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NEW ROLL-SPIRAL COUPLING MODE CRITERION FOR HIGH-SPEED VEHICLES

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Abstract

Design methods and dynamic characteristic study of high-speed vehicles is a hot research topic, nowadays. The lateral motion is one of important contents in characteristic study. Roll-spiral coupling mode may take place on high-speed vehicles, so when the coupling mode happens is a valuable content. Most of the conclusions about the coupling mode are based on the assumption of low angle of attack, which are not applicable for large angle of attack. In this paper, a new coupling criterion is deduced, considering angle of attack, while influences of aerodynamic parameters on the coupling mode is analyzed. All conclusions are validated by examples.

Keywords: High speed vehicles; Dutch roll mode; Roll-spiral coupling mode; Criterion; Dynamic characteristic

1. General Introduction

Due to the strategy, many countries are actively engaged in the research of high-speed vehicle. Unique configurations and special environment of near space high speed vehicles will cause some special flight dynamics effect compared with conventional vehicles, including dynamic instability, unsteady and strong coupling characteristics [1~7].

The weak damping and the lower aerodynamic/inertia ratio of high-speed vehicle make the classical characteristics of the near space flight mode change, and the mode coupling is easy to occur. Richard in a NASA report pointed out that the space shuttle met roll-spiral coupling mode in high speed and supersonic reentry stage. The high Mach number coupling dynamics problems are difficult to control [8].

In the analysis of the characteristics of the vehicles, the eigenvalues of the state matrixes or the characteristic roots of the characteristic equations are often used to analyze the modes in the motion, including roll-spiral coupling mode. In the design specification, more attention is paid to the damping characteristics ($\xi_{RS}\omega_{RS}$) of the mode, and there is little attention to the appearance condition of the mode.

Hua Yuguang and Li Lin et al. used the roll-spiral mode as the necessary content of flight quality analysis and gave the necessary and sufficient condition for the emergence of the coupling mode of flying wing aircrafts [12~13].

$$\left(-V\bar{L}_p\bar{N}_\beta - g\bar{L}_\beta\right)^2 - 4gV\bar{L}_\beta\bar{N}_\beta\bar{N}_r < 0 \tag{1}$$

Zheng Benwu et al. analyzed the influence of aerodynamic parameters on the roll-spiral coupling mode, proposed suggestions for fighter modification, and also deduced the necessary and sufficient conditions for the emergence of the coupling mode [14].

$$\bar{L}_\beta\bar{N}_r - \bar{N}_\beta\bar{L}_p < 0 \tag{2}$$

W. D. Grantham and T. Cvrlje et al. analyzed the influence of parameters on the roll-spiral coupling mode for many various aircraft layouts, simplified roll existence condition of the roll-spiral coupling mode, and got the necessary and sufficient condition for the coupling mode [14~16].

$$\left(\bar{L}_p+\left(\frac{g}{V}-\bar{N}_p\right)\frac{\bar{L}_\beta}{\bar{N}_\beta}\right)^2-4\bar{N}_r\frac{g}{V}\frac{\bar{L}_\beta}{\bar{N}_\beta}<0 \quad (3)$$

We can see Equ (1) is the simplification of Equ. (3) ignoring a cross derivative ($\bar{N}_p$).

These criteria are obtained under the hypothesis of small angle of attack and may not be applicable to the case of large angle of attack. This paper derives the roll-spiral coupling mode criterion for the flying of the large angle of attack.

Firstly, the paper introduces a conventional analysis method for lateral-directional modes, which can be used to verify the criterion of coupling mode appearance. Secondly, a new criterion of the roll-spiral coupling mode is deduced, and the influence of aerodynamic parameters on mode coupling is analyzed. Lastly, two high speed fight states of a vehicle are used to verify the validity and superiority of the new criterion.

2. Lateral-directional Mode Analysis

Lateral-directional mode analysis is usually carried out based on the linear small-disturbance state-space equation. By calculating the eigenvalues of the state matrix, the modes and their dynamic characteristics in the motion are analyzed [7]. The lateral-directional linear state-space equation as follows

$$\dot{x}=\begin{bmatrix}\dot{\beta}\\\dot{p}\\\dot{r}\\\dot{\varphi}\end{bmatrix}=\begin{bmatrix}\bar{Y}_\beta & \bar{Y}_p+\sin\alpha_0 & \bar{Y}_r-\cos\alpha_0 & g\cos\theta_0/V_0\\\bar{L}_\beta & \bar{L}_p & \bar{L}_r & 0\\\bar{N}_\beta & \bar{N}_p & \bar{N}_r & 0\\0 & 1 & \tan\theta_0 & 0\end{bmatrix}\begin{bmatrix}\beta\\p\\r\\\varphi\end{bmatrix}+\begin{bmatrix}0 & \bar{Y}_{\delta r}\\\bar{L}_{\delta a} & \bar{L}_{\delta r}\\\bar{N}_{\delta a} & \bar{N}_{\delta r}\\0 & 0\end{bmatrix}\begin{bmatrix}\delta_a\\\delta_r\end{bmatrix} \quad (4)$$

where, state variable $x=[\beta \quad p \quad r \quad \phi]^T$, input variable $u=[\delta_a \quad \delta_r]^T$. δ_a and δ_r are respectively aerodynamic aileron and rudder. The elements of the state matrix are obtained under reference condition.

Generally, surface symmetric vehicles have two real eigenvalues and a pair of complex conjugate eigenvalues, called: roll subsidence mode, spiral mode and dutch-roll oscillation mode.

Sometimes, eigenvalues of the state matrix are two pairs of complex conjugate eigenvalues, which are called dutch-roll oscillation mode and roll-spiral coupling mode. Therefore, the lateral-directional mode analysis can be used to verify the coupling mode criteria.

Based on the understanding of the basic characteristics of the above three modes, the fourth-order characteristic equation can be transformed into:

$$D(\lambda)=\left[\lambda^2+2\zeta_d\omega_d\lambda+\omega_{nd}^2\right]\left[\lambda-\lambda_R\right]\left[\lambda-\lambda_S\right]=0 \quad (5)$$

The parameters in the equation are often in the form of simplified expressions and are used as mode criteria, specifically as:

$$\begin{cases}\omega_{nd}^2\approx\bar{N}_\beta\\2\zeta_d\omega_{nd}\approx-\left(\bar{Y}^\beta+\bar{N}_r\right)+\dfrac{\bar{L}_\beta}{\bar{N}_\beta}\left(-\bar{N}_p+\dfrac{g}{V_0}\right)\end{cases} \quad (6)$$

$$\lambda_R=\bar{L}_p+\frac{\bar{L}_\beta}{\bar{N}_\beta}\left(-\bar{N}_p+\frac{g}{V_0}\right) \quad (7)$$

$$\lambda_S=-\frac{g}{V}\frac{\bar{N}_\beta\bar{L}_r+\bar{L}_\beta\bar{N}_r}{-\bar{N}_\beta\bar{L}_p+\bar{L}_\beta\bar{N}_p} \quad (8)$$

It is generally believed that if $\omega_{nd}^2>0$ and $2\zeta_d\omega_{nd}>0$,the dutch roll mode is stable; if $\lambda_R<0$, the roll convergence mode is stable; if $\lambda_S<0$, the spiral mode is stable.

3. Roll-spiral Coupling Mode Criterion

Generally speaking, poor aerodynamic damping and strong lateral static stability can result in roll subsidence mode and spiral mode coupling, which is known as roll-spiral coupling mode or lateral-directional long-period mode. Roll-spiral coupling mode is mainly characterized by the slow change of the roll angle and the yaw angle, so roll rate and yaw rate equations are kept. But the sideslip angle cannot be simply omitted because the aerodynamic moments depend on the sideslip angle. In addition, the lateral component of gravity is also the basic cause of yaw motion. Therefore, when the characteristic equation is simplified, the lateral force derivative is neglected, and the lateral inertial force and the gravity lateral component are considered to be balanced. The specific simplification is:

$$\begin{vmatrix} 0 & \sin\alpha_0 & -\cos\alpha_0 & g\cos\theta_0/V_0 \\ \bar{L}_\beta & \bar{L}_p-\lambda & \bar{L}_r & 0 \\ \bar{N}_\beta & \bar{N}_p & \bar{N}_r-\lambda & 0 \\ 0 & 1 & \tan\theta_0 & -\lambda \end{vmatrix}=0 \tag{9}$$

The characteristic equation is expended to:

$$a\lambda^2+b\lambda+c=0$$

where,

$$A=\left(\cos\alpha_0-\frac{\bar{L}_\beta}{\bar{N}_\beta}\sin\alpha_0\right) \tag{10}$$

$$B=\left(\cos\alpha_0\left(\frac{\bar{L}_\beta}{\bar{N}_\beta}\bar{N}_p-\bar{L}_p\right)+\sin\alpha_0\left(\frac{\bar{L}_\beta}{\bar{N}_\beta}\bar{N}_r-\bar{L}_r\right)\right)-\frac{g}{V_0}\left(\frac{\bar{L}_\beta}{\bar{N}_\beta}\cos\theta_0+\sin\theta_0\right) \tag{11}$$

$$C=\frac{g}{V_0}\left[\cos\theta_0\left(\frac{\bar{L}_\beta}{\bar{N}_\beta}\bar{N}_r-\bar{L}_r\right)+\sin\theta_0\left(\bar{L}_p-\frac{\bar{L}_\beta}{\bar{N}_\beta}\bar{N}_p\right)\right] \tag{12}$$

When the eigenvalue is a pair of conjugate complex roots, the two real root modes of roll and spiral become roll-spiral coupling mode, so the condition for the coupled mode is:

$$B^2-4AC<0$$

Assuming that the aircraft is in a straight level fight, $\theta\approx\alpha$, the above inequality can be simplified as

$$\left[\bar{L}_p+\left(\frac{g}{V_0}-\bar{N}_p-\bar{N}_r\tan\alpha\right)\frac{\bar{L}_\beta}{\bar{N}_\beta}+\left(\frac{g}{V_0}+\bar{L}_r\right)\tan\alpha\right]^2-4\frac{g}{V_0}\left[\left(\frac{\bar{L}_\beta}{\bar{N}_\beta}\bar{N}_r-\bar{L}_r\right)-\tan\alpha\left(\frac{\bar{L}_\beta}{\bar{N}_\beta}\bar{N}_p-\bar{L}_p\right)\right]\left(-\frac{\bar{L}_\beta}{\bar{N}_\beta}\tan\alpha+1\right)<0 \tag{13}$$

When the angle of attack is small, ($\alpha\approx\tan\alpha\approx 0$) and the cross derivative $\bar{L}_r$ is ignored, the above inequation can be simplified to the condition of coupling mode occurrence obtained by T. Cvrlje[16], as shown in Equ. (3).

The above criteria can be organized into a quadratic polynomial of the angle of attack.

$$f(w)=\left(a-c\frac{g}{V}+w\left(b-\frac{g}{V}\right)\right)^2-4(1-cw)\frac{g}{V}(b-aw)=\left[\left(b-\frac{g}{V}\right)^2-4ac\frac{g}{V}\right]w^2+2\left(a+c\frac{g}{V}\right)\left(b+\frac{g}{V}\right)w+\left[\left(a-c\frac{g}{V}\right)^2-4b\frac{g}{V}\right] \tag{14}$$

where, $w=\tan\alpha$, $a=\frac{\bar{L}_\beta}{\bar{N}_\beta}\bar{N}_p-\bar{L}_p$, $b=\frac{\bar{L}_\beta}{\bar{N}_\beta}\bar{N}_r-\bar{L}_r$, $c=\frac{\bar{L}_\beta}{\bar{N}_\beta}$

The discriminant of Euqation $f(w)=0$ is

$$\Delta=4\left(a+c\frac{g}{V}\right)^2\left(b+\frac{g}{V}\right)^2-4\left[\left(b-\frac{g}{V}\right)^2-4ac\frac{g}{V}\right]\times\left[\left(a-c\frac{g}{V}\right)^2-4b\frac{g}{V}\right]=16\frac{g}{V}(ac+b)\left[\left(b-\frac{g}{V}\right)^2+\left(a-c\frac{g}{V}\right)^2\right] \tag{15}$$

According to the theory of quadratic equations, the following conclusions can be drawn.

① When $\Delta \le 0$, there is always $f\left(w\right) \ge 0$,and there is no roll-spiral coupling mode at each angle of attack.

$\Delta \le 0$ is equivalent to the following formula

$$ac + b = \frac{\bar{L}_\beta}{\bar{N}_\beta}\left(\frac{\bar{L}_\beta}{\bar{N}_\beta}\bar{N}_p - \bar{L}_p\right) + \left(\frac{\bar{L}_\beta}{\bar{N}_\beta}\bar{N}_r - \bar{L}_r\right) \le 0 \quad (16)$$

② When $\Delta > 0$, it is necessary to judge whether $f\left(w\right) < 0$ is satisfied. If it is satisfied, the vehicle will have a roll-spiral coupling mode, otherwise the coupling mode will not appear.

In summary, a simple and practical criterion can be obtained. When the vehicle parameters meet the following condition, the aircraft will not have a roll-spiral coupling mode.

$$\frac{\bar{L}_\beta}{\bar{N}_\beta}\left(\frac{\bar{L}_\beta}{\bar{N}_\beta}\bar{N}_p - \bar{L}_p\right) + \left(\frac{\bar{L}_\beta}{\bar{N}_\beta}\bar{N}_r - \bar{L}_r\right) \le 0 \quad (17)$$

Further analysis is made on the above criterion:

(1) When the cross derivatives are not considered, the criterion is

$$\frac{\bar{L}_\beta}{\bar{N}_\beta}\left(\bar{N}_r - \bar{L}_p\right) \le 0$$

The condition for the coupling obtained by the above equation is consistent with Equ. (2). High speed vehicles generally satisfy the above inequalities, so when the cross-derivatives are not considered, the roll-spiral mode generally does not occur in the mode analysis of high-speed vehicles.

(2) Considering the quadratic term and the zeroth term of $\bar{L}_\beta/\bar{N}_\beta$, the larger $\bar{N}_p$ is and the smaller $\bar{L}_r$ is, the more the roll-spiral coupling tends to occur.

(3) Considering the primary term of $\bar{L}_\beta/\bar{N}_\beta$, when the two directions satisfy the static stability condition, namely $\frac{\bar{L}_\beta}{\bar{N}_\beta} < 0$, the smaller $\bar{N}_r$ is and the larger $\bar{L}_p$is, the more the tendency of the coupling mode occurs.

(4) High speed vehicles generally satisfy $\bar{N}_r - \bar{L}_p < 0$. If $\bar{N}_p > 0$, the larger the roll-to-sideslip ratio $\left|\frac{\varphi}{\beta}\right| \approx \left|\frac{\bar{L}_\beta}{\bar{N}_\beta}\right|$ is, the more the tendency.

From the above analysis, the steps to determine whether there is a roll-spiral coupling mode are:

- It is judged whether the formula (17) is satisfied. If it is satisfied, there must be no roll-spiral coupling mode. Otherwise, the coupling mode may occur and further analysis is needed.
- If Eq. (17) is not satisfied, it is judged whether the inequality $f(w) < 0$ is satisfied. If the inequality is satisfied, the coupling mode must appear, otherwise the coupling mode does not appear.

4. Case Analysis

4.1 Dynamics Analysis

This paper is based on a blended-wing-body high speed vehicle, and its shape is shown in Fig. 1.

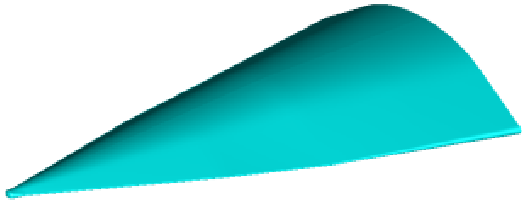

Figure 1 – A high-speed vehicle

Flight state A: H=40km, Ma=18.5, and α=13.2° . The lateral state matrix in this state is

$$A_{hc}=\begin{bmatrix}-0.0054 & 0.2288 & -0.9735 & 0.0016\\ -178.7170 & -0.0699 & 0.0183 & 0\\ 1.0537 & 0.0007 & -0.0081 & 0\\ 0 & 1 & 0.2351 & 0\end{bmatrix}$$

By directly obtaining the eigenvalues of the state matrix, two modes can be obtained in the state, which are: -0.0350±6.4750i (Dutch roll mode) and -0.0066±0.0033i(roll-spiral coupling mode).

Flight state B: H=30km, Ma=17.9, and α=7.4° . The lateral state matrix in this state is

$$A_{hc}=\begin{bmatrix}-0.0062 & 0.1288 & -0.9917 & 0.0017\\ -90.2900 & -0.0551 & 0.0102 & 0\\ 8.3922 & -0.0011 & -0.0087 & 0\\ 0 & 1 & 0.1299 & 0\end{bmatrix}$$

Similarly, using the method of obtaining the eigenvalues of the state matrix, three modes can be obtained, which are normal lateral modes: -0.0149±4.4666i(Dutch roll mode), -0.0388(Roll subsidence mode) , -0.0014（Spiral mode）

4.2 Coupling criterion analysis

The coupling of roll mode and spiral mode in State A and State B is analyzed by using the coupling criteria in various literatures and this paper. The validity of these criteria is analyzed base on the mode analysis results of the state matrix in Section 4.1.

The specific results are shown in Table 1. Criterion 1 in the table is the criterion given in References [9] and [13], namely Eq.(1) in this paper. Criterion 2 is the criterion given in Reference [14], namely Eq. (2) in this paper. Criterion 3 is the criterion given in Reference [16], namely Eq.(3) in this paper. Criterion 4 is given in this paper, namely Eq. (17).

Table 1 – Coupling mode criterion of vehicles

	Fight State A （Coupling mode）	**Fight State B** （No coupling mode）
Criterion 1	Cr=0.1263>0， There is no coupling mode. （Criterion invalid）	Cr=0.3353>0， There is no coupling mode. （Criterion valid）
Criterion 2	Cr=1.5152>0， There is no coupling mode. （Criterion invalid）	Cr=1.2480>0， There is no coupling mode. （Criterion valid）
Criterion 3	Cr=0.0417>0， There is no coupling mode. （Criterion invalid）	Cr=0.0067>0， There is no coupling mode. （Criterion valid）
Criterion 4	Cr=10.7047>0， Coupling modes may occur. Then $f(w)=-0.0744<0$	Cr=-0.6412<0， There is no coupling mode. （Criterion valid）

	There is a roll-spiral coupling mode （Criterion valid）	

It can be seen from Table 1 that all criteria can correctly analyze whether the coupling mode occurs in State B. All criteria show State B does not exhibit a coupling mode, which is the same as the analysis result of the state matrix.

For flight state A, only criterion 4 says State A is likely to have a coupling mode. Other criteria say that State A does not exhibit a coupled mode, while state matrix analysis results in State A having a roll-spiral coupling mode.Therefore, Criterion 4 can effectively analyze the coupling of the roll mode and spiral mode of the high-speed vehicle. Other criteria are slightly insufficient when performing high speed roll-spiral coupling mode analysis.

5. Conclusions

In this paper, Considering the angle of attack, a new coupling mode criterion is derived, and the applicability and advantages of the criterion are verified. Through the analysis of this paper, the following conclusions can be drawn.

(1) For high altitude and high-speed flight, poor aerodynamic damping, and rolling-spiral coupling mode may occur in lateral motion.

(2) In the case of no cross derivative, the roll-spiral coupling mode will not appear in the high-speed vehicle.

(3) The larger $\bar{N}_p$ is and the smaller $\bar{L}_r$ is, the larger the roll-spiral coupling trend is.

(4) When two directions satisfy the static stability condition, namely $\frac{\bar{L}_\beta}{\bar{N}_\beta} < 0$, the smaller $\bar{N}_r$ is and the larger $\bar{L}_p$ is, the larger the tendency of the coupling mode will be.

(5) The high-speed vehicles generally satisfy $\bar{N}_r - \bar{L}_p < 0$. If $\bar{N}_p > 0$, the larger the roll-to-sideslip ratio is, the larger the trend of coupling mode will appear.

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7. Copyright Statement

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