



RESEARCH ON UNSTEADY AERODYNAMIC MODELING METHOD OF PREPOSITIVE WING - MAIN WING COMPOSITE SYSTEM AT HIGH ATTACK ANGLE

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Abstract

The prepositive elliptical wing- main wing composite configuration forms an active flow control system through the disturbance between the two wings, which improves the unsteady aerodynamic efficiency of high attack angle. Based on the physical mechanism of flow field interference between two wings, an unsteady aerodynamic state space model of the prepositive wing - main wing composite system at high attack angle is constructed. By analyzing the correlation between the disturbance parameters of the prepositive wing and the main wing, the separation point equation of the state space equation is redefined. The model is established based on the physical mechanism, and the method is verified by CFD simulation. The result shows that model has good aerodynamic prediction ability and certain extrapolation generalization characteristics for the composite system. The maximum root-mean-square error of backtracking prediction for lift coefficient, drag coefficient and moment coefficient of the original sample data is less than 0.01, and the maximum root-mean-square error of generalization prediction for other composite systems after parameterization is less than 0.1. It can be seen that this modeling method overcomes the limitations of existing high attack angle aerodynamic modeling techniques in terms of applicability to complex systems. It provides a comprehensive description of the aerodynamic characteristics of composite systems, considering both physical significance and practical value in flow field control. It also provides new insights into the unsteady flow field dynamics and comprehensive flow field control research of compound systems.

Keywords: prepositive elliptical wing - main wing multi-body composite system, state space model, unsteady aerodynamic modeling, computational fluid dynamics (CFD)

1. Introduction

With the development of aviation technology and the constant changes of the battlefield situation, the new generation of highly maneuverable aircraft pays more attention to the flight capability in high attack angle states. In the process of high attack angle maneuvers, the aerodynamic forces and moments of the aircraft are coupled with the flight altitude, leading to strong nonlinear instantaneous aerodynamic forces and moments [1]-[3], directly affecting the flight envelope expansion, air combat effectiveness and survival characteristics of the aircraft. Therefore, how to accurately predict the unsteady aerodynamic force at high attack angle is of great significance to improve the flight performance of the aircraft and ensure its flight safety. Although the existing unsteady aerodynamic modeling technology has been able to characterize the dynamic aerodynamic force at high angles of attack, it still needs to be further improved and optimized in terms of generality, physical meaning characterization, and generalization characteristics. In the flight states of high attack angle, the unsteady aerodynamic force caused by the separated flow will seriously reduce the aerodynamic performance of the aircraft. In general, the active and passive flow control technology [4]-[6] with components installed on the wing is needed to improve the characteristics of the large separation flow field so as to improve the unsteady aerodynamic performance of the aircraft at high attack angle. After that, the wing and the flow control components jointly form a multi-body composite system. The change of its design parameters is highly correlated with the flow control effect under different

working conditions, which in turn affects the unsteady aerodynamic changes of the whole aircraft. Therefore, it is of great significance to establish the unsteady aerodynamic modeling method of multi-body composite system for the study of the aerodynamic performance of aircraft at high attack angle. Compared with other modeling methods, the state space model takes the generation and rupture of eddy current as the cause of aerodynamic changes, constructs eddy current description variables, and takes the influence of eddy current into the aerodynamic model, which has certain physical significance. Moreover, a definite aerodynamic model can be obtained through parameter identification, avoiding the influence of artificial setting of polynomial model and the unclear physical meaning of intelligent methods such as fuzzy and neural network. But in the actual applications, the traditional modeling method generally considers the geometry, motion and aerodynamic properties of the aircraft configuration as a whole, takes the macroscopic flow field mechanism as the starting point, and considers the aerodynamic hysteresis characteristics only as the factor of attack angle. Therefore, it is difficult to make a reasonable and clear description of the complex flow field interference between the components of the multi-body composite system and the influence of the design parameters on the aerodynamic characteristics, and it is also impossible to predict the unsteady aerodynamic force after the change of the system design parameters [7]-[9]. And the unknown parameters that need to be identified in the original state space equation have been too many, and the design parameter combination of the composite system is also very large. If the influence relationship on the aerodynamic characteristics is directly added to the state space model, the solution speed and prediction accuracy of the model will be greatly affected. In summary, how to efficiently extract the key parameters of the composite system in the process of maneuvering motion at high attack angle, reasonably and accurately describe the influence of the design parameters on the physical characteristics of the flow field, and then introduce it into the internal state quantity of the state space equation is the key to realize the nonlinear aerodynamic modeling of the multi-body composite system at high attack angle.

In this paper, based on a flow control configuration of prepositive elliptical wing - main wing [10], the unsteady aerodynamic state space modeling method of multi-body composite system at high attack angle is studied. Combined with the results of numerical simulation, the orthogonal experiment method is used to select the representative design parameter combinations, and then the influence equation of the parameter value on the time-averaged aerodynamic force of the composite system is constructed. Based on the principle of the basic state space equation, the physical quantity of the design parameters of the prepositive wing - main wing composite system is added to the internal state quantity that characterizes the aerodynamic separation characteristics of the maneuvering motion at high attack angle. The differential equation is used to express the development and change process of the unsteady flow field separation, and the state space model suitable for the composite system is established. Finally, the prediction ability of the established model is verified.

2. Prepositive Elliptical Wing - Main Wing Composite System

2.1 Prepositive Elliptical Wing - Main Wing Composite Flow Control Configuration

The prepositive elliptical wing - main wing composite system consists of two parts: the main wing and the elliptical wing set in front of its leading edge point. As shown in Figure 1, the flow control effect is mainly dependent on the slot formed between the two and the dynamic wake generated by the elliptical wing. The main influencing parameters include relative geometric parameters, position parameters and motion parameters of the elliptical wing and main wing. When the prepositive elliptical wing - main wing composite system is maneuvering at high angle of attack, the flow field will show obvious "space-time" interference phenomenon, and the change of its design parameters will also cause different interference to the unsteady aerodynamic force of the composite system. For the prepositive elliptical wing, the subsonic airflow disturbance generated by the downstream main wing will propagate upstream to affect the aerodynamic change of the elliptical wing itself, and the gap between the elliptical wing and the main wing will also change the flow behind the elliptical wing. And the flow field of the main wing is greatly affected by the disturbance of the elliptical wing. On the one hand, the gap formed between the two changes the flow quality of the leading edge of the main wing. On the other hand, the unsteady wake of the elliptical wing will also affect the flow of the upper surface of the main wing in real time. The overall aerodynamic characteristics of the composite system are closely related to the mutual interference between components, and the aerodynamic and motion are highly coupled.

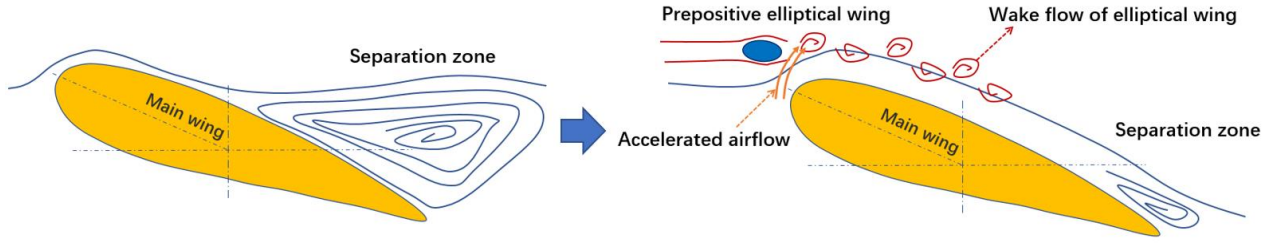


Figure 1 – Flow control mechanism of prepositive wing - main wing composite configuration.

2.2 Design Parameters of Prepositive Elliptical Wing - Main Wing Composite System

In this paper, the relative position parameters of the prepositive elliptical wing - main wing composite system are studied. The typical NACA0012 airfoil is selected as the main wing, the chord length is c , the average angle of attack of pitch oscillation is α_0 , the amplitude is α_m , the dimensionless angular velocity is ω , and the motion law is $\alpha = \alpha_0 + \alpha_m \sin(\omega t)$. The long axis length of the prepositive elliptical wing is $0.02c$, and the short axis length is $0.01c$, which is relatively static with the main wing during the pitching motion, as shown in Figure 2. The position parameters include the attack angle α_{pre} of the elliptical wing, the horizontal distance L_H and vertical distance L_V between the center of the elliptical wing and the leading edge of the main wing, as shown in Figure 3.

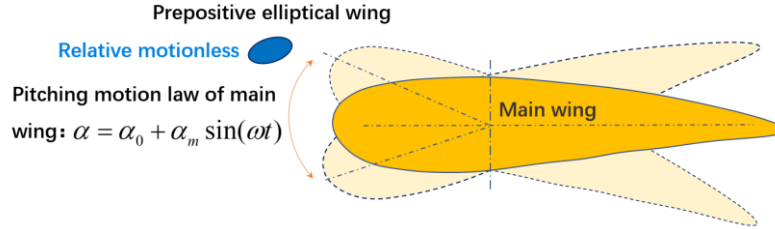


Figure 2 – Illustration of prepositive elliptical wing - main wing composite configuration scheme.

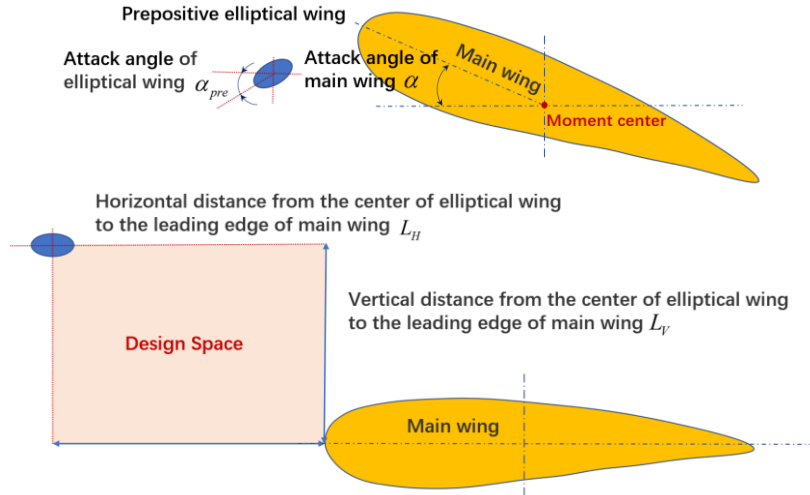


Figure 3 – Position parameters of prepositive elliptical wing - main wing composite configuration.

Table 1 shows the design parameters of the relative position of the prepositive elliptical wing -main wing composite system and their respective value ranges. In order to further explore the influence of the correlation between the elliptical wing and the main wing, study its objective laws, and improve the efficiency of analysis and modeling, the design parameters are dimensionless.

Table 1 - Position design parameters of prepositive elliptical wing -main wing composite system

Design parameter	Sign	Value range	Nondimensional parameter
Attack angle of prepositive elliptical wing	α_{pre}	$90^\circ \sim -90^\circ$	$\overline{\alpha_{pre}} = \alpha_{pre} / \alpha_0$
Horizontal distance between the center of	L_H	$0 \sim 0.1c$	$\overline{L_H} = L_H / c$

the elliptical wing and the leading edge of
the main wing

Vertical distance between the center of
the elliptical wing and the leading edge of
the main wing

$$L_V$$

$$0 \sim 0.1c$$

$$\overline{L_V} = L_V / c$$

3. Unsteady Aerodynamic State Space Model

3.1 Goman state space model

The basic assumption of the state space model is that the main reason for the aerodynamic hysteresis effect of the aircraft at high angle of attack is the occurrence of flow separation and vortex breakdown during motion. Goman proposed the position of airfoil separation point as the internal state variable of the system, and established the unsteady aerodynamic state space model at high attack angle [11]-[13]. The state variable is defined as a dimensionless quantity $\bar{x} = x/c \in [0,1]$, where x represents the distance from the flow separation point of the airfoil to the leading edge, and c is the chord length of the airfoil. This dynamic characteristic is described by a differential equation:

$$\tau_1 \frac{dx}{dt} + x = x_0(\alpha - \tau_2 \dot{\alpha}) \quad (1)$$

Where, τ_1, τ_2 are the time constant, τ_1 represents the formation process of unsteady equilibrium separated flow, and τ_2 represents the separation lag characteristic of flow.

The relationship between the position of the flow separation point and the angle of attack in the steady flow state can be set as Eq. (2).

$$x_0(\alpha) = \frac{1}{1 + e^{\sigma(\alpha - \alpha^*)}} \quad (2)$$

Where, σ is the slope factor, and α^* corresponds to the Angle of attack when the position of the separation point reaches the midpoint of the airfoil chord in the steady state.

Combined with the modeling principle of state space model, the unsteady aerodynamic force at high angle of attack can be written as a function of internal state quantity $\bar{x}$ and multi-order quantities of flight state parameters $(\alpha, \dot{\alpha}, \dots, q, \dot{q}, \dots)$. In the study of this paper, the incoming flow velocity is constant, so $q = \dot{\alpha}$, while omitting the higher-order terms that have less influence, Eq. (3) can be obtained:

$$\begin{aligned} C_l &= C_l(\bar{x}, \alpha, \dot{\alpha}) \\ C_d &= C_d(\bar{x}, \alpha, \dot{\alpha}) \\ C_m &= C_m(\bar{x}, \alpha, \dot{\alpha}) \end{aligned} \quad (3)$$

The Eq. (3) is expanded by Taylor to take the second-order term and carry out dimensionless:

$$C_i(\bar{x}, \alpha, \dot{\alpha}) = C_{i0} + C_{i\alpha}\alpha + C_{i\dot{\alpha}}\hat{\dot{\alpha}} + \frac{1}{2}[C_{i\alpha^2}\alpha^2 + 2C_{i\alpha\dot{\alpha}}\alpha\hat{\dot{\alpha}} + C_{i\dot{\alpha}^2}\hat{\dot{\alpha}}^2] \quad (4)$$

Where, $\hat{\dot{\alpha}} = \dot{\alpha}t^*$ is the dimensionless form of the pitch angle rate, $t^* = c/2V$ represents the characteristic time of the flow, V is the velocity of incoming flow, and the partial derivatives of each term in the Eq. (4) are approximated by quadratic polynomials related to $\bar{x}$. The state space model of unsteady aerodynamic coefficient at high attack angle can be obtained by combining the expression of internal state quantity and the expression of aerodynamic coefficient. Its mathematical form is as Eq. (5).

$$\tau_1 \frac{dx}{dt} + x = x_0(\alpha - \tau_2 \dot{\alpha})$$

$$x(\alpha, \dot{\alpha}) = \frac{1}{1 + e^{\sigma(\alpha - \alpha^* - \tau\dot{\alpha})}}$$

$$C_l(\bar{x}, \alpha, \dot{\alpha}) = C_{i0} + C_{i\alpha}\alpha + C_{i\dot{\alpha}}\dot{\alpha} + \frac{1}{2}[C_{i\alpha^2}\alpha^2 + 2C_{i\alpha\dot{\alpha}}\alpha\dot{\alpha} + C_{i\dot{\alpha}^2}\dot{\alpha}^2] \quad (5)$$

Where, C_{i0} is a constant, and other partial derivatives of the form are as follows:

$$C_{i\alpha}(\bar{x}) = a_1 + b_1\bar{x} + c_1\bar{x}^2$$

$$C_{i\dot{\alpha}}(\bar{x}) = a_2 + b_2\bar{x} + c_2\bar{x}^2$$

$$C_{i\alpha^2}(\bar{x}) = 2(a_3 + b_3\bar{x} + c_3\bar{x}^2)$$

$$C_{i\alpha\dot{\alpha}}(\bar{x}) = a_4 + b_4\bar{x} + c_4\bar{x}^2$$

$$C_{i\dot{\alpha}^2}(\bar{x}) = 2(a_5 + b_5\bar{x} + c_5\bar{x}^2)$$

In the high attack angle unsteady aerodynamic state space model, the input variables are $\alpha, \dot{\alpha}$, the model output is the aerodynamic force C_l , the internal state quantity is $\bar{x}$, and the unknown parameters include $\tau_1, \tau_2, \sigma, \alpha^*, a_i, b_i, c_i (i=1 \sim 5)$, all of which are obtained by parameter identification method.

3.2 Parameter Identification

In this paper, the parameter identification method based on nonlinear regression analysis is selected. This method has the advantages of strong applicability, controllable complexity and strong anti-noise ability, which can meet the requirements of unknown parameter identification of state space model. The root mean square error e_{RMS} and relative error e are used as the parameter identification criteria. The calculation formula is as follows:

$$e_{RMS} = \sqrt{\frac{1}{n} \sum_{i=1}^n (\hat{y}_i - y_i)^2} \quad (6)$$

$$e = \frac{\sqrt{\frac{1}{n} \sum_{i=1}^n (\hat{y}_i - y_i)^2}}{\sqrt{\frac{1}{n} \sum_{i=1}^n y_i^2}} = \frac{e_{RMS}}{\sqrt{\frac{1}{n} \sum_{i=1}^n y_i^2}} \quad (7)$$

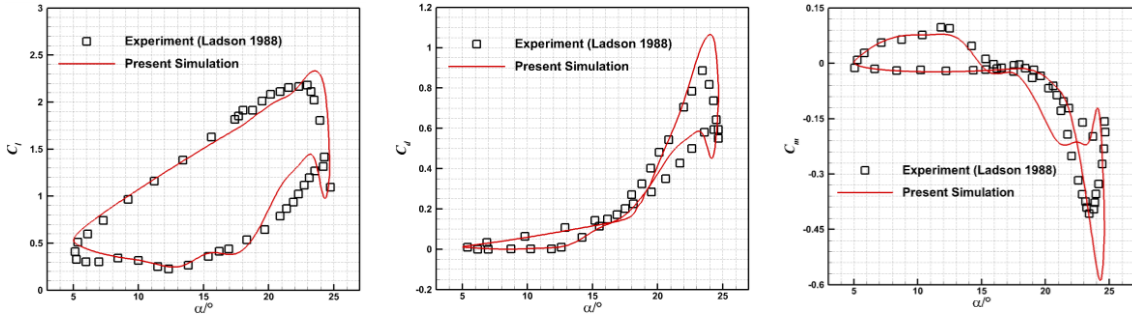
Where, y_i is the calculated value obtained by CFD method, $\hat{y}_i$ is the corresponding output value obtained by unsteady aerodynamic model, and n is the total amount of sample data.

4. Improvement of Unsteady Aerodynamic Modeling for Composite System

4.1 Numerical Simulation of Unsteady Aerodynamic Force of Composite System

The $k - \omega$ SST turbulence model is used to simulate the pitching motion process of NACA0012 airfoil at high angle of attack, and compared with the experimental results [14]. The experimental state parameters are: $\alpha_0 = 14.91^\circ$, $\alpha_m = 9.88^\circ$, $k = 0.151$, $Ma = 0.283$, $Re = 3.45 \times 10^6$. The variation of the unsteady aerodynamic coefficients of the airfoil with the angle of attack are shown in Figure 4. The calculated values are in good agreement with the experimental values, and the trend is consistent, indicating that the numerical simulation method used in this paper is reliable.

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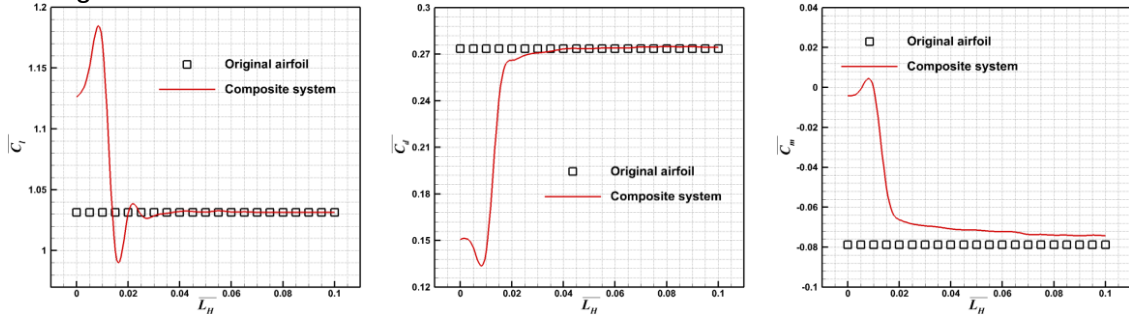
(a) Lift coefficient

(b) Drag coefficient

(c) Pitch moment coefficient

Figure 4 - Comparison of numerical simulation results and experimental results.

The numerical simulation analysis of the prepositive elliptical wing - main wing composite system is carried out, and the calculation condition is same as the verification case. The initial dimensionless design parameters of the composite system are $\overline{L}_H = 0.01$, $\overline{L}_V = 0.045$ and $\overline{\alpha}_{pre} = -2$. The unsteady aerodynamic force C_l, C_d, C_m and the time-averaged aerodynamic force $\overline{C}_l, \overline{C}_d, \overline{C}_m$ of the composite system with different parameters under high attack angle pitching oscillation are obtained by using the control variable method to explore the influence of single parameter on the aerodynamic performance of the composite system. The influence of composite configuration design parameters on flow field characteristics is divided into macroscopic and microscopic considerations. First, the time-averaged aerodynamic force of the composite system and the original airfoil are compared, as shown in Figure 5-7.

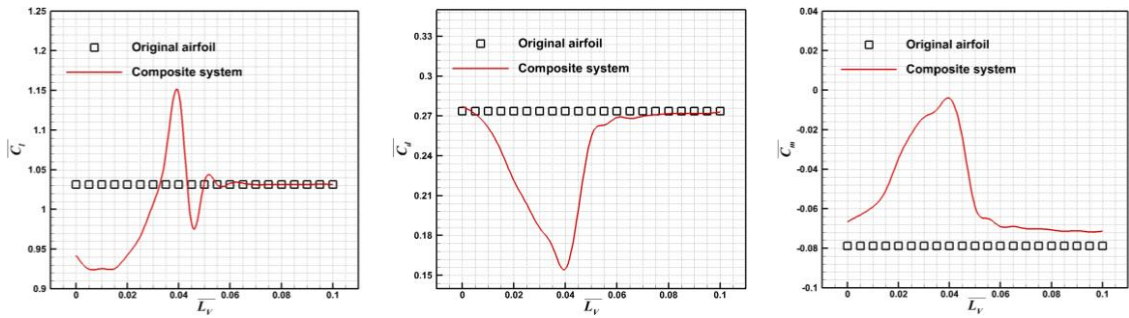


(a) Lift coefficient

(b) Drag coefficient

(c) Pitch moment coefficient

Figure 5 - $\overline{L}_H$'s influence on time-averaged aerodynamic force.

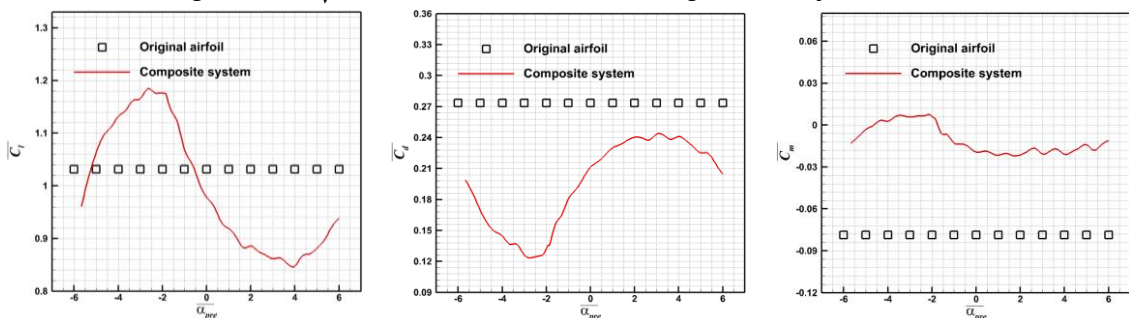


(a) Lift coefficient

(b) Drag coefficient

(c) Pitch moment coefficient

Figure 6 - $\overline{L}_V$'s influence on time-averaged aerodynamic force.



(a) Lift coefficient

(b) Drag coefficient

(c) Pitch moment coefficient

Figure 7 - $\overline{\alpha_{pre}}$'s influence on time-averaged aerodynamic force.

It can be seen that both $\overline{L_H}$ and $\overline{L_V}$ have a certain effective value range, within which the time-averaged aerodynamic force of the composite system will have one or more peaks with the increase of a single design parameter. When their values exceed this effective range, the time-averaged aerodynamic force of the composite system begins to infinitely approach a constant of time-averaged aerodynamic force close to the original airfoil. It can be considered that the mutual interference between the prepositive elliptical wing and the main wing is small enough to be ignored. This mathematical property can be described by Gaussian mixture model (GMM) which is obtained by superimposing multiple Gaussian distributions. Compared with $\overline{L_H}$ and $\overline{L_V}$, $\overline{\alpha_{pre}}$ is the secondary influencing factor of the aerodynamic performance of the composite system. Under the fixed values of $\overline{L_H}$ and $\overline{L_V}$, the time-averaged aerodynamic force of the composite system fluctuates around an intermediate value with the change of $\overline{\alpha_{pre}}$. Its mathematical characteristics can be approximated by sine function. Based on the above analysis, a set of equations containing unknown parameters are initially constructed to describe the influence of a single design parameter on the time-averaged aerodynamic force of the composite system:

$$\begin{cases} f_1(\overline{L_H}) = \delta_3(1 - (\delta_1\overline{L_H} - \delta_2))^2 e^{-(\delta_1\overline{L_H} - \delta_2)^2} \\ f_2(\overline{L_V}) = \delta_6(1 - (\delta_4\overline{L_V} - \delta_5))^2 e^{-(\delta_4\overline{L_V} - \delta_5)^2} \\ f_3(\overline{\alpha_{pre}}) = \delta_7 + \delta_8 \sin(\overline{\alpha_{pre}} + \delta_9) \end{cases} \quad (8)$$

Where, δ_1, δ_2 are used to correct the value range of $\overline{L_H}$ and determine the peak position, δ_3 is used to control the peak height of the function, and the form of polynomial is used to change the shape of the peak to adapt to different aerodynamic changes. Similarly, δ_4, δ_5 are used to correct the value range of $\overline{L_V}$, and δ_6 is used to control the peak height of the function. While δ_7 is the intermediate value of the sine function, δ_8 and δ_9 are the amplitude and initial phase respectively.

In fact, the aerodynamic characteristics of the composite system will be affected by the values of the three design parameters at the same time, and there is complex mutual interference among these parameter combinations, rather than simple linear superposition. Therefore, Eq. (8) needs to be further modified, and $\overline{L_H}, \overline{L_V}, \overline{\alpha_{pre}}$ are used as interrelated independent variables to construct the equation. As shown in Eq. (9).

$$\begin{cases} \overline{C_i} = F(\overline{L_H}, \overline{L_V}, \overline{\alpha_{pre}}) = (a + b - c)d \\ a = \delta_3(1 - (\delta_1\overline{L_H} - \delta_2))^2 e^{-(\delta_1\overline{L_H} - \delta_2)^2 - (\delta_4\overline{L_V} - \delta_5 + 1)^2} \\ b = \delta_6(\delta_1\overline{L_H} - \delta_2 - (\delta_1\overline{L_H} - \delta_2)^3 - (\delta_4\overline{L_V} - \delta_5)^5) e^{-(\delta_1\overline{L_H} - \delta_2)^2 - (\delta_4\overline{L_V} - \delta_5)^2} \\ c = \delta_{10} e^{-(\delta_1\overline{L_H} - \delta_2 + 1)^2 - (\delta_4\overline{L_V} - \delta_5)^2} \\ d = \delta_7 + \frac{\delta_8}{\tan(\pi(\overline{L_H} + \overline{L_V}) / 0.8)} \sin(\overline{\alpha_{pre}} + \delta_9) \end{cases} \quad (9)$$

Eq. (9) is based on Gaussian distribution, polynomial and elliptic shape, and consists of four parts. The correlation of design parameters $\overline{L_H}, \overline{L_V}, \overline{\alpha_{pre}}$ is considered comprehensively. It can characterize the overall flow control effect of the composite configuration on the separation of the upper surface of main wing and the macroscopic influence on the aerodynamic coefficient of the system under different design parameter combinations. Here, the unknown parameters $\delta_1, \delta_2, \delta_4, \delta_5, \delta_7, \delta_8, \delta_9$ have the same meaning as Eq. (8), $\delta_3, \delta_6, \delta_{10}$ are used to control the height of the peak or trough, and the polynomial

part leads to the change of the shape of the peak or trough. a, b and c together constitute a three-dimensional surface with multiple peaks in the local region, and $\overline{L}_H, \overline{L}_V$ are associated by elliptic equations and polynomials. Formula d adds the influence of $\overline{\alpha}_{pre}$ on this basis, and believes that when $\overline{L}_H + \overline{L}_V > 0.2$, that is, the value of $\overline{L}_H$ and $\overline{L}_V$ exceed the design space, the influence brought by the change of $\overline{\alpha}_{pre}$ will be greatly weakened.

Secondly, the unsteady aerodynamic force of the composite system during pitching motion is closely related to the instantaneous angle of attack, and the design parameter combination has different influence on the aerodynamic change of the composite system at different angles of attack. Figure 8 compares the aerodynamic force change of the composite system and the original airfoil with the value of the initial design parameter. Due to the prepositive elliptical wing plays a certain control effect on the flow separation on the main wing surface, the shape of the hysteresis loop of the composite system changes greatly compared with that of the original airfoil at high angles of attack, while the flow on the main wing surface is the attached flow at small angles of attack, and the unsteady aerodynamic force of the composite system is close to that of the original airfoil. To sum up, in order to accurately describe the flow control effect of the composite system on the main wing during the change of attack angle, it is necessary to focus on the change of flow separation point position caused by the change of design parameters when establishing the state-space model of the composite system.

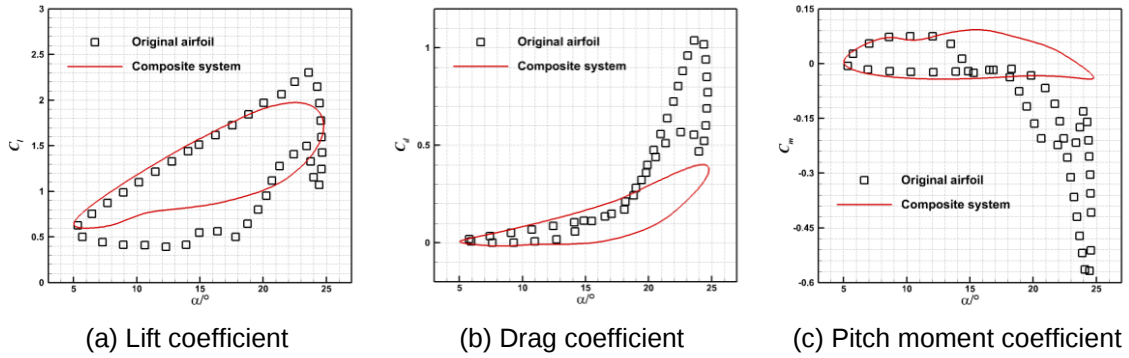


Figure 8 - Unsteady aerodynamic comparison between the composite system and the original airfoil.

4.2 Orthogonal Experimental Design

In the process of modeling the unsteady aerodynamic force of the composite system, the unknown parameters δ_i in Eq. (9) need to be identified according to the numerical simulation results. In order to reduce the amount of calculation, save the calculation time, and efficiently extract the influence of different design parameter combinations on the changes of flow field characteristics of the composite system during the maneuvering motion of the high attack angle, according to the existing calculation results, an orthogonal table $L_{36}(6^3)$ with three factors and six levels is constructed by selecting the parameter combinations in the feasible design space through the orthogonal experimental design method, as shown in Table 2.

Table 2 - Factors and levels of orthogonal experiment.

Factors	$\overline{L}_H$	$\overline{L}_V$	$\overline{\alpha}_{pre}$
	A	B	C
Level 1	0	0.015	6
Level 2	0.01	0.03	4
Level 3	0.02	0.045	2
Level 4	0.03	0.06	0
Level 5	0.04	0.075	-2
Level 6	0.05	0.09	-4

The 36 groups of composite systems with different parameter combinations selected by orthogonal experimental method are calculated, and the corresponding values $\overline{C}_l, \overline{C}_d, \overline{C}_m$ are obtained. Using these data, the parameters in Eq. (9) are identified respectively, and the results are shown in Table

3. Among them, the relative error of $\overline{C_l}$ and $\overline{C_d}$ are less than 5%, while the error of $\overline{C_m}$ is slightly larger, but still within the acceptable range. In order to reduce the difficulty of identification caused by the increase of unknown parameters in the state space equation, the unsteady aerodynamic modeling of the composite system is carried out with the following parameter values.

Table 3 - Parameter identification results.

Dependent variable	Parameters										Error
	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8	δ_9	δ_{10}	e
$\overline{C_l}$	-21.409	1.672	0.847	39.439	2.119	2.006	0.963	0.001	1.645	0.447	3.8%
$\overline{C_d}$	-41.741	0.049	0.027	54.005	1.583	0.349	1.017	0.006	-7.483	0.076	4.2%
$\overline{C_m}$	55.397	0.535	0.209	-48.99	-1.32	-0.66	0.813	0.028	-1.074	0.611	6.1%

4.3 Unsteady Aerodynamic Modeling of Composite System

Based on the principle of the basic state space equation, the aerodynamic separation characteristics of the composite system under high angle of attack maneuvering are expressed by the internal state quantity x . Eq. (9) is added to the internal state quantity, and the influence of each parameter combination on the aerodynamic force and flow field characteristics of the composite system is introduced. The development and change process of unsteady flow field separation is expressed by differential equation. In particular, it is considered that the influence of $F(\overline{L_H}, \overline{L_V}, \overline{\alpha_{pre}})$ on α^* is a quadratic relationship to highlight the influence of design parameters on the position of separation point. By establishing the state space equation of the aerodynamic force of the prepositive elliptical wing - main wing composite system and combining it with the output equation, the physical characteristic characterization model of the state space equation of the multi-body composite system can be obtained. As shown in Eq. (10).

$$\tau_1 \frac{dx}{dt} + x = x_0(\alpha - \tau_2 \dot{\alpha})$$

$$x(\alpha, \dot{\alpha}, \overline{L_H}, \overline{L_V}, \overline{\alpha_{pre}}) = \frac{1}{1 + e^{\sigma(\alpha - \alpha^* F(\overline{L_H}, \overline{L_V}, \overline{\alpha_{pre}}) - \tau \dot{\alpha}) F(\overline{L_H}, \overline{L_V}, \overline{\alpha_{pre}})}}$$

$$C_l(\overline{x}, \alpha, \dot{\alpha}) = C_{i0} + C_{i\alpha} \alpha + C_{i\dot{\alpha}} \dot{\alpha} + \frac{1}{2} [C_{i\alpha^2} \alpha^2 + 2C_{i\alpha\dot{\alpha}} \alpha \dot{\alpha} + C_{i\dot{\alpha}^2} \dot{\alpha}^2] \quad (10)$$

In the unsteady aerodynamic state space model of the prepositive elliptical wing - main wing composite system at high angle of attack, the input variables are $\alpha, \dot{\alpha}$ and the three design parameters $\overline{L_H}, \overline{L_V}, \overline{\alpha_{pre}}$ of the composite system, the model output is the aerodynamic force C_l , the internal state quantity is $\overline{x}$, and the unknown parameters include $\tau_1, \tau_2, \sigma, \alpha^*, a_i, b_i, c_i (i = 1 \sim 5)$, which is consistent with the original state space model.

5. Verification of Composite System State Space Model

The aerodynamic example of the pitching motion of the prepositive elliptical wing - main wing composite system calculated by the CFD method is used to test the rationality of the structure of the state space model of the multi-body composite system and the accuracy and applicability of the model. Three sets of cases with different design parameter values are selected to test the basic prediction ability of the model. And other three sets of cases with changing single parameter values and one set of case with random values are selected to test the generalization ability of the model. The calculation conditions are the same as the verification case.

5.1 Verification of Basic Prediction Ability of the Model

For the prepositive elliptical wing - main wing composite system, three groups of harmonic motion cases with different design parameters are selected to verify the basic prediction ability and prediction

accuracy of the established model.

Case 1: $\overline{L_H} = 0.01$, $\overline{L_V} = 0.045$, $\overline{\alpha_{pre}} = -2$

Case 2: $\overline{L_H} = 0.03$, $\overline{L_V} = 0.065$, $\overline{\alpha_{pre}} = -2$

Case 3: $\overline{L_H} = 0.01$, $\overline{L_V} = 0.045$, $\overline{\alpha_{pre}} = -4$

Figure 9-Figure 11 show the comparison between the calculated and predicted values of the aerodynamic coefficients of the composite system in three cases. It can be seen that the state space model well fits the overall trend of the unsteady aerodynamic coefficient hysteresis loop of the composite system, and the predicted value is almost consistent with the calculated value, indicating that the model established in this paper can complete the prediction of the composite system with different parameters, and the accuracy is high.

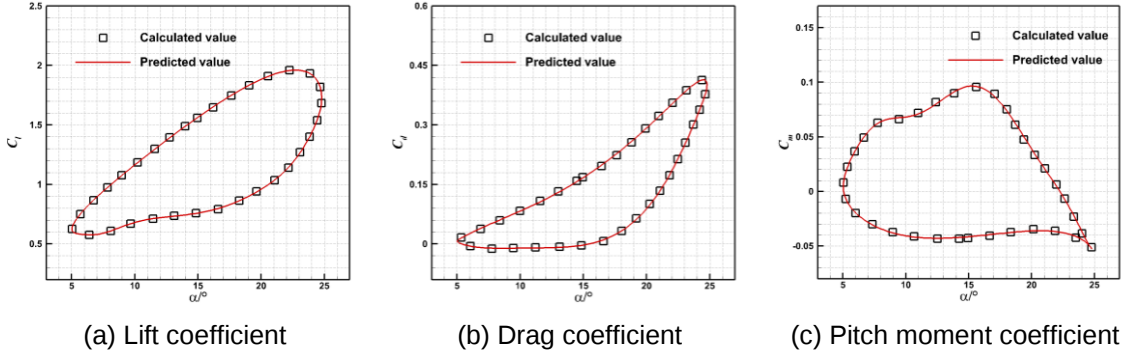


Figure 9 - The aerodynamic coefficient prediction results of Case 1.

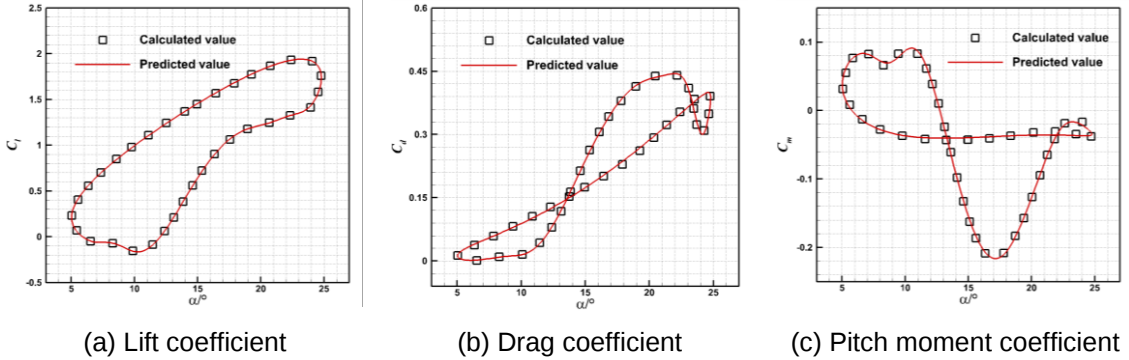


Figure 10 - The aerodynamic coefficient prediction results of Case 2.

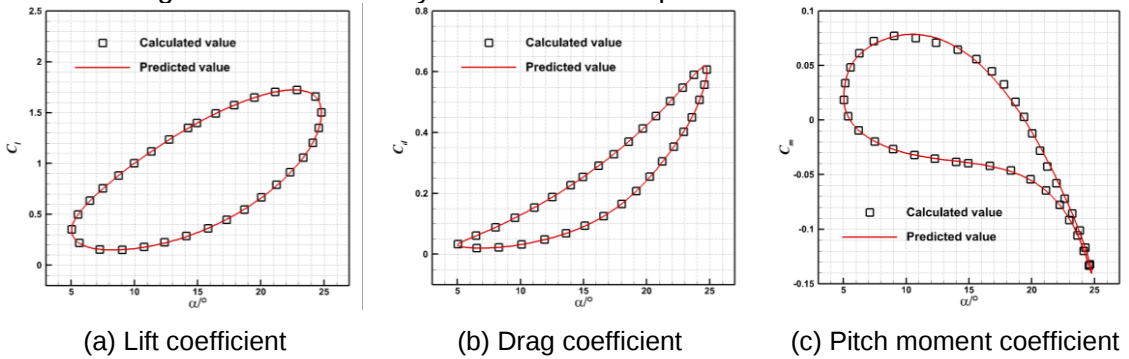


Figure 11 - The aerodynamic coefficient prediction results of Case 3.

The root mean square error and relative error of the aerodynamic coefficient prediction results of the composite system state space model are shown in Table 4. For the lift coefficient and drag coefficient of the composite system under different parameter combinations, the relative error between the predicted value and the calculated value is basically less than 1.5 %, while the error of the pitching moment coefficient is slightly larger. Due to the limited accuracy of the parameter identification method and the small magnitude of the absolute value of the pitch moment coefficient, a completely accurate pitch moment coefficient model cannot be obtained. The original Goman state space model also has the same problem when predicting the aerodynamic coefficient. In general, the state space model of the composite system has basic predictive ability, and the modeling results are ideal.

Table 4 - Comparison of aerodynamic coefficient prediction errors.

Aerodynamic coefficient	C_l		C_d		C_m	
	e_{RMS}	e	e_{RMS}	e	e_{RMS}	e
Case 1	0.0025	0.2%	0.0015	0.72%	0.0017	2.14%
Case 2	0.0052	0.43%	0.0034	1.32%	0.0031	4.55%
Case 3	0.0026	0.25%	0.0004	0.13%	0.0025	2.84%

5.2 Verification of Generalization Ability of the Model

After determining the prediction accuracy of the model, four groups of cases are used to verify the generalization ability of the model. Each case contains two sets of training data and one set of test data.

Case 4:

The design parameters of the training data are:

$$\begin{aligned}\overline{L_H} &= 0.02, \overline{L_V} = 0.045, \overline{\alpha_{pre}} = -2 \\ \overline{L_H} &= 0.04, \overline{L_V} = 0.045, \overline{\alpha_{pre}} = -2\end{aligned}$$

The design parameters of the test data are:

$$\overline{L_H} = 0.03, \overline{L_V} = 0.045, \overline{\alpha_{pre}} = -2$$

Case 5:

The design parameters of the training data are:

$$\begin{aligned}\overline{L_H} &= 0.01, \overline{L_V} = 0.05, \overline{\alpha_{pre}} = -2 \\ \overline{L_H} &= 0.01, \overline{L_V} = 0.07, \overline{\alpha_{pre}} = -2\end{aligned}$$

The design parameters of the test data are:

$$\overline{L_H} = 0.01, \overline{L_V} = 0.06, \overline{\alpha_{pre}} = -2$$

Case 6:

The design parameters of the training data are:

$$\begin{aligned}\overline{L_H} &= 0.01, \overline{L_V} = 0.045, \overline{\alpha_{pre}} = 0 \\ \overline{L_H} &= 0.01, \overline{L_V} = 0.045, \overline{\alpha_{pre}} = -2\end{aligned}$$

The design parameters of the test data are:

$$\overline{L_H} = 0.01, \overline{L_V} = 0.045, \overline{\alpha_{pre}} = -4$$

Case 7:

The design parameters of the training data are:

$$\begin{aligned}\overline{L_H} &= 0.01, \overline{L_V} = 0.045, \overline{\alpha_{pre}} = -2 \\ \overline{L_H} &= 0.03, \overline{L_V} = 0.065, \overline{\alpha_{pre}} = -2\end{aligned}$$

The design parameters of the test data are:

$$\overline{L_H} = 0.03, \overline{L_V} = 0.045, \overline{\alpha_{pre}} = -2$$

The comparison between the predicted results and the calculated results of the above Cases are shown in Figure 12-Figure 15. Among them, Cases 4 and 5 are interpolation predictions, and Cases 6 and 7 are extrapolation predictions. It can be seen that the aerodynamic coefficient output by the state space model of the composite system is close to the calculated value, and the trend of the hysteresis loop rising or falling is accurately predicted, indicating that the physical meaning of the improved state space model is further enhanced, and the improved method is reasonable.

RESEARCH ON UNSTEADY AERODYNAMIC MODELING METHOD OF PREPOSITIVE WING - MAIN WING COMPOSITE SYSTEM AT HIGH ATTACK ANGLE

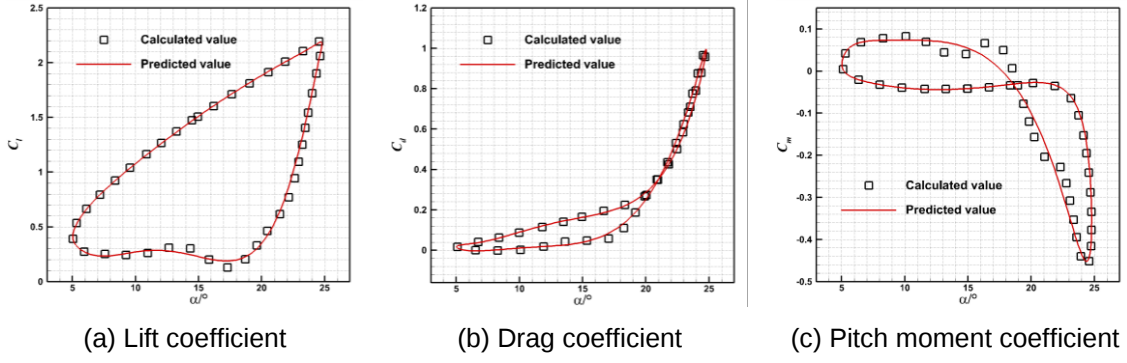


Figure 12 - The aerodynamic coefficient prediction results of Case 4.

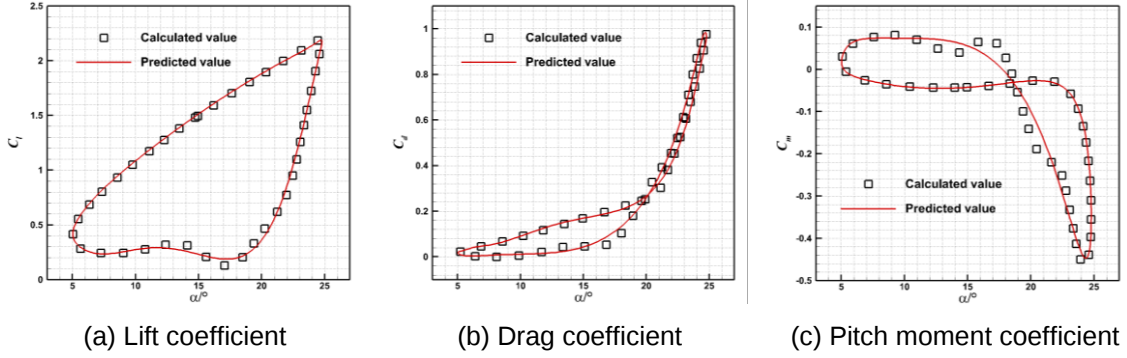


Figure 13 - The aerodynamic coefficient prediction results of Case 5.

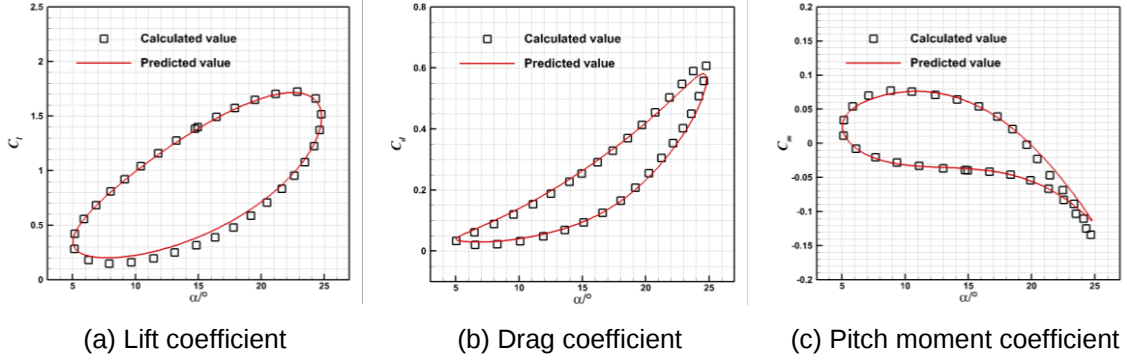


Figure 14 - The aerodynamic coefficient prediction results of Case 6.

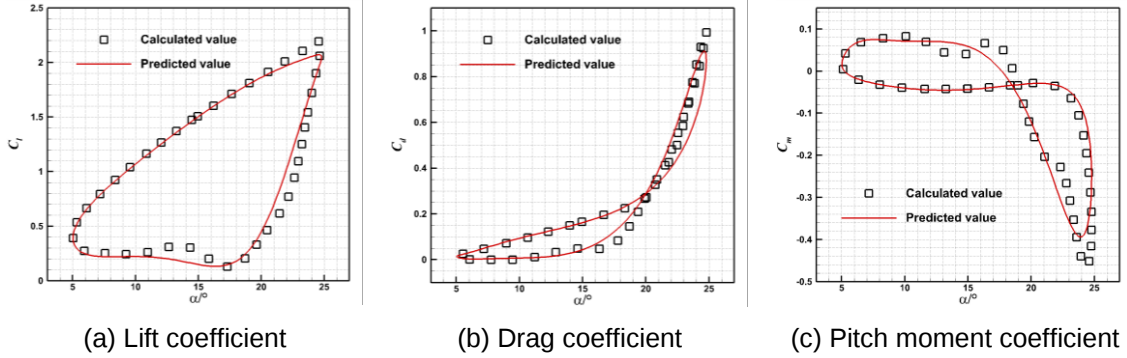


Figure 15 - The aerodynamic coefficient prediction results of Case 7.

The root mean square error and relative error of the aerodynamic coefficient prediction results of the composite system state space model in the four groups of examples are shown in Table 5. Compared with the original Goman state space model, the improved state space model proposed in this paper includes additional design parameters of the composite system, which can predict the unsteady aerodynamic force of the composite system with different parameter values. The error of model extrapolation prediction is larger than that of interpolation prediction. And the error mainly comes from the prediction of aerodynamic coefficient at high angle of attack. At this time, the flow field on the airfoil surface is complicated, the hysteresis effect is enhanced, and the aerodynamic nonlinearity is more prominent. Therefore, it is difficult to accurately predict aerodynamic coefficient through state space model extrapolation. In general, the state space model of the composite system has certain

generalization ability, and the prediction error is within the acceptable range.

Table 4 - Comparison of aerodynamic coefficient prediction errors.

Aerodynamic coefficient	C_l		C_d		C_m	
	e_{RMS}	e	e_{RMS}	e	e_{RMS}	e
Case 4	0.0272	2.16%	0.0161	3.79%	0.0169	5.91%
Case 5	0.0244	1.93%	0.0132	3.15%	0.0159	6.89%
Case 6	0.0348	3.32%	0.0174	5.35%	0.0096	8.37%
Case 7	0.0861	6.83%	0.0629	12.05%	0.0391	17.29%

6. Conclusions

Aiming at the flow control configuration of the prepositive elliptical wing- main wing composite system, the unsteady aerodynamic modeling method of the multi-body composite system with high angle of attack is studied in this paper, and the basic prediction ability and generalization ability of the state space model of the composite system are verified.

(1) Based on the principle of Goman state space equation and unsteady separation flow mechanism, the state space model of composite system has clear physical significance, and has wide application prospect and reference value in unsteady aerodynamic modeling of multi-body composite system.

(2) When establishing the state space model of the composite system, the influence of parameters between the prepositive elliptical wing and the main wing is considered comprehensively, the design parameters are added to the model as input variables, and the unknown parameters in the equation are identified successively, which effectively reduces the difficulty of parameter identification of the state-space model of the composite system.

(3) The state space model of the composite system has reasonable structure, strong applicability, high prediction accuracy, and certain generalization ability. The maximum root-mean-square error of backtracking prediction for lift coefficient, drag coefficient and moment coefficient of the original sample data is less than 0.01, and the maximum root-mean-square error of generalization prediction for other composite systems after parameterization is less than 0.1.

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