
# ON AN EFFICIENT GLOBAL/LOCAL STOCHASTIC METHODOLOGY FOR FAILURE PREDICTION OF AIRCRAFT COMPOSITE STRUCTURES

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## Abstract

The quantification of uncertainties in the mechanical response of composite structures can be a demanding task from a computational point of view. This is due both to the number of uncertain parameters in a real study case and the complexity of the model to be analyzed. In this paper, an efficient global/local approach to estimate the uncertainties of the quantities of interest in specific regions of interest with limited computational effort is proposed. This is achieved by refining only locally the model taking advantage of Refined Structural Theories. At the same time, since the variance of the uncertain parameters is usually relatively small, the stochastic analysis is dealt with a sensitivity study carried out both in the global and in the local model. In this way, it is possible to assess the influence of global and local uncertain parameters in the same sub-modeling analysis. The results of the applications shown focus on obtaining an accurate probabilistic distribution of the stress field that can be later used in failure criteria to evaluate the subsequent distribution of the failure index. A good correlation with the reference Monte Carlo simulations is shown. For a more detailed description of the followed steps and results, the reader is encouraged to refer to [1].

**Keywords:** Global/Local Analysis, Stochastic Finite Element Method, Carrera Unified Formulation, Uncertainty Quantification and Propagation

## 1. Introduction and theoretical background

The possibility of quantifying the uncertainty in engineering design is of fundamental importance for the Aerospace industry. An uncertainty quantification study generally requires three different steps [2]:

1. Identification of the sources of uncertainty present in the system
2. The definition of the computational model.
3. The propagation of the uncertainties through the model.

The focus of this paper is on the uncertainty propagation part, with the aim to upscale it to situations of industrial interest. The main drawback of probabilistic studies consists in the fact that they tend to be computationally expensive for any real case scenario. This is due on one hand to the number of random variables involved, and on the other hand to the number of degrees of freedom.

In this study, this aspect is counteracted combining together three different frameworks: refined structural theories using the Unified Formulation [3] (Section 1.1), global/local approaches (Section 1.2) and a Stochastic Finite Element Analysis based on a perturbation approach [2] (Section 1.3). In the following section a brief theoretical background of these frameworks is given. Then, the main points of the methodology used are outlined (Section 2.), followed by some applications (Section 3.).

## 1.1 Refined Structural Theories

In Finite Element Analysis (FEA), achieving accurate stress fields for assessing damage initiation and propagation typically requires fine solid discretization, which increases the computational cost. By employing structural elements like plates and shells, their kinematic properties can be utilized to develop specific structural theories. However, classical structural theories offer a relatively limited displacement field representation. Neglected displacement components become significant, especially when estimating the three-dimensional stress field is crucial, such as for local effect assessments and precise damage and failure analysis. Therefore, refined theories with enhanced interpolation capabilities have been developed.

Implementing these refined structural theories can be challenging due to the different interpolation methods of the displacement field compared to standard solid models. To address this, a framework based on the Carrera Unified Formulation (CUF) can be used. Although this work focuses on higher-order plate theories, similar formulations exist for beams and shells [3]. Within the CUF framework, the displacement field $u$ for a plate is expressed as follows:

$$u(x,y,z) = F_\tau(z)N_i(x,y)q_{\tau i} \tag{1}$$

where summation over repeated indices is implied. The shape functions $N_i(x,y)$ are used for in-plane discretization, while $F_\tau(z)$ is used for the through-thickness direction. The array $q_{\tau i}$ contains the degrees of freedom. From the Principle of Virtual Displacements (PVD), it can be shown that the internal elastic work $W_{int}$ is written in this compact form:

$$\delta W_{int} = \delta q_{sj}^\top k^{\tau sij} q_{\tau i} \tag{2}$$

where $\delta(\bullet)$ denotes the virtual variation and $k^{\tau sij}$ is a $3\times 3$ matrix known as the Fundamental Nucleus (FN), which is independent of the specific in-plane and out-of-plane expansions used. By knowing $k^{\tau sij}$, elements of user-defined order can be constructed using simple nested loops, as illustrated in Figure 1 [3].

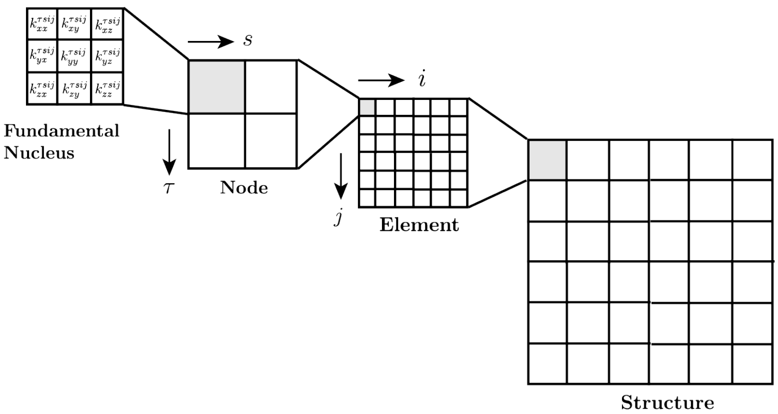


Figure 1 – *CUF assembly procedure* [3].

For layered composite plates, two distinct approaches can be identified: Equivalent Single Layer (ESL) and Layer-Wise (LW) models.

In the ESL approach, the displacement field is defined across the entire thickness, regardless of the number of layers. Various methods can be used, but here a Taylor-like expansion is considered:

$$\begin{aligned} u_x(x,y,z) &= q_{x0}(x,y) + zq_{x1}(x,y) + z^2 q_{x2}(x,y) + \cdots \\ u_y(x,y,z) &= q_{y0}(x,y) + zq_{y1}(x,y) + z^2 q_{y2}(x,y) + \cdots \\ u_z(x,y,z) &= q_{z0}(x,y) + zq_{z1}(x,y) + z^2 q_{z2}(x,y) + \cdots \end{aligned} \tag{3}$$

Thus, the set of expansion functions used in this case is: $F_\tau = \{1, z, z^2, \cdots\}$. Since this expansion spans the entire thickness, the number of element degrees of freedom remains independent of the number of layers because each ply contributes to the same Fundamental Nucleus (FN).

LW theories, on the other hand, assign degrees of freedom to each individual layer. Consequently, these models are generally more computationally intensive than ESL models because adding layers increases the size of the structural matrices. In this paper, a Lagrange expansion is used for LW discretization. This approach is beneficial for modeling because it only involves displacement degrees of freedom, making it straightforward to impose essential boundary conditions.

## 1.2 Global/Local Methods

Global/local techniques are used to refine a model locally, as fine discretization over a large domain is generally impractical. In this study, a sub-modelling approach is employed: the displacements from the global solution are imposed on the boundary of a local region. Hence, the global and local models are treated as separate entities, allowing flexible refinement of the global solution in regions of interest after its initial completion. This study focuses on failure initiation, assuming no significant stiffness change between global and local solutions. Therefore, a straightforward one-way global/local approach is considered sufficiently accurate. Otherwise, corrections to the global results would be necessary, as discussed in [4]. In this context, the global models will utilize Equivalent Single Layer (ESL) and First Order Shear Deformation (FSDT) theories, while the local discretizations will employ Layer-Wise (LW) models with Lagrange shape functions. Since there is generally no direct correspondence of degrees of freedom on the boundary between the models, the displacement field of the global model must be interpolated at the locations of the local degrees of freedom. This interpolation is necessary for both in-plane and out-of-plane directions. For instance, in the case of FSDT plate elements (with displacement field defined as in [5]), using linear in-plane interpolation (4-node elements), the displacements to enforce on the local boundary are computed as shown in Figure 2.

$$\begin{cases} u = u^* + z\phi_x^* \\ v = v^* + z\phi_y^* \\ w = w^* \end{cases} \tag{4}$$

with $[u^*, v^*, w^*, \phi_x^*, \phi_y^*]^\top$ linearly interpolated between $[u_1, v_1, w_1, \phi_{x1}, \phi_{y1}]^\top$ and $[u_2, v_2, w_2, \phi_{x2}, \phi_{y2}]^\top$. Note that appropriate rotations should be carried out depending on the reference systems used for the global and local models.

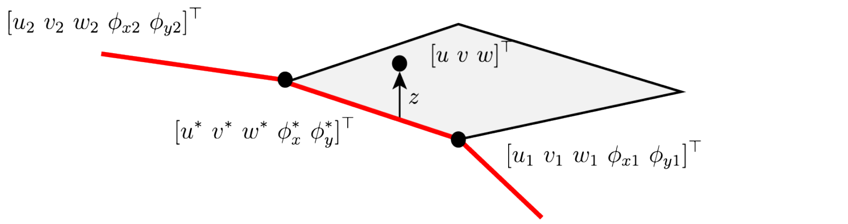


Figure 2 – *Interpolation of degrees of freedom in submodelling approach.*

## 1.3 Uncertainty Quantification with Perturbation Method

The response of a system for uncertainty quantification analysis can be derived from its Taylor expansion around the mean values of the random variables. This concept underpins the Perturbation Method, which has been explored by various authors and is sometimes referred to as Probabilistic FEM (PFEM) [6] or Stochastic FEM (SFEM) [7, 8]. The primary advantage of this method is its simplicity compared to other techniques, relying on first and second-order derivatives with respect to the random variables. These sensitivities can be efficiently computed using standard approaches, as will be detailed later.

However, the main limitation of this method is that it typically can only handle random variables with relatively small coefficients of variation (around 10-15%). For higher variances, the results may become inaccurate. Despite this limitation, the Perturbation Method is considered a robust methodology in many engineering applications where the variances are generally small.

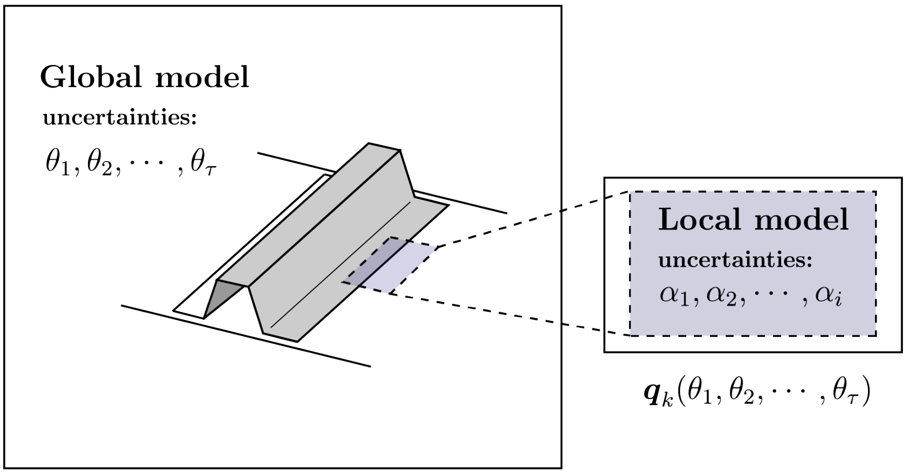


Figure 3 – *Global model uncertainties $\theta_\tau$ and local model uncertainties $\alpha_i$.*

According to [9], consider a generic linear system $Kq = f_{ext}$, which depends on random variables $\alpha_i$ with zero mean. The model can be approximated using a second-order expansion around these random variables:

$$K = K^0 + \sum_i \frac{\partial K}{\partial \alpha_i}\alpha_i + \frac{1}{2}\sum_{ij}\frac{\partial^2 K}{\partial \alpha_i \partial \alpha_j}\alpha_i\alpha_j + o(\|\alpha\|^2)$$

$$q = q^0 + \sum_i \frac{\partial q}{\partial \alpha_i}\alpha_i + \frac{1}{2}\sum_{ij}\frac{\partial^2 q}{\partial \alpha_i \partial \alpha_j}\alpha_i\alpha_j + o(\|\alpha\|^2) \quad (5)$$

$$f_{ext} = f_{ext}^0 + \sum_i \frac{\partial f_{ext}}{\partial \alpha_i}\alpha_i + \frac{1}{2}\sum_{ij}\frac{\partial^2 f_{ext}}{\partial \alpha_i \partial \alpha_j}\alpha_i\alpha_j + o(\|\alpha\|^2)$$

where $(\bullet)^0$ represents the mean value (that is, for $\alpha_i = 0$) and the derivatives are evaluated at $\alpha_i = 0$. Now, substituting these expansions in $Kq = f_{ext}$ and equating terms of the same order it is possible to obtain:

$$q^0 = (K^0)^{-1} f_{ext}^0$$

$$\frac{\partial q}{\partial \alpha_i} = (K^0)^{-1}\left(\frac{\partial f_{ext}}{\partial \alpha_i} - \frac{\partial K}{\partial \alpha_i}q^0\right) \quad (6)$$

$$\frac{\partial^2 q}{\partial \alpha_i \partial \alpha_j} = (K^0)^{-1}\left(\frac{\partial^2 f_{ext}}{\partial \alpha_i \partial \alpha_j} - \frac{\partial K}{\partial \alpha_i}\frac{\partial q}{\partial \alpha_j} - \frac{\partial K}{\partial \alpha_j}\frac{\partial q}{\partial \alpha_i} - \frac{\partial^2 K}{\partial \alpha_i \partial \alpha_j}q^0\right)$$

And the displacement field can now be expressed as a second order function of the random variables from Eq.(5b).

## 2. Methodology for Stochastic Global/Local Analysis

In this section, the workflow of the proposed stochastic global-local analysis is detailed. Since this is a first-order method based on a perturbation approach, only the sensitivities with respect to the random variables are needed. Section 2.1 describes how these sensitivities can be efficiently calculated for the global model. Subsequently, Section 2.2 presents the solution for the local model, taking into account the uncertainties in both the global and local models.

As illustrated in Figure 3, a generic structural system representing the global model is influenced by uncertainties $\theta_\tau$. The local model, which enforces displacements $q_k(\theta_\tau)$ on its boundaries from the global simulation, is characterized by uncertainties $\alpha_i$. It is important to note that the uncertainties $\theta_\tau$ and $\alpha_i$ can be either of the same type or different types. This allows for differentiation of uncertainties between the global and local models, enabling to differentiate uncertainties for features modeled locally but not globally, such as holes and details of connections.

## 2.1 Global sensitivity study

Restricting the Perturbation Method explained in Section 1.3 to a linear expansion, only the sensitivities of the displacement field w.r.t. the random variables are necessary. Therefore, it is possible to employ standard methods to efficiently obtain the sensitivities without relying on finite differences, which generally lead to more expensive computations and to problems related to the step size to be used. The so-called Direct Differentiation Method (DDM) [8] is an efficient framework to employ and couple with the present stochastic analysis. The focus here will be limited to linear and non-linear elastic systems and the procedure employed to compute the sensitivities follows the explanation given in [8]. Considering the global model, we aim to compute its sensitivities w.r.t. the random variables to be later used for the local analysis. Employing a Newton-Raphson scheme, the residual at the $i^{\text{th}}$ iteration of pseudo-time $t+\Delta t$ is:

$$ {}^{t+\Delta t}R_i({}^{t+\Delta t}q_i) = {}^{t+\Delta t}f_{ext} - f_{int}({}^{t+\Delta t}q_i) \tag{7} $$

assuming that the external forces $f_{ext}$ do not depend on the degrees of freedom $q$, while the internal forces $f_{int}$ do. The displacement field is updated as:

$$ {}^{t+\Delta t}K_i^{(T)} \Delta q_{i+1} = {}^{t+\Delta t}R_i \tag{8} $$

with ${}^{t+\Delta t}K_i^{(T)}$ the tangent matrix given by:

$$ {}^{t+\Delta t}K_i^{(T)} = -\frac{\partial^{t+\Delta t}R_i}{\partial q} = \frac{\partial^{t+\Delta t}f_{int}({}^{t+\Delta t}q_i)}{\partial q} \tag{9} $$

At convergence of the Newton-Raphson scheme the residual is approximately null. Differentiating w.r.t. $\theta_\tau$, since ${}^{t+\Delta t}R$ depends both implicitly and explicitly on $\theta_\tau$, it is possible to obtain

$$ {}^{t+\Delta t}K^{(T)} \frac{\partial^{t+\Delta t}q}{\partial \theta_\tau} = \frac{\partial^{t+\Delta t}f_{ext}}{\partial \theta_\tau} - \left.\frac{\partial^{t+\Delta t}f_{int}}{\partial \theta_\tau}\right|_{q \neq q(\theta_\tau)} \tag{10} $$

where the notation $(\bullet)|_{q\neq q(\theta_\tau)}$ means that the partial differential of the internal forces has to be computed considering $q$ as independent from $\theta_\tau$.

## 2.2 Local sensitivity study

Extending the ideas introduced in Section 1.3, we consider the application of the Perturbation Method for the local study. The FE discretized linear system for the local model is:

$$ \begin{bmatrix} K_{kk} & K_{ku} \\ K_{uk} & K_{uu} \end{bmatrix} \begin{bmatrix} q_k \\ q_u \end{bmatrix} = \begin{bmatrix} f_{ext_k} \\ f_{ext_u} \end{bmatrix} \tag{11} $$

with $(\bullet)_k$ the *known* degrees of freedom (all assumed to be on the boundary and enforced from the global solution) and $(\bullet)_u$ the *unknown* degrees of freedom. The dependence upon the random variables is modelled as in Eq.(12) with $\alpha_i$, $\theta_\tau$ zero-mean random variables and all the derivatives are assumed to be evaluated at $\alpha_i, \theta_\tau = 0$.

$$ \begin{aligned} K &= K^0 + \sum_i \frac{\partial K}{\partial \alpha_i}\alpha_i + \frac{1}{2}\sum_{ij}\frac{\partial^2 K}{\partial \alpha_i \mathrm{d}\alpha_j}\alpha_i\alpha_j && \Big\}\ \text{function of } \alpha_i \\ q_k &= q_k^0 + \sum_\tau \frac{\partial q_k}{\partial \theta_\tau}\theta_\tau + \frac{1}{2}\sum_{\tau s}\frac{\partial^2 q_k}{\partial \theta_\tau \partial \theta_s}\theta_\tau\theta_s && \Big\}\ \text{function of } \theta_\tau \\ f_{ext_u} &= f^0_{ext_u} + \sum_i \frac{\partial f_{ext_u}}{\partial \alpha_i}\alpha_i + \frac{1}{2}\sum_{ij}\frac{\partial^2 f_{ext_u}}{\partial \alpha_i \partial \alpha_j}\alpha_i\alpha_j && \Big\}\ \text{function of } \alpha_i \end{aligned} \tag{12} $$

The unknown degrees of freedom are a function of the global and local uncertainties as:

$$
\begin{aligned}
q_u = q_u^0 + & \overbrace{\sum_i \frac{\partial q_u}{\partial \alpha_i}\alpha_i + \sum_\tau \frac{\partial q_u}{\partial \theta_\tau}\theta_\tau}^{\text{linear dependencies on random variables}} + \\
& \underbrace{+\frac{1}{2}\sum_{ij}\frac{\partial^2 q_u}{\partial \alpha_i \partial \alpha_j}\alpha_i\alpha_j + \frac{1}{2}\sum_{\tau s}\frac{\partial^2 q_u}{\partial \theta_\tau \partial \theta_s}\theta_\tau\theta_s + \frac{1}{2}\sum_{i\tau}\frac{\partial^2 q_u}{\partial \alpha_i \partial \theta_\tau}\alpha_i\theta_\tau}_{\text{quadratic dependencies on random variables}}
\end{aligned} \tag{13}
$$

The unknowns of the problem are $q_u^0$, $\mathrm{d}q_u/\mathrm{d}\theta_\tau$, $\mathrm{d}q_u/\mathrm{d}\alpha_i$, $\mathrm{d}^2q_u/\mathrm{d}\theta_\tau\mathrm{d}\theta_s$, $\mathrm{d}^2q_u/\mathrm{d}\alpha_i\mathrm{d}\alpha_j$, $\mathrm{d}^2q_u/\mathrm{d}\alpha_i\mathrm{d}\theta_\tau$, i.e., the deterministic solution, the sensitivities w.r.t. the random variables, and the second-order derivatives. Similarly as before, substituting into the equilibrium equations:

$$K_{uk}q_k + K_{uu}q_u = f_{ext_u} \tag{14}$$

it is possible to equate terms with the same order.
Once all the unknowns are computed, the full displacement field can be directly related to the random variables using Eq.(13). It is important to note that no assumptions were made regarding the form of $\alpha_i$ and $\theta_\tau$, allowing for the simulation of any type of distribution. Since the degrees of freedom are now known as a quadratic function of the random variables, the output distribution can theoretically be computed in closed form using standard approaches if the input distributions are known [10]. However, because we are usually interested in the distributions of the stress field or other quantities of interest, it is more practical to perform a Monte Carlo Simulation (MCS) with the expression for displacement given in Eq.(13). While this slightly increases the computational effort during post-processing, the cost is typically negligible since Eq.(13) is a simple polynomial.
The methodology presented so far would be second-order accurate, requiring the second-order derivatives of the displacement field of the global solution (to determine $\partial^2 q_k/\partial\theta_\tau\partial\theta_s$). Generally, these quantities are not readily available and can be computationally expensive to obtain, especially when dealing with multiple random variables. Moreover, in many engineering systems, variability is quite limited, making a first-order expansion sufficient and allowing the direct use of results from the global sensitivity study.
This workflow is evaluated by comparing the resulting probability density function (PDF) of the quantities of interest with those obtained from MCS, either with or without submodelling, depending on the computational cost. In the first scenario, the full refined model is simulated multiple times with randomly sampled uncertain parameters. In the second scenario, both the global and local models are simulated each time, with the local model driven by the global results. It is crucial that the same sampled value for the uncertain material parameter is used in both models, if it is common to both.

## 3. Application

In this section, the framework presented before is applied to a simple plate with an opening, being both the global and local models linear elastic. The focus will be in obtaining the probabilistic distributions of the stresses and the corresponding failure indices for the chosen failure criterion.

Table 1 – *IM7-8552 elastic, strength and interface properties* [11], [12].

| $E_1$, GPa | $E_2$, $E_3$ GPa | $G_{12}$, GPa | $G_{23}$, GPa | $\nu_{12}$ | $\nu_{23}$ |
|---|---|---|---|---|---|
| 171.42 | 9.08 | 5.29 | 3.92 | 0.32 | 0.487 |
| $X_T$, MPa | $X_C$, MPa | $Y_T$, MPa | $Y_C$, MPa | $S_L$, MPa | $S_T$, MPa |
| 2323.5 | 1017.5 | 62.3 | 253.7 | 89.6 | 62.3 |
| $Y_{BC}$, MPa | $Y_{BT}$, MPa | $G_{Ic}$, kJ/m$^2$ | $G_{IIc}$, kJ/m$^2$ | $\eta$ (BK) | |
| 600.0 | 38.7 | 0.28 | 0.79 | 1.45 | |

The properties of IM7-8552 (Table 1) were used.
The plate with a circular opening is studied as detailed in Figure 4 where the boundary conditions used are specified. On the upper edge a uniform displacement along the $y$-axis $\bar{u}_y$ is imposed. A uniform pressure $p$ is also applied on the whole plate domain. The methodology described in the

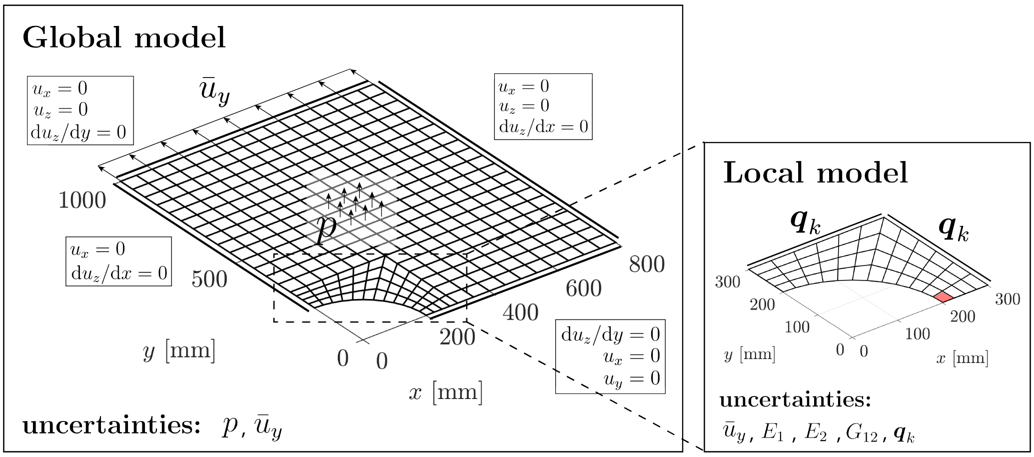


Figure 4 – *Panel with opening under tension and out-of-plane pressure. The element of interest in the local model is highlighted in red.*

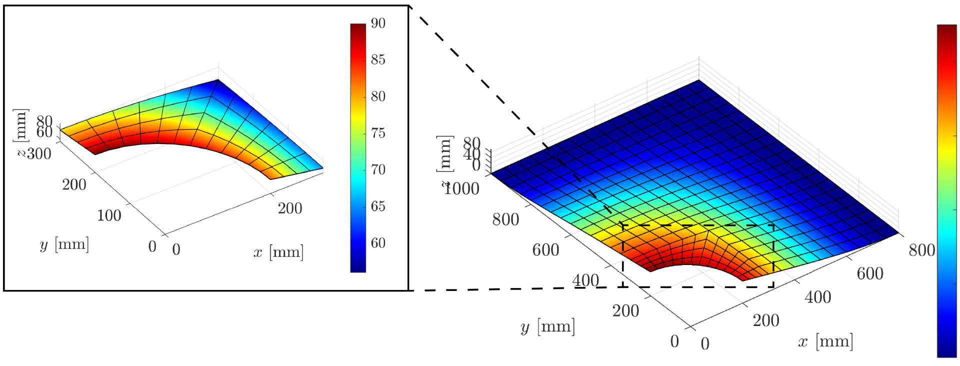


Figure 5 – *Deformed shapes for global and local models for the study case of the plate with opening.*

previous section is applied for this linear elastic study case to obtain stochastic information of the stress field near the opening.

Taking advantage from the global/local framework described earlier, the analysis is refined in a specific region of interest around the opening. This refinement is achieved in two ways. First, the computational model of the local domain is made finer and has higher interpolation capabilities compared to the global model. For the global model, an Equivalent Single Layer (ESL) plate model with a linear expansion for each displacement component (6 degrees of freedom per node) is used. In contrast, the local model employs a Layer-Wise (LW) plate model with a cubic Lagrange expansion for each ply. Second, the local model is refined in terms of the random variables considered. In the global domain, only the applied boundary conditions ($p$ and $\bar{u}y$) are treated as stochastic. In the local domain, however, some elastic properties ($E_1$, $E_2$, $G12$) are also considered random variables. This simplification reduces the computational effort required for the global stochastic analysis. Note that the applied pressure affects both the local and global domains, so this random variable is consistent across both regions. A summary of the model specifications is provided in Table 2, and details about the random variables used are specified in Table 3.

Table 2 – *Model specifications for the study case of the plate with opening.*

| | Global model | Local model |
|---|---|---|
| Layup | $[0/45/-45/90]_s$ | |
| Ply thickness | 0.125 mm | |
| Model | ESL (6 DOF per node) | LW (cubic Lagrange per ply) |
| Random variables | $p$, $\bar{u}_y$ | $q_k$, $p$, $E_1$, $E_2$, $G_{12}$ |

Table 3 – *Random variables for the study case of the plate with opening.*

| Random variable | Mean | Coefficient of Variation | Distribution |
|---|---|---|---|
| $p$ | $5\,10^{-5}$ MPa | 0.1 | Normal |
| $\bar{u}_y$ | 1 mm | 0.1 | Normal |
| $E_1$ | 171.42 GPa | 0.1 | Normal |
| $E_2$ | 9.08 GPa | 0.1 | Normal |
| $G_{12}$ | 5.29 GPa | 0.1 | Normal |

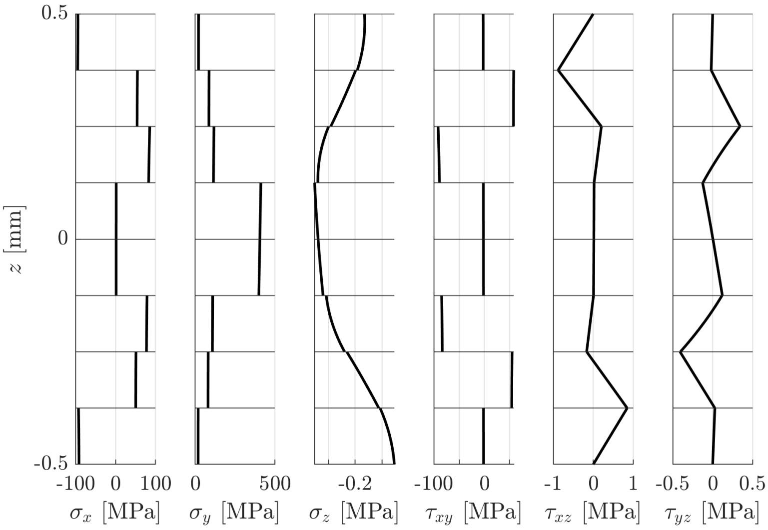


Figure 6 – *Deterministic through-thickness stress distribution for the element of interest.*

The results of the deterministic analysis are reported in Figure 5, where the deformations are shown, and in Figure 6 where the stress distribution at the centroid of the element of interest (Figure 4) is presented. Note that, since the pressure is applied only on the upper surface of the plate, from equilibrium $\sigma_z$ should be null on the lower surface. However, due to the fact that the model is not refined enough, this effect cannot be captured.

It is worth to mention that the way the displacements are enforced on the local domain boundary can be similarly followed also for the sensitivities, hence building the terms $\partial q_k/\partial\theta_\tau$. For the current study case, the displacement field of the global model is expressed as a function of the random variables as:

$$q = q^0 + \frac{\partial q}{\partial p}p + \frac{\partial q}{\partial \bar{u}_y}\bar{u}_y \tag{15}$$

At the same time, the first order ESL plate model allows to write:

$$\begin{aligned} u_x(x,y,z) &= q_{x0}(x,y) + z\,q_{x1}(x,y) \\ u_y(x,y,z) &= q_{y0}(x,y) + z\,q_{y1}(x,y) \\ u_z(x,y,z) &= q_{z0}(x,y) + z\,q_{z1}(x,y) \end{aligned} \tag{16}$$

where, for example:

$$q_{x0} = q_{x0}^0 + \frac{\partial q_{x0}}{\partial p}p + \frac{\partial q_{x0}}{\partial \bar{u}_y}\bar{u}_y \tag{17}$$

So the displacement is expressed as (taking only $u_x$ as illustrative example):

$$\begin{aligned} u_x &= \left(q_{x0}^0 + \frac{\partial q_{x0}}{\partial p}p + \frac{\partial q_{x0}}{\partial \bar{u}_y}\bar{u}_y\right) + z\left(q_{x1}^0 + \frac{\partial q_{x1}}{\partial p}p + \frac{\partial q_{x1}}{\partial \bar{u}_y}\bar{u}_y\right) \\ &= \underbrace{(q_{x0}^0 + z q_{x1}^0)}_{q_x^0} + \underbrace{\left(\frac{\partial q_{x0}}{\partial p} + z\frac{\partial q_{x1}}{\partial p}\right)}_{\partial q_x/\partial p} p + \underbrace{\left(\frac{\partial q_{x0}}{\partial \bar{u}_y} + z\frac{\partial q_{x1}}{\partial \bar{u}_y}\right)}_{\partial q_x/\partial \bar{u}_y} \bar{u}_y \end{aligned}$$

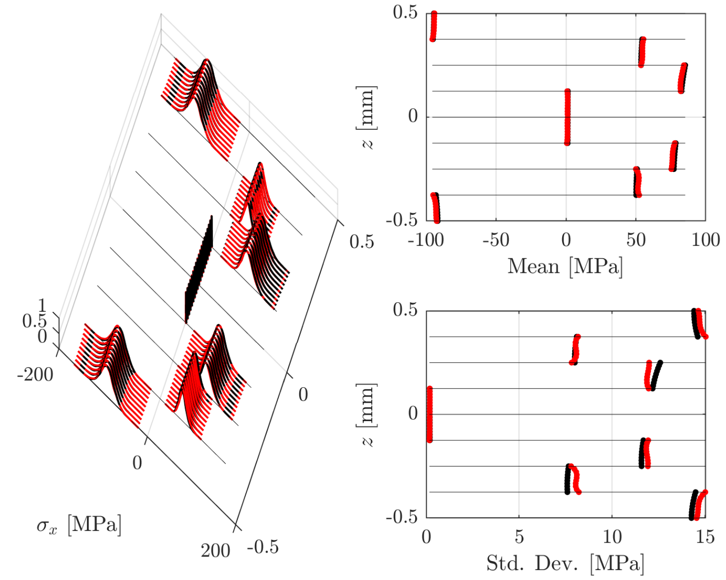


Figure 7 – *Probabilistic distribution of $\sigma_x$ for the element of interest highlighted in Figure 4. Reference results are highlighted with red colors.*

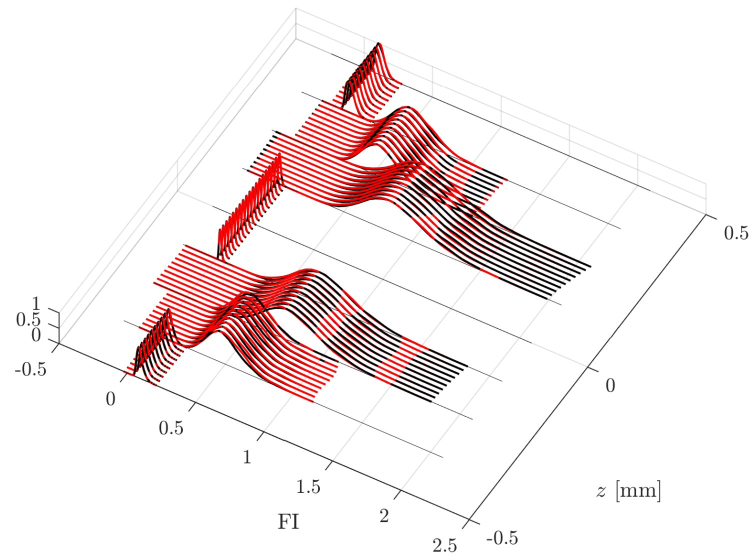


Figure 8 – *Probabilistic distribution of failure index along the thickness for the element of interest. Reference results are highlighted with red colors.*

where $q_x^0$, $\partial q_x/\partial p$, $\partial q_x/\partial \bar{u}_y$ are the components of the degrees of freedom $q_k$ which have to be enforced on the boundary of the local domain.

The proposed global/local stochastic methodology was applied in this study case, assigning each random variable a Coefficient Of Variation (COV) of 0.1. The results of the stochastic analysis are depicted in Figure 7, showing for the sake of brevity only the distributions of the in-plane stress $\sigma_x$ at the centroid of the element highlighted in Figure 4 across the thickness. In Figure 7, the distributions from the stochastic analysis are represented in black, while the results from a reference Monte Carlo Simulation (MCS) are shown in red for comparison. To ensure clarity, the probability density functions are normalized to unity. Additionally, for a more quantitative comparison, the means and standard

deviations are also presented in the same graph.
Based on these results, the following observations can be made:

- Even for a relatively high COV the results agree fairly well with the reference MCS. Simulations (not reported here for the sake of brevity) have been carried out also for lower variances obtaining similar results. From these results, it is observed that, for an accurate stochastic stress analysis, a linear expansion around the random variables is enough.
- The MCS have been performed with a fully LW model, so without employing the sub-modelling step.

For design purposes, it is often more practical to assess a failure index (FI) derived from a specific failure criterion rather than examining individual components of the stress tensor. The distribution of this FI can be obtained similarly to that of the stress tensor, by simulating the displacement field using Monte Carlo methods based on random variables. As an example, the three-dimensional invariant-based failure criterion proposed in [12] was employed here, utilizing the strength properties detailed in Table 1. According to this criterion:

$$\mathrm{FI} = \varepsilon_1/\varepsilon_1^T \tag{18}$$

with $\varepsilon_1^T$ the allowable strain in the fibre direction. The failure condition for matrix failure and fibre failure in compression can be expressed as a function of some invariants as follows:

$$\mathrm{FI} = \alpha_1 I_1 + \alpha_2 I_2 + \alpha_3 I_3 + \alpha_{32} I_3^2 \tag{19}$$

where the coefficients $\alpha_1$, $\alpha_2$, $\alpha_3$, $\alpha_{32}$ depend on the strength material properties, and the invariants $I_1$, $I_2$, $I_3$ can be computed from the three-dimensional stress tensor. More details about their derivations and their explicit expressions can be found in [12]. It is noteworthy that considering randomness in the material strength properties adds no complexity, as the distribution of the FI is directly computed from the expansion of the displacement field around the random variables. However, in this example, the material strengths are assumed deterministic. The distribution of the FI for the element of interest is illustrated in Figure 8. Notably, the tails of the distribution for certain plies exceed $\mathrm{FI} = 1$, indicating that for the level of randomness assumed in the elastic properties, the design is susceptible to failure. A more rigorous reliability analysis can be conducted by integrating the Probability Density Function (PDF) of this distribution to compute the corresponding probability of failure.

## 4. Conclusions

This work proposes a methodology to propagate uncertainties in multiscale structural analysis, accounting for both global and local random variables. The standard submodelling technique for refining a Finite Element analysis locally is combined with a perturbation-based uncertainty quantification study. The model is linearly expanded around the uncertain parameters, which implies that large variances cannot be considered. However, in many engineering scenarios, this is not a significant limitation due to the relatively small variations typically encountered. Additionally, the proposed methodology allows for the use of standard sensitivity analysis techniques, making the overall approach computationally efficient.
Using the Direct Differentiation Method (DDM), sensitivities are directly computed from the tangent stiffness matrix at the last load step. Particularly for cases focused on uncertainty in material parameters, it is possible to exactly differentiate the structural matrices, avoiding issues related to finite differences and step size.
In this framework, a study was conducted on accurate stress analysis for failure prediction of laminated composites. The local discretization approach, based on Layer-Wise (LW) plate models, allowed for the retrieval of the full stress tensor and the associated probabilistic distributions. The results can hence be applied to a three-dimensional failure criterion, obtaining the distribution of failure indices for subsequent reliability analysis.
Although the methodology was evaluated using simple use cases, it is considered suitable for real-world problems encountered by stress engineers. The main limitations of the current framework are

that it was applied to relatively simple analyses. Further development is needed for history-dependent problems, where sensitivities must be incrementally updated. The submodelling technique used means that the local model does not communicate with the global one. Improvements could include feeding back the reaction forces and related sensitivities to the global model, thus developing a fully coupled global/local stochastic model, as demonstrated in [13].

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## Copyright Statement



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