



REQUIREMENTS UNCERTAINTY PROPAGATION IN CONCEPTUAL DESIGN USING BAYESIAN NETWORKS

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Abstract

This paper presents the application of a Bayesian Network as a tool for propagating the uncertainty between the aircraft-level design and the component-level design. The framework is applied to an example case in UAV design for payload transport. By querying the model, we demonstrate its ability to capture the casual relationships of the design problem and propagating the effects of design decisions on other parameters and requirements.

Keywords: Requirements uncertainty, Machine learning, Bayesian networks, UAV Conceptual design

1. Introduction

Concept design generation is characterized by the identification of the correct values for the design parameters such that the requirements are sufficiently satisfied. This procedure is traditionally an iterative trial error, often relying on the design experience of the engineering team. Different empirical approaches are used, such as the House of Qualities, to select from a pool of candidate concepts the best solution. The output of this process is a specification of these design parameters which would act as functional requirements to the component design teams. Often, margins are provided to compensate for uncertainties in the component design step [1].

The conceptual design team makes the most important decisions regarding the direction of the project. Lack of foresight and mistakes may lead to rework, incrementing the cost and time of product development [2]. Mapping the causal relationship between the conceptual design parameters and the component design may be beneficial in understanding the impact a decision has on the rest of the design. Additionally, the designers would also estimate the ability to satisfy the requirements given a range of design parameters.

The authors have previously explored this last issue by developing a framework for design space exploration based on probabilistic surrogate modelling [3]. The methodology calculates the probability of satisfying the requirements in each pocket of the design space. This is useful as a first pass of the design but may not be sufficient when requirements change. Component design aspects may be introduced if the design model includes them, but no causality is present. Uncertainty was handled by relying on Set-Based Design principles [4, 5], through the elimination of the unfeasible design spaces. The remaining designs were guaranteed to be feasible only for a fixed set of requirements. Sub-optimal points would be kept as ready alternatives in case of changes in the specification, rendering them robust to a restriction in the requirements. Nonetheless, the approach is not sufficient for wide changes in any direction, unless an interval-based approach is used. Covering this gap requires implementing a method for propagating requirement uncertainties on the design parameters.

The work presented in this study is an ongoing research effort in applying Bayesian Networks to engineering design problems. The possible areas of application are the propagation of uncertainties on requirements, propagation of decision-making between disciplines, and general rapid scenario analysis for setting up detailed analysis and optimisation studies. Additionally, causal inference is being explored for enhancing Bayesian Optimisation to exploit the underlying causal structure to reduce computational cost [6, 7].

2. Methodology

A new approach based on Bayesian Networks (BN) is proposed to capture the causality between the requirements, design parameters, and quantity of interests (QoI). BNs are graphical models that represent the probabilistic relationship among a set of variables. They are used to model uncertainty and dependencies between different factors in a system and perform reasoning under uncertainty [8]. The network consists of nodes, representing the variables, and directed edges, which represent the probabilistic dependency between the variables.

Each node encodes a conditional probability distribution between the input variables and the node variable. This function provides the probability of any value of the node given any value of the input variables. Usually, BNs use discrete variables, hence nodes are described with conditional probability tables (CPTs). While they can be manually filled, CPTs can be learned directly from data using parametric learning algorithms [9]. The casual structure of the network can also be learned from data with structure learning algorithms.

Once trained, a BN fully encodes the relationship between its variables and can be used to answer any probabilistic query about them. Given a set of observed variables, known as evidence, the BN infers on any of the remaining unobserved variables. It is possible to produce a probability distribution on the query variables and select those with the highest probability for further evaluation or design selection.

This flexibility in probabilistic reasoning brings forward the following advantages over the original PDOPT method:

- Mapping can be performed in any direction: for instance, from requirements to QoI, from input parameters to requirements, and so on. PDOPT supports only mapping requirements onto the input design space for the process of set elimination.
- Intrinsic support for requirement uncertainty. In PDOPT, requirements are cast as constraint equations $g(x) < c$, where the right-hand side is fixed. Only the uncertainty from finding the decision boundary in the input space is present. BNs can support changing RHS if the training data includes changing boundary values.
- System structure representation and modularity. The variables in the BN can be grouped based on their system/component membership. Additionally, it is possible to connect different networks only by specifying a CPT between two nodes. In contrast, PDOPT has a monolithic structure, where the design model is used as a black box and ignores any structure between the variables or requirements.
- Casual dependency between requirements is captured. In contrast, PDOPT assumes each requirement to be casually independent of each other: the total probability of a design being desirable and feasible is just the product of the probabilities of all the requirements. This may lead to distortions if the input space is more sensible to one of the requirements over the others.

In the same spirit of PDOPT, data from the design simulation model is used to train the parameters of the BN. This would ensure the model is unbiased and reflects the physical description in the simulation. The generation of the training dataset begins with the definition of the distributions of the input parameters and requirements, which are then sampled with a Latin Hypercube method. These data points are evaluated with the simulation model to produce the QoIs. Requirements are evaluated by comparing the sampled constraint values to the respective QoIs, generating a boolean value for satisfaction. If the computational model for the source of data is computationally expensive, it is possible to augment the data using a surrogate model, such as Kriging, for insampling. Data is then binned into discrete ranges for the training of the CPTs from data. When carrying out inferences, a de-discretisation routine is used to convert back into continuous values. Figure 1 shows the data generation and pre-processing for training the BN. In this publication, the BN structure is provided in advance and uses discrete quantities. Thus, all the variables (input, QoI, requirements) are discretised into levels when training the network. The Python library `pgmpy` is adopted for the BN training and inference.

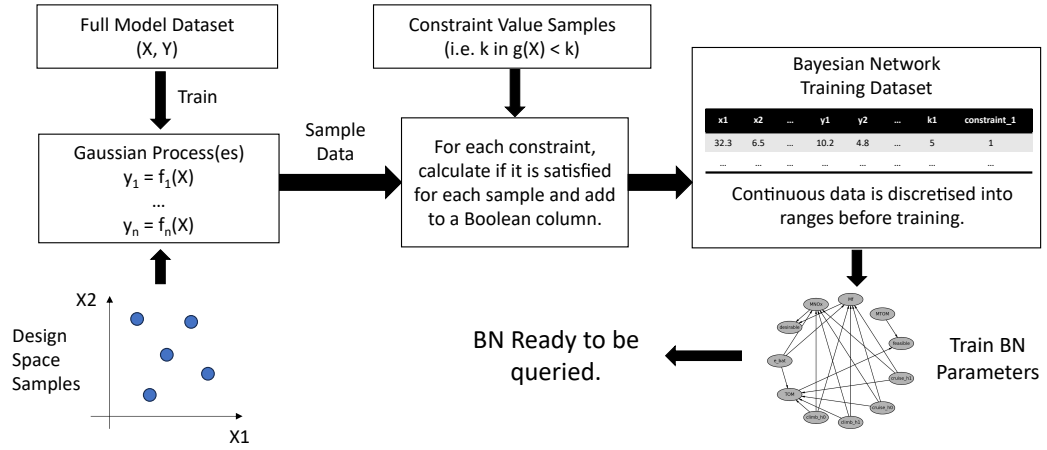


Figure 1 – Training procedure for the Bayesian Network.

3. Application Scenario

The test case in this paper is a simulacrum for a typical aircraft design process, where system-level conceptual design interacts with the component-level design processes. The first iteration of a transport UAV design is modelled, with an emphasis on wing aerodynamics. Figure 2 shows the parameters of the system design and the ones specific to the wing design within the BN.

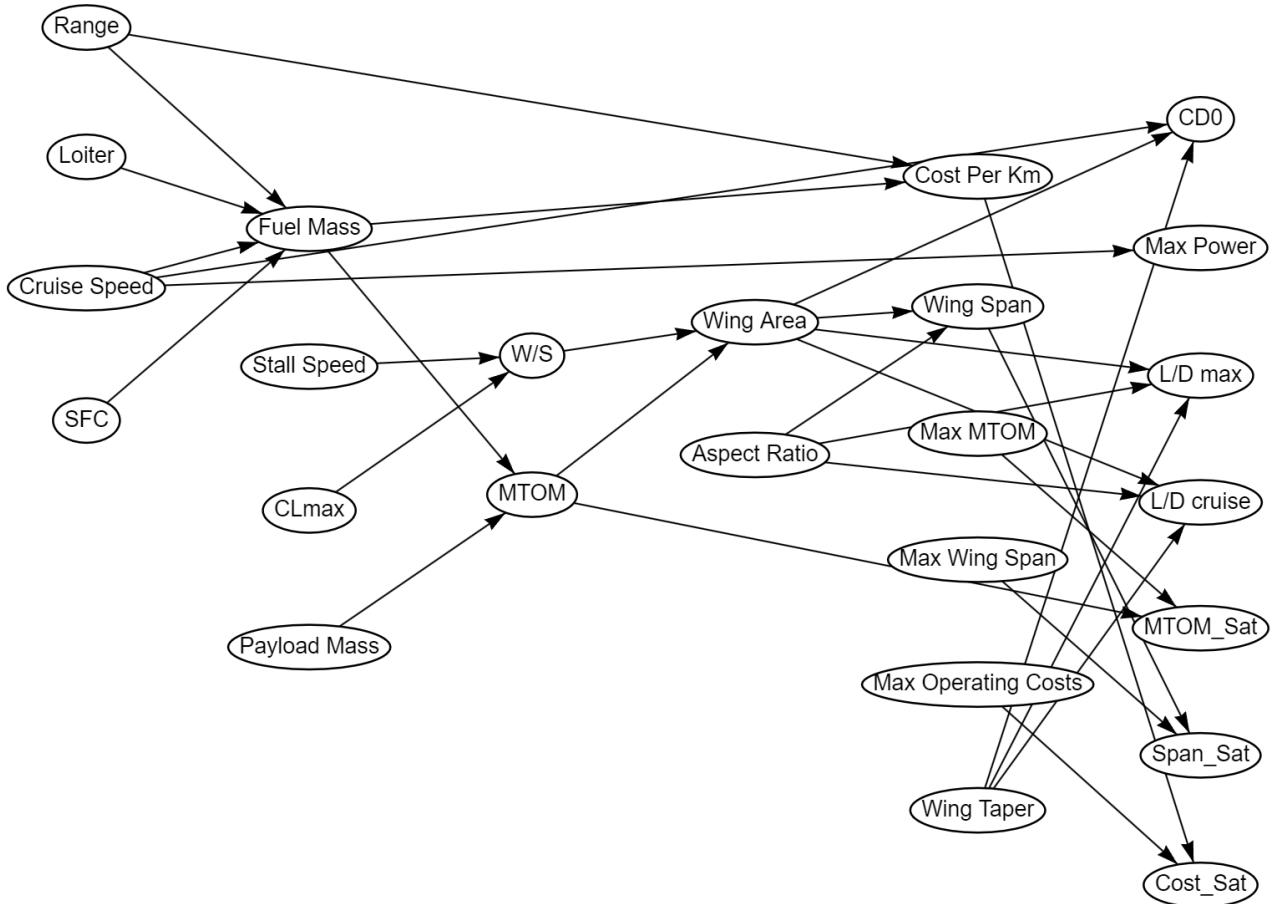


Figure 2 – Bayesian Network of the UAV Design Problem

The vehicle is tasked with transporting from a hub a payload and dropping it to the target area at a safe altitude. The mission is parametrised such that the range and payload mass are variables of the problem. Its profile is shown in Figure 3, where range R is the maximum delivery distance. The UAV configuration is the traditional tube and wing layout with a rear conventional tail. The flow

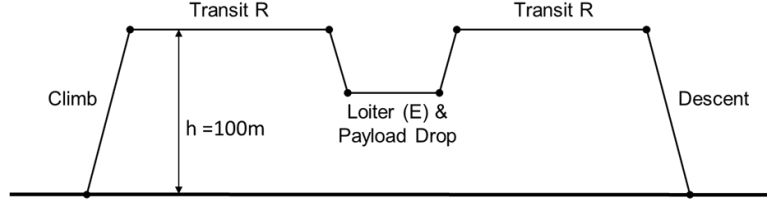


Figure 3 – UAV Delivery Mission

condition is assumed to be subsonic and incompressible, with a maximum elevation of 120m ASL. It is assumed the application will be in a flat region at sea level, and the operating regime is fully turbulent.

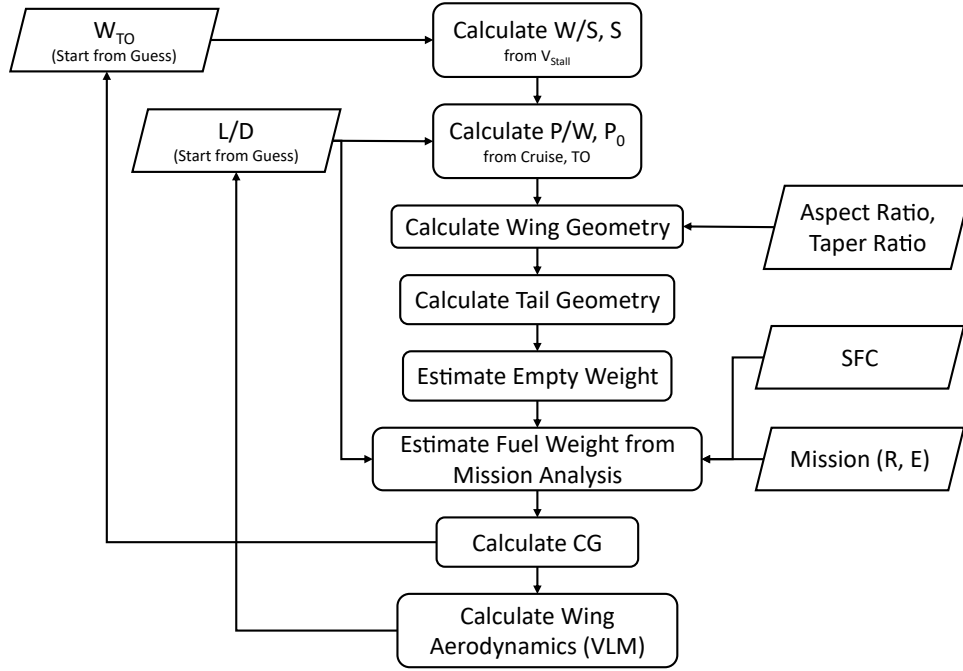


Figure 4 – UAV Sizing Flowchart

Figure 4 presents the flowchart for sizing the UAV. The wing area is calculated from the stall requirements, while the maximum power is estimated at 1.5 times the cruise-required power. Fuel mass for the mission is estimated through an iterative mission analysis as described by Roskam [10], using statistical mass data from Gundlach [11]. The fuselage is assumed to be a cylindrical pod with hemispheric ends, 50cm in length and 15cm in diameter. The tail surfaces are estimated with the tail volume method, and the wing is sized for analysis with the vortex lattice code AVL [12]. Viscous drag is estimated through the wetted areas method. Finally, operating costs are calculated as pounds sterling per km, assuming a fuel cost of 1.5 GBP per Litre. The bounds for each input variable for data generation are shown in Table 1.

Feasibility requirements have been introduced with a uniform distribution centred on the requirement with a variance of 20%, presented in Table 2. The data is discretised into 10 levels for the purpose of training the BN.

4. Results

Results indicate the model is capable of capturing the causal relationships between the parameters, including the satisfiability of the requirements. Figure 5 presents the impact of the three requirements on the Payload-Range diagram. These were produced by conditioning the Bayesian Network with the statement "Requirement must be satisfied". Mathematically this can be expressed as:

$$P(R, M_{payload} | \text{MTOM Satisfied} = \text{True}, \text{Wingspan Satisfied} = \text{True}, \text{Costs Satisfied} = \text{True}) \quad (1)$$

Parameter	Lower Bound	Upper Bound
Delivery Transit Range [km]	1	200
Delivery Loiter Time [min]	5	15
Cruise Speed [m/s]	20	50
Stall Speed [m/s]	10	25
Payload Mass [kg]	1	10
$C_{L,max}$	1.0	1.3
Specific Fuel Consumption [kg/kWh]	0.25	0.45
Aspect Ratio	6	10
Wing Taper	0.5	1

Table 1 – Ranges of input parameters of the UAV design model.

Requirement	Sign	Constraint Value
Maximum Take-Off Mass	<	10 ± 2 [kg]
Wing Span	<	2.5 ± 0.5 [m]
Operating Costs	<	0.015 ± 0.003 [GBP/km]

Table 2 – Feasibility requirements for the UAV problem.

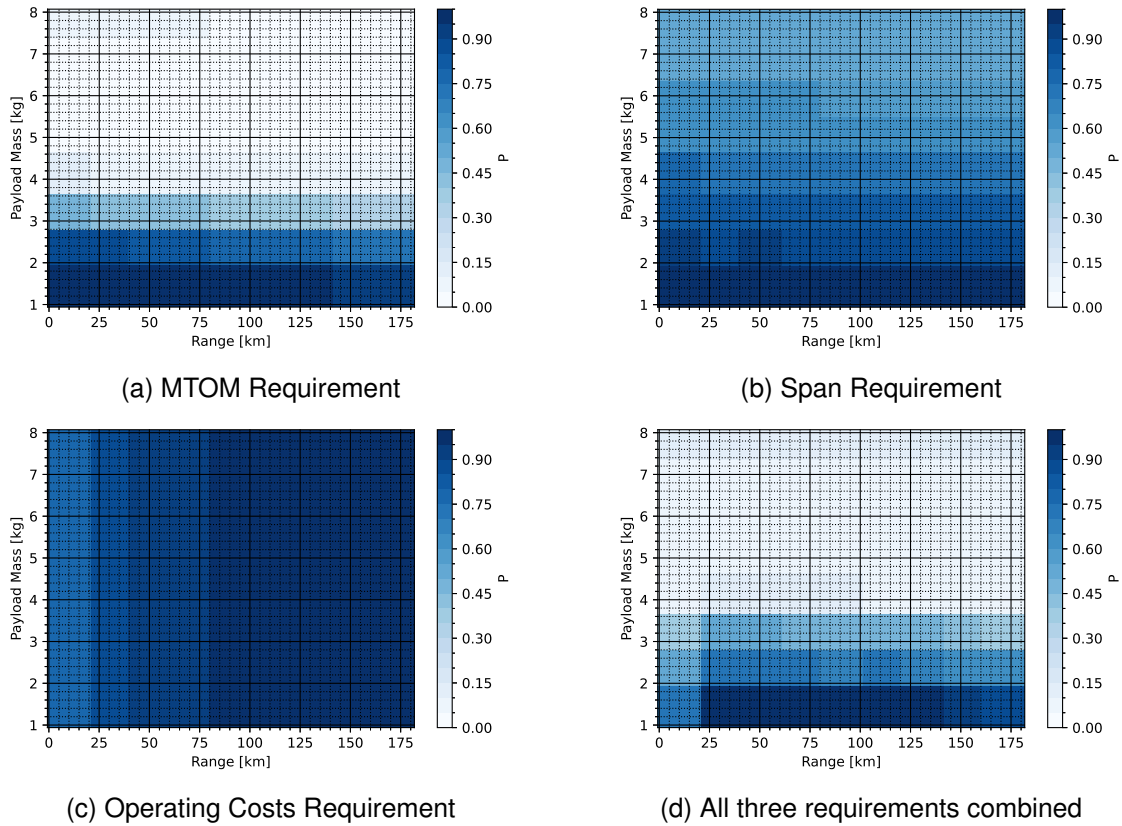


Figure 5 – Payload-Range Heatmaps of the three requirements and their combination. The Bayesian Network was queried by conditioning each requirement to be satisfied. (Values normalised between 0-1)

where the three constraints satisfaction statements (maximum take-off mass, wing span and operating costs) are set to true to satisfy the three requirements, while range R and $M_{payload}$ are evaluated with a sweep of all the possible values. The result is the probability that the statements in the parenthesis are true, that is *what is the likelihood that Range and Payload have a specified value given that the requirements are satisfied*.

The heatmap values have been normalised to the 0-1 range for ease of interpretation: these values

would be a discrete distribution across the sampled space. The MTOM requirement affects the maximum payload, suggesting a maximum of less than 4 kg, and partly the range producing a typical payload-range diagram shape where the payload reduces at high range. The operating cost requirements affect the range, as it was calculated as cost per kilometre. The combination of the three requirements point at an interval of 1-3 kg for payload and 25-140 km for range. The BN handled the uncertainty of the requirements, producing these heatmaps indicating to the designer the intervals to choose for a robust design. This area of the design space can be then further explored with optimisation algorithms to retrieve individual candidate designs satisfying the constraints.

Another possibility is querying the BN with different values for a parameter. Figure 6 presents an example using different values of the Aspect Ratio and Taper Ratio. These can be mathematically formulated as:

$$P(CD_0, L/D_{max} | AR = a, \lambda = b) \quad (2)$$

where $a = [6.5, 8, 10]$ and $b = [0.5, 0.7, 0.9]$. The expression evaluates the likelihood of all the possible values of minimum drag CD_0 and L/D_{max} given an observed value of wing aspect ratio AR and wing taper ratio λ .

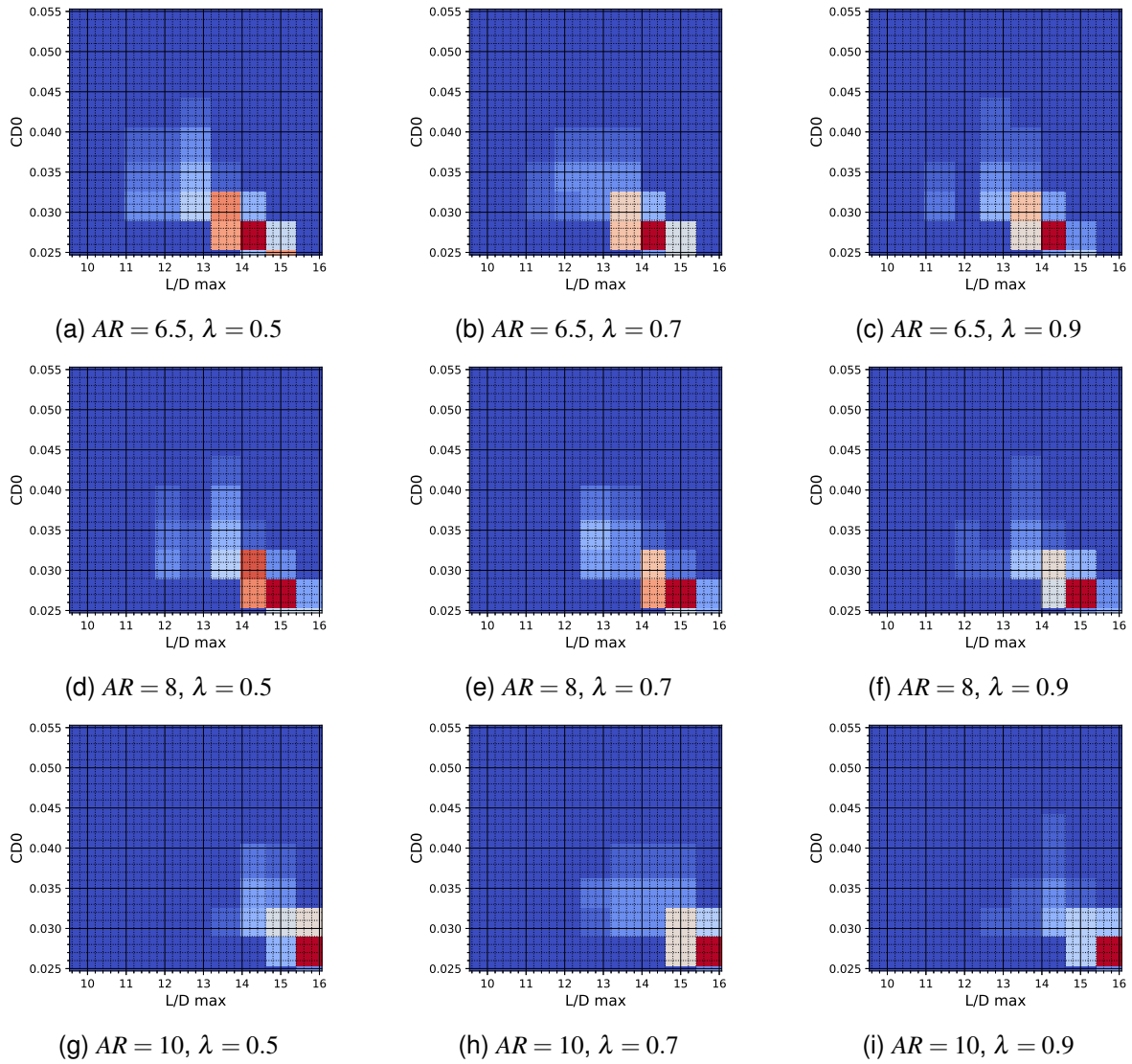


Figure 6 – Effects on CD_0 and max L/D of Aspect Ratio and Taper Ratio. Red indicates an high likelihood value.

The Aspect Ratio drives both the reduction in drag and the increment of maximum L/D , with different distribution spread depending on the value of Taper ratio. Specifically, at $\lambda = 0.5$, the effect of Aspect

Ratio is stronger, while it is less evident with a straight wing.

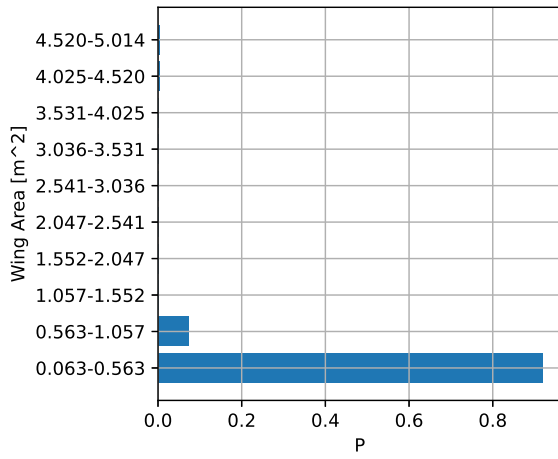
As a final example, the geometric wing parameters are extracted after selecting from Figure 5d a design range of 100 km and a payload mass of 1 kg, which is within the most promising design space area. First, the wing area S_{wing} is estimated with an inference, assuming the requirements are satisfied and with the selected design point:

$$P(S_{wing} | M_{payload} = 1 \text{ kg}, R = 100 \text{ km}, \text{Requirements Satisfied}) \quad (3)$$

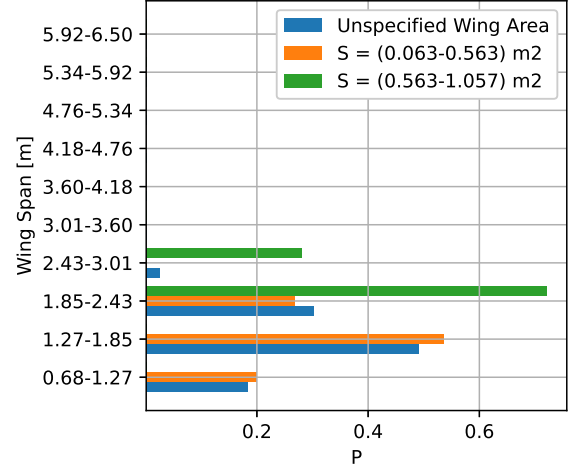
with the resulting conditional probability distribution shown in Figure 7a. The statement "Requirements Satisfied" in Eq. 3 is a shorthand for the three constraints that were imposed as satisfied in Eq. 1. Other wing geometry parameters that are queried using the same evidence of Eq. 3 are the wing span b , the wing aspect ratio AR , and the wing taper ratio λ (Eq. 4).

$$P(b | M_{payload} = 1 \text{ kg}, R = 100 \text{ km}, \text{Requirements Satisfied})$$

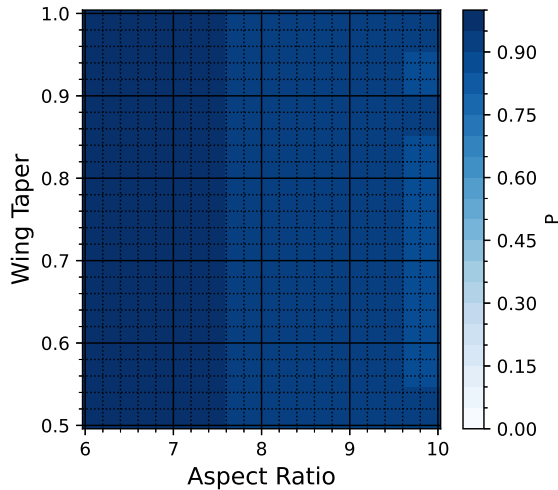
$$P(AR, \lambda | M_{payload} = 1 \text{ kg}, R = 100 \text{ km}, \text{Requirements Satisfied}) \quad (4)$$



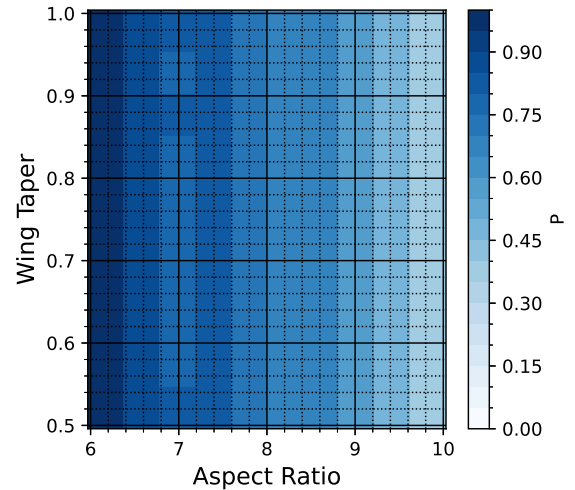
(a) Wing Area Distribution of the design point.



(b) Wingspan Distribution of the design point.



(c) Wing Geometry parameters for $S = (0.063 - 0.563) \text{ m}^2$



(d) Wing Geometry parameters for $S = (0.563 - 1.057) \text{ m}^2$

Figure 7 – Wing geometry parameters of the design point with $R = 100 \text{ km}$ and $M_{payload} = 1 \text{ kg}$.

The highest probability (91%) is within the 0.063-0.563 m² range, with the 0.563-1.057 m² range coming second. Reasonably, with a small payload the wings should not be oversized. Figure 7b

presents the predicted distribution of wingspan according to three cases: without specifying the wing area and with the wing area specified according to the values in Fig. 7a. The general distribution has a wider range, while conditioning by the selected wing area, collapses the distributions around less possible values. As should be expected, the wingspan is greater the greater the wing area, and the ranges fall within the constraint of Table 2.

Figures 7c and 7d present the distribution of aspect ratio and wing taper ratio for both selected wing areas, after performing an inference on the previous design point. There is notably indifference on the wing taper value, while low aspect ratios are preferred, especially with a larger wing area. The reason is in the wingspan constraint that has to be satisfied. On the other hand, the taper ratio is not affecting any requirement, hence its distribution is uniform. According to the Bayesian Network (Fig. 2), only C_{D0} and L/D are affected by this parameter.

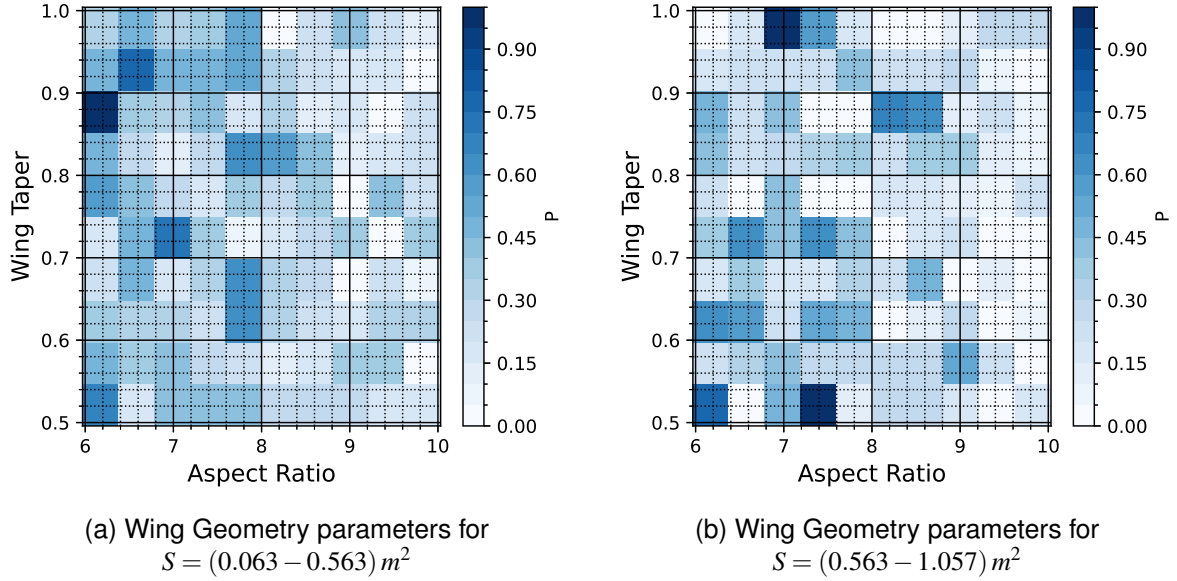


Figure 8 – Wing geometry parameters with added requirement of $L/D_{cruise} = 10$.

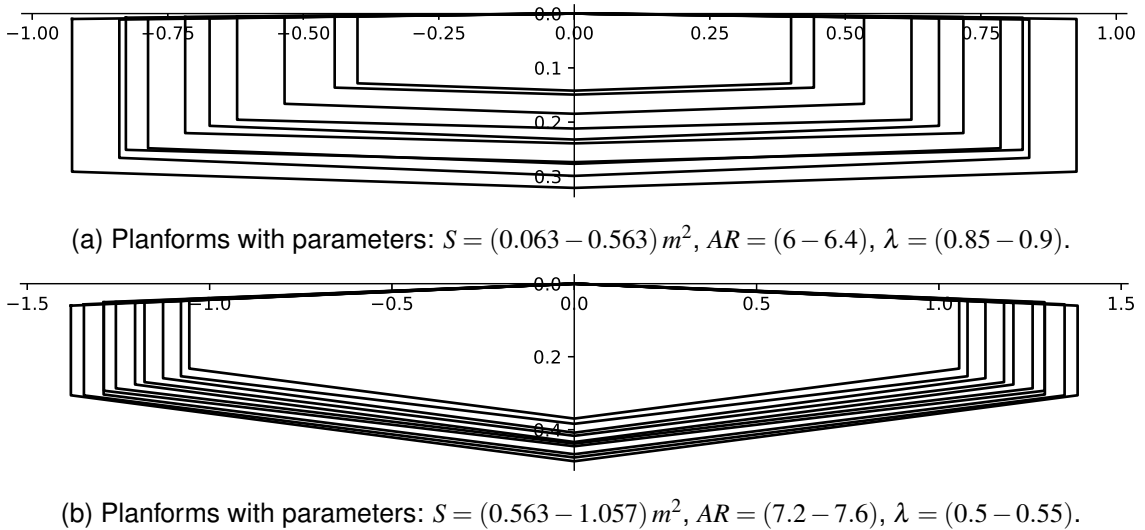


Figure 9 – Wing planforms of the most likely configurations in Fig. 8

As an example, the design space was again queried with the additional requirement of $L/D_{cruise} = 10$. The resulting values of the Aspect Ratio and Wing Taper of the two previous cases are shown in Figures 8a and 8b. The design space is now fragmented with clear indication of the highest likelihood configurations. The most likely portion of the design space was selected and sampled to produce

the wing planforms shown in Figures 9a and 9b. The result is a family of similar configurations which enable the designer to narrow rapidly the final design. Indeed, the engineer can already visualise the geometry of the UAV before carrying out detailed analysis; geometry that has been inferred from requirements and is expected to be able to satisfy them in an optimisation problem.

5. Conclusions

The potential use of Bayesian Networks for design decision-making has been presented in this article. Uncertainty of the requirements can be modelled and its impact estimated on the design space and quantities of interest. Any quantity can be queried, resulting in a potentially powerful what-if scenario analysis tool.

An example application has been shown using a UAV conceptual design example, combining semi-empirical methods with a low-fidelity aerodynamic calculation. Queries were used to find the promising combinations of the design space that satisfied a maximum take-off mass constraint, a wing span constraint, and a maximum operating cost constraint. Through propagation of these requirements, which had a 10% uncertainty on their value, a promising combination of Range and Payload mass was found. After selecting a design point from the indicated range, further inference was carried out to explore the geometric configurations that would satisfy the requirements. Two families of wing configurations were produced after adding a performance requirement of L/D .

All the inferences produced in this paper were rapidly obtained after training the model. With a properly generated dataset, Bayesian Network models could accelerate the development of models by narrowing the design space to the ranges that most likely would satisfy the requirements. Additionally, uncertainty from the requirements can be handled within the model. Further expected developments of this methodology include the integration in the PDOPT framework and streamlining of definition of the Bayesian Network structure. Future demonstrations should include an optimisation analysis to confirm the successful identification of the feasible design space.

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