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MIXED INTEGER NONLINEAR PROGRAMMING FOR EVTOL AIRCRAFT DESIGN AND OPTIMIZATION

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Abstract

Electric vertical take-off and landing (eVTOL) aircraft has great advantages in speed, efficiency and carbon emissions as compared to conventional helicopters or fixed-wing aircraft for urban air mobility (UAM), which seems to revolutionize both aviation industry and our daily commuting. As a key element of UAM, efficient eVTOL aircraft design and optimization is very important to the success of its adoption. The goal of this paper is to apply new design and optimization approach to eVTOL aircraft design for better energy efficiency. To enhance the overall aircraft level performance of the baseline eVTOL, we employ Mixed Integer Nonlinear Programming (MINLP) to include both continuous design variables and discrete design variables for eVTOL design space exploration. Our study has shown that for a given flight mission, the MINLP optimization can result in 25.5% total energy usage as compared to initial eVTOL design.

Keywords: eVTOL, aircraft design, MINLP, carbon emission, urban air mobility

1. Introduction

Personal air vehicle (PAV) or flying car has been an old dream for human beings since early 1900s [1]. However, till today there are no such products that can be used in our daily commuting after almost one century attempts. In recent years, PAV is reviving due to the advent of advanced composite materials, unmanned flight control technology, advanced electric propulsion technology such as electric motor and lithium battery. With the above features, such kind of flying vehicles usually refer to electric vertical take-off and landing (eVTOL) aircraft, which are often associated with urban air mobility (UAM) [2]. According to FAA[1], UAM builds mainly upon eVTOL aircraft to realize “a safe and efficient aviation transportation system” to transport commuters or cargo at certain altitudes and areas.

With the increasing congestion and commuting time in large cities, there is a growing desire to expand commuting transport from the ground to the air. EVTOL is as such is receiving great attention from academics, industry, investors, government departments and other stakeholders [3–6]. As a new mode of transport, eVTOL is considered as revolution in both aviation and daily commuting. Currently, there are hundreds of eVTOL program in development worldwide. The Vertical Flight Society (VFS) has collected most publicly released eVTOL program in its website[2], where many are in concept design phase or in early prototype phase. The VFS website has divided the eVTOL configuration into 4 different categories[2], i.e. Vectored Thrust such as tilt-wing, tilt-rotor, tilt-duct;

[1] https://www.faa.gov/uas/advanced_operations/urban_air_mobility
[2] https://evtol.news/aircraft

Lift+Cruise with independent thrust sources for VTOL and cruise; Hover Bikes or Personal Flying Devices such as with multicopper-type wingless configurations; Electric Rotorcraft such as electric helicopter or electric autogiro.

As mentioned above, there are hundreds of eVTOL aircraft in progress with different vehicle configurations and flight mission scenario assumptions. For efficient UAM operations, eVTOL aircraft need to outperform existing light helicopters in terms of economy, safety and to reach a level at least close to that of existing ground vehicles, especially when considering the increasing international requirements for environmental friendliness in aviation, including carbon emissions and noise. As such, it is of great significance to find proper or optimal eVTOL vehicle configurations. Generally speaking, aircraft configuration design and optimization is never easy task. For conventional aircraft design, the vehicle configuration is usually selected or determined based on historical data and/or designer's experience. The long-existing tube-and-wing configuration is also partly due to the performance feature of gas-turbine engines, i.e. larger gas-turbine engine has better performance. With electric propulsion, the propulsion scaling effects is significantly diminished. With multiple small distributed propulsion instead of one or two large engines being applied to eVTOL aircraft, the design space of vehicle configuration can be greatly extended.

In order to fully exploit the design space of eVTOL aircraft (e.g. the choice of different overall configurations, the number of engines, the choice of electric propulsion system, etc.) need to be considered as design variables. The above design variables are no longer continuous design variables compared to conventional design parameters such as wing aspect ratio, engine by-bass ratio, etc. Therefore, integer-based optimization needs to be introduced. As a result, the eVTOL aircraft design problem changes from a continuous nonlinear programming (NLP) problem to a mixed integer nonlinear programming (MINLP) problem. As an important class of optimization problems, MINLP algorithms have been well developed for solving certain aerospace design problems [7–10]. In this paper, we will utilize MINLP-based design methods to carry out an eVTOL aircraft design problem. For simplicity, we apply a baseline tilt-duct eVTOL aircraft developed by NASA for our study [11]. In addition, the design variables are only confined to ducted fan parameters. In our study, the overall eVTOL aircraft performance are firstly investigated with respect to different ducted-fan parameters. Then, MINLP optimization is carried out to include both continuous and discrete ducted-fan design variables.

2. Methods and Data

2.1 Baseline eVTOL aircraft

Ducted fan has advantages in thrust performance, noise emissions and safety as compared to open propeller due to aerodynamic benefits of the shrouded ducts [12–14]. The baseline eVTOL aircraft is in this study is taken from Six-Passenger NASA UAM tilt-duct eVTOL reference vehicle [11]. Figure 1 shows the geometric layout of the reference tilt-duct eVTOL aircraft from Ref. [11], which is reproduced using NASA OpenVSP tool. Table 1 gives a short summary of the top-level aircraft parameters of the reference eVTOL aircraft of Ref. [11]. It has to noted that only parameters that will be used in the following simulations are listed here.

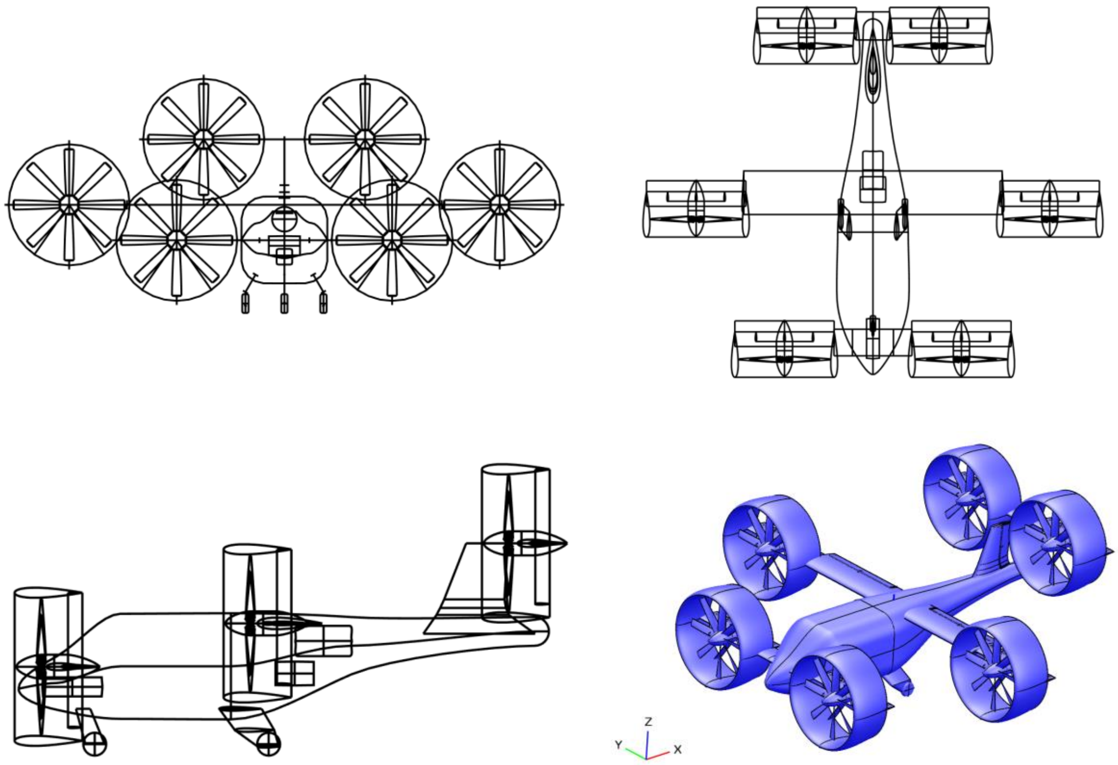


Figure 1 3-D views of NASA UAM tilt-duct eVTOL aircraft, reproduced with NASA OpenVSP, data based on Ref. [11]

Table 1 Top-level aircraft parameters of NASA UAM tilt-duct eVTOL reference vehicle, data based on Ref. [11]

Parameter	Value
Design gross weight	7089 lb (3215.5 kg)
Payload	1200 lb (544.3 kg)
Ducted-fan diameter	7.08 ft (2.16 m)
Hover disk loading	30 lb/ft^2 (146.5 kg/m^2)
Cruise airspeed	151 kt (77.7 m/s)
Block speed	115 kt (59.2 m/s)
Cruise lift to drag ratio	9.1
Cruise wing area	229 ft^2 (21.3 m^2)
Flight range	75 NM (138.9 km)
Cruise reserve	20 minutes

2.2 Ducted fan analysis

In our study, the open-source DFDC code that has been developed by Drela and Youngren [15], is employed for ducted fan analysis. Using the lifting-line to represent the rotor blade and axisymmetric panel to represent the duct and centerbody, the fidelity of DFDC code is between classic blade-element methods and a general 3-D panel method, which makes it an ideal code for fast and effective ducted fan design and analysis [15]. To verify the accuracy of the DFDC code, the experimental results of a benchmark ducted fan from NASA [16] are selected for comparison. Table 2 summarizes the geometric data of the benchmark ducted fan of Ref. [16]. Figure 2 shows the comparison of

DFDC calculated results with wind tunnel experimental results from literature [16]. As can be seen in the figure, the prediction of DFDC satisfies well with wind tunnel experiment, i.e. the deviations are acceptable for conceptual aircraft design and optimization. Therefore, we select DFDC for our further ducted fan analysis, as a tradeoff between computation time and accuracy.

Table 2 Key geometric data of the ducted fan, data from Ref. [16]

Parameter	Value
Duct inside diameter at propeller, inch	84.75
Duct Maximum external diameter, inch	101.56
Duct chord, inch	49
Duct exit diameter, inch	93.3
Duct net exit area, ft^2	44.08
Duct net area at propeller, ft^2	37.84
Duct rotation station, percent of duct chord	64
Propeller station, percent of duct chord	28.6
Propeller diameter，ft	7
Tip clearance, in	0.4
Number of blades	3
Integrated design lift coefficient	0.43

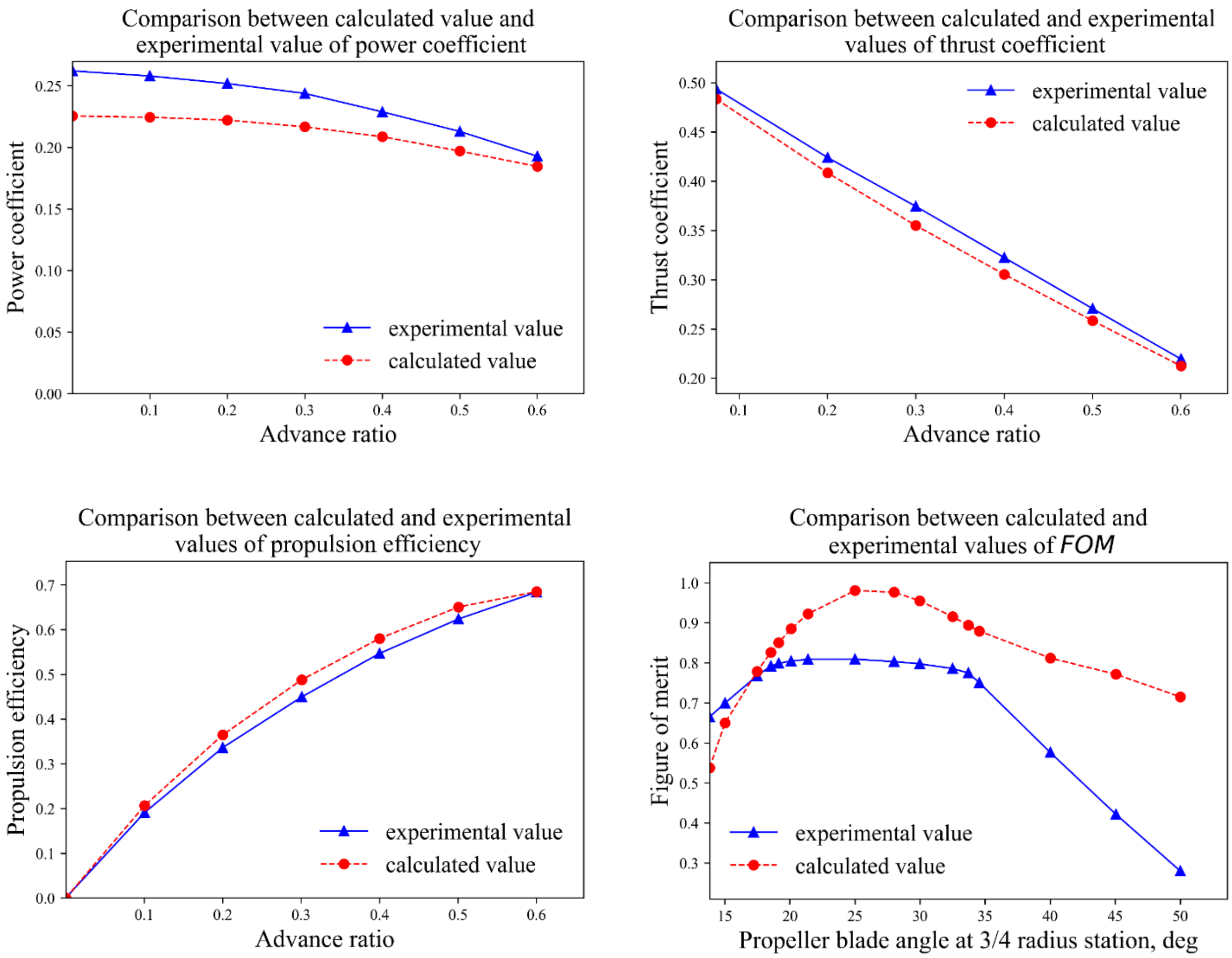


Figure 2 Comparison of DFDC calculated results with wind tunnel experimental results

2.3 The overall analysis and optimization process

Figure 3 shows the overall eVTOL design and optimization process in our study. As most disciplinary data are directly taken from literature [11], the design variables are only from ducted fan parameters. The design objective is total energy used in a predefined flight mission. As mentioned previously, the ducted fan parameters include both continuous and discrete variables. The Bayesian Optimization (BO) approach [10] is adopted for current eVTOL aircraft design and optimization.

The test results of Bayesian Optimization (BO) under different benchmarks are shown in Tables 3 and 4, where Table 3 describes information on the benchmarks and Table 4 summarizes the results of BO. The results show that the BO can give a good optimization solution even with a reduced number of optimization steps, indicating that makes BO good competitive when dealing with computationally expensive optimization problems with both continuous and discrete design variables.

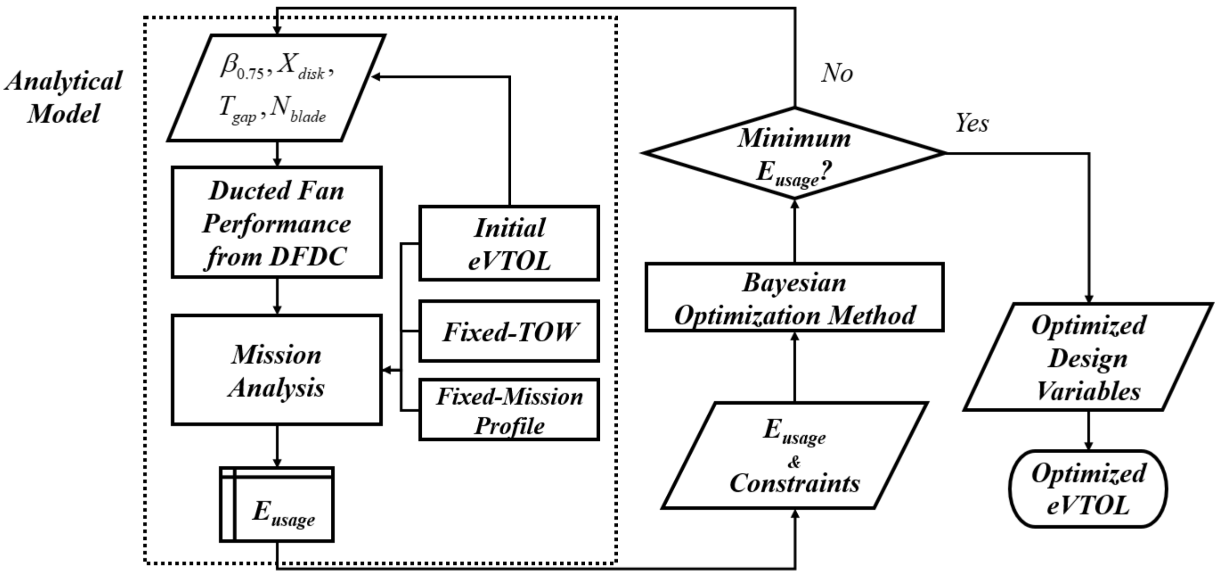


Figure 3 The overall design and optimization process

Table 3 Description of the benchmarks

Optimization benchmark 1 **Discrete Beam Design**	
Minimize $f(A_s, b, h) = 0.6bh + 29.4A_s$ **Subject to** $g_1 = \frac{h}{b} - 4 \le 0$ $g_2 = \frac{7.375A_s^2}{b} + 180 - A_s h \le 0$	**Where** $A_s \in \{6.0, 6.16, 6.32, 6.6, 7.0, 7.11, 7.2, 7.8, 7.9, 8.0, 8.4\}$ $h \in \{28, 29, 30, ..., 39, 40\}$ $b \in [5, 10]$
Optimization benchmark 2 **Discrete Rosenbrock Function**	
Minimize $f(x_1, x_2, x_3) = (1-x_1)^2 + 100(x_2 - x_1{}^2)^2 + (1-x_2)^2 + 100(x_3 - x_2{}^2)^2$	**Where** $x_1, x_2, x_3 \in Z$ $x_1, x_2, x_3 \in [-10, 10]$
Optimization benchmark 3 **Spring Design**	
Minimize $f(w, d, L) = (L+2)w^2 d$ **Where** $w \in [0.05, 2.0]$ $d \in [0.25, 1.3]$ $L \in [2.0, 15.0]$	**Subject to** $g_1 = 1 - \frac{d^3 L}{71785 w^4} \le 0$ $g_2 = 1 - \frac{140.45 w}{d^2 L} \le 0$ $g_3 = \frac{2(w+d)}{3} - 1 \le 0$ $g_4 = \frac{d(4d-w)}{w^3(12566-w)} + \frac{1}{5108 w^2} - 1 \le 0$

Table 4 Results of the benchmarks

Benchmark		**Solution from [17]**	**Solution by BO**
Discrete Beam Design	Step	1050	**60**
	Result	(6.32,8.5,34)	**(7.11,8.51,32)**
		359.2080	**366.3058**
Discrete Rosenbrock Function	Step	156	**10**
	Result	(1,1,1)	**(1,1,1)**
		0	**0**
Spring Design	Step	1000	**300**
	Result	(0.05159,0.35675,11.28713)	**(0.05,0.37334,12.40672)**
		0.012665	**0.013446**

3. Results

3.1 Sensitivity study of ducted fan parameters

Before carrying out the optimization study, the sensitivity analysis of propulsive efficiency, figure of merit (FOM) and the pressure distribution on duct surface with respect to advance ratio, tip gap, distance between rotor disk and leading edge, number of blades is investigated. The baseline calculation conditions for the sensitivity study of ducted fan parameters are listed in Table 5.

Table 5 Baseline calculation conditions for ducted fan

Parameters		Value	Unit
Number of blades		8	-
Tip gap		10.00	mm
Blade angle at 75% radius	Pressure distribution analysis	35	°
	Propulsion efficiency analysis	18	°
	Hover efficiency analysis	18	°
Distance between rotor disk and leading edge		0.408	m

For tilt-duct eVTOL, one of the most important parameters is ducted fan propulsive efficiency. Figure 4 shows the sensitivity study of propulsive efficiency depending on advance ratio, tip gap, distance between rotor disk and leading edge, number of blades. As can be seen in the figure, the propulsive efficiency increases gradually with improving advance ratios. To better represent the impacts of coupling impacts, the relative gain in efficiency is plotted versus corresponding advance ratios and other ducted fan parameters.

Similarly, the FOM and pressure distribution on duct surface sensitivity studies as functions of advance ratio, tip gap, distance between rotor disk and leading edge, number of blades are carried out (cf. Figure 5 and Figure 6, respectively). The trends indicate that a more comprehensive optimization study is necessary to find the desired the ducted-fan propulsive efficiency, FOM and pressure distribution.

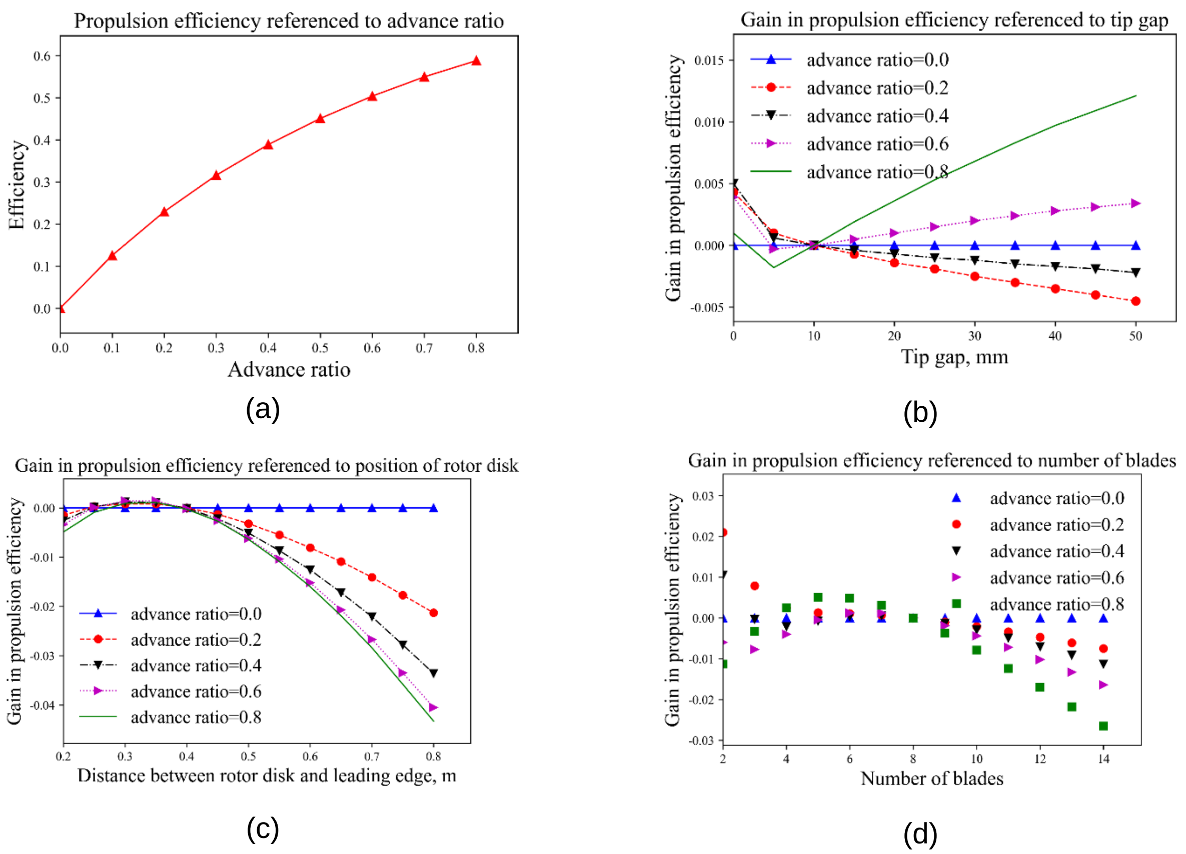
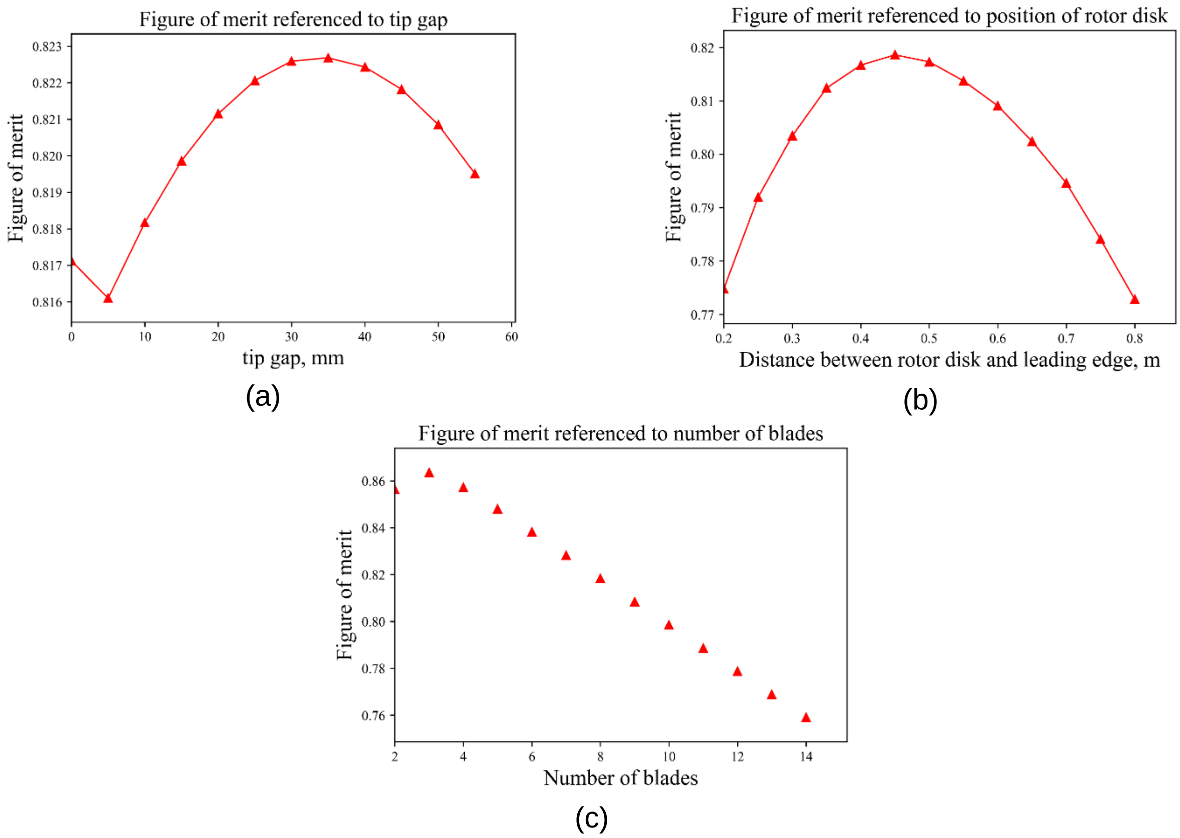
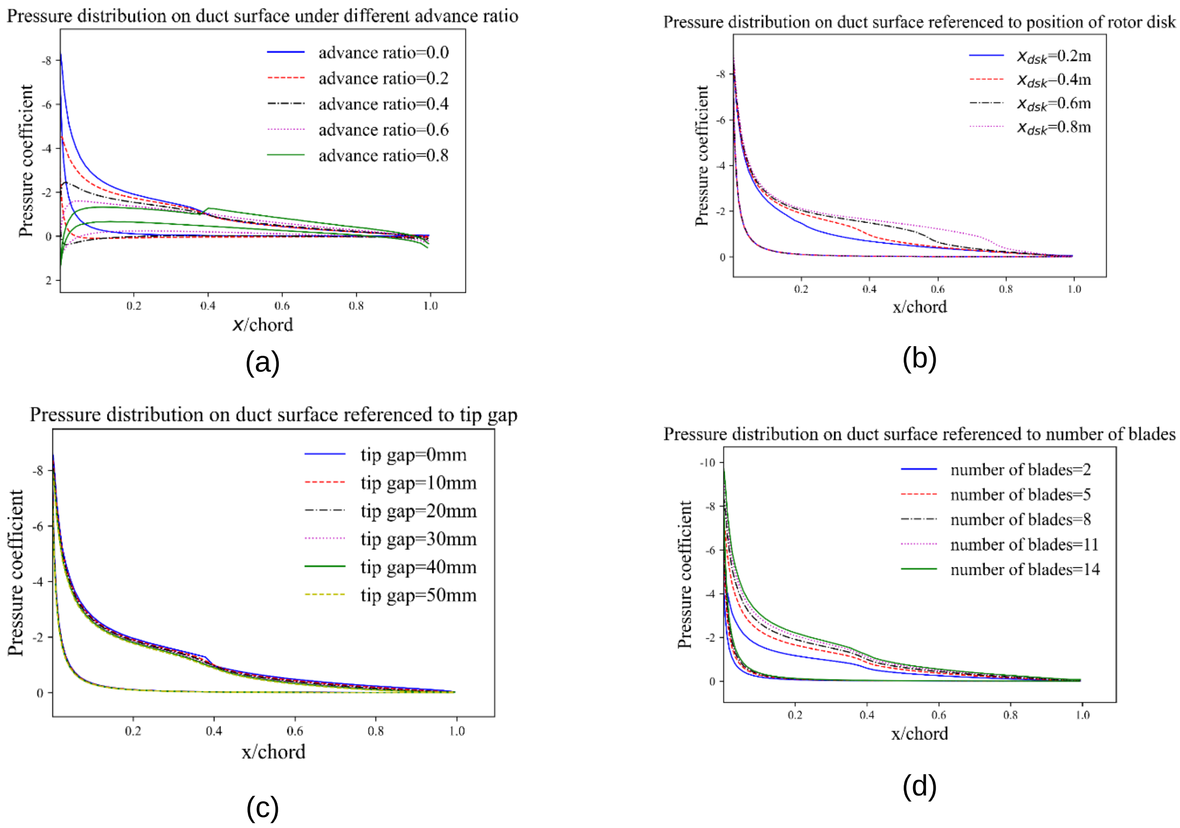


Figure 4 Sensitivity study of propulsive efficiency depending on advance ratio, tip gap, distance between rotor disk and leading edge, number of blades



(a) (b) (c)

Figure 5 Sensitivity study of FOM depending on tip gap, distance between rotor disk and leading edge, number of blades



(a) (b) (c) (d)

Figure 6 Pressure distribution on duct surface with respect to advance ratio, tip gap, distance between rotor disk and leading edge, number of blades

3.2 Optimization of ducted fan parameters considering the mission

Based on the sensitivity study results shown in previous section, we carry out the eVTOL aircraft performance optimization. The full optimization formulation is listed in Table 6, with total energy usage as design objective, blade angle at 75% radius, tip gap, distance between rotor disk and leading edge and number of blades as design variables, and take-off weight and the rate of climb as design constraints.

The optimization results in Table 7 show that the reduction of tip gap and number of blades is favorable to reducing the total energy usage. About 25.5% total energy usage can be reduced via optimization as compared to initial eVTOL case.

Figure 7 shows the ducted fan performance in cruise, climb, descent and hover conditions. Comparison of ducted fan performance during cruise, a 55% improvement in efficiency and 46% reduction in power can be achieved. In climbing conditions about 4% increase in ducted fan efficiency and 4% decrease in power can be achieved. In vertical descent condition, the ducted fan efficiency is increased by 5% while the power is slightly decreased by 2%. When coming to the hovering condition, thrust coefficient and power coefficient increase while the ducted fan efficiency increases by 15% and the power decreases by 15%.

Table 6 Formulation of the tilt-duct optimization problem

	Function/variable	Nature	Quantity	Range
Minimize	Total Energy Usage	cont	1	
With respect to	Blade angle at 75% radius	cont	1	[30., 50.]
	Disk axial location (m)	cont	1	[0.2,0.8]
	Trailing edge gap (mm)	cont	1	[5.,50.]
	Number of blades	discrete	1	[2,12]
	Total variables		**4**	
Subject to	TOM≤MTOW	cont	1	
	Rate of climb > 4.572 (m/s)	cont	1	
	Total constraints		**2**	

Table 7 Result of the tilt-duct optimization

	Variables				Results	
	$\beta_{0.75}$	$X_{disk}(m)$	$T_{gap}(mm)$	N_{blade}	$E_{usage}(MJ)$	$\downarrow$ %
Initial	35.00	0.408	20.00	8	2368.6	**25.5%**
Optimized	**36.92**	**0.691**	**5.02**	**2**	**1763.5**	

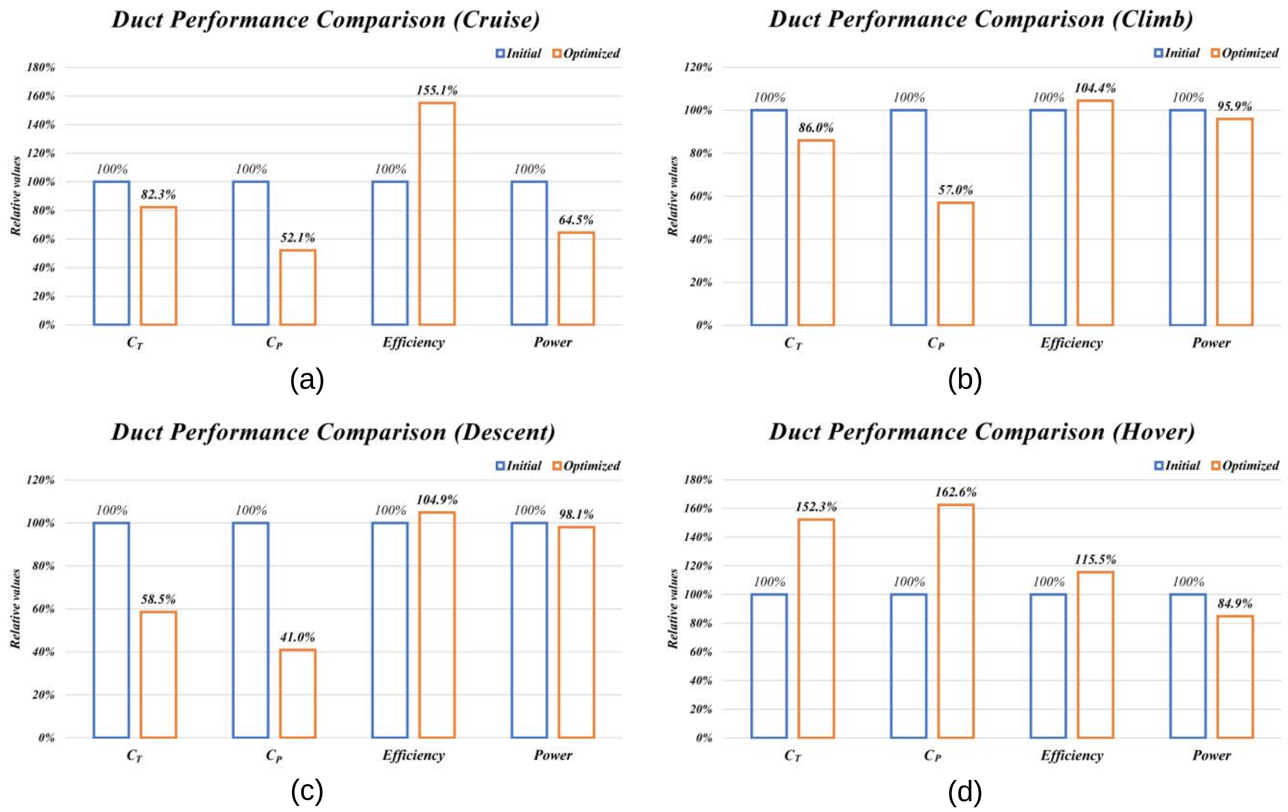


Figure 7 Duct performance comparison in different scenario

Figure 8 shows the comparison of pressure distribution on duct surface in hovering and cruise state before and after optimization. The changes in pressure distribution due to changes in design variables are also in line with the trends given in the previous sensitivity analysis.

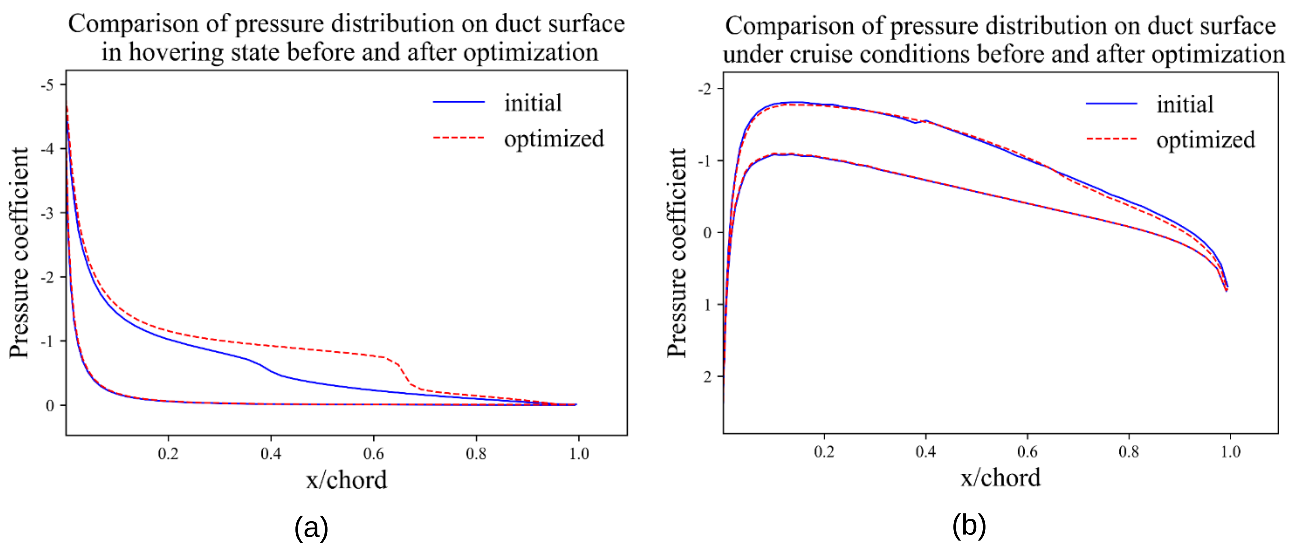


Figure 8 Comparison of pressure distribution on duct surface in hovering and cruise state before and after optimization

4. Conclusion

To investigate the potential and further to improve eVTOL aircraft, we have carried out detailed flight mission analysis and optimization based on the ducted fan performance calculated from DFDC and MINLP based on BO. Main summary and conclusion are summarized as follows.

1. To figure out the ducted fan parametric impacts on the ducted fan performance, a detailed

sensitivity study on the propulsion efficiency, figure of merit, pressure distribution on duct surface with the advance ratio, tip gap, distance between rotor disk and leading edge, number of blades have been carried out.

2. For the tilt-duct eVTOL aircraft performance optimization, the MINLP approach has been applied to cope with both continuous and discrete ducted-fan design variables. The optimization results show that the efficiency of the ducted fan in the cruise condition is significantly improved after optimization throughout the mission, while the power is also significantly reduced. There were also small improvements in ducted fan efficiency and power in the other flight conditions. These changes resulted in a 25.5% reduction in the total energy use of the optimized eVTOL aircraft compared to the initial tilt-duct eVTOL aircraft.

Nomenclature

C_T = thrust coefficient
C_P = power coefficient
E_{usage} = total energy usage
N_{blade} = number of blades
L/D = lift to drag ratio
T_{gap} = tip gap
X_{disk} = distance between rotor disk and leading edge
$MTOW$ = maximum take-off weight
R = cruise distance
TOW = take-off weight
$\beta_{0.75}$ = blade angle at 75% radius

Copyright Statement

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