuchi N, Hagiwara I. Structural topology and shape optimization for a frequency response problem. Computational Mechanics 1993; 13(3):157-174.

[24]Liu H, Zhang W, Gao T. A comparative study of dynamic analysis methods for structural topology optimization under harmonic force excitations. Structural and Multidisciplinary Optimization 2015; 51(6):1321-1333.

[25]Yoon GH. Structural topology optimization for frequency response problem using model reduction schemes. Computer Methods in Applied Mechanics and Engineering 2010; 199(25):1744-1763.

[26]Demmel JW. Applied numerical linear algebra. SIAM; 1997.

[27]Bai Z, Demmel J, Dongarra J, Ruhe A, van der Vorst H. Templates for the solution of algebraic eigenvalue problems: a practical guide. SIAM; 2000.

[28]Sittl C, Marburg S, Wagner M. Application of a Krylov subspace method for an efficient solution of acoustic transfer functions. Mechanical Systems and Signal Processing 2021; 148:107135.

[29]Panagiotopoulos D, Deckers E, Desmet W. Krylov subspaces recycling based model order reduction for acoustic BEM systems and an error estimator. Computer Methods in Applied Mechanics and Engineering 2020; 359:112755.