MACRO, MESO AND MICRO SCALE ENERGY EVALUATION IN LAMINATED AND COMPOSITE STRUCTURES USING THE COMPONENT-WISE APPROACH

S. Saputo[1], R. Augello[1] & E. Carrera[1]

[1]Mul^2 Lab, Department of Mechanical and Aerospace Engineering, Politecnico di Torino. Corso Duca degli Abruzzi 24, 10129 Torino, Italy.

Abstract

This manuscript examines the deformation energy of fiber-reinforced composites at different scales, from macro-, to meso- and micro- ones. Different scales of the composite material are analyzed using the Equivalent Single-Layer (ESL), the Layer-Wise (LW) and the Component-Wise approaches. To face the problem of developing numerical models of composite structures at different scales, this work makes use of the Carrera Unified Formulation (CUF) provides hierarchical higher-order structural models with flexible expansion polynomials, thereby facilitating the formulation of different models of composite structures. In the context of strain dissipation energies, various integral assessments of deformation energy are introduced to identify the distribution of in-plane and out-of-plane strain dissipation energy regions. These integral quantities are computed using 3D integration sub-domains that span the macro- and micro-volumes of the structure, and can be directly computed for fiber and matrix domains. Through numerical analysis and comparison with Abaqus solid finite element models, the study demonstrates that while the total energy absorbed by a structure under certain loading conditions remains constant, modeling the structure at different scales offers valuable insights into the energy distribution.

Keywords: Energy evaluation; Macro-, meso- and micro- scales; Composite structures; Carrera Unified Formulation

1. Introduction

Composite materials offer significant advantages in performance, efficiency, and cost-effectiveness, leading to their increasing use in aerospace, automotive, and marine engineering. Despite their widespread adoption, unresolved issues remain that necessitate further research to improve design methodologies. Fiber-reinforced composites exhibit multiple length scales, including sub-lamina (fiber and matrix), lamina, and laminate levels. Accurate modeling of these scales and their interactions is crucial for reliable stress field predictions and structural integrity assessments.

Traditional methods like first-order shear deformation theory and higher-order Equivalent Single Layer (ESL) models are effective for macro-scale analysis. In contrast, Layer-Wise (LW) [3] theories provide detailed mesoscale insights, accurately representing displacement and stress distribution through the material's thickness. Numerous microscale theories address challenges in composite design, such as material failure, using techniques like representative volume element (RVE)[9] virtual testing and variational asymptotic methods. However, integrating these theories across scales is complex, often requiring simplifications in the mathematical models.

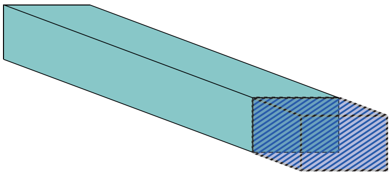
Sophisticated techniques for analyzing laminates include higher-order models [4], trigonometric theories, zigzag models [5], mixed variational theories, and LW methods. While these methods yield accurate macro- and mesoscale stress solutions, they struggle with microscale phenomena. Multiscale approaches, combining macro-scale analysis with detailed micromechanical analysis, have become essential for studying damage and failure in composite materials. Examples include the

Generalized Method of Cells (GMC), the Representative Volume Element (RVE), and the Mechanics of Structure Genome (MSG)[10]. Accurate strain energy evaluation is essential for understanding mechanical behavior and predicting failure modes. Although evaluating the total strain energy absorbed by a structure provides valuable information, it may not reveal detailed structural behavior. A method that includes a detailed analysis at the component level is necessary to identify areas prone to high strain energy accumulation, which often precedes structural failure[7].
Effective management of model sizes to balance accuracy with computational feasibility is crucial. In finite element methods, strain energy is a key metric for assessing structural integrity and identifying stress hotspots. This is vital for designing safer components and predicting material failure under various loading conditions. The Component-Wise (CW) [6] method and advanced 1D hierarchical models are sophisticated tools for multiscale analysis of composite structures. The CW method models different composite components separately but cohesively, using a unified formulation. Initially developed within the Carrera Unified Formulation (CUF) [8], this methodology applies to composite laminates, including layers, fibers, and matrix materials. It has been extended to multi-component aircraft structures, civil structures, and the assessment of failure parameters in composites. CUF methodology enables the evaluation of integral quantities like strain dissipation energy in composite structures, focusing on the integration domain. By dividing the structure into distinct components and analyzing their individual contributions to overall strain energy, CUF provides detailed insights into energy distribution and dissipation.
This study focuses on evaluating strain energy, specifically how energy, calculated as the integral of the product of stress and strain over an elementary volume, behaves under different conditions. Practical examples, such as bending, stretching, and torsion of isotropic beams, are used to validate the approach. Following successful validation against Abaqus software results, the study proceeds to investigate composite structures.

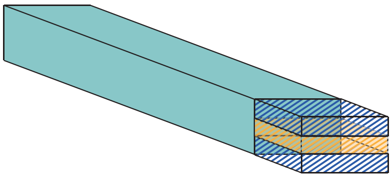
1.1 Energy Evaluation

To accurately identify strain dissipation energy locations within composite structures, it is crucial to assess integral quantities within sub-domains, represented as lines, areas, or volumes encompassing components or interfaces between laminae and fiber-matrix layers. These integral quantities are derived from stress and strain distributions, relying heavily on the precision of these fields. Achieving accurate evaluations of three-dimensional stress and strain fields is challenging, especially with Finite Element (FE) methods, which may yield inaccuracies under complex boundary conditions. Strain energy, a measure of the internal energy stored due to deformation, is an important parameter to evaluate for assessing structural integrity in composites. However, FE models often use homogenized, equivalent elements, which can overlook the distinct properties and failure mechanisms of fibers and matrix materials, leading to inaccurate strain energy distribution predictions. One strategy to improve accuracy is using solid models, but this significantly increases computational demands.
The Carrera Unified Formulation (CUF) methodology allows for representing different modeling scales, depicting a layered structure using Equivalent Single Layer (ESL) Fig. 1a, Layer-Wise (LW) Fig. 1b, and Component-Wise (CW) Fig. 1c approaches. The ESL model treats a multilayered plate as a monolayer plate, while the LW model retains each layer with increased computational demands. The CW model, however, facilitates the prediction of three-dimensional stress fields down to the microscale, enabling reliable evaluations. It preserves the geometrical and material characteristics of each component. Unlike classical approaches, which reduce the structure to a single equivalent entity, the CW method retains the stiffness matrix elements of different components, superimposing them only at the interface to ensure displacement continuity. This study focuses on evaluating integral quantities, such as strain energy, in both global and local regions of the structure. Each independent subdomain of the structure is denoted as V_i, and the strain energy in V_i can be computed as follows:

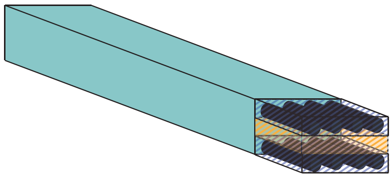
$$
\begin{aligned}
E_i &= \int_{V_i} \sigma^T \varepsilon \, dV_i \\
E_{ip} &= \int_{V_i} \sigma_n \varepsilon_n \, dV_i \qquad n = xx, yy, xy \\
E_{op} &= \int_{V_i} \sigma_m \varepsilon_m \, dV_i \qquad m = zz, xz, yz
\end{aligned}
\tag{1}
$$



(a) Model ESL



(b) Model LW



(c) Model CW

Figure 1 – ESL, LW, and CW approaches offer flexibility in defining distinct sub-volumes. While the ESL model allows defining sub-volumes along the beam axis, the LW model extends this capability to include sub-volumes within various layers, in addition to along the beam axis. On the other hand, the CW model enables the definition of sub-volumes for both the fiber and the matrix, as well as along the beam axis.

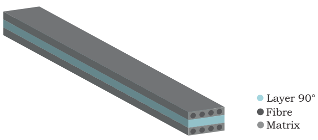
where E_i ,E_{ip}, E_{op} are respectively the total strain energy, the in-plane strain energy and out-plane $(E_o p)$ strain energy contributions evaluated in each defined subvolumes V_i

2. Numerical Results

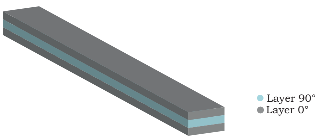
This section introduces the same composite beam discussed in [1], with a stacking sequence of [0/90/0], a length of 40 mm, a height of 0.6 mm, and a width of 0.8 mm. The central layer is treated as homogeneous across all models. Four different models are considered, and they are:

- **Model 0 ESL**: This model involves a macro-scale approach of the complete beam. (Fig. 2a).
- **Model 1 LW**: At the meso-scale level, this model treats the top and bottom layers as homogeneous (similar to the central layer) but with a 90-degree variation in material orientation, utilizing the Layer-Wise (LW) approach (Fig. 2b).
- **Model 2 CW**: This model represents the most refined approach using the Component-Wise (CW) method. In this model, the fibers and the matrix are discretized with independent kinematics. (Fig. 2c).
- **Model 3 LW-CW**: This hybrid model (LW-CW) uses a micro-scale approach for a single subvolume of the lower lamina, while the majority of the volume is discretized using a meso-scale approach (Fig. 2d).

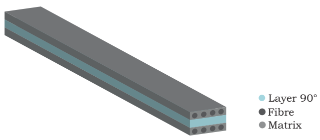
The results are achieved using quadratic elements to individually represent each component of the structure over the cross-section, while FE along the beam axis adopt cubic shape functions.



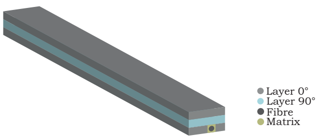
(a) Model 0 ESL



(b) Model 1 LW



(c) Model 2 CW



(d) Model 3 LW-CW

Figure 2 – [0/90/0] cases

We assume that the fiber material exhibits transverse isotropy, meaning its material properties vary only in the plane perpendicular to the fiber axis. Conversely, the matrix material is considered isotropic, implying its properties are uniform in all directions. The engineering constants for the equivalent layer are then calculated using the established hybrid Rules of Mixtures approach (Table 1). As boundary conditions constrain, the beam is clamped at one end, with a force of 1 N applied at the

Table 1 – Material Properties

	Material Properties								
	E_1 [GPa]	E_2 [GPa]	E_3 [GPa]	v_{12}	v_{23}	v_{13}	G_{12} [GPa]	G_{13} [GPa]	$G23_{23}$ [GPa]
Fibre	202.038	12.134	12.134	8.358	8.358	47.756	0.2128	0.2128	0.2704
Layer	103.173	5.145	5.145	2.107	2.107	2.353	0.2835	0.2835	0.3124
Matrix		3.252			0.355			1.2	

other end along the beam axis at the center of the cross-section. To compare the proposed approach with commercial finite element software, a second-order 3D elements model with equivalent geometric and physical characteristics to those described for Model 2 was implemented in Abaqus. The Abaqus model was discretized into 320 divisions along the beam axis, compared to the 40 elements used for the CUF approach across all models.

3. Bending case

Table 2 proposes the results obtained for the various models and compares them with the Abaqus 3D model when bending force is applied. Specifically, stress values are calculated at the point with coordinates l/2, h/6, b/8, where l, h and b represents the length, height and width, respectively. The displacement values are evaluated at the point with coordinates l, h/2, b/2. Additionally, the table reports the strain energy values (for the entire beam) and the number of degrees of freedom for each model.

Table 2 – Displacement and Stresses of crossply beam for different CW and 3D model.

	Abaqus	Model 2	Model 0	Model 1	Model 3
$u_z \times 10^4$ m	-15.53	-15.527	-15.468	-14.91	-14.95
$\sigma_{yy} \times 10^{-6}$ Pa	-5.8597	-5.86478	-5.8488	-2.87962	-5.66517
$\sigma_{yz} \times 10^{-8}$ Pa	-2.2523	-2.5598	-5.7719	-1.67354	-2.81767
Total Energy 10^{-3} J	7.7653	7.7633	7.7342	7.4580	7.4973
Dof	4932459	313995	5445	22869	59169

The stress point values for all models show good approximation to those obtained from the Abaqus model, except for Model 1, which, even if it follows the same trend, is less accurate than the other models. To examine the stress distribution in detail, the stresses were evaluated along the path indicated by the dashed red line in Fig. 3. As shown in the figure, this path, which runs along the middle axis of the beam, intersects two fibers in Models 0 and 2 and a single fiber in Model 3. The results of these stress trends are presented in Fig. 4. The component-wise approach (Model 2 and the 3D Abaqus model) reveals that the most critical area for shear stress is located at the interface between the bottom zone of the matrix and the top zone of the fiber, rather than in the central zone or at the interface of the layers. A comparison of the shear stress trends, σ_{yy} and σ_{yz}, obtained from the CUF-CW model and the Abaqus-3D model, shows a perfect overlap in both trend and values. In contrast, the LW model, although it exhibits a similar trend to the reference, fails to reach the same point values, underestimating the stress magnitudes.

The LW-CW model underscores the importance of a CW model. Given the beam's symmetry with respect to its mid-plane and the applied load, the lack of symmetry in the results highlights the necessity of the CW model for achieving more accurate outcomes.

The stress trend σ_{yy} depicted in Fig. 4a shows that Model 0 can predict stress patterns comparable to more sophisticated models like Model 2 and the reference Abaqus model. However, the stress trend σ_{yz} illustrated in Fig. 4b does not include results for Model 0. This omission is intentional because the reference values are overestimated compared to the results of the other models, which would obscure the clarity of the graph. This behavior is well-documented and emphasized in [2].

While the numerical models yield similar values, there are variations in energy distribution across the beam section and in the energy absorbed by its components. Table 3 outlines these variations,

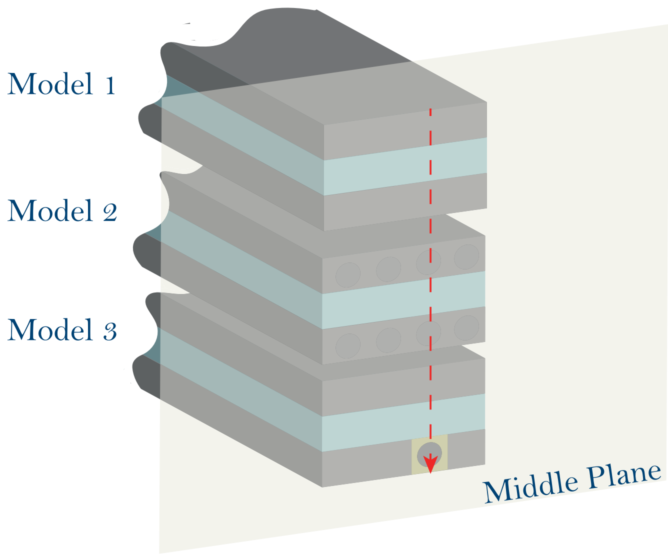
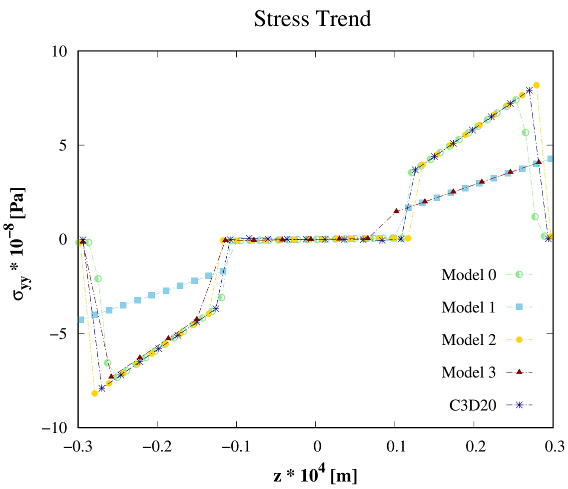
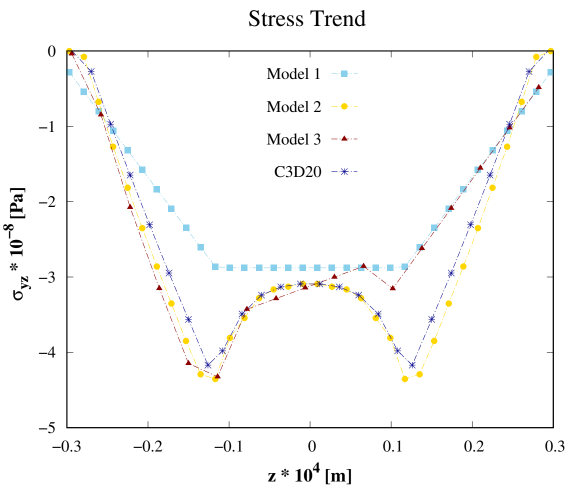


Figure 3 – Path cut for stress trend



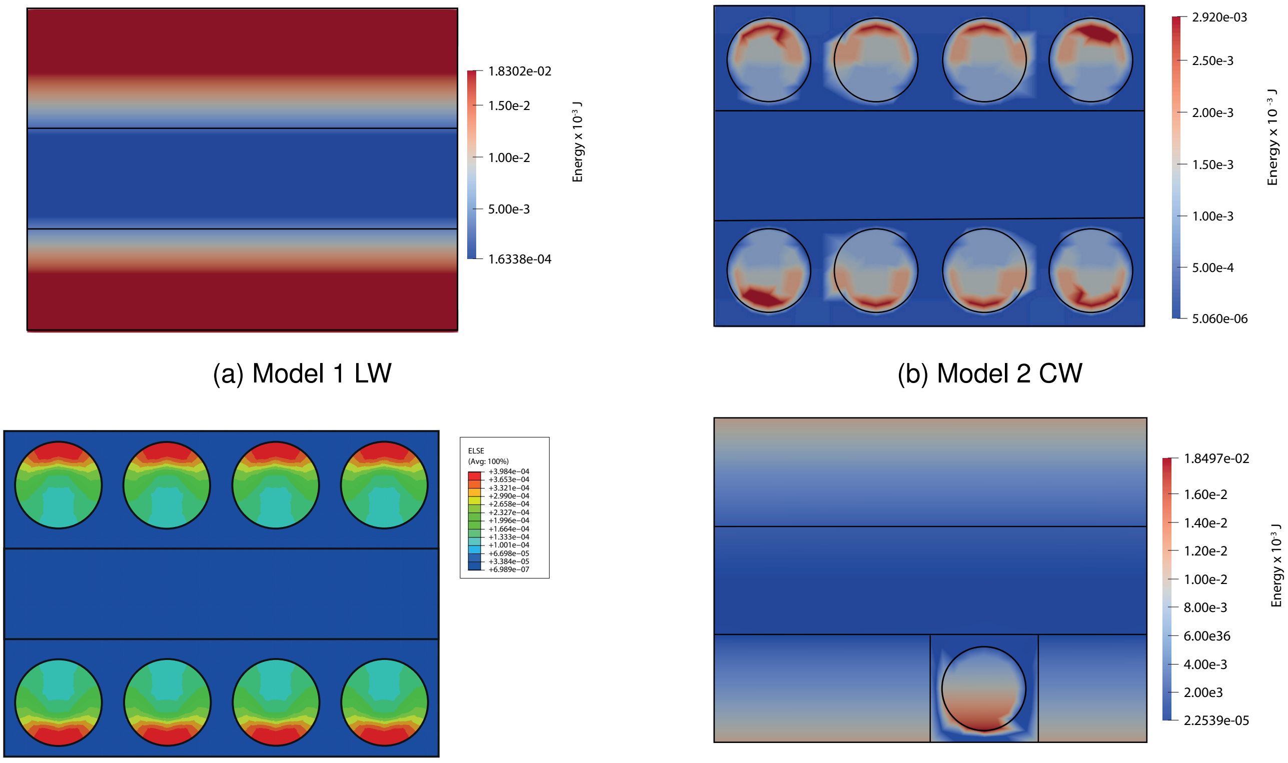
(a) σ_{yy}



(b) σ_{yz}

Figure 4 – Longitudinal and shear stress along the thickness

depicting the trend in energy absorption for Model 1 across its layers, for Model 0 and Model 2 focusing on fiber absorption, and for Model 3 considering absorption by fibers, matrix, and layers. In the CW model, fiber absorption is enumerated clockwise from left to right across the beam section, while in the LW-CW approach, fibers in layer 3 are reported last, except for matrix and fiber discretization. Given the loading conditions and resulting beam deformation, energy absorption from layers or matrices perpendicular to the beam axis is negligible compared to that absorbed by the fibers.
To assess section distribution, a beam cross-section at the beam's midspan (Fig. 5) is considered. While this slicing placement may not highlight maximum energy absorption values, it offers insight into section distribution.



(a) Model 1 LW

(b) Model 2 CW

(c) Abaqus Model

(d) Model 3 LW-CW

Figure 5 – Strain dissipation energy for a section slice

Utilizing a microscale approach (Model 2 and Abaqus model) enables a detailed examination of energy distribution among different components, revealing that deformation energy is primarily concentrated within the fibers, especially in the sub-volumes near the top and bottom of the beam. Similarly, Model 1 demonstrates a symmetric distribution of deformation energy with a linearly increasing gradient, reaching its minimum at the center of the beam section and its maximum at the top and bottom edges. Model 3 exhibits an intermediate trend, validating observations from Models 1 and 2. Choosing an LW strategy over a CW model results in an inaccurate redistribution of deformation energy due to the homogenization of layers, causing the sub-volumes intended for matrix material to absorb more energy than they actually do. Tables 3 and 4 displays global energy absorption values, highlighting which components and models are most significantly involved.

4. Conclusion

This paper provides a comprehensive investigation into the strain energy absorption of fiber-reinforced composite structures using various model scales. Mathematical models if layered beams at different scales, evaluating the results in terms of strain energy integral quantities, are proposed. Macro-, meso- and micro-scales are given by mathematical models based on equivalent single-layer, layer-wise and component-wise approaches, respectively. Every model is formulated by means of the Carrera unified formulation, which makes it possible to derive models at different scale using one-dimensional finite elements, overcoming the high computational cost often related to solid elements of commercial software. The component-wise results exhibit remarkable alignment with those obtained from solid models, demonstrating impressive accuracy while significantly reducing computational costs. Notably, the total degree of freedom (DOF) count in the 1D CW models is approximately fifteen to fourteen times fewer than that of solid models. The CUF model's ability to separately identify the contributions of different stress components to the overall deformation energy is invaluable. This detailed insight can significantly enhance the understanding of complex physical behaviors within composite materials, such as delamination. By isolating the stress components that induce the highest deformation energy, researchers can better understand and predict areas prone to failure. This

	Energy Distribution							
	Model 0 $\times 10^{-3}$ [J]							
Energy					7.7342			
	Model 1 $\times 10^{-3}$ [J]							
	Layer 0°				Layer 90°			Layer 0°
Energy	3.714				0.0302			3.714
	Model 2 $\times 10^{-3}$ [J] CW							
	Fibre 1	**Fibre 2**	**Fibre 3**	**Fibre 4**	**Fibre 5**	**Fibre 6**	**Fibre 7**	**Fibre 8**
Energy	0.9526	0.9526	0.9526	0.9526	0.9526	0.9526	0.9526	0.9526
	Matrix 1	**Matrix 2**	**Matrix 3**	**Matrix 4**	**Matrix 5**	**Matrix 6**	**Matrix 7**	**Matrix 8**
Energy	0.0178	0.0178	0.017	0.0178	0.0178	0.0178	0.0178	0.0178
	Model Abaqus $\times 10^{-3}$ [J] CW							
	Fibre 1	**Fibre 2**	**Fibre 3**	**Fibre 4**	**Fibre 5**	**Fibre 6**	**Fibre 7**	**Fibre 8**
Energy	0.9528	0.9528	0.9528	0.9528	0.9528	0.9528	0.9528	0.9528
	Matrix 1	**Matrix 2**	**Matrix 3**	**Matrix 4**	**Matrix 5**	**Matrix 6**	**Matrix 7**	**Matrix 8**
Energy	0.0178	0.0178	0.0178	0.0178	0.0178	0.0178	0.0178	0.0178
	Model 3 $\times 10^{-3}$ [J]							
	Layer 0°			**Layer 90°**	**Layer 90°**		**Fibre**	**Matrix**
Energy	3.752			0.0310	2.8221		0.8831	0.0774

Table 3 – Energy Distribution for LW model, CW model, LW-CW model

	Energy Distribution			
	Fibre 1-4	**Fibre 2-3**	**Fibre 5-8**	**Fibre 6-7**
E_{ip}	0,95184	0,95178	0,88285	0,95178
E_{op}	0,00928	0,00936	0,00945	0,00936
	Matrix 1-4	**Matrix 2-3**	**Matrix 5-8**	**Matrix 6-7**
E_{ip}	0,017204	0,017401	0,017204	0,017401
E_{op}	0,000596	0,000494	0,000596	0,000494

Table 4 – In plane and Out of plane strain energy contribution for CW model

capability is crucial for advancing the study of delamination and local phenomena.

5. Copyright Statement

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