33RD CONGRESS
OF THE INTERNATIONAL COUNCIL
OF THE AERONAUTICAL SCIENCES
STOCKHOLM, SWEDEN, 4-9 SEPTEMBER, 2022

ICAS
2022
SWEDEN

Towards Virtual Fatigue Testing: developments and challenges for metallic and composites aerostructures

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Abstract

A successful completion of structural testing campaign is a key milestone to obtain the certification of the aircraft structure according to airworthiness regulations. The full testing campaign for a new aircraft program has a significant impact in terms of project costs, usually corresponding to more than 5% of the non-recurrent costs of the aircraft development.

Thanks to the emergence and consolidation of advanced computer simulations for structural mechanics, a significant effort is being made in the recent years by the industry to reduce the required number of structural physical tests replacing them by virtual tests. The ultimate goal of this process is to achieve a fully predictive virtual testing technology applicable to major airframe components and aircraft full-scale structural testing.

In the development roadmap of the predictive virtual testing technology, one of the most challenging areas is fatigue virtual testing. Being able to predict the fatigue behavior of a real aircraft structure requires the implementation and validation of a simulation technology package able to reproduce with high-fidelity different physical phenomena such as fatigue crack initiation, developed at material microstructure level, crack propagation, developed at local macroscopic level, and structure instability, which is produced at structure component level. The simulation of these different phenomena must be fully integrated to properly reproduce the actual airframe behavior during fatigue testing, leading to a multi-scale simulation requirement.

In addition to this multi-scale dimension, the intrinsic characteristics of the different materials used in aircraft structures must be also reproduced. This fact has become quite relevant since the introduction of composites at primary structure elements and the progressive introduction of Additive Manufacturing. Different materials and manufacturing processes lead to quite different structural behaviors and typical failure modes which also introduce a multi-damage scale to the virtual testing technology.

In this paper, an overview of the latest developments carried out at Airbus (Defence and Space) towards virtual fatigue testing will be presented emphasizing the most relevant specific needs in terms of virtual fatigue testing technology for both metallic, manufactured by conventional or Additive Manufacturing (AM) processes, and composite structures, and also an identification of the main developments to be addressed for virtual fatigue testing effective implementation in aircraft design and certification process.

Keywords: fatigue, virtual testing, structures, CDM, XFEM

1. Introduction

Aviation industry is characterized by the pursue of a continuous improvement in its safety standards, what has allowed an important reduction in the rate of accidents in the last 60 years while experimenting an exponential growth in the aircraft daily operations (Figure 1:)

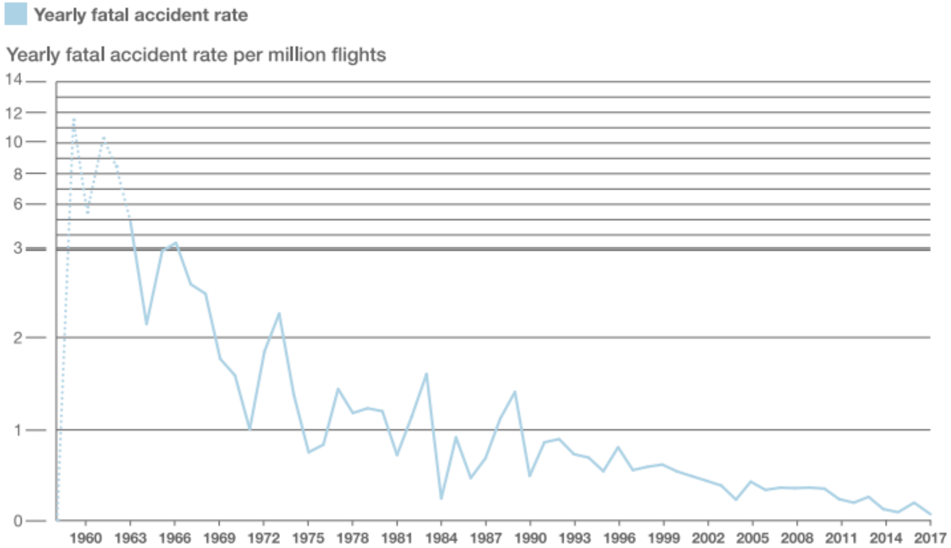


Figure 1: Fatal accident rate per year [1]

These safety standards involve all the different actors that take part in the aviation world: airlines, aircraft manufacturers, airworthiness authorities, maintenance organizations… Focusing on the aircraft design process performed by the manufacturers, the quality in terms of level of safety of the aircraft is ensured by the demonstration of compliance with a wide set of airworthiness requirements during the certification process. In the case of large transport aircraft, for example airliners, these requirements are compiled in regulations from different airworthiness authorities, such as the European Aviation Safety Agency [2] and the Federal Aviation Agency [3], which share many of the regulation requirements.

For the aircraft operational safety, the structural capability of the airframe plays a key role. This capability to sustain the aircraft operational loads must be maintained during the whole service life of the aircraft. This necessity is expressed in the regulations mentioned above through their paragraph 25.571 “Damage tolerance and fatigue evaluation of structure” from which an extract that summarizes the philosophy of the requirement is shown below:

“An evaluation of the strength, detail design, and fabrication must show that catastrophic failure due to fatigue, manufacturing defects, environmental deterioration, or accidental damage, will be avoided throughout the operational life of the aeroplane”

According to these regulations, test evidence is required in order to support the fatigue analyses that are the basis for the establishment of the aircraft structural maintenance program:

“Inspections for fatigue damage or replacement times must be established as necessary. These actions must be based on quantitative evaluations of the fatigue characteristics of the structure. In general, analysis and testing will be required to generate the information needed. The applicant should perform crack growth and residual strength testing to produce the design data needed to support crack growth and residual strength analyses. Full-scale fatigue test evidence is required to support the evaluation of structure that is susceptible to WFD. Test evidence is needed to support analysis used to establish safe-life replacement times”

Thus, the main advantage of fatigue tests is their reliability in order to establish the evidence of the global life of any design, accounting for all the complexity of an aerostructure. But the use of tests has also several drawbacks, mainly linked to their duration and the limited number of tests, which implies that the gaps between assumed test load spectrum and the real aircraft usage –or between tested and real configuration– have to be covered by an extensive interpretation of the results. Actually, one of the main roles of analysis is to conduct test interpretation by enabling the possibility to conduct efficiently different trials that cover the variation of the relevant input test parameters.

However, the scope of fatigue life analysis focuses on potentially critical areas only. In addition, the analysis systems used today, despite their sophistication, do not guarantee 'per se' the accuracy of the results, so the resolution and accuracy of tests cannot be equalled by analysis.

Therefore, symbiosis between analysis and test is still necessary today. This relationship is actually shifted towards the prevalence of tests as primary evidence to calculate airframe life estimates, as both civil and military regulations mandate these tests for aircraft certification.

However, this status quo is prone to experience a quick change. Fuelled by the successes of simulation in other areas and by the constant improvements in the capabilities of fatigue analysis (new numerical techniques, new lifing models, etc), a new analysis framework able to manage the fatigue damage phenomenon from a comprehensive and holistic perspective is being created, with the aim of gradually removing the need of physical testing in the development of aircraft structures, allowing their certification by analysis only ([4] and [6]). This new, ongoing paradigm is called *Virtual Fatigue Testing* (VFT) in the literature.

2. Virtual Fatigue Testing

At some extent, the term 'Virtual Fatigue Testing' is linked to that of 'Digital Twin'. The idea of the Digital Twin (DiTw) was initiated by USAF and NASA in the sense of a high-fidelity model of an as-built system covering its whole lifecycle. Since then it has become a widely adopted concept in the literature but with very different –sometimes even contradictory– interpretations.

For the purpose of this work, Digital Twin (or Fatigue Digital Twin, FDiTw, hereafter) will be defined as the organized collection of high-fidelity models used to mimic the condition of a fatigue test specimen. These models will be specialized into several tasks (structural response, fatigue damage/crack initiation, crack growth, residual strength, and stochastic response) that need to cooperate in order to achieve the common goal, like an ecosystem (see Figure 2:). Analogously, Virtual Fatigue Testing will be understood as the a priori high-accuracy simulation of the response of Fatigue Digital Twins in their test environment (loading, temperature, etc). The details of the definition of FDiTw will be focused here on metallic test articles, but many of the results can be extrapolated to composites also. An initial definition of the characteristics and capabilities required for this FDiTw to be implemented in Airbus (Defence and Space) were discussed in [5].

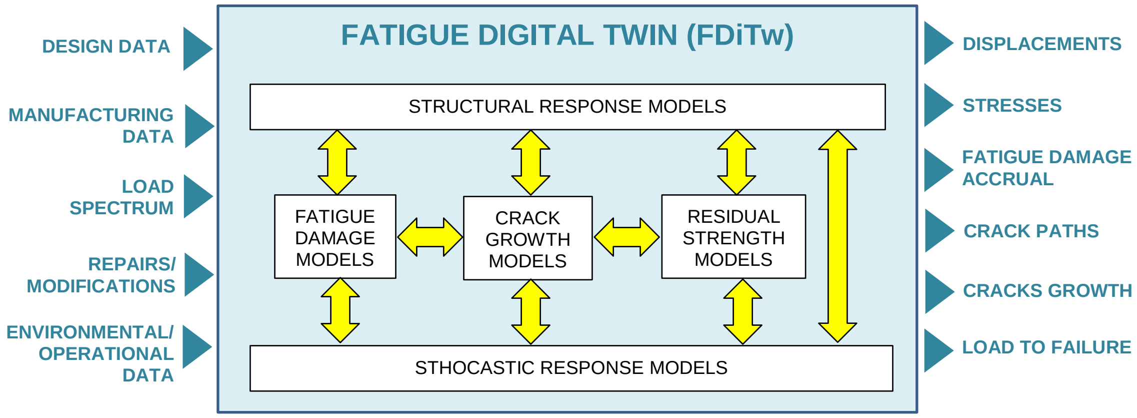


Figure 2: Fatigue Digital Twin (FDiTw) ecosystem models

The success of VFT will depend on the selection, integration and certification of high-fidelity models for the FDiTw that consistently forecast the results of their physical equivalents with the required level of reliability. The individual technologies that enable those models will be evaluated taking into account the following criteria:

- *Level of fidelity*: in the classical definition applicable to the Finite Element Method (FEM), a high-fidelity model is expected to contain more degrees of freedom and more elements compared to a low-fidelity model [7]. In a broader sense, here it will be considered that 'high-fidelity' implies that the FDiTw models include all necessary features of the physical specimen (displacements, stresses, fatigue damage evolution, crack growth, residual strength, etc), and all of them reproduce consistently real outcomes of the fatigue phenomenon.
- *Range of applicability*: modes for the use in VFT will include a clear evaluation of the expected range of input and output values, in order to avoid the unreliability inherent to the operation outside these ranges.
- *Cost of model development*: due to their sophisticated nature, the maturity level of the models may not be the required for a practical implementation, thus needing particular development work that needs to be considered as a factor in their selection.
- *Autonomy*: it is expected that virtual fatigue test will run with the minimum human intervention possible. For example, the representation of the crack growth should be done by the model itself without any external help (e.g., remeshing). At the same time, determination of structure condition at any evaluated point must require the minimum engineering interpretation.
- *Integration with other modules*: in current standard simulation environment, is acceptable the use of different techniques that are not always compatible, as the analyst will manage the transitions between one technique and the other. However, in VFT this is not desirable, as it can be a source of errors or deviations with respect to the reality.
- *Certificability*: all the models need to have a sound theoretical basis that can be checked against a set of analytical and real test results in order to prove their accuracy. Practices such as a posteriori addition of factors or use of equivalent/apparent properties to fit specific results are not allowed.

3. Simulation procedures for Virtual Fatigue Testing

The development of the required high-fidelity models to reproduce the behavior of a fatigue specimen is directly linked to the necessity of implementing advanced simulation approaches able to translate the physical phenomena linked to fatigue degradation into a set of numerical equations that can be solved by a computerized system. This physical-into-numerical translation can be done using 3 main different models:

- *White Box models* → based on pre-defined physical models. The underlying formulation driving the response of the system is imposed in the model. The main advantage of these models is that they do not require previous data or results about the problem to be solved as the mathematical relation between inputs and outputs is known. The main disadvantage is that they require "a-priori" knowledge of the governing physical model for the specific problem.

- *Black Box models* → based on Artificial Intelligence methods. In these models the mathematical relation between inputs and outputs is autonomously derived by the model using previously known data and results for similar problems to the one that needs to be resolved. These models require these previous data to be trained.

- *Grey Box models* → these models are a combination of White and Black Box models, using White Box when the physical behaviour of the system is known and Black Box models for unknown areas of the system behaviour.

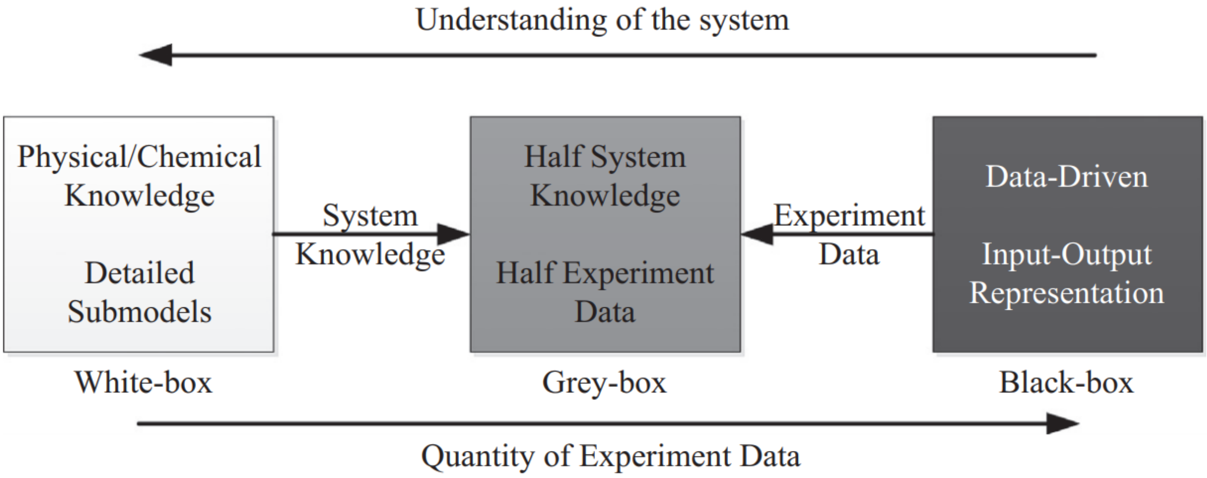


Figure 3: Main modelling approaches for advanced simulations [8]

For the development of a FDiTw in Airbus (Defence and Space), a White Box approach is selected. The main reason for that is that the physical behavior of the structure is known in terms of material response to applied loads at design detail level (stress-strain relation and material failure criteria and models). From this knowledge of the basic behavior at local level, the global behavior of the fatigue test specimen can be derived by using a Finite Element Method (FEM) approach.

This White Box model is developed using Simulia Abaqus software as simulation core with several ad-hoc user subroutines and software developed in Airbus to implement the behavioral modelling techniques required to properly cover the different phases of the fatigue phenomenon. These behavioral modelling techniques for each fatigue phase and for the different material concepts used in aircraft structures will be detailed in the following chapters. Additionally, mathematical models able to reproduce the stochastic nature of the fatigue phenomenon, in order to quantify and manage uncertainty, must be implemented in the simulation scheme in order to develop a Virtual Fatigue Testing approach.

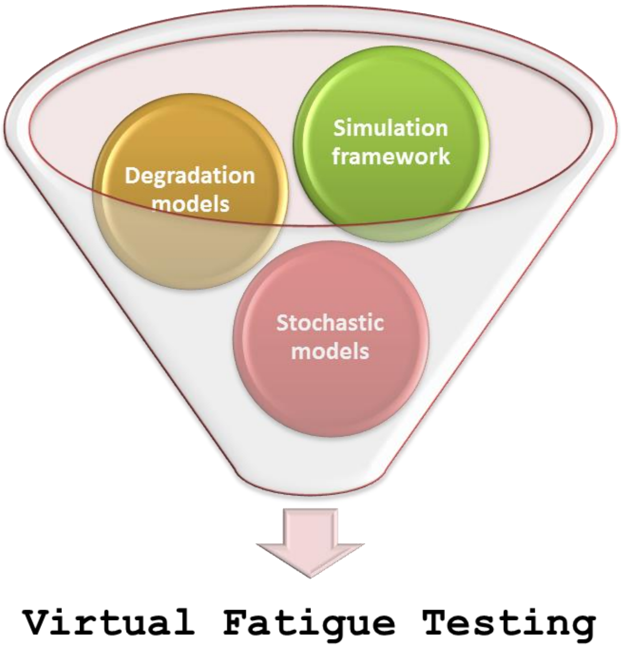


Figure 4: Simulation needs for Virtual Fatigue Testing

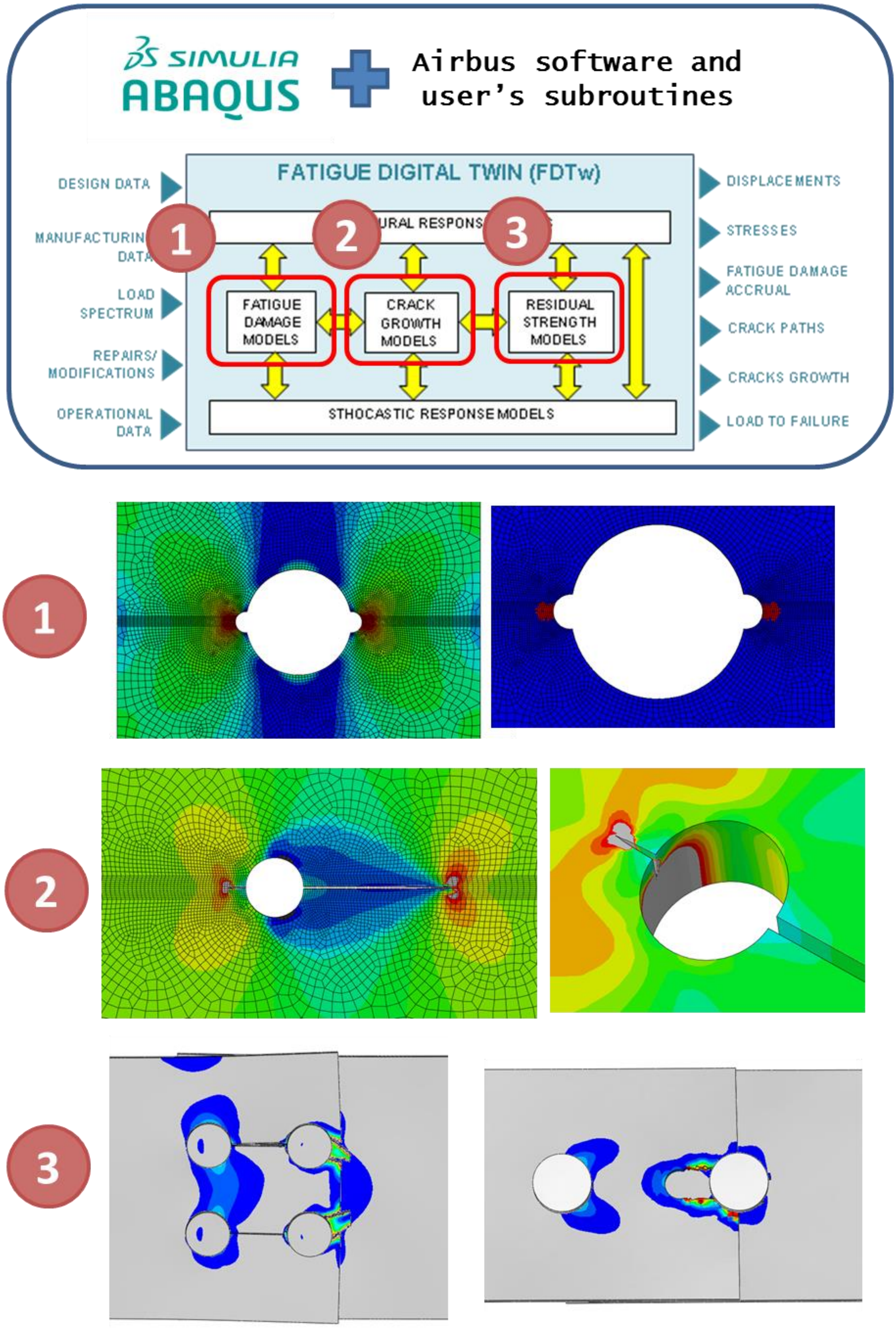


Figure 5: Simulation ecosystem for a Virtual Fatigue Testing approach
(1) Fatigue damage initiation simulation. Stress plot (LH) and Fatigue Damage plot (RH).
(2) Fatigue crack growth simulation. Stress fields around crack tips.
(3) Residual strength simulation. Damage plots for two different scenarios.

The combination of these 3 elements, simulation framework, material degradation models and stochastic response models, allows the generation of a simulation ecosystem able to reproduce the behavior of a fatigue test specimen during the different phases of the fatigue phenomenon (fatigue damage accumulation, fatigue crack propagation and residual strength failure) as shown in Figure 5:

In the next chapters, the specific characteristics and needs for each of these fatigue phenomenon phases will be discussed. The state-of-the-art in each of these phases for both metallic and composite materials within Airbus (Defence and Space) will be presented.

4. Virtual Testing for fatigue damage initiation

Fatigue damage initiation process is probably the most complex material phenomenon to be address for Virtual Fatigue Testing. The reason for that are the following:

- Damage initiation occurs at microscale level as it is a consequence of the nucleation and coalescence of “micro cracks” appearing at material microstructure level. Therefore the high-fidelity simulation of the process would need to assess the problem at micro scale.

- Micro scale simulation techniques for fatigue damage initiation are not mature enough to be implemented for industrial purposes such as Virtual Fatigue Testing; therefore alternative phenomenological approaches that can be implemented in a meso or macro scale are required.

- Fatigue damage initiation is highly sensitive to several parameters that cannot be easily considered in a phenomenological approach. For example:
    - Material parameters such as microstructure or surface quality,
    - Stress conditions such as stress triaxiality or residual stress fields
    - Variable amplitude loading sequences

  As a consequence of this sensitivity to all these parameters which are not easily quantifiable in many cases, fatigue initiation lives show a significant scatter due to the effect.

In the next sections, the approaches followed at Airbus (Defence and Space) to simulate fatigue damage initiation for metallic and composite materials for Virtual Fatigue Testing purposes are discussed.

4.1 Damage initiation in metallic materials

For Virtual Fatigue Testing of fatigue damage initiation in metallic materials, the conventional fatigue calculation approach based on the use of stress concentration factors and experimental Stress-Life curves was discarded by Airbus (Defence and Space), as the goal was to develop a simulation approach able to mimic the actual structural behavior in terms of damage initiation, not just a correlation of lives up to failure, in order to be able to deal with a high level of confidence with the virtual testing of structural specimens for which non-standard configurations and stress conditions exists.

To allow this behavioral modelling of the damage initiation, a simulation approach able to reproduce the material degradation at design detail level is required. The approach selected is the Continuum Damage Mechanics (CDM).

CDM is a relatively new field developed in engineering mechanics and deals with mechanical behavior of a deteriorating medium at the continuum scale. The concept of damage mechanics was first introduced by Kachanov in 1958 for failure analysis of metals under creep conditions. It was developed for structural fatigue applications in the 70’s by Lemaitre and Chaboche, however, it was not applied in the aeronautical industry up to recent dates due to unpractical computing costs for such complex simulations.

CDM models the development of cracks, voids or cavities in each scale that lead to deterioration of mechanical properties of materials. Damage in materials could be divided into different scales (micro, meso and macro scale). Referring to micro scale, microstructure effects may be analyzed (micro voids, micro cracks and decohesion of planes). In contrast, macro scale allows to study visible or near visible discrete damage manifestations studied in fracture mechanics. The mesoscale is a building block of CDM in which discrete phenomena can be smeared into average effects. Most damage models have been developed and formulated using mesoscale concept.

CDM divides material into small elements with homogeneous properties. A mesoscale volume called Representative Volume Element (RVE) is defined. The main feature of this RVE is that the material structural discontinuities can be assumed to be statistically homogeneous and the corresponding mechanical state of the material can be represented by the statistical average of the mechanical variables in that volume.

Damage variable is the average material degradation in the RVE. In this project, a scalar damage variable has been selected, which is defined in each model.

$$D = \frac{\delta S_D}{\delta S} \qquad (1)$$

Where δS is the undamaged area of the intersection of the plane in the RVE and δS_D is the damaged area of the intersection of all micro cracks or micro voids in that plane in the RVE.
At this point, it is possible to define an effective stress $\tilde{\sigma}$, it represents the increase of load due to decrease of the area and appearance of damage.

$$\tilde{\sigma} = \frac{\sigma}{1 - D} \qquad (2)$$

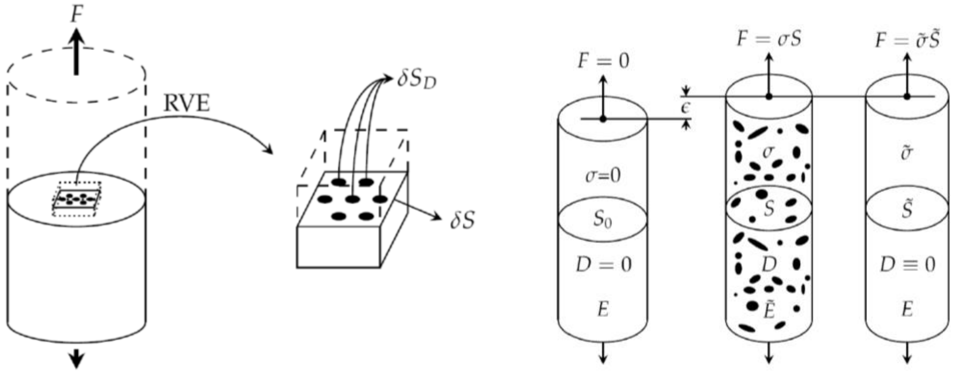


Figure 6: Damage and RVE concept for CDM

This technique gives the opportunity to convert a damaged problem to undamaged fictitious problem.
Additionally, damage quantification can be based on degradation of elastic modulus E of the material. It presents notable advantages, which are shown in the following sections of this project. When elements are loaded, they suffer degradation that affect to their stiffness. When one element fails, its load must be shared between the adjacent elements. This continuum process originates the catastrophic failure. The damage variable (D) is bound by 0 and 1 ($0 \leq D \leq 1$). Where $D = 0$ means undamaged material and $D = 1$ means total failure for each element.

$$D = 1 - \frac{\tilde{E}}{E} \qquad (3)$$

Where E is the elastic modulus of the undamaged material and $\tilde{E}$ is the elastic modulus of the damaged material.

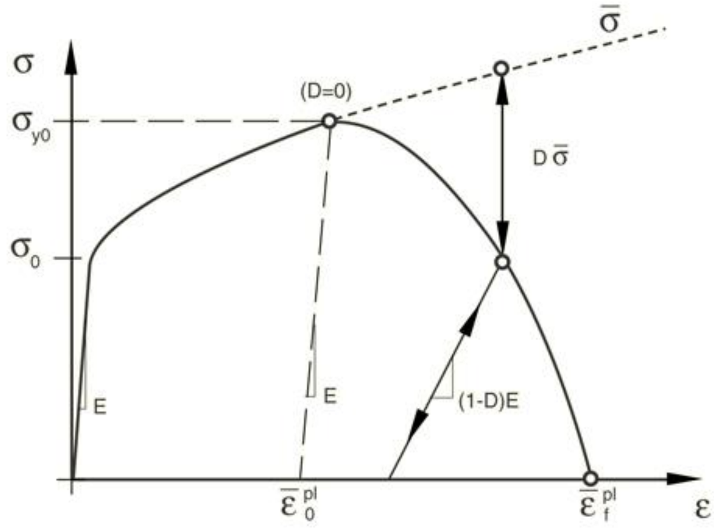


Figure 7: Effect of damage over material constitutive behavior

Several phenomenological models have been published for the characterization of fatigue damage initiation [9]. The authors of this paper have performed an evaluation of several of them selecting the Peerlings' fatigue model [10] as baseline for fatigue damage initiation simulation due to the combination of accuracy in the prediction of fatigue lives and easy material parameter calibration from existing physical test data.

Peerlings' model is a phenomenological model for predicting the onset of high cycle fatigue. This theory is based on strains (in contrast for example to Chaboche's model, which is based on stress). That point offers the possibility to add micro-plasticity effects to this model.

The damage accumulation is defined by the following equation, where N is the applied number of cycles, N_F the critical number of cycles for fatigue crack initiation and α is a material parameter.

$$D = -\frac{1}{\alpha}\ln\left(1-(1-e^{-\alpha})\frac{N}{N_F}\right) \qquad (4)$$

The number of cycles for crack initiation can be calculated by integration of the following equation up to damage = 1, where ε_a is the applied alternating strain and β and C are also material parameters (which can be calibrated using Stress-Life curves)

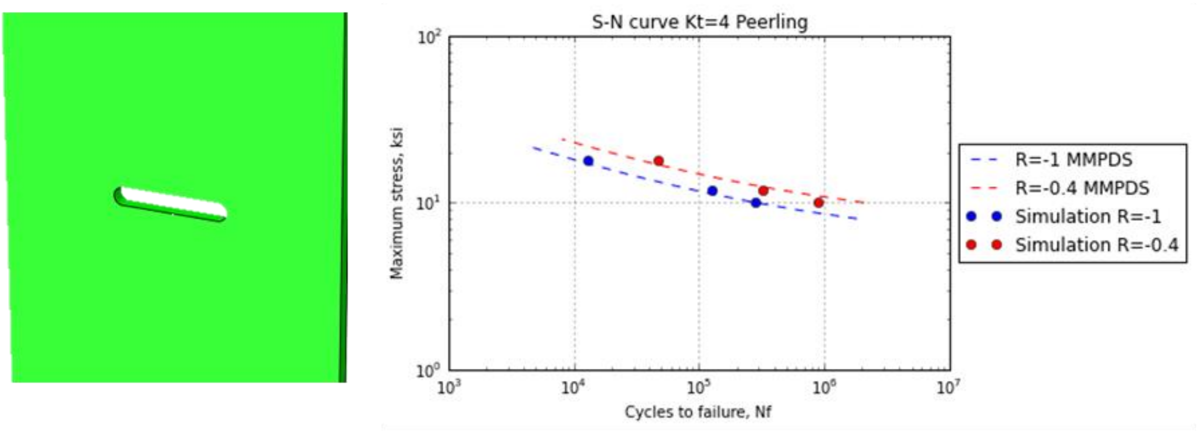
$$N_F = \frac{\beta+1}{2\,\alpha\,C}\,(1-e^{-\alpha})\,\varepsilon_a{}^{-(\beta+1)} \qquad (5)$$

Although Peerlings' fatigue model is originally formulated for a constant amplitude fully-reverse strain loading, the authors of this paper have adapted the formulation using mean-strain corrections and non-linear damage accumulation techniques under variable amplitude to extend its applicability for any applied sequence.

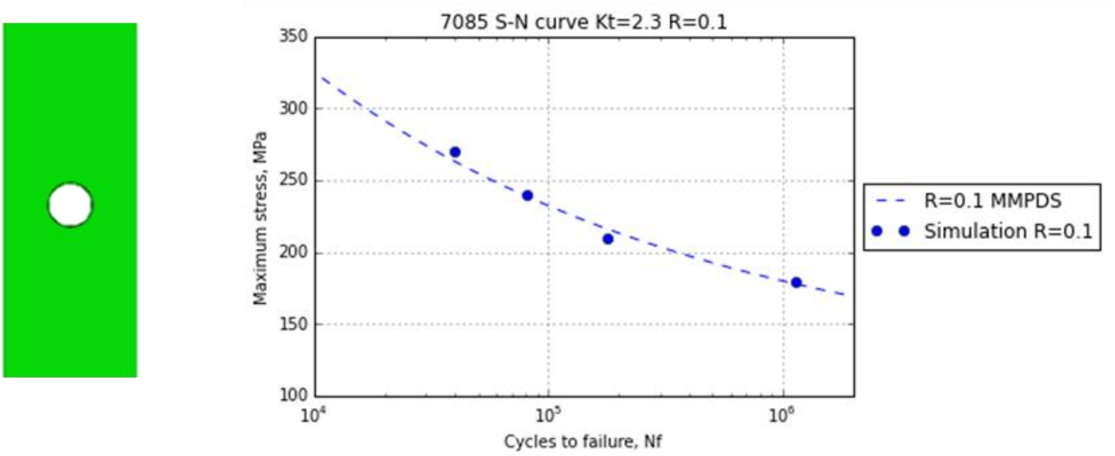
This modified Peerlings' model has been implemented by the authors in an Airbus software named CraViT which includes the fatigue damage model into an Abaqus USDFLD user subroutine to be delivered into the Virtual Fatigue Testing simulation ecosystem based in Abaqus (Figure 5:).

To validate this Virtual Testing technique a set of Stress-Life curves for different metallic alloys and geometric notch configurations have been derived using VFT simulation. The obtained virtual curves have been compared with experimental curves from [19] obtaining a good correlation as shown in Figure 8:

2024-T3. Kt = 4.0



7085-T7651. Kt = 2.3



7050-T7451. Kt = 3.0

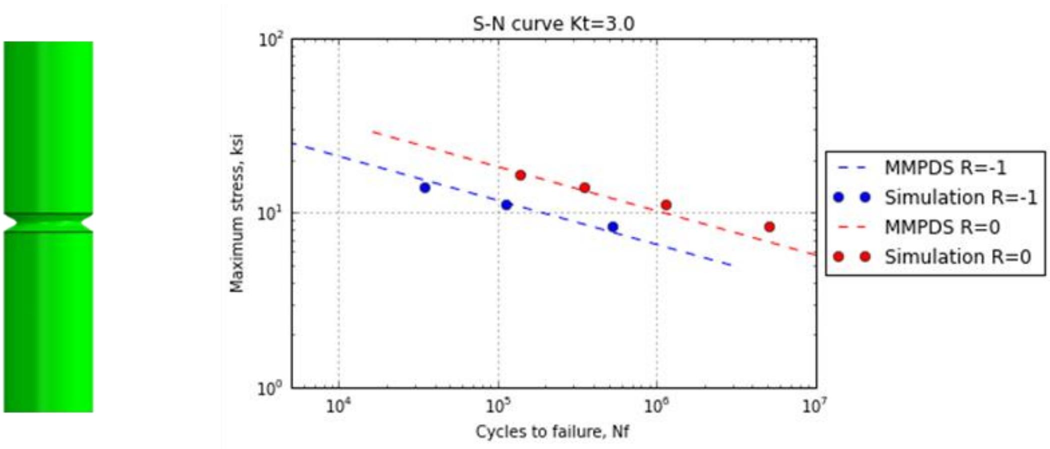


Figure 8: Virtual Fatigue Testing for Stress-Life curves (different materials and geometries)

The accuracy of the VFT technique for variable loading spectra tests has been checked performing a VFT for a specimen representing a typical single shear riveted joint at which a variable loading spectra representative of aircraft wing skin is applied.

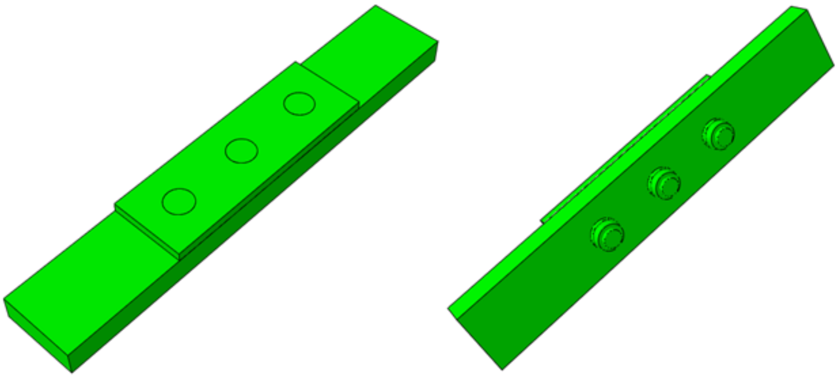


Figure 9: Single shear joint coupon

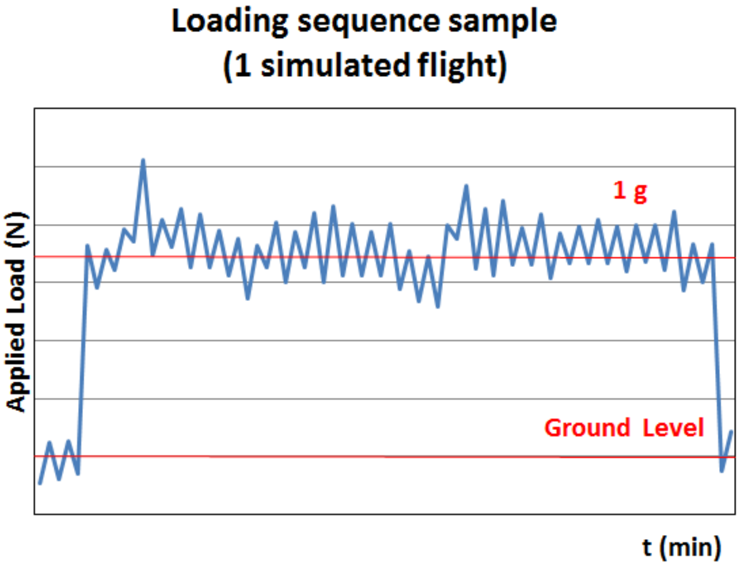


Figure 10: Sample of the applied load sequence (representative for aircraft wing skin)

Virtual Testing results have been compared with the physical testing of 10 coupons (for 5 coupons the rivet holes are drilled using standard quality and for the other 5 coupons, the holes are drilled with improved quality by means of a reaming process). A good correlation between virtual and physical lives up to failure is obtained as shown in Figure 11:

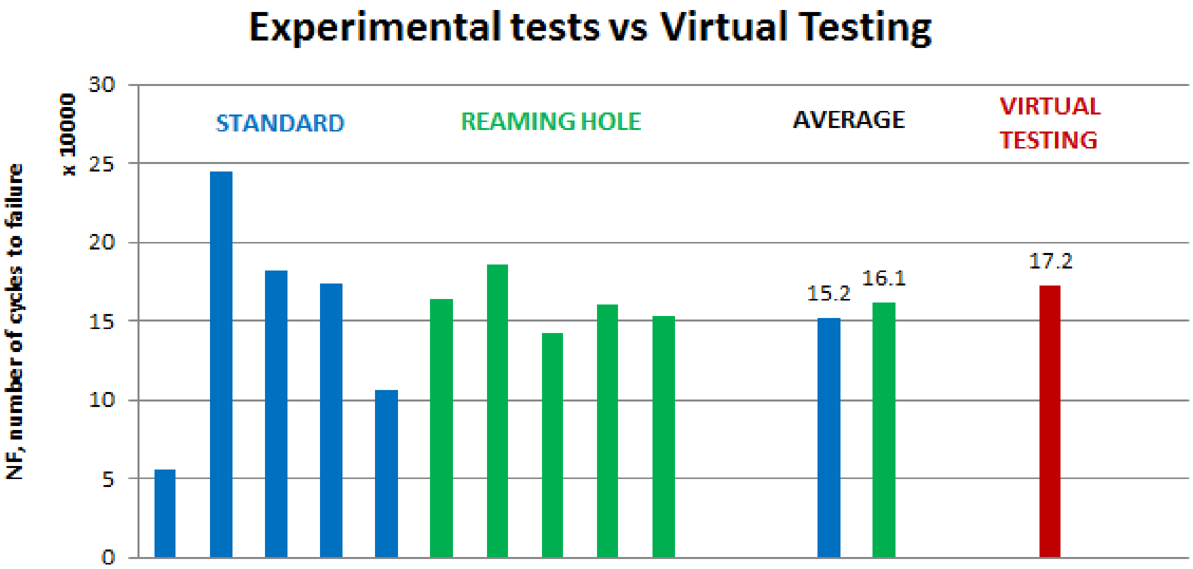


Figure 11: Correlation between virtual and physical testing results

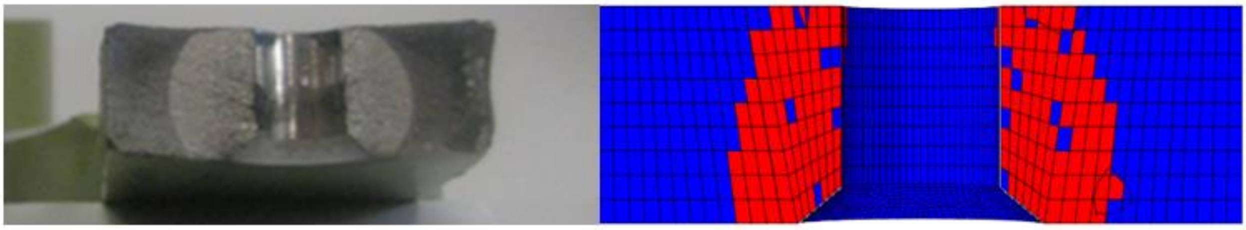


Figure 12: Fatigue crack correlation between virtual and physical test specimen. Physical crack surface (LH) against damage variable plot (RH).

4.2 Damage initiation in metallic materials for Additive Manufacturing parts

One of the tasks to be accomplished at the development of a new technology is to identify the defects related to it. The defects have been proved to be the main drive of the fatigue life.

From one side, to assess the effects of these defects a new analysis methodology has to be developed as for the conventional analysis which cover metallic parts do not consider such a level of defects and for the ones which cover composite parts are designed for not having fatigue issues.

From the other side, as the Additive Manufacturing (AM) pursues designs for weight savings and optimization, the geometries to be analyzed are not the typical ones for metal or composite. More complex parts are designed and manufactured and therefore, the load paths and stress concentrations as well as residual stresses and critical zones are more complex to determine.

Furthermore, the variability of defects and material properties detected at the parts depending on the process parameters and design of the part makes it harder to define a deterministic methodology for analysis.

For all these reasons a method based on finite element analysis using probabilistic fracture mechanics is proposed. The stochastic approach is nowadays recognized to be a proper methodology to take into account all the possible variabilities and together with the current computational capabilities it has the opportunity to become a more extended methodology.

One of the approaches that are being applied for AM simulations is the Probabilistic Fracture Mechanics (PFM). This is based on Stochastic Finite Element Method (SFEM) which is an extension of the classical deterministic finite element approach.

For AM simulations, the authors have implemented a Montecarlo Simulation Module to the CraViT software initially developed for deterministic crack initiation virtual testing (see section 4.1). Using this stochastic implementation, the effect of typical AM defects, such as pores and lacks of fusion, over the fatigue life of an AM part can be assessed.

VFT must assess multiple configurations in order to obtain the parameters which solve the physics of these phenomena. Each configuration is defined by random distribution of defects. Some configurations are presented in Figure 13: with different levels of defect density.

In AM simulations, defects (pores and lacks of fusion) have been represented by ellipsoids of revolution characterized by the ellipsoid ratio, ratio of smaller radius to larger radius and are randomly located. Size, position, volume, number of defects, minor and major ellipsoid ratio and critical defects have been the variables assessed and are considered also as probabilistic variables.

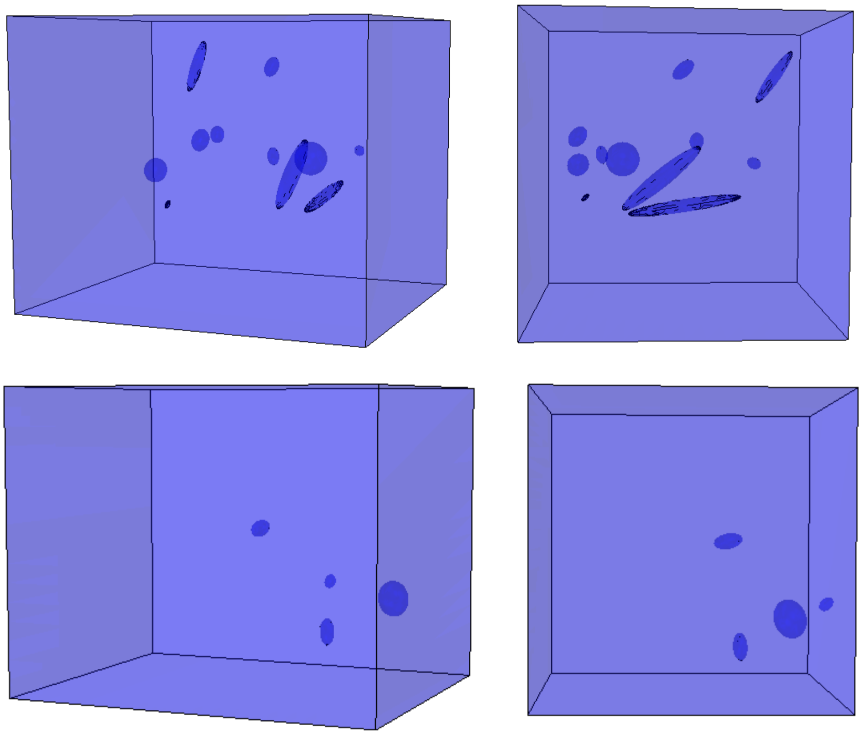


Figure 13: Example of some configurations with high and low density of defects.

In the following figure, the damage evolution is presented as well as the starting point in the most critical defect of the previous figure examples. Only the damaged defects appear in the image. The first case with a high density of defects and with several lacks of fusion, the damage begin in one of them. Not only these defects are the origin of the damage, but other smaller defects also start to be damaged in the same way. In the case with low density of defects, the damage begins in the most critical defect and progress to the rest of them.

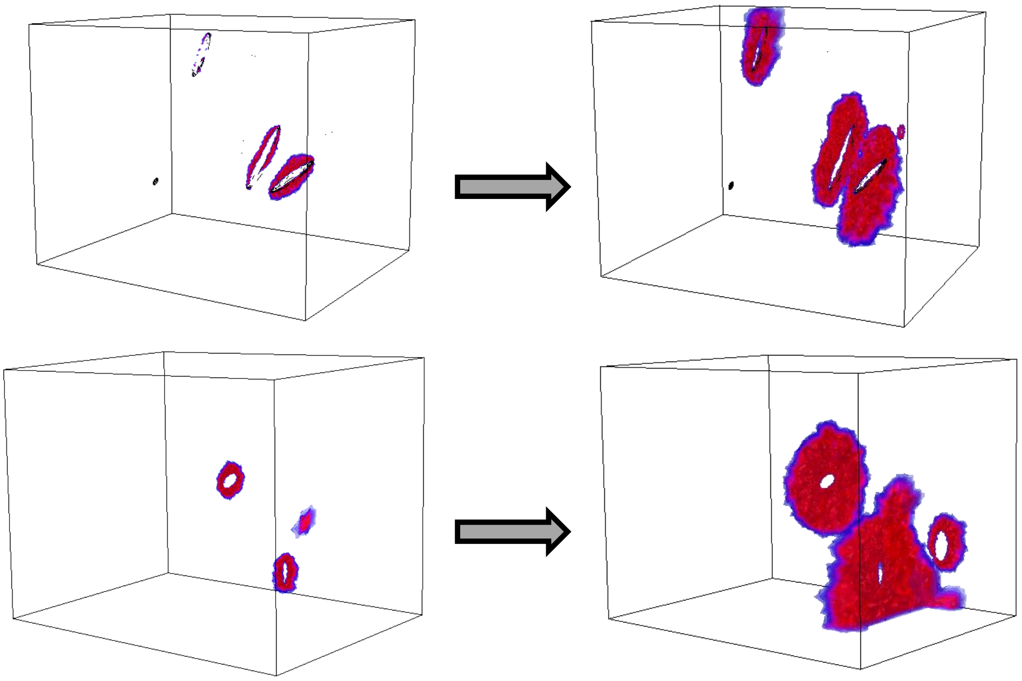


Figure 14: Damage progression. Damage variable plots obtained in the Statistical Volume Element simulation.

Due to the high computational costs of 3D simulations, the following tests have been done in 2D. For instance, some 2D configurations are presented below:

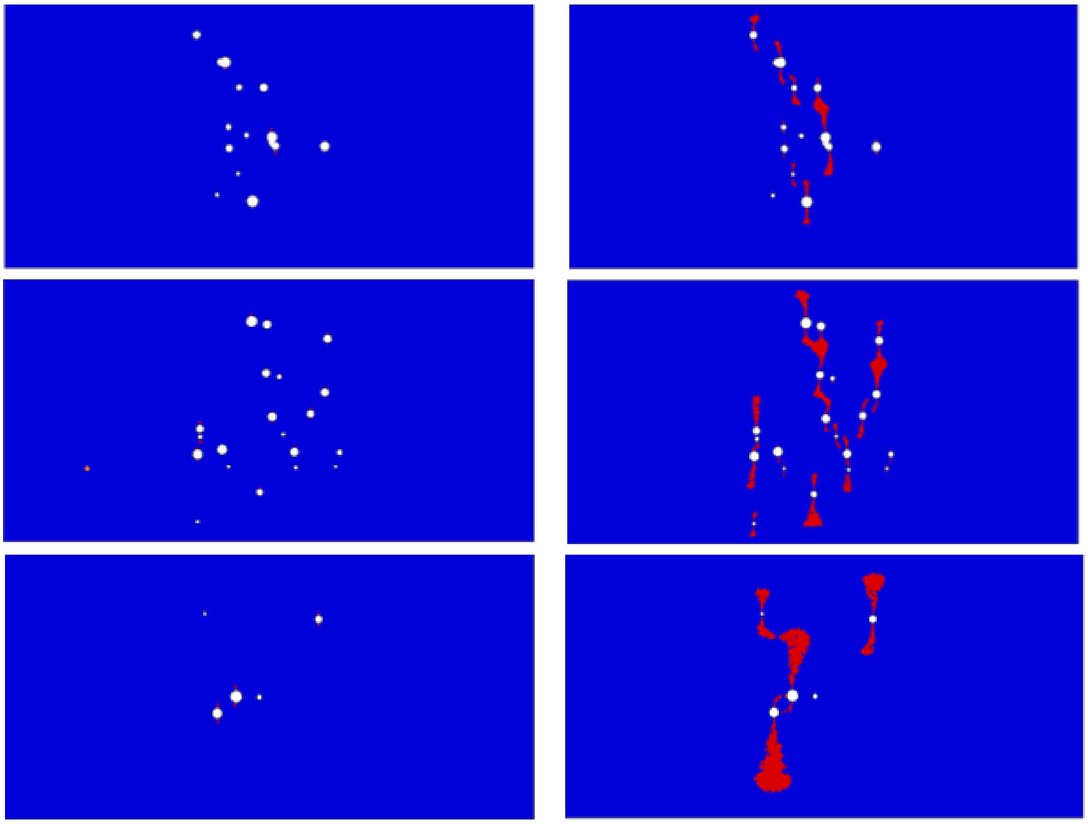
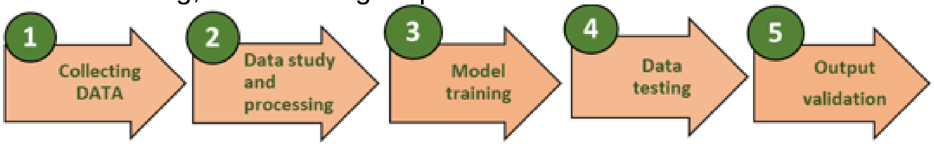


Figure 15: Crack initiation from defects at AM part. Damage variable plot.

In the previous figure, it can also be seen the damage progression across the material and where the damage starts. Red zones show propagation of cracks. In the first case, it is observed how there are several defects that collapse between then generating a larger and more irregular one. These are the origin of the damage and progresses to medium-sized defects. In the second case, despite having a high density as well, most of the defects have a similar size, so the damage propagates through all of them, spreading later between them. The same occurs when the density is lower, the larger defects are the origin of the damage. Parts fail when material degradation is so high that loads are not withstood.

Once all the cases have been computed for different load levels (enough cases to obtain the key variables that govern the physics of the problem), machine learning techniques are applied to the results. The objective is to obtain the global damage law. In this way, the computational costs could be reduced as the evaluation of new cases could be performed through this global damage law.

To apply machine learning, the following steps have to be followed:



During the data study and processing phase, the variable of study (in this case, the number of cycles) and the rest of the problem variables are selected. Furthermore, the possible outliers must be detected and eliminated.

In the model training phase, among other tasks, the data is split to training and test sets. Some of the cases have been also taken from these two sets to perform a blind test at the validation phase. The selection of the algorithm parameters is also performed in this step for the correct performance of the algorithm. Besides, a feature selection is carried out in order to obtain the most important variables of the problem which will reduce the computational time and avoid overfitting the model. This feature selection can be seen in Figure 16:

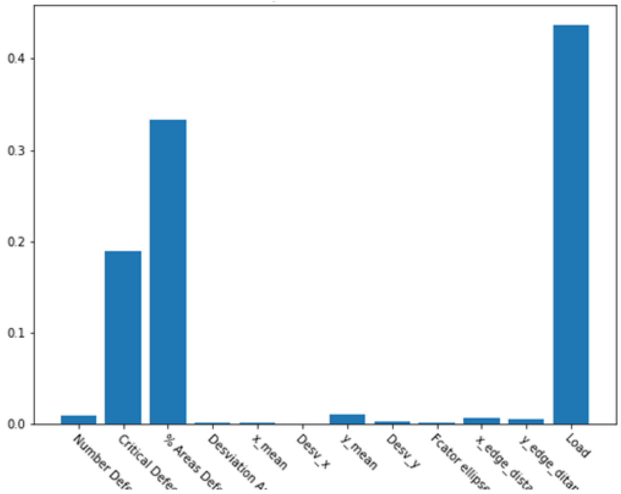


Figure 16: Feature importance of the problem variables

The next step is the data testing phase in which the test data set is checked with the model obtained from the training phase. A representation of the study variable as a function of the area of defects percentage for the different load levels is shown in Figure 17:

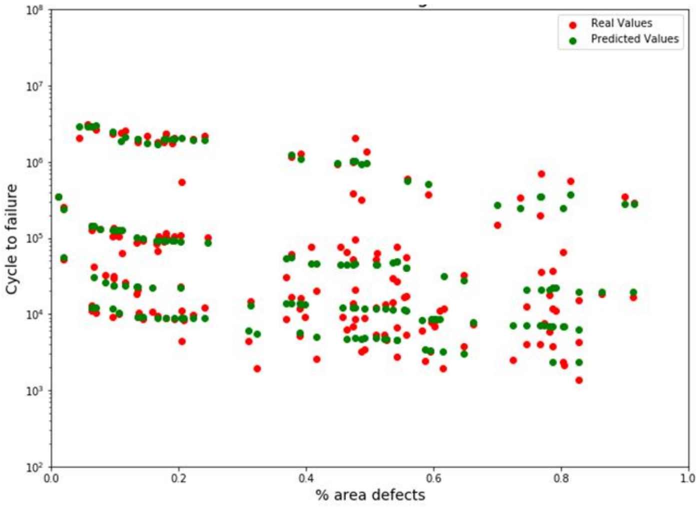


Figure 17: Predicted and real values of the test data set

The last step of the machine learning process is the validation of the results and the accomplishment of blind tests. For the validation part, the representation of the errors is useful. These results can give an initial estimation of the scatter factor that could be applied depending on the load for fatigue analysis.

4.3 Damage initiation in composite materials

Regarding Virtual fatigue testing for composite materials, the state of the art is especially immature more so than in the case of metallic materials. The complexity of composite materials makes for its failure and degradations mechanisms to be not fully understood yet, since it involves fiber, laminar and interlaminar cracks at microstructural level.

Despite, several methodologies have flourished in order to overcome this challenge. One of the most notables being Cohesive Zone Models introduced in 1960 by Dugdale [11] and Barenblatt [12], developing the concept of cohesive cracks.

Although based partially on Fracture Mechanics theories, since CZM are able to simulate damage initiation and propagation, they can also be included inside the CDM framework.

Cohesive damage models use interface elements along the predefined propagation path. What makes

them very suitable for delamination simulations.
The behavior of these elements is not linear; they follow a special law relating element tractions with nodal separations.

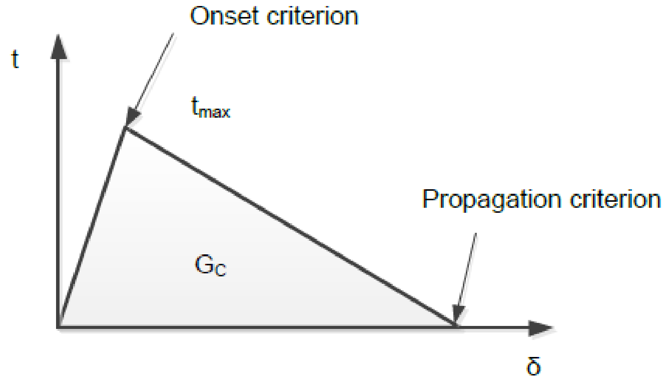


Figure 18: CZM traction separation law

For damage initiation phase, cohesive damage can develop within the interface elements reducing its mechanical properties according to a predefined relation. Damage onset on these models is introduced when the maximum interface strength is overcome. At this point interface stiffness is reduced and eventually set to zero.

The area defined by the traction separation law is the critical strain release rate SERR. When SERR (G) is equal to critical value the traction strength becomes zero and new crack surfaces are created. Cohesive traction separation laws are result of modelization criteria and not derived from actual material behavior.

Considered cohesive law shapes in the literature have adopted numerous shapes. Alfano [13] conducted an investigation to clarify the influence of the softening shape. The exponential law was found to be optimal in terms of accuracy while the bilinear law represented the best compromise between accuracy and computational cost.

Cohesive linear damage evolution is described according to a combination of normal and shear solicitations this notation was introduced by Camanho and Davila [14]. Where the linear softening damage parameter is defined as:

$$D = \frac{\delta_f(\delta - \delta_o)}{\delta(\delta_f - \delta_o)} \qquad (6)$$

Where δ_o is the nodal displacement at damage onset, δ_f displacement at failure and δ is the actual effective displacement according to the given solicitations:

$$\delta = \delta_{eff} = \sqrt{\langle\delta_n\rangle^2 + \delta_s^2 + \delta_t^2} \qquad (7)$$

This softening parameter is incorporated at the second slope of the bilinear traction separation law resulting in the progressive element degradation region (see Figure 36:).

Airbus (Defence and Space) has applied this formulation to a selection of practical applications with successful outcomes. Predictions in terms of load at damage onset and location of the delamination initiation have been accurate in multiple scenarios.

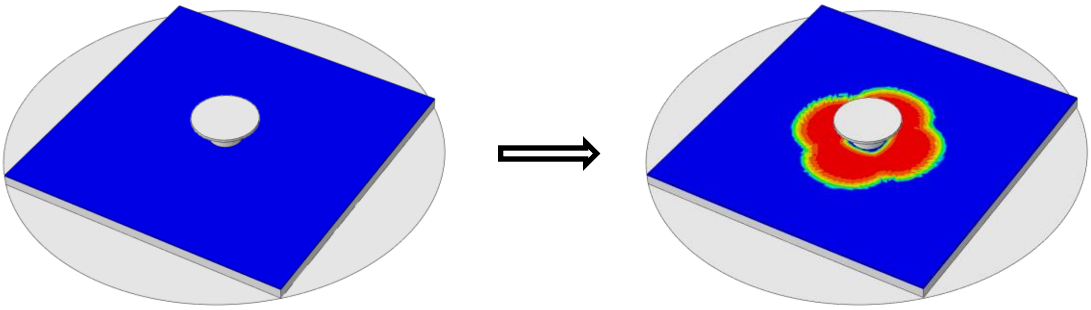


Figure 19: Pullout test Delamination Initiation Cohesive Degradation. Damage variable plot.

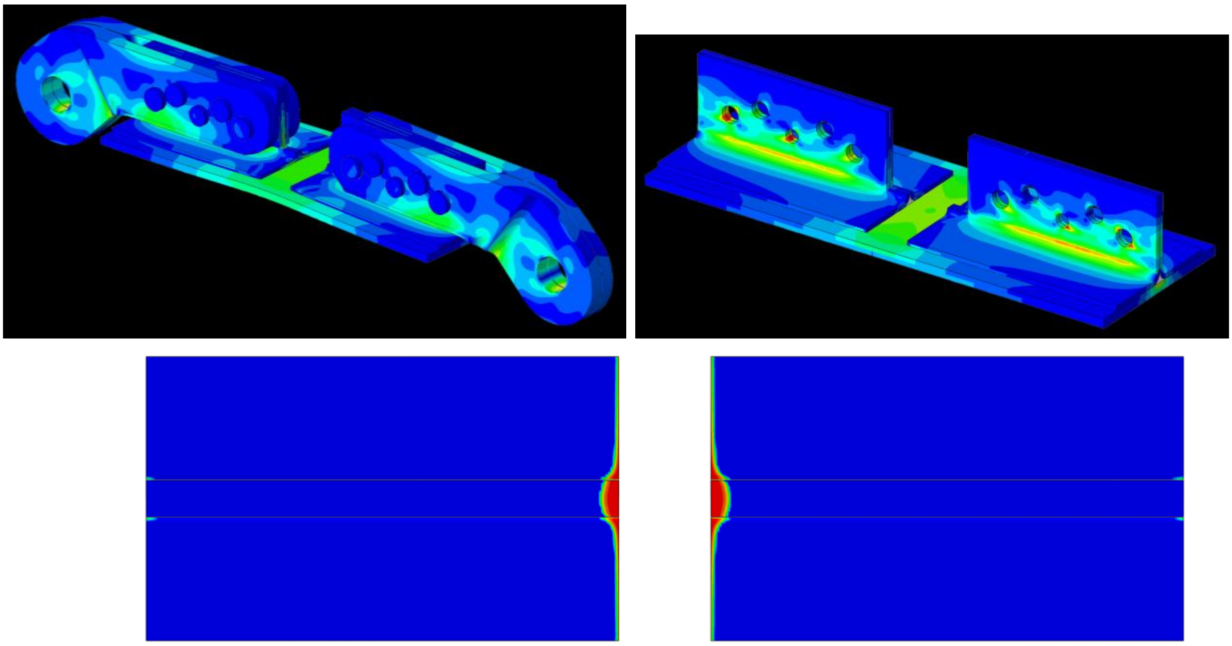


Figure 20: CFRP Infusion test specimen Delamination Initiation Cohesive degradation. Stress plot (up) and damage variable plot (down)

Apart from this baseline approach, several authors have proposed different modifications to the methodology. With the objective to address better the physics behind composite fatigue damage phenomenon.

Harper and Hallet [15] opted for a decomposed damage parameter that accounted separately for static and fatigue damage. The static damage parameter is defined as:

$$D_S = \frac{\delta - \delta_o}{\delta_f - \delta_o} \qquad (8)$$

Whereas the fatigue damage parameter is defined by the ratio of the undamaged fatigue SERR to the totally damaged one.

$$D_F = \frac{G_T}{G_{TC}} \qquad (9)$$

Then:

$$D_T = D_S + D_T \qquad (10)$$

Moroni and Pirondi [16] proposed a model that incorporates CDM principles, based on the work of Turon [18]. In this model damage is represented by the nominal area ratio of the undamaged and damage RVE.

$$D = \frac{A_D}{A} \tag{11}$$

Where A is the nominal RVE area and A_D is the damaged RVE area, affected by the presence of micro cracks. Such approach is already applied for damage initiation in metallic materials (see chapter 4.1).

Khoramishad and Crocombe [17] proposed a strain based degradation law only dependant on the adhesive system and not on joint configuration:

$$\frac{\Delta D}{\Delta N} = \begin{cases} \alpha(\varepsilon_{max} - \varepsilon_{th})^{\beta}, & \varepsilon_{max} > \varepsilon_{th} \\ 0, & \varepsilon_{max} \leq \varepsilon_{th} \end{cases} \tag{12}$$

Initial steps for the implementation of CZM into fatigue damage initiation regime have been made at Airbus (Defence and Space). For that purpose system architecture defined in chapter 3 is adopted.

5. Virtual Testing for fatigue damage propagation

The simulation of damage propagation through a continuum solid is a significant challenge taking into account that the damage represents a clear discontinuity in the behavior of the model, while the foundation of the structural simulation is the Finite Element Method, which requires continuity through the finite elements.

In this chapter, the different simulation techniques deployed by the authors for the implementation of fatigue damage propagation capabilities in the Virtual Fatigue Testing ecosystem are discussed.

5.1 Damage propagation in metallic materials

For metallic materials, traditionally, the fatigue crack propagation simulation has been addressed by applying re-meshing techniques in order to adapt the original finite element mesh to the evolution of the crack path due to the damage propagation. The main drawback of these approaches is the significant increment of the computational costs due to the necessity of producing a very fine mesh around the crack tip in order to properly capture the quasi-asymptotic stress field at this area.

The alternative used by Airbus (Defence and Space) is the eXtended Finite Element Method (XFEM) [20], which over the years has become a very efficient tool for solving crack arbitrary propagation problems. From a mathematical point of view, XFEM is based on the enrichment of solution-type functions by adding new degrees of freedom to the predefined traditional FE mesh. Thus, the equation representing the crack growth as a discontinuity is:

$$\mathbf{u}^h(\mathbf{x}) = \sum_{i \epsilon I} \phi_i(\mathbf{x})\,\mathbf{u}_i + \sum_{j \epsilon J} \phi_j(\mathbf{x})\,\mathbf{b}_j H(\mathbf{x}) + \sum_{k \epsilon K} \phi_k(\mathbf{x}) \left(\sum_{l=1}^{4} \mathbf{c}_k^l F_l(\mathbf{x}) \right) \tag{13}$$

where $\mathbf{u}_i$ is the nodal parameters of the entire set of nodes (Set I), $\phi(\mathbf{x})$ the corresponding shape functions, $\mathbf{b}_j$ the nodal enriched degrees of freedom for the nodes of the elements that are fully cut by the discontinuity (Set J), H($\mathbf{x}$) the jump-function, $\mathbf{c}_k^l$ the nodal enriched degrees of freedom of the nodes of the element where the crack tip is located, and $F_l(\mathbf{x})$ adequate asymptotic functions for the displacement field near the discontinuity tip (Set K).

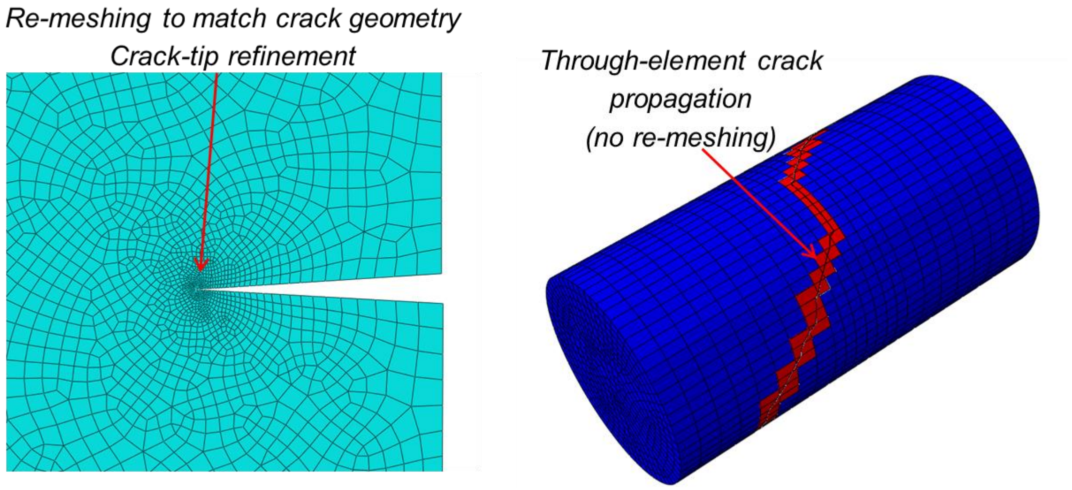


Figure 21: Comparison between re-meshing technique and XFEM approach for fatigue damage propagation

Currently, XFEM has already been implemented in several FE commercial packages. However, XFEM is not in itself a crack propagation tool, but just a numerical method especially designed for treating discontinuities. Therefore, the authors have developed an evolution of XFEM by means of a code named iCracx, capable of performing crack propagation simulations without any limitation in terms of contacts, material model, loading spectra complexity, etc. This software is included in the Virtual Fatigue Testing simulation ecosystem shown in Figure 5:

iCracx allows the computation of the evolution of the Stress Intensity Factor, J-integral and crack propagation direction resulting in an autonomous prediction of the crack trajectory and crack propagation rates based on Linear Elastic Fracture Mechanics.

The accuracy of iCracx XFEM implementation can be tested by comparison against validated analytic solutions for Stress Intensity Factors for example. In the next figures a comparison between iCracx simulation results and analytic solution for typical crack growth scenarios from a hole with combinations of corner and through cracks is shown (scenario HC1 in NASGROv8).

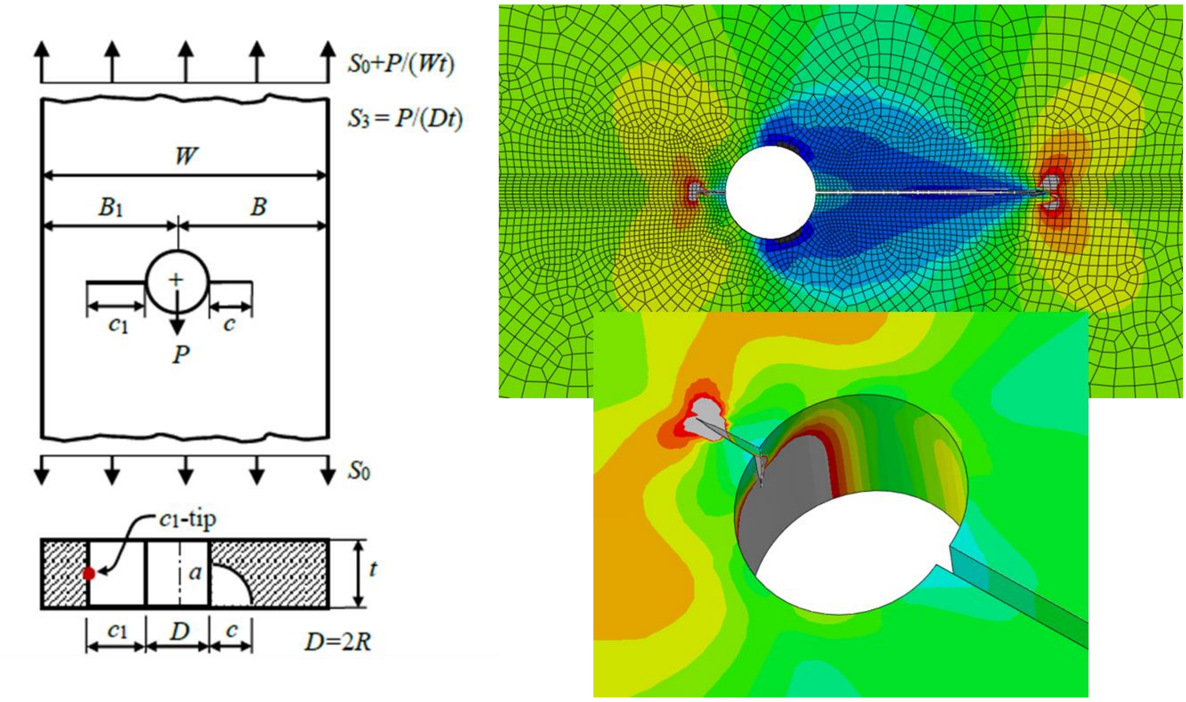


Figure 22: Crack scenario for the case study. Combination of corner and through crack at a hole. Stress fields around crack tips.

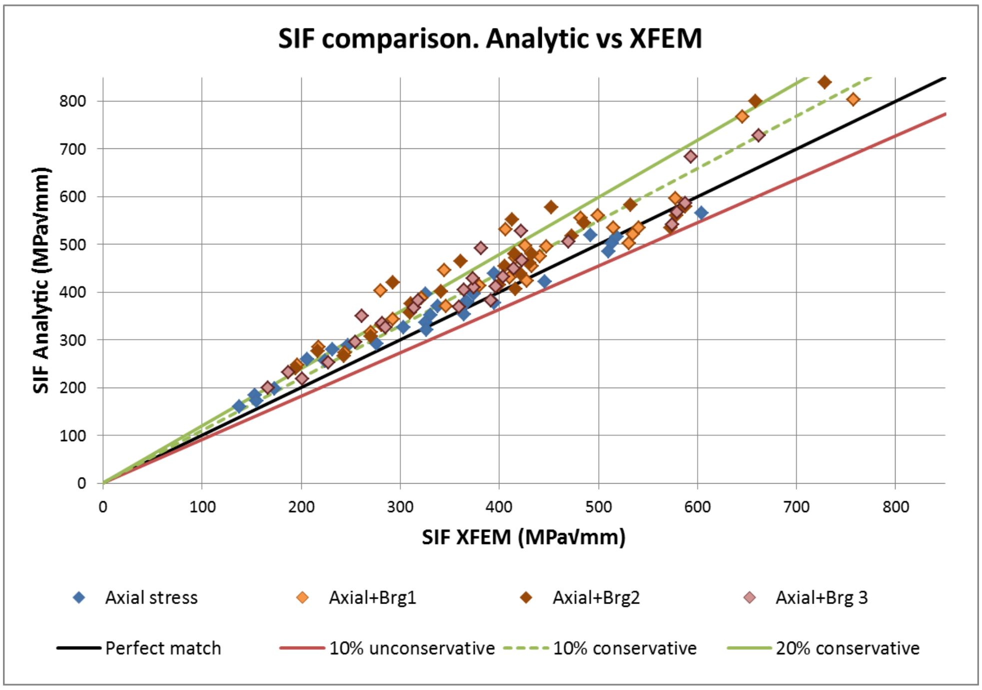


Figure 23: Correlation of Stress Intensity Factors. Analytic vs iCracx.

This software has been applied in Airbus (Defence and Space) to multiple structural cases with good correlation against physical testing and in-service experience [21] [22]. In the next figure a sample of a correlation against an actual in-service finding performed in Airbus (Defence and Space) during this year is included.

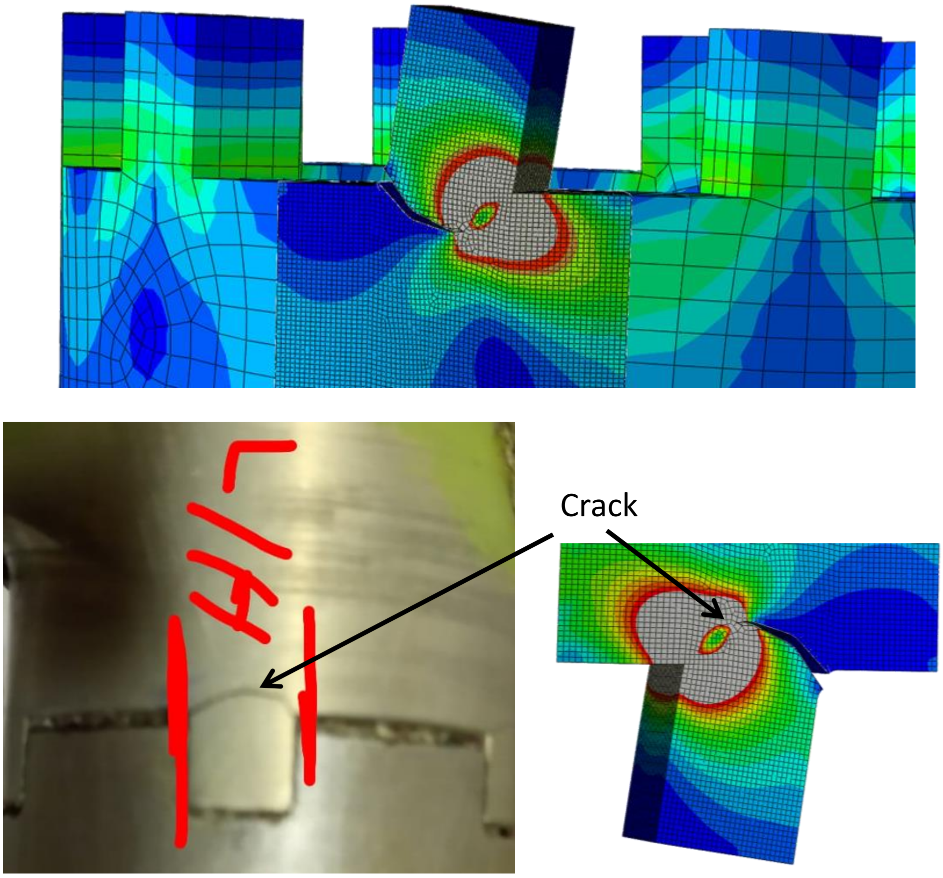


Figure 24: Correlation between iCracx simulation and in-service fatigue damage finding for a castellated torque shaft. Stress field plots at crack area.

As an alternative to XFEM, the application of Continuum Damage Mechanics for fatigue crack propagation, as done for fatigue damage initiation (chapter 4.1), provides the capability to link both phases of the fatigue phenomenon. In the case of CDM, the fatigue crack surface is modelled by the complete degradation of the elastic modulus of the cracked elements. CDM allows the prediction of the structural hot spots at which fatigue cracks will nucleate, crack propagation paths and crack propagation rates.

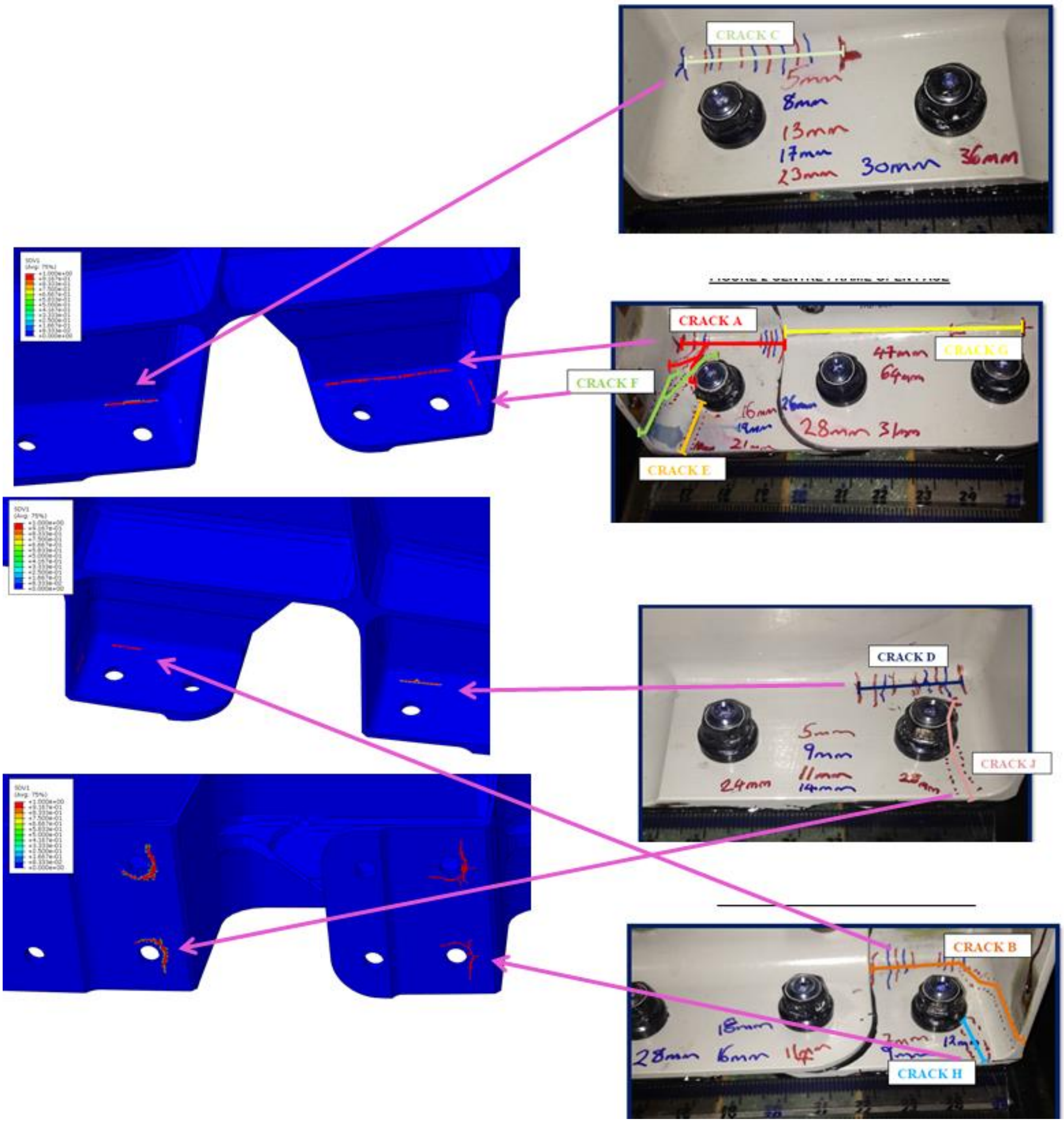


Figure 25: Correlation of crack propagations between CDM virtual testing and physical testing. Damage variable plot (LH) vs physical crack surfaces (RH).

5.2 Damage propagation in metallic materials for Additive Manufacturing parts

Fatigue crack propagation for additive manufacturing parts is linked to the study of the possible defects to be found in a part. The initial crack length to be taken into account is connected to initial defects size and crack initiation which is presented in chapter 4.2.

Due to the complexity of the parts manufactured with AM and taking advance of the accuracy and flexibility of the developed capabilities with iCracx (detailed in previous chapter) this methodology is the one chosen to analyze crack propagation for AM.

Crack size for crack initiation phase is taken and introduced in iCracx as initial crack to perform the complete crack propagation analysis. To be able to perform this stage of the analysis the da/dN curves must be available for the material to be studied.

5.3 Damage propagation in composite materials

Traditionally composite materials initiation and damage propagation phases are not simulated. Certification authorities' ([2], [3]) requirements establish that damage must not occur nor propagate in composite structures. Compliance demonstration is assured by physical tests which are remarkably expensive.

Two different scenarios are identified to be potential sources of damages. Unconventional service loading like out plane loading which is controlled by the weak interlaminar strength and discrete damage sources like impact accidents and manufacturing defects.

Typical simulations of composite materials are impact simulations and loading after impact in that sense the purpose of this simulation is to determine the residual strength of this materials and their damage tolerance.

Since there is no slow-crack propagation criteria applicable to composite materials no propagation simulation can be of any use, in contrast to the case of metallic materials. But the development of virtual testing FEM capabilities can introduce the adoption of less restrictive criteria for the certification of composite structures.

As with metallic materials the adoption of a damage tolerant design criteria instead of Fail-Safe, Safe life criteria has given the possibility of further optimization of structures. With composite structures able to deal with damages and manage a controlled propagation can provide a significant advantage in certification and operator costs.

6. Virtual Testing for residual strength evaluation

The Residual Strength evaluation consists on the determination of the remaining load-carrying capability of a damaged or cracked structure up to the static failure. Residual strength failure is evaluated as quasi-static phenomenon however its numerical simulation requires the capability to model the sudden failure of the structure which implies sudden changes in the model stiffness.

6.1 Residual strength evaluation in metallic materials

For metallic structures, conventional structural simulations based on the Implicit Finite Element Method present significant difficulties to properly capture this kind of phenomena as it is required to obtain a converged solution at each time step of the simulation. The process to achieve convergence in the solution can require a very large number of iterations with very small time steps what can result in unpractical computational costs.

This limitation in the simulation of residual strength failures can be overcome by using the Explicit Finite Element Method. Explicit Simulations are conceived for highly dynamic phenomenon, however quasi-static failures can be simulated by ensuring that kinetic energy remains negligible (appropriate definition of analysis times and mass scaling).

The material failure can be modelled using a Continuum Damage Mechanics approach, as it was discussed for fatigue damage initiation in section 4.1. Using a ductile damage model added to the material property set in the simulation; autonomous failure simulation capability is added to the Virtual Fatigue Testing ecosystem. The selected ductile damage model is based on the equivalent strain to failure as a function of stress triaxiality ([23], shown in Figure 26:).

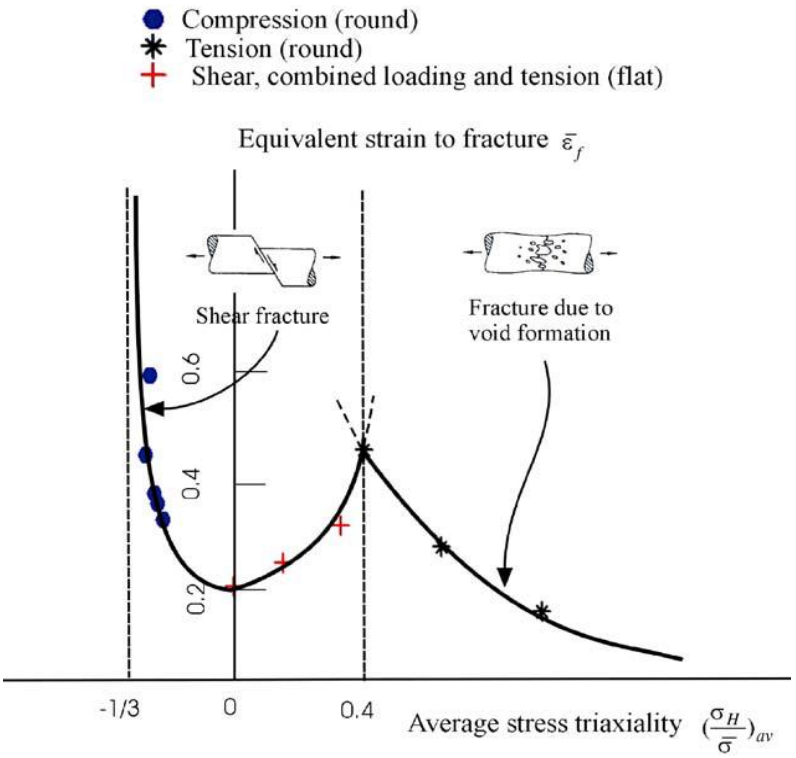


Figure 26: Equivalent strain to fracture - triaxiality ([24])

Stress triaxiality is defined as the ratio of hydrostatic pressure (or mean stress, σ_H) to the von Mises equivalent stress ($\bar{\sigma}$). The triaxiality of the stress state is known to influence the amount of plastic strain which a material experiences before ductile failure occurs. As it can be seen in the previous figure, different fracture mechanisms are considered, this curve had been obtained experimentally and is based on shear fracture mechanism, the void growth and the combination of shear decohesion and void growth ([24]).

The relation between equivalent fracture strain ($\bar{\varepsilon}_f$) and stress triaxiality for the corresponding material of the structure can be obtained by dedicated coupon tests or by the adjustment of existing analytic or empirical formulations for similar materials to match the actual behavior of the material in terms of structural strength based on known design allowables such as the ultimate tensile stress (F_{tu}), the ultimate bearing allowable (F_{bru})…

This ductile damage model embedded in the simulation controls the degradation in the material stiffness due to the damage initiation and evolution prior to final failure as shown in Figure 7: The residual stiffness at each finite element of the model during the damage progression is calculated as (1-D)xE where E is the elastic modulus of the material and D is the cumulated damage in the element, driven by the ductile damage model introduced in the simulation.

An initial validation of the ductile failure model implementation can be easily done by virtual testing of standard design allowable coupons (tensile, bearing, compact-tension...) and correlation against physical specimen results.

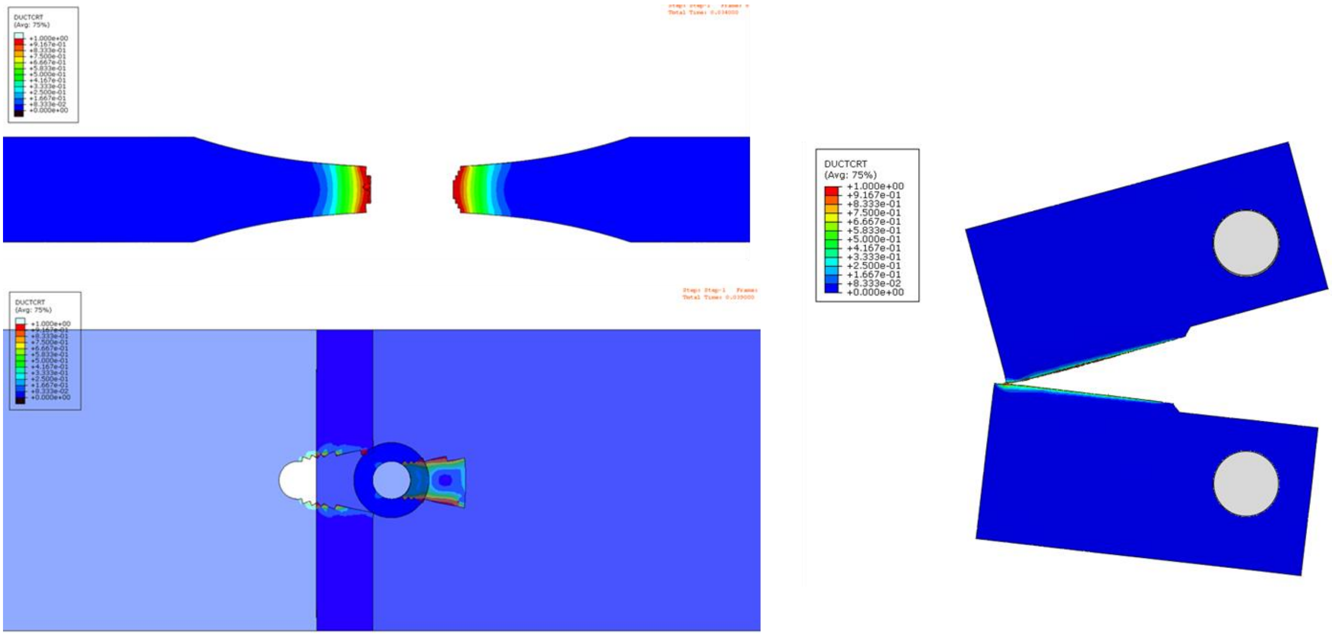


Figure 27: Virtual Testing of standard coupons for static and residual strength. Damage variable plots.

This methodology has been successfully applied in Airbus (Defence and Space) [25] to predict different failure modes using numerical simulations what allows a direct comparison with physical testing as shown in Figure 28: . The applicability to more complex structures, representative of actual aircraft structure subcomponents, has also been verified (Figure 29:)

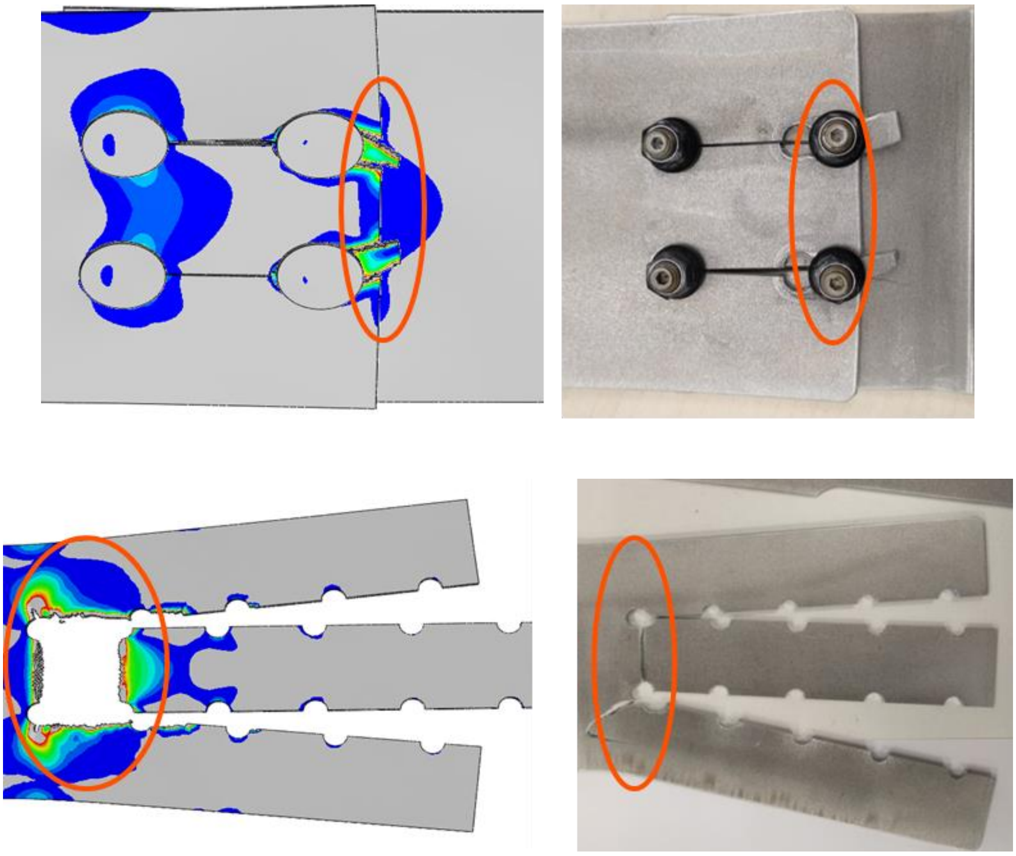


Figure 28: Virtual vs physical testing correlation for Residual Strength at metallic structures. Damage variable plots (LH) against physical fracture surfaces (RH).

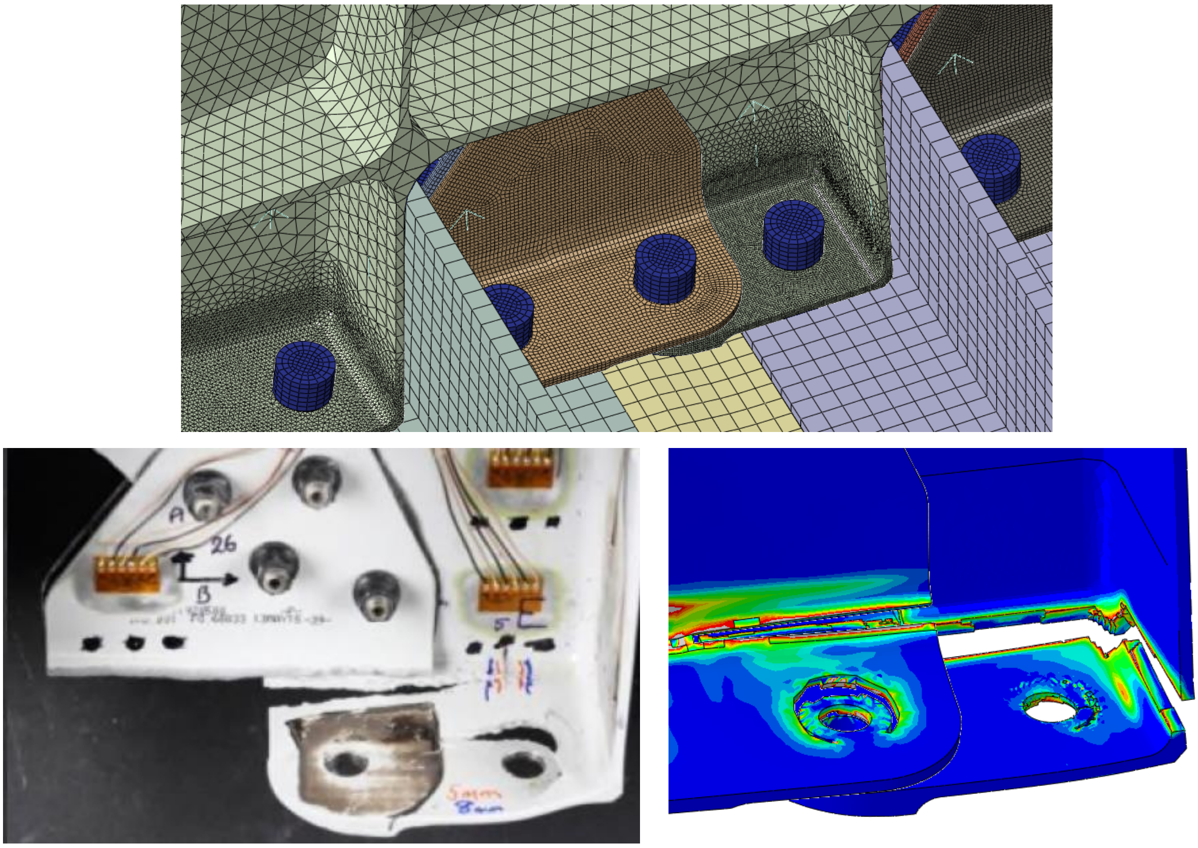


Figure 29: Virtual vs physical testing correlation for Residual Strength at metallic structures. Complex structures. Physical fracture surfaces (LH) against damage variable plots (RH).

6.2 Residual strength evaluation in metallic materials for Additive Manufacturing parts

The approach described in section 6.1 for conventional metallic materials is considered applicable to metallic Additive Manufacturing parts, and particularly suited to this manufacturing group of technologies, since complex and topologically optimized structures are one of the major fields of application of AM and the complexity (e.g. multiaxiality) could be challenging for conventional analysis methodologies.

As described in section 4.2, AM technology at the current state of the art has particularities if compared to conventional manufacturing processes at some respects, including for example the presence of higher number of internal defects or residual stresses.

The approach for AM requires a certain level of maturity and understanding of both the manufacturing process and the materials involved, many of them specifically tailored for this technology and showing differences in the macroscopic behavior if compared to the same materials conformed by means of conventional processes.

Residual strength evaluation for AM manufactured parts therefore implements Probabilistic Continuum Damage Mechanics and Fracture Mechanics (Section 4.2) for defect modeling in combination with the methodology for conventional metallic materials (Section 6.1) and material data particularized for each Additive Manufacturing process if necessary.

6.3 Residual strength evaluation in composite materials

The prediction of the residual strength capacity of a delaminated element has been studied with different methodologies, being the most relevant ones the fracture mechanics methods such as Virtual Crack Closure Technique (VCCT, which assumes that the energy needed to open and propagate a crack a certain amount is the same as the energy required to close it) and the Cohesive Zone Models (CZM, FEM based method where the delamination grows along interfaces using special cohesive elements whose behavior is given by a traction separation law).

On the one hand, VCCT have been successful predicting delamination growth, although remeshing is required as the crack advances. On the other hand, CZM have been able to predict both static and fatigue delamination growth and it avoids the need for re-meshing along a pre-defined crack path. Furthermore, CZM can deal with delamination growth that is non-uniform along the delamination front, or the investigation of planar delamination growth. Thus CZM are very suitable in the simulation interlaminar cracks and therefore, this work is only focused on this method.

Whereas the growth of the interlaminar crack is preceded by the formation of a damaged zone ahead of the crack tip, at this stage the evolution of the crack depends on the fracture toughness of the resin and the stress state.

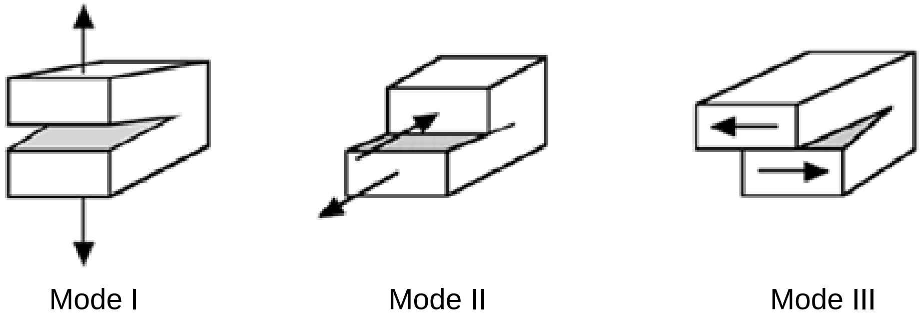


Figure 30: Crack or delamination opening modes

Crack propagation in brittle matrix systems submitted to mode I loading is produced due to coalescence of micro-cracks formed just ahead of the crack tip. Mode II and mode III loading submit the crack tip to shear stresses that form oblique micro-cracks at a considerable distance from the crack tip. These micro-cracks develop until they reach the fibre/matrix interface, thus the coalescence of these micro cracks occurs at the interface.

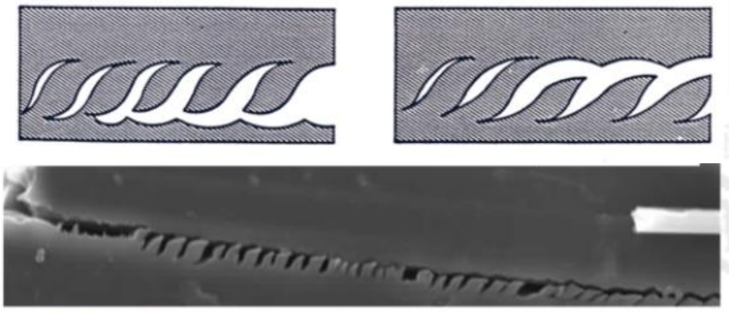


Figure 31: Mode II interlaminar micro-crack coalescence

The delamination resistance testing consists in a series of tests that are aimed to determine the fracture toughness of materials, i.e. its resistance to fracture and delamination. In case of composite materials, fracture or delamination is denoted by the critical SERR (Strain Energy Release Rate) or G_C.

For UD fiber reinforced composites, ASTM and other sources provide standards for determining mode I fracture toughness and mode II fracture toughness.
The opening mode I fracture toughness G_{IC} can be determined by the double cantilever beam (DCB)

test. DCB consists on a rectangular uniform thickness, UD laminated composite specimen containing a non adhesive insert on the midplane that represents the initial delamination.

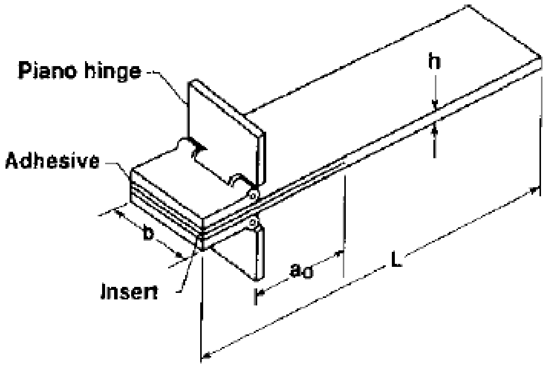


Figure 32: DCB test specimen

The opening mode II fracture toughness G_{IIC} can be obtained with the end notch flexure (ENF) test. The tested specimens present a similar configuration to those of DCB specimens. Test configuration is based on two cylindrical supports and a central upper one where the load is introduced, a record of the applied force versus centre roller displacement is to be obtained to compute mode II fracture toughness.

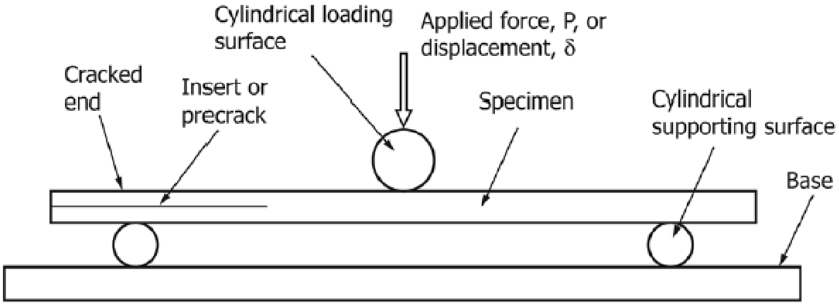


Figure 33: ENF specimen and test rig

In contrast to mode I and mode II delamination fracture toughness, pure mode III delamination is not easy to achieve in a test configuration. Testing methods proposed by different authors do not achieve a pure mode III, since the contribution of other delamination modes is still considerable. As a result there is not a standardized method available to measure mode III interlaminar fracture toughness yet.

Delamination mode mix has also been studied with some dedicated test configurations. In particular with Mix Mode Bending (MMB) test, the main characteristic of the test rig is the ability to alter the mode mix by changing the length of the lever arm.

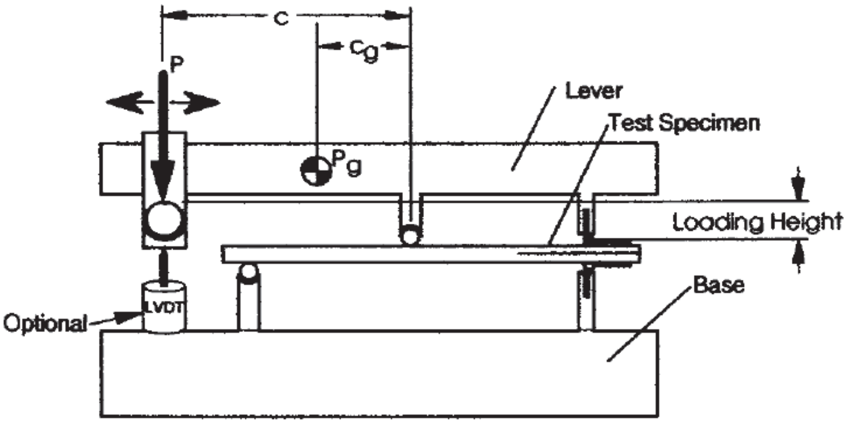


Figure 34: MMB specimen and test rig

Cohesive damage models use interface elements or nodal surface based interactions along the predefined propagation path, where the delamination is expected to grow. Their behavior is not linear; they follow a special law relating element tractions with nodal separations, (see Figure 18:).

When applying traction separation laws to actual structures, different cohesive behaviors can be clearly differentiated from one region to another:

- Elastic zone regions are those where damage onset has not yet occurred and elastic interface material properties have not been degraded.
- Cohesive zone regions or stress transfer zone are usually referred as areas where damage onset has already occurred but interface stiffness is not completely reduced.
- Zero stress regions are those areas where fracture criterion has been met and their material properties have been completely hindered so that they are part of the delamination discontinuity.

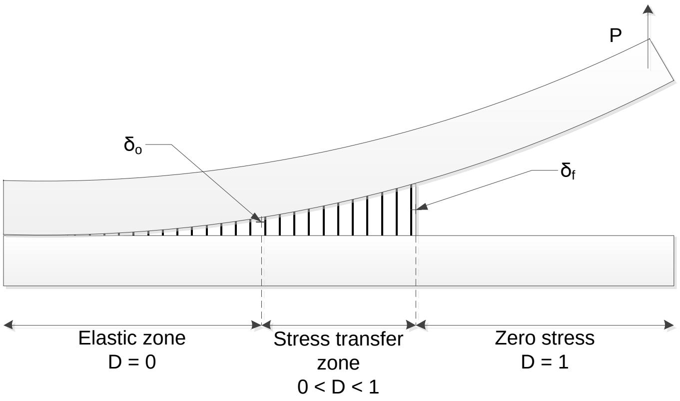


Figure 35: Cohesive damage zones

The parameters needed to define a bilinear traction separation law are traction stresses, interface element stiffness and the maximum nodal separations at which damage onset and propagation are produced.

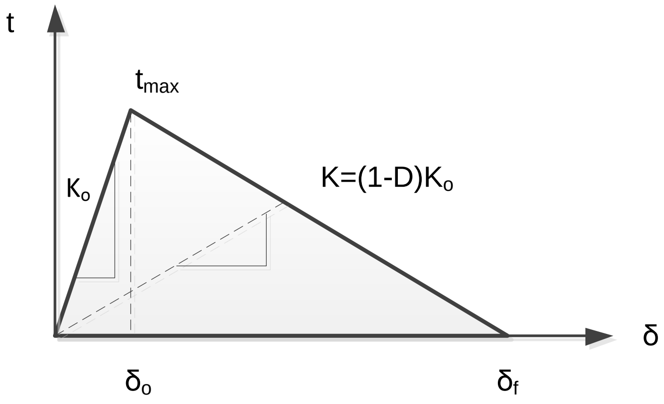


Figure 36: Cohesive traction separation law

Therefore, three different regions are defined depending on the actual nodal separations:

$$\begin{aligned} t &= K_o\delta \quad if \quad 0 \le \delta \le \delta_o \\ t &= K\delta \quad if \quad \delta_o \le \delta \le \delta_f \\ t &= 0 \quad if \quad \delta_f \le \delta \end{aligned} \qquad (13)$$

Where K_o is the penalty stiffness of the undamaged cohesive element. As the faces of the element separate, stiffness evolves as a function of the damage parameter D. Interface stiffness or K is called penalty stiffness and it is a property of the model rather than a material property.

For a damage evolution based on displacement with lineal softening damage parameter D can be obtained with the expression in (6)

In case of damage evolution based on energy dissipated as damage develops, proposed criteria incorporate fracture toughness and fracture energy relations. The most common criteria are the following:

- Mode independent: Fracture occurs when SERR is equal to the pure mode fracture toughness.

$$\frac{G}{G_c} = 1 \tag{14}$$

- Power law: Failure under mix mode conditions is governed by a power law interaction of the energies required to cause failure in the individual modes

$$\left\{\frac{G_I}{G_{IC}}\right\}^{\alpha} + \left\{\frac{G_{II}}{G_{IIC}}\right\}^{\alpha} + \left\{\frac{G_{III}}{G_{IIIC}}\right\}^{\alpha} = 1 \tag{15}$$

- Benzeggagh-Kenane (B-K) [26]: Particularly useful when the critical fracture energies during deformation purely along the first and the second shear directions are the same.

$$G_c = G_{IC} + (G_{IIC} - G_{IC})\left(\frac{G_{II}}{G_I + G_{II}}\right)^{\mu} \tag{16}$$

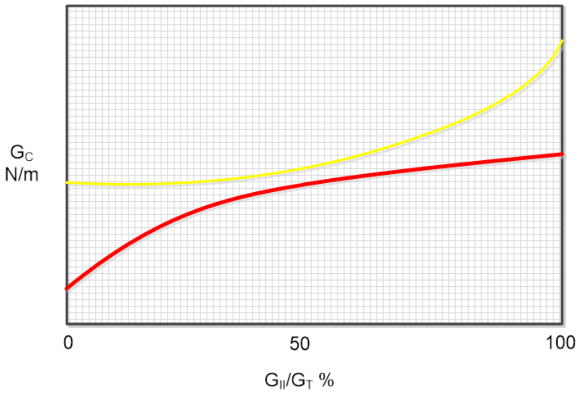


Figure 37: Example mixed mode fracture energy evolution

Several virtual tests have been successfully validated by Airbus (Defence and Space) with CZM. Pure opening test modes mentioned before are complemented with mix mode opening tests:

Target	**Test**
Determination of mode I interlaminar fracture toughness	DCB – Double Cantilever Beam
Determination of mode II interlaminar fracture toughness	ENF – End Notch Flexural
	C-ELS – End Loaded Split
Determination of mix mode behavior	MMB – Mix Mode Bending
Determination of shear strength	SLS – Single Lap Shear
	DLS – Double Lap Shear
Determination of apparent interlaminar shear strength	4PB – Four Point Bending

The results obtained with CZM simulations have been correlated with its corresponding physical tests.

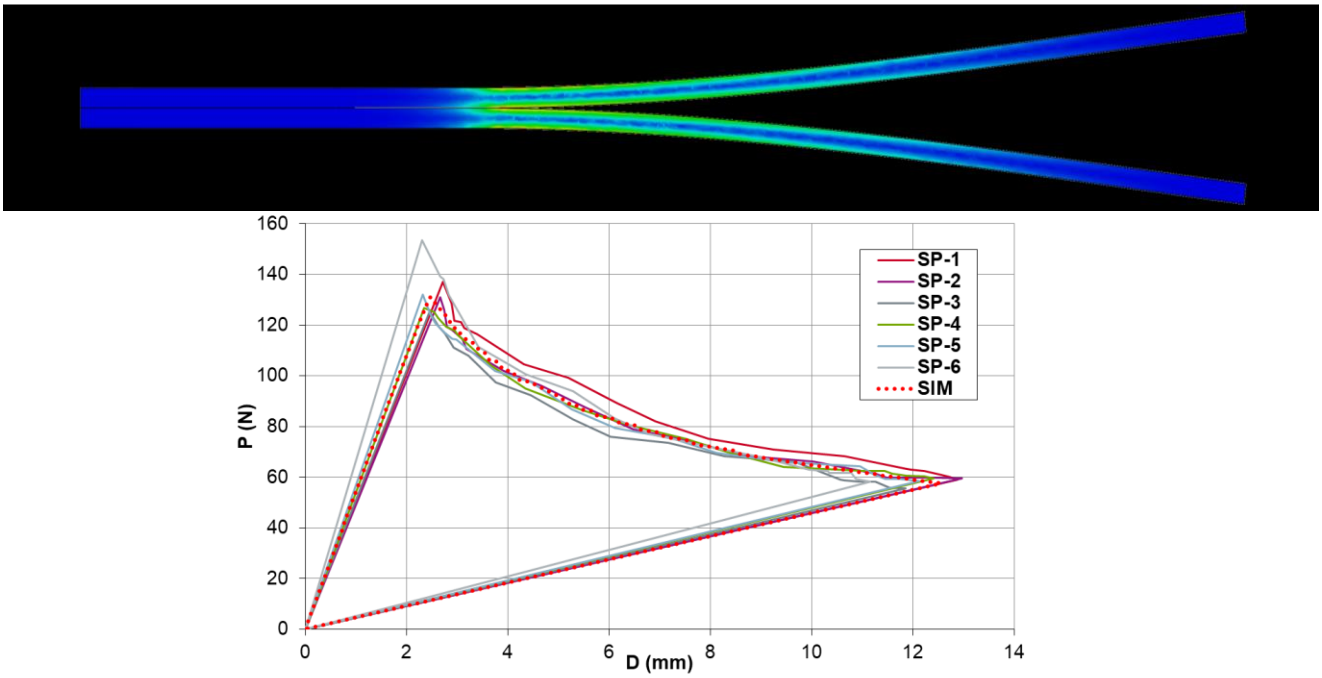


Figure 38: DCB virtual testing

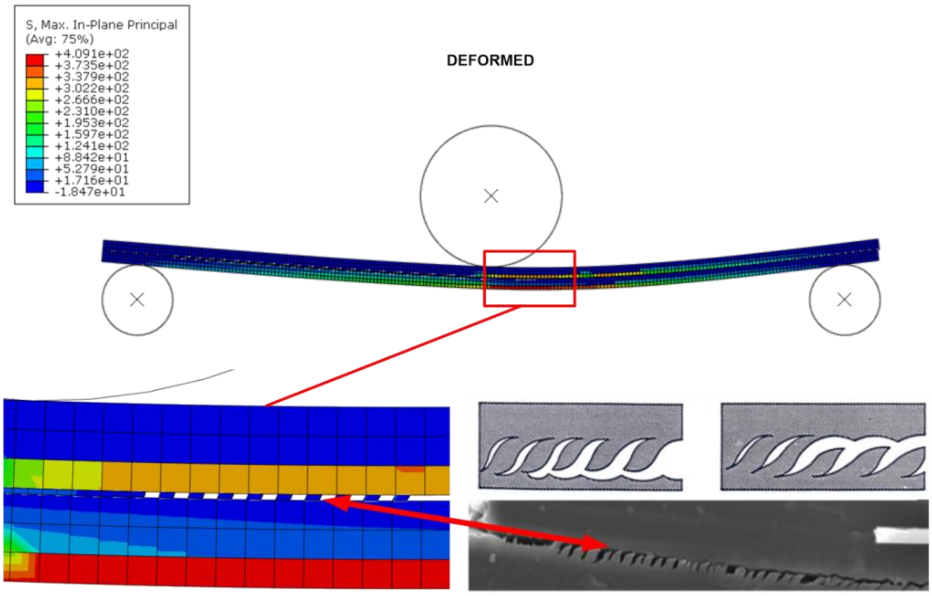


Figure 39: ENF virtual testing. Stress plot (LH) against physical failure surface (RH).

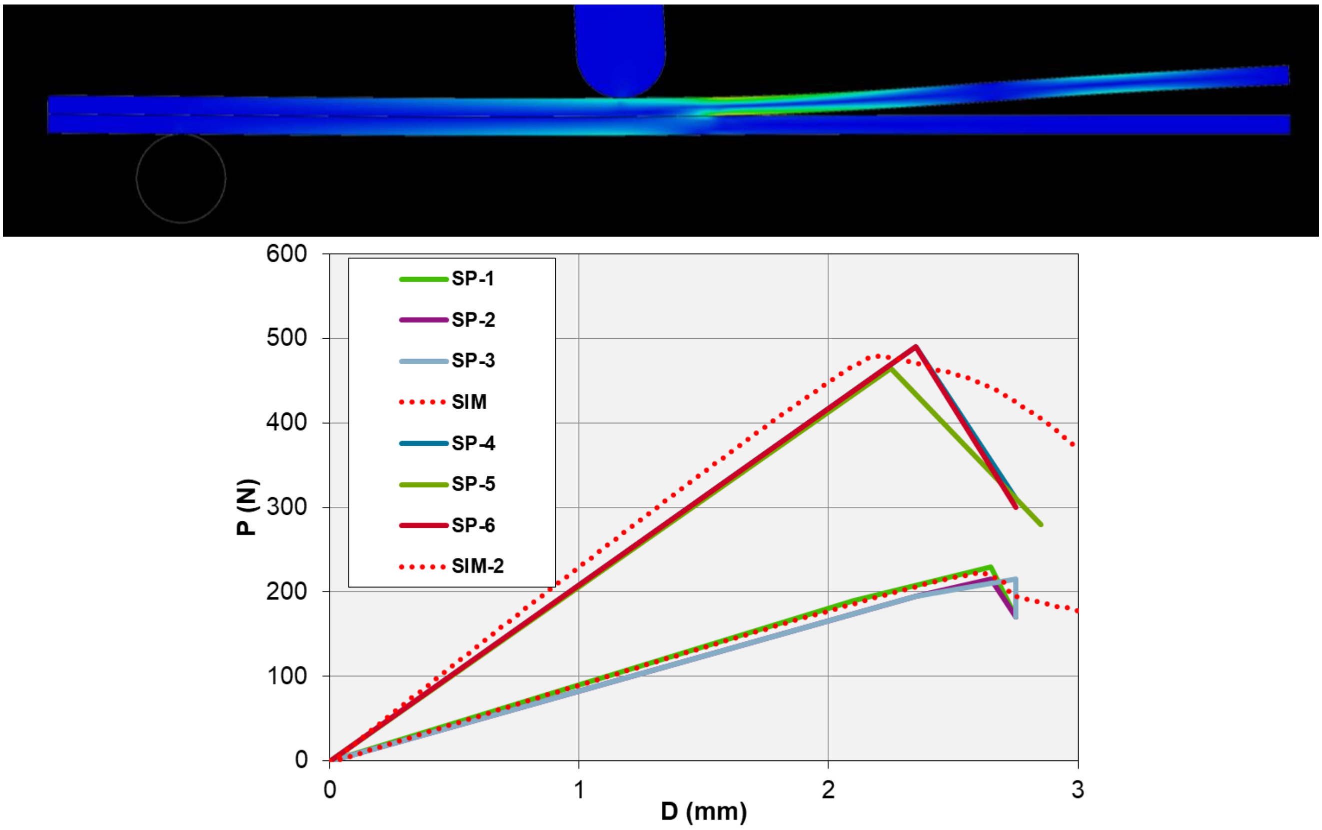


Figure 40: *MMB virtual testing*

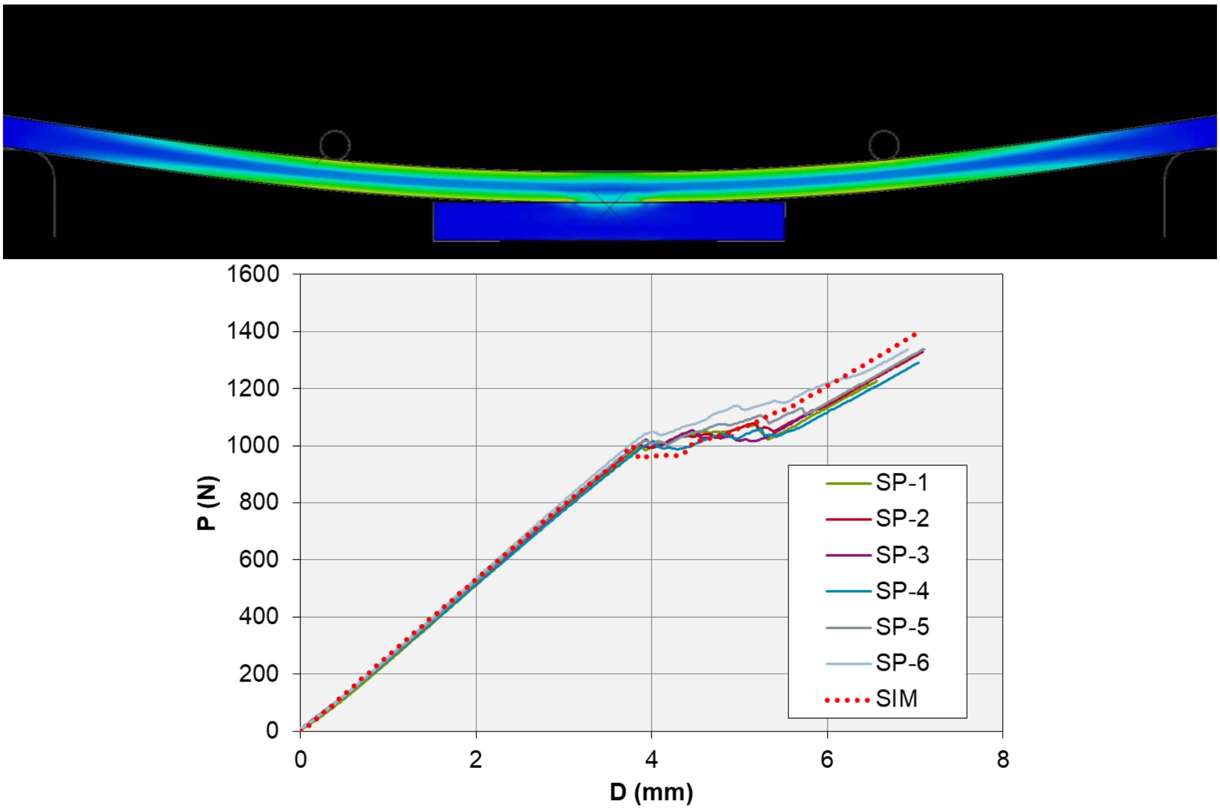


Figure 41: *4PB virtual testing*

7. Conclusions and way forward

As it was stated in the initial chapters of this paper, the development of a full Virtual Fatigue Testing environment requires a combination of a simulation framework, robust and versatile material behavioral models and uncertainty quantification through stochastic modelling.

Throughout the paper, a wide set of behavioral models for the different phases of the fatigue phenomenon have been presented. These models cover the two main material families which are currently applied in the aviation industry: metallic alloys, both in conventional manufacturing and additive manufacturing, and composites. It can be seen in the paper that the state of development of these models in Airbus (Defence and Space) is not the same for each material family.

For metallic materials, virtual testing solutions are able to cover the three phases of the fatigue phenomenon (crack initiation, crack propagation and residual strength) in a wide variety of structural configurations including complex cases such as full aircraft components. In the other hand, for additive manufacturing parts and composite structures, the level of development for virtual testing solutions is still at coupon and structural element level.

The main reasons for this status are two:

- A historical reason, based on the fact that metallic parts manufactured by conventional machining were introduced to aircraft primary structure many years before carbon fiber composites and AM parts (introduction of AM parts in aircraft is very recent).
- A phenomenological reason, based on the higher susceptibility of metals to fatigue in comparison with composites due to the different design philosophies applied for both kinds of structures. The current status can be visualized in Figure 42:

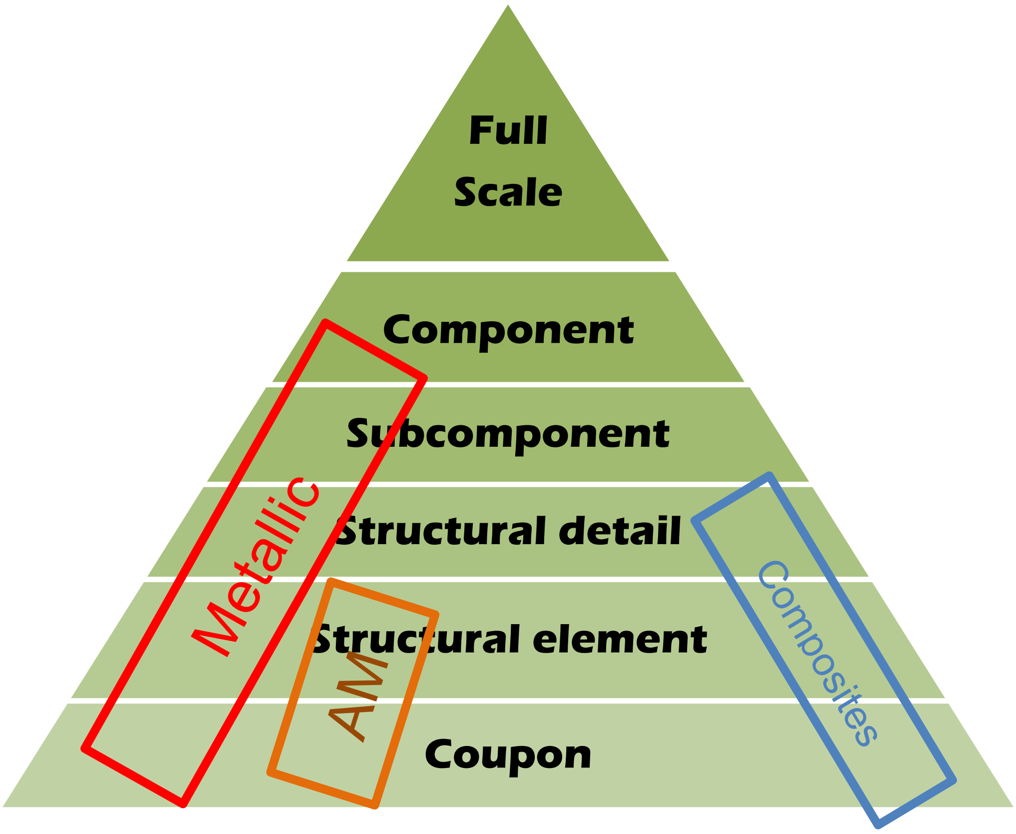


Figure 42: Applicability of Virtual Fatigue Testing solutions

In the next years, virtual testing developments will be focused on four main points:

- Implementation of additional material behavioural models in the virtual testing environment, able to deal with more complex phenomena such as multiaxial fatigue, stress corrosion…

- Extension of current virtual testing capabilities to higher levels of the test pyramid.
- Improvement of the digital continuity between the different phases of the virtual fatigue testing, from coupon to full scale level.
- Implementation of uncertainty quantification and management capabilities through stochastic modelling to include the effect of the variability in material properties, manufacturing processes, aircraft roles and environments…

To meet these goals, several challenges must be faced:

- Improved characterization of material properties in order to feed the advanced material damage models.
- Management of complex loading spectra.
- Developments in high performance computing to allow applications at full scale level with acceptable computational costs.
- Robust verification and validation approaches to ensure simulation credibility.

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9. Copyright Statement

10. Acknowledgment

The authors want to recognize the continuous support provided by their colleagues from Airbus Defence and Space, especially from Fatigue and Damage Tolerance team and Stress Methods team in Getafe. Without their contribution it would not have been possible to achieve the goal of this work.

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