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UPDATING OF JET TRAINER AIRCRAFT DYNAMIC MODEL TO RESULTS OF GROUND VIBRATION TEST

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Abstract

This paper discusses updating of a jet trainer aircraft dynamic model according to the results of ground vibration test. The paper outlines the process of flutter analysis with the special regard to the role of the ground vibration test. Next, the Bayesian parameter estimation method, which was employed for updating, is outlined. Finally, the application example is provided. The results of the symmetric model are shown, evaluated and the conclusions are formulated.

Keywords: aeroelasticity, model updating, Bayesian Method

1. Introduction

Computational (FEM) models of aircraft structures used for aeroelastic analyses must be validated according to experimental results. The most important are modal characteristics, which represent the main input data for flutter analyses. Flutter analyses have ultimate character and accuracy, and reliability of flutter results are strongly dependent on the accuracy and reliability of the input modal data. Therefore, the model updating according to the ground vibration test (GVT) results is required. Various applications of model updating are documented in references [1-10]. Model updating is used usually for the models of larger aircraft, where the aircraft development process is longer and the certification procedure is more complex and may include further modifications, parametric studies, multi-mass configurations, etc., which would not be feasible using the GVT-based modal data directly as is usual for the general aviation aircraft [11] [12].

2. Theoretical Background

Updating is a problem of multidisciplinary optimization (MDO), i.e., of seeking the optimal combination of parameters to minimize the objective function respecting specified constraining functions and boundary conditions. For the effective model updating, the Bayesian Least Squares Estimation Method [13] is frequently employed.

The objective function (OBJ) is expressed as:

$$OBJ = \{\Delta R\}^T [W_R] \{\Delta R\} + \{\Delta P\}^T [W_P] \{\Delta P\} \quad (1)$$

It represents the weighted sum of the error in design responses **{ΔR}** and the difference in design variables **{ΔP}**. **[W_P]** and **[W_R]** are then diagonal scatter matrices for design variables and for design responses, respectively. The solution is iterative, expressed as:

$$\{P_u\} = \{P_0\} + [G]\{-\Delta R\} \quad (2)$$

where **{P_u}** is the vector of design variables after updating; **{P_0}** is the vector of design variables before updating; **{ΔR}** is the design response change vector and **[G]** is the gain matrix calculated according to Bayesian Estimation Theory.

Provided that the number of design responses is higher compared to the number of design variables, **[G]** is expressed as:

$$[G] = ([W_P] + [S]^T[W_R][S])^{-1}[S]^T[W_R] \tag{3}$$

In the more frequent case, in which the number of design variables is higher compared to the number of design responses, **[G]** is expressed as:

$$[G] = [W_P]^{-1}[S]^T([W_R]^{-1} + [S][W_P]^{-1}[S]^T)^{-1} \tag{4}$$

[S] is the sensitivity matrix representing rates of design response changes with respect to change in design variables, expressed as:

$$[S] = \left[\frac{\partial R_i}{\partial P_j}\right] \tag{5}$$

Response modal parameters **[R]** include GVT-based natural frequencies and mode shapes. Correlation of natural frequencies is considered as the relative frequency error, which is expressed as:

$$\{\varepsilon_f\} = \left[\frac{f_{FEM} - f_{GVT}}{f_{GVT}}\right] \tag{6}$$

Correlation of mode shapes **{Ψ}** is considered in the form of the Modal Assurance Criterion **(MAC),** expressed as:

$$MAC(\Psi_{FEM}\,,\Psi_{GVT}\ ) = \frac{|(\{\Psi_{FEM}\}^T\{\Psi_{GVT}\})|^2}{\left((\{\Psi_{FEM}\}^T\{\Psi_{FEM}\})(\{\Psi_{GVT}\}^T\{\Psi_{GVT}\})\right)} \tag{7}$$

where subscripts in eqn. 6 and 7 denote for an analytical (FEM) model and for an experimental (GVT) data, respectively.

Sensitivity coefficient for a natural frequency can be expressed as:

$$\frac{\partial R_i}{\partial P_j} = \frac{\partial f_i}{\partial P_j} = \frac{\{\Psi_i\}^T\left(\frac{\partial[K]}{\partial P_j} - (2\pi f_i)^2\ \frac{\partial[M]}{\partial P_j}\right)\{\Psi_i\}}{8\pi^2 f_i(\{\Psi_i\}^T[M]\{\Psi_i\})} \tag{8}$$

Solution of eqn. 8 is semi-analytical as the derivatives of stiffness **[K]** and mass **[M]** matrices with respect to design variables $\mathbf{P_j}$ are approximated using Finite Difference Method.

Sensitivity coefficients for MAC can be expressed as:

$$\frac{\partial R_i}{\partial P_j} = \frac{\partial MAC(\Psi_{FEM}\,,\Psi_{GVT})}{\partial P_j} = 2\frac{\{\Psi_{GVT}\}^T\{\Psi_{FEM}\}\{\Psi_{GVT}\}^T\ \frac{\partial\{\Psi_{FEM}\}}{\partial P_j}}{\{\Psi_{GVT}\}^T\{\Psi_{GVT}\}\{\Psi_{FEM}\}^T\{\Psi_{FEM}\}} - \frac{(\{\Psi_{GVT}\}^T\{\Psi_{FEM}\})^2\{\Psi_{FEM}\}^T\ \frac{\partial\{\Psi_{FEM}\}}{\partial P_j}}{\{\Psi_{GVT}\}^T\{\Psi_{GVT}\}\,(\{\Psi_{FEM}\}^T\{\Psi_{FEM}\})^2} \tag{9}$$

Linear linkage condition including an arbitrary combination of both design variables and design responses is applicable as:

$$P_k = C_0 + \sum_i C_i\,P_i + \sum_j C_j\,R_j \tag{10}$$

where **C**-terms are factors of a linear combination. Finally, side constraints for both design variables and design responses are also applicable:

$$P_i^L \le P_i \le P_i^U$$

$$R_j^L \le R_j \le R_j^U \tag{11}$$

where **L** and **U** superscripts denotes for lower and upper bound, respectively.

3. Methodology of Model Updating

Updating of an aircraft structure model is very complex problem. It requires good knowledge regarding the possible error sources, accuracy and reliability of the input data and the dynamical behavior of the structure. Updating is usually performed in several steps in which the strategy is appropriately modified according to the situation. The key issue is the appropriate selection of design variables and design responses and setting of scatter values [5] [6].

3.1 Analytical Model

FEM model has a character of a dynamic stick model. Such models are usually used for flutter analyses of ordinary aircraft structures. Contrary to the detailed models, stick models are suitable for updating as there is relatively simple relation between design variables and design responses. Stiffness model includes mass-less beam-like elements (structural parts) and scalar springs (specific connections, control surface actuation, etc.). Inertia model includes lumped mass elements with the appropriate moments of inertia. The model usually includes a single side only with either symmetric or antisymmetric boundary condition. The example of structural model is shown in figure 1.

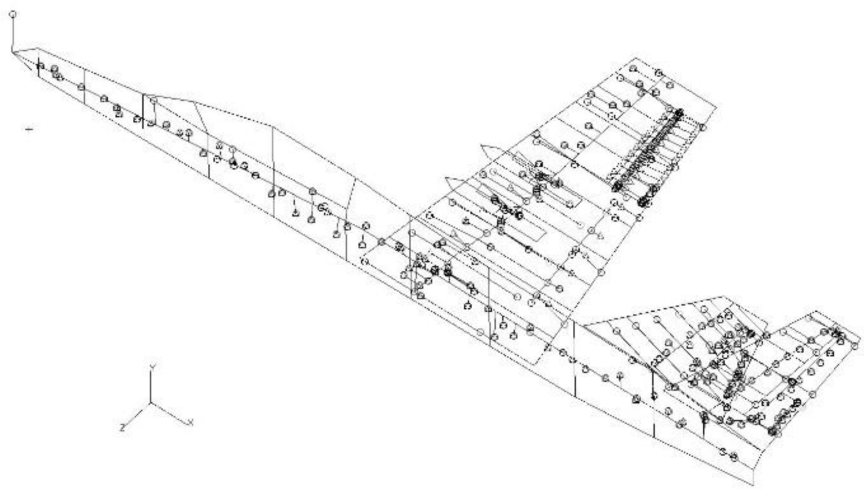

Figure 1 – FEM model (jet trainer aircraft).

3.2 Experimental Model

GVT data are reduced to the modes used for updating. The grid of measurement points is adjusted and reduced to those ones that are used for updating. Provided that uniaxial sensors are used, the deformations are to be recalculated to the triaxial scheme.

The appropriate selection of points is important as it affects the MAC-values, i.e., the correlation criterion of mode shapes. Figure 2 shows grid of experimental points, which includes main structural parts and control surfaces. Alternatively, the reduced grid of main structural parts only may be used as well. FEM node – GVT point pairing is then based on topology.

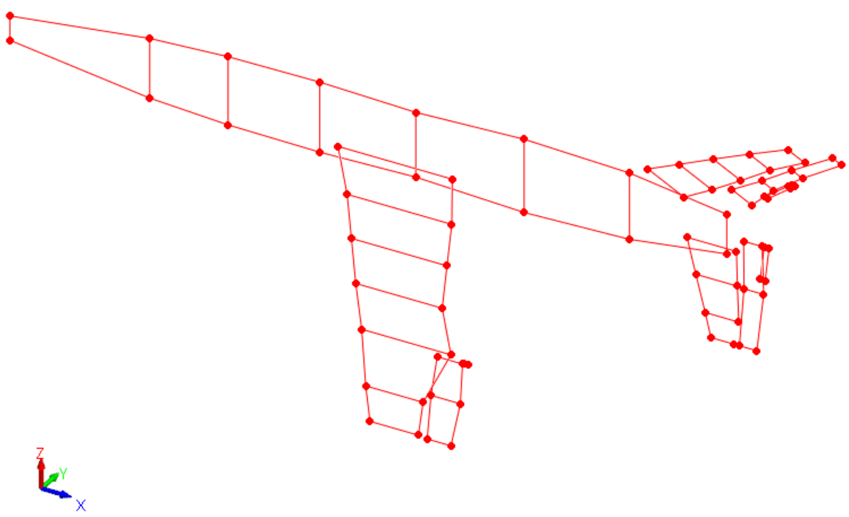

Figure 2 - GVT model (jet trainer aircraft)

3.3 Preparatory Activities

Design variables for updating are the stiffness data, i.e., beam-like elements stiffness and scalar spring elements stiffness. Compared to the inertia data, the stiffness data based on the virtual prototype are considered as less accurate and reliable.

Therefore, the data, which are not considered as design variables must be validated and adjusted prior the updating. The preparatory activities include the adjustment of control surfaces and tabs mass data according to the weighing. In addition, the total inertia data are adjusted according to the prototype weighing. Finally, effective stiffness of tabs actuation is updated according to the static stiffness measurements as the GVT-based tab flapping modes result data are somewhat unreliable due to the high natural frequencies and complicated identification of tab flapping modes.

3.4 Main Phase

Design variables include beam-like elements vertical bending stiffness, in-plane bending stiffness and torsional stiffness and scalar spring stiffness modeling control surface actuations and structural part connections. Beam-like elements design variables include two choices (global level, i.e., scale factor for a group of elements and local level, i.e., independent change of any single variable).

Design responses (i.e., natural frequencies, MAC-values) include bending and torsional modes of the main structural parts and flapping modes of control surfaces. Modes are split into symmetric and antisymmetric modes and the updating is performed for both groups separately. Therefore, separate models with the diverse final values of design variables for symmetric and antisymmetric case are obtained.

Mode pairing (FEM and GVT) is performed manually by a visual comparison of mode shapes using the specific graphic format showing node lines and modal deformation of structural parts. Although MAC-values are used as design responses, automated pairing of modes according to MAC-values is not applicable as it may lead to inappropriate pairing, because the aircraft structure is very complicated dynamical system with the high modal density.

First, updating of the baseline configuration is performed. As the next step, correlation analysis of the updated model for additional mass configurations with the corresponding GVT data is performed and, provided that the results are not satisfactory, further updating using additional design variables is performed. As the result, the diverse models for each mass configuration may be obtained.

Note that the possibility to include the GVT data of multi-mass configuration into updating process and take a single model for multiple mass configurations is not recommended. Such an approach makes the design space more extensive and consequently, such updating gives worse results.

Contrary to that, updating considering only a subset of major modes, contributing to a specific flutter instability, is feasible.

4. Application Example

Model updating is demonstrated on the example of the new Czech jet trainer aircraft (wingspan 9.4 m, maximal take-off weight 5800 kg). GVT of the aircraft prototype has been accomplished by the VZLU GVT-lab team in 2019 [17]. GVT included a single (baseline) mass configuration for which a complete set of modes has been measured. Additional configurations included specific pod-based configurations or specific conditions of the control system. For these additional configurations, just appropriate modes were measured, e.g., pod-modes, control system transfer functions, etc.

4.1 Global Updating

As the example, updating of the symmetric model is presented here. Experimental results of the baseline configuration included 16 symmetric modes, which would be applicable for updating. From these modes, 12 symmetric modes were selected. The list of the selected experimental modes is shown in table 1.

Tab.1 - Experimental modes (symmetric)

#	title	f_0 [Hz]
01	1st symmetric wing bending	14.603
02	Symmetric aileron flapping	14.970
03	1st fuselage vertical bending	18.130
04	Symmetric elevator flapping (fixed stick)	24.101
05	1st symmetric tailplane bending	27.979
06	2nd fuselage vertical bending	35.263
07	1st symmetric wing torsion	38.461
08	2nd symmetric wing bending	51.943
09	1st symmetric wing in-plane bending	60.224
11	2nd symmetric wing torsion	70.131
12	1st symmetric tailplane in-plane bending	76.146
14	1st symmetric tailplane torsion	87.698

Comparison of the initial and final pairing of modes is shown in figures 3 and 4. Pair numbers correspond to the GVT-mode numbers according to table 1. Figure 3 demonstrates relative error in natural frequencies. The final errors are less than 4.5 %. This is excellent result. Ordinarily, the errors up to 5% are considered as good, up to 10 % as acceptable. Figure 4 shows a comparison of the initial and final state in terms of MAC values. The results are also good, all MAC values increased or remained. The only exception is the mode # 01 (1st symmetric wing bending) for which the low MAC value is caused by the aileron points. Nevertheless, provided that the MAC is considered excluding the aileron points, the value increase to 97.6 %. The reason is the cross-influence of 1st symmetric wing bending and aileron flapping modes, the frequencies of which are very close one another.

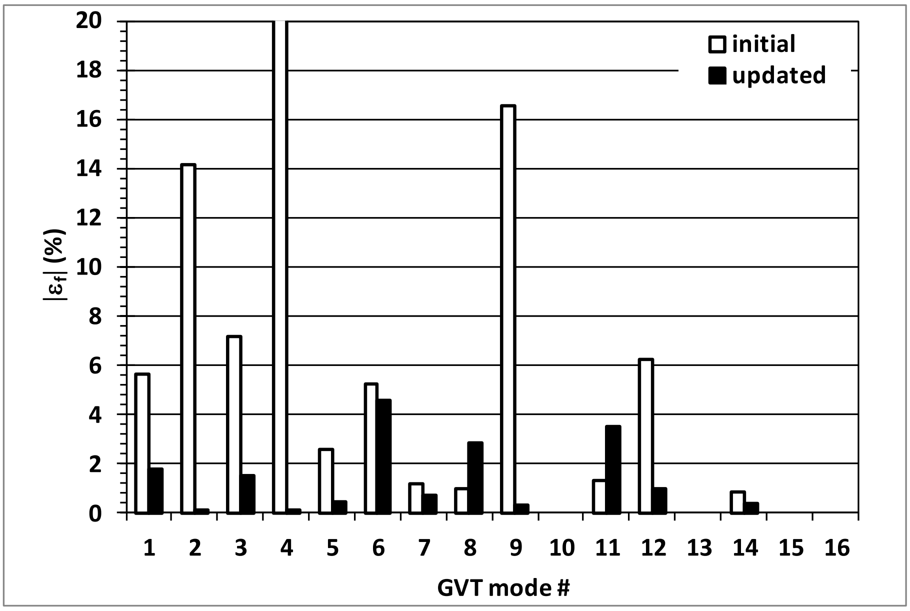

Figure 3 - Comparison of initial and final model, baseline configuration, frequency error

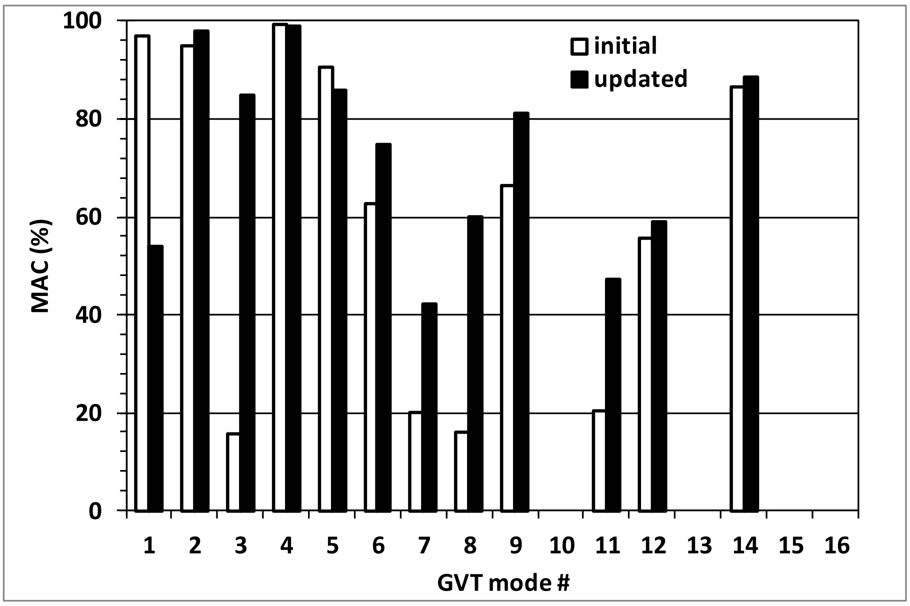

Figure 4 - Comparison of initial and final model, baseline configuration, MAC values

4.2 Wing Modes Updating

In addition, updating to the subset of four wing modes (1st wing bending, 1st fuselage vertical bending, 2nd fuselage vertical bending and 1st wing torsion), which are the main modes contributing to the wing flutter, was also performed. As the initial state, the model updated for the baseline configuration was used, except for the wing stiffness, for which the initial stiffness was used. The results are shown in figures 5 and 6. The improvement of the model agreement with the GVT results is not as significant as for the previous example. The main advantage here is the much lower change in wing stiffness parameters compared to the global updating of the baseline configuration (see section 4.3).

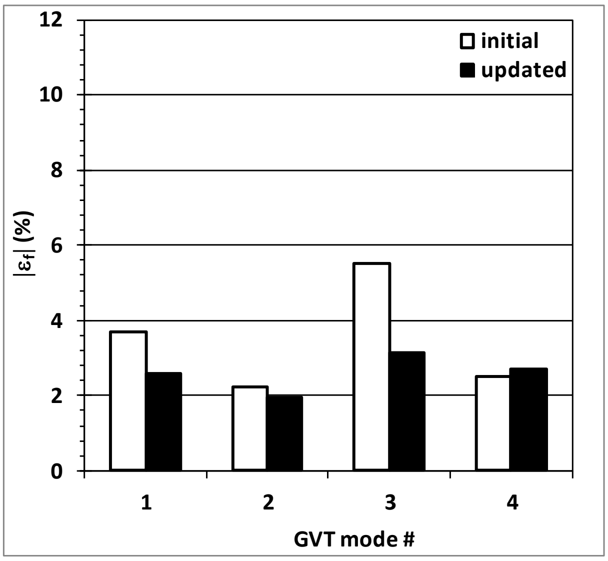

Figure 5 - Comparison of initial and final model, wing modes, frequency error

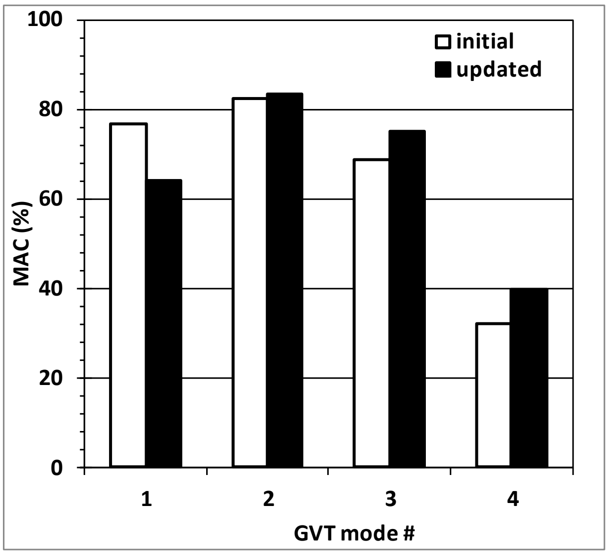

Figure 6 - Comparison of initial and final model, wing modes, MAC values

4.3 Changes in Design Variables

Changes in design variables during updating are presented in figures 7 and 8. Both figures show wing stiffness distribution in the spanwise direction expressed as the cross-sectional inertia. Figures present stiffness for initial state and for the two presented updated states. Figure 7 shows torsional stiffness while figure 8 shows vertical bending stiffness. As apparent from both figures, the changes in design variables for the global updating is very significant, especially in the root area in which the influence of the local flexibility of the wing and fuselage connection is simulated. Also, stiffness hump roughly at the 1/3 of spanwise station is significant. This hump is caused by 2nd bending and torsional modes, which are included into the design space for the global updating.

Contrary to that, the changes of the wing stiffness parameters for updating to the wing modes are low and character of stiffness spanwise distribution was kept. The reason is that just 1st wing bending, and torsional modes were included into this updating.

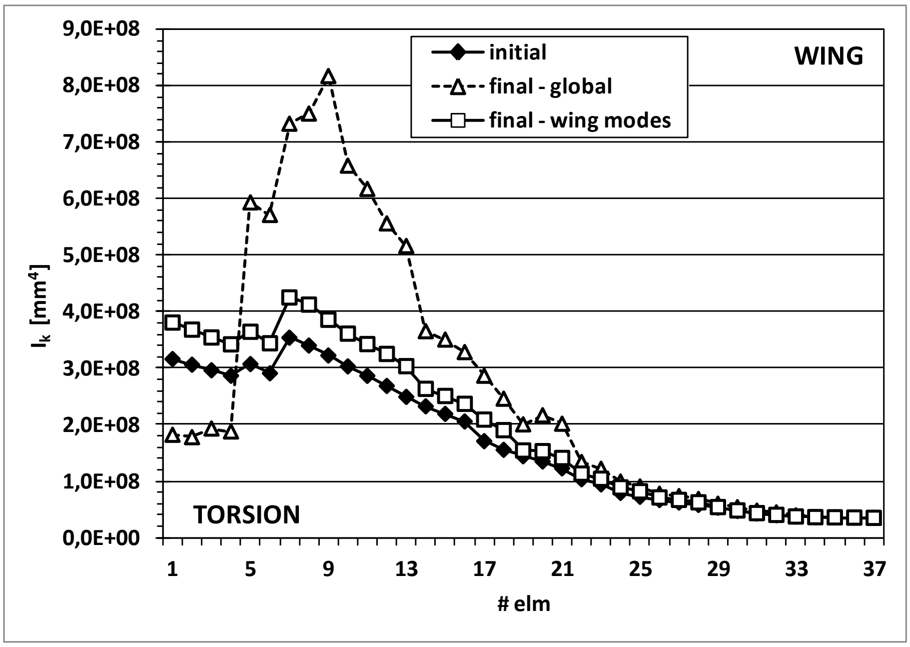

Figure 7 - Design variables change, wing torsional stiffness, initial state, final state – global (see section 4.1), final state – updating to wing modes (see section 4.2)

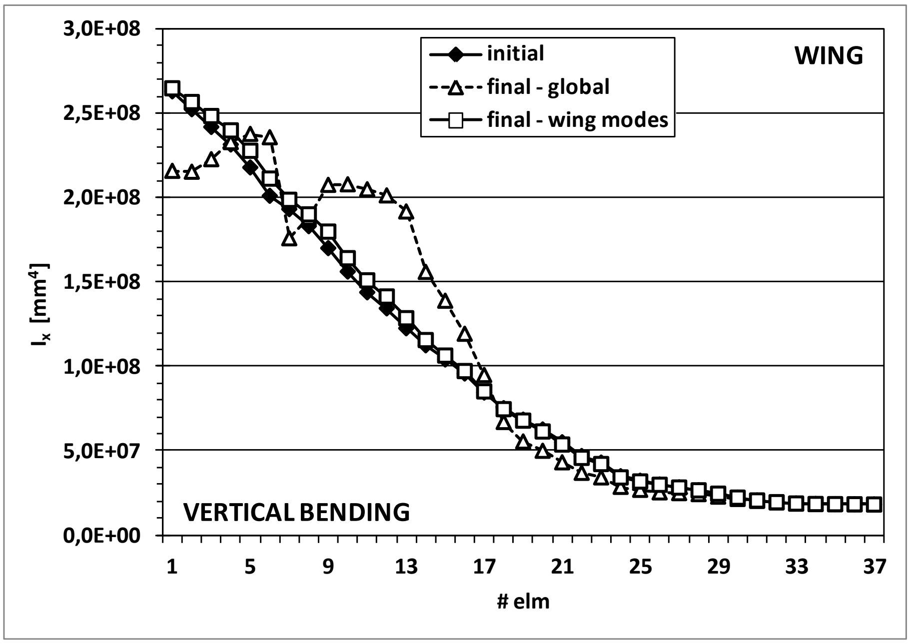

Figure 8 - Design variables change, wing vertical bending stiffness, initial state, final state – global (see section 4.1), final state – updating to wing modes (see section 4.2)

5. Conclusion

The paper describes the updating of structural parameters of the aircraft structure dynamic FEM model in order to match the results of the GVT. The paper is focused on a jet trainer aircraft. The paper describes theoretical background and the methodology, which is demonstrated on the example of the new Czech jet trainer aircraft. The results of the symmetric FEM model for two cases of updating are presented. Modal parameters of updated models got much closer to the target GVT data. Updated models are prepared for the final phase of flutter calculations of the subjected aircraft.

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7. Copyright Statement

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