A dynamic self-control ERR strategy for bi-directional evolutionary structural optimization

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Abstract

The element removal ratio is a crucial parameter in bi-directional evolutionary structural optimization (BESO), which is of significance for the stability and efficiency of the optimization process and the final topology optimization results. The traditional BESO method uses a fixed evolution rate (the ERR method), which may ruduce inefficiency. And the existing self-updating evolution strategy ignores the influence of the initial element removal ratio and regulates the evolution rate to a higher level, which results in more potential unstability. In view of the above, this paper used the relative standard deviation (RSD) representing the disperesion degree in statistics to define structural performance index, measured the uniformity of the structural element sensitivity, and analyzed the influence of the initial element removal ratio and the ideal change law of ERR in the process of structural optimization. Therefore, BESO was divided into two processes, the fast evolution phase and the smooth evolution phase, and it switched automatically from the fast one to the other according to the PI. The relative index of the structural performance fusing all iterative information was used as the control factor. Using this control factor, this paper constructed the self-control function of ERR under two stages separately, which control the change of the evolution rate in real time. This method can obtain a faster optimization process on a stable basis. Finally, the implementation process of the BESO method with dynamic self-control ERR was put forward. The comparison with related calculation examples showed that the method in this paper improved the optimization efficiency while taking into account its stability under similar optimization quality.

Keywords: Element removal ratio (ERR); Bi-directional evolutionary structural optimization (BESO); Structural performance index (PI); Dynamic self-control ERR (DSC-ERR).

1. Introduction

Structural optimization occupies an important position in structural design, which can help designers find the optimal design plan that satisfy various constraints such as a given amount of material. Structural optimization has three types include size, shape and topology optimization. The structural topology optimization roughly determines the final optimized structure, which has been extensively investigated by many researchers around the world. Various topology optimization methods have been proposed over the past few decades. Among them, the evolutionary structural optimization (ESO) is one of the most popular methods that gradually removes inefficient material from a structure. However, this method is prone to accidental deletion and if the element removed by mistake cannot be restored, it may lead to a large deviation in the optimization results. The ESO method was developed for the bidirectional evolutionary optimization (BESO)[1,2], which allows the removal of inefficient materials while adding materials to the structures. The BESO method is proved to be more robust than the ESO method[3,4].

In the BESO method, the element removal ratio is one of the key factors affecting optimization efficiency, quality and stability. Querin[1] defined the element removal ratio as the sum of the initial element removal rate and the evolutionary rate. Chu gave the recommended range of the element removal ratio of 1%-4%[5], and proposed that the remaining structural volume is determined by the product of the element removal ratio and the number of structural element remaining in this iteration[6] and its expression is

$$V_{i+1} = V_i(1 - ERR_i) \quad (1)$$

$$ERR_i = ERR_{i-1} + ER \quad (2)$$

Where, V_{i+1}、V_i are the structural volumes of the $(i+1)\text{th}$、$i\text{th}$ iteration. ERR_i is the element removal rate, ER is the evolutionary rate. It can be seen that ERR_i would have been increased, it gradually improve the efficiency of the algorithm. However, the larger ERR_i values may result in elements mis-deleted and non-convergence of solution, there may be an oscillation state. Moreover, the values of the initial element removal rate and the evolution rate are subjectively affected, and this enhances the instability of the structure topology optimization. To comprehensively consider the efficiency and stability of the algorithm, a reasonable element removal strategy is to increase the element removal rate with a large amount of redundancy at the beginning. As the optimization progresses running, the structural stress distribution tends to be uniform, and the element removal rate should be reduced. As a result, scholars began to study the dynamic element removal rate that can adapt to structural changes. Zhang[7] constructed a real-time adjustment element removal rate function according to the structural stress. The removal rate function is defined as

$$ERR_i = \left[1 - \min\left(\frac{\sigma_{ave}}{\sigma_{\max} - \sigma_{\min}}, 0.9\right)\right]^t * ERR_0 \quad (3)$$

Where σ_{ave} is the average stress of the structure, $\sigma_{\max}$、$\sigma_{\min}$ are the maximum and minimum stresses of the structure respectively, ERR_0 is the maximum recommended value, $t \in [0,1]$ is a parameter that is greatly affected by subjectivity. To improve the optimization solution process more effectively, Luo Jing[8] proposed a dynamic element removal function with fewer artificial control parameters. The removal rate is adjusted by the ratio of the maximum and minimum element stress of the structure, defined as:

$$ERR_i = \kappa \cos\left(\frac{\pi}{2} * \frac{\alpha_{\min}}{\alpha_{\max}}\right) \quad (4)$$

Although this method obtains relatively better optimization results, the stress information used is only limited to a certain iteration step, and parameters κ are needed to assist in the change of the element deletion rate. Based on this, Kuang Bing[9] constructed a relative index β_i of the degree of homogenization of elemental sensitivity number under all iterative step information. The removal rate function is as follows:

$$ERR_i = ERR_0 + ERR_1 \times \cos(\pi \times \beta_i) \quad (5)$$

Where ERR_0 is the initial element removal rate, $ERR_1 = 0.04 - ERR_0$ is the fluctuation range of the element removal rate. Although this method constructs an adaptive removal rate change function with no empirical parameters, the element removal rate obtained by the calculation formula (4) is close to 0.04, ignoring the influence of the initial element removal rate. In this article, this method is called the self-updating ERR method.

Based on the above-mentioned dynamic element removal rate strategy, this paper proposes a dynamic self-control element removal rate function, which realizes self-control of the removal rate while considering the influence of the initial removal rate. Firstly, a structural performance indicator was constructed to reflect the homogenization of structural elemental sensitivity number. Secondly, the optimization process of the BESO method was divided into the fast deletion phase and the smooth deletion phase, and switched automatically from the fast deletion phase to the smooth deletion phase according to the performance indicator. Finally, the performance indicator is used to construct the self-control function of the element removal rate under the two stages, and the element removal rate is dynamically controlled. In the following, this method will be referred to simply as the DSC-ERR method.

2. Problem statement and the sensitivity number

2.1 Problem statement

In this paper, the topology optimization is to seek the minimum structural stiffness with a fixed target material volume. The BESO method works by removing and adding elements, so the design variable can be the element itself. To obtain a nearly solid-void design, the BESO method often combines with the SIMP method that is a material interpolation schemes with a penalization. The structural topology optimization can be expressed as:

$$\begin{aligned} &\text{Find}: X = \{x_1, x_2, ..., x_n\} \\ &\min \quad C = \frac{1}{2}u^T K u \overset{SIMP}{=} \frac{1}{2}\sum_{i=1}^{n} x_i^p u_i^T k_0 u_i \\ &\text{s.t.} \quad v^* - \sum_{i=1}^{N} v_i x_i = 0 \\ &\qquad F = KU \\ &\qquad x_i = x_{\min} \ or \ \ 1 \end{aligned} \tag{6}$$

Where C is the mean compliance, F and U are the structural force matrix and the displacement matrix, K is the structural stiffness matrix, k_0 is the stiffness of the solid element and p the penalty exponent, v^* is the prescribed volume of the final structure and v_i the volume of an individual element. The design variable x_i is the material density of the *i*th element. The value of x_i is taken as 0.001 for the void element to avoid the singularity of the stiffness matrix.

2.2 The sensitivity number

In the BESO method, because the sensitivity number indicates the changes in the overall stiffness due to removal of an element, it is used to express the contribution degree of the element to the structure. The sensitivity of the *i*th element is defined as the partial derivative of the objective function to the density of the *i*th element. For the void elements, the sensitivity numbers can be obtained through a filter scheme that smooths the sensitivity in the entire design domain. The elemental sensitivity number is so significant because its order determines the addition and removal of elements. The sensitivity can be simplified to:

$$\alpha_i = -\frac{1}{p}\frac{\partial C}{\partial x_i} = \frac{1}{2}u_i^T k_0 u_i \tag{7}$$

2.3 The convergence criterion

In the BESO method, the stopping of the cycle not only reach the objective volume, but also meet the convergence criterion, as follows:

$$\frac{\left|\sum_{j=1}^{N}\left(C_{k-j+1} - C_{k-N-j+1}\right)\right|}{\sum_{j=1}^{N} C_{k-j+1}} \le \tau \tag{8}$$

Where k is the current iteration number, τ is allowable convergence tolerance and N is an integer number. In this article $\tau = 0.001$ and $N = 5$.

3. A dynamic self-control ERR strategy

The core idea of the BESO method is to add high-sensitivity elements while removing low-sensitivity elements. When the structural elemental sensitivity number differs greatly, the structure has more redundant components, so the element removal rate can be appropriately increased. In other words, the degree of homogenization of elemental sensitivity number can indicate the pros and cons of the structure. This paper constructed structural performance indicators express the degree of homogenization of structural elemental sensitivity number and explored the relationship between the degree of homogenization of structural elemental sensitivity number and the element removal rate, so as to determine the element removal rate by structural performance indicators.

3.1 Definition and change law of structural performance index

This paper calculated the relative standard deviation of the structural elemental sensitivity number as the structural performance index. Relative standard deviation is usually used in statistics to indicate the degree of dispersion of measurement data. The smaller the value, the lower the degree of dispersion and more average the measurement data. The structural performance index is calculated as

$$\beta_i = \frac{S}{\bar{x}} \times 100\% = \frac{\sqrt{\dfrac{\sum_{i=1}^{n}\left(\alpha_i - \bar{\alpha}\right)^2}{n-1}}}{\bar{\alpha}} \times 100\% \tag{9}$$

Where β_i is structural performance index, S 、 $\bar{x}$ are the standard deviation and average of the elemental sensitivity number respectively.

In the initial stage of structural topology optimization, with the reasonable removal of structural elements, the uniformity of the sensitivity number of structural elements increases, that is, the structural performance index may decrease. At this time, the structure has more inefficient elements that should be removed, so the efficiency of the algorithm needs to be considered first. And the efficiency depends on the number of elements removed. The increase in the number of elements removed can be achieved by the increase in the element removal rate. In other words, the efficiency of the algorithm can be improved by increasing the element removal rate. Afterwards, the optimal iteration approaches a steady state, the element removal rate is not suitable to continue to increase, otherwise it will cause structural instability. Therefore, the element removal rate can be increased in the initial stage of optimization, and appropriately decreased in the relatively stable stage. In the optimization process, the influence of initial element removal rate and evolution rate should also be considered. Generally, the smaller the initial element removal rate and evolution rate, the better the topology can be obtained, but the calculation efficiency is lower. If they are too large, although the calculation efficiency will increase, it is easy to remove element by mistake resulting in wrong optimization results.

In short, the ideal change rule of the removal rate should be based on a reasonably low initial element removal rate, first continuously increase the element removal rate to quickly reduce the inefficient elements, and then slowly reduce the removal rate to maintain the stability of the optimized results. Therefore, this paper divides the process of structural topology optimization into two phases: the fast deletion phase and the smooth deletion phase. According to the performance index and the recommended range of the element removal rate, it will automatically move from the fast deletion phase to the smooth deletion phase and control the increase or decrease of the element removal rate.

3.2 Dynamic self-control element removal rate function construction

Through large number of calculation examples, Chu[5] have proved that satisfactory optimization results can be obtained when the element removal rate is between 0.01 and 0.04. Luo[8] recommends that the initial element removal rate should be a smaller value, such as 0.02, and pointed out that a fixed evolution rate may be not get the best optimization results. This article constructed a dynamic control function for the fast deletion phase and the smooth deletion phase, and determine the evolution rate according to the performance index of the structure. First, when the performance index β_i drops to a certain value $\tilde{\beta}$ or the element removal rate ERR_{i-1} exceeds 0.04, it considers entering the smooth deletion stage from the rapid deletion stage. Secondly, when determining the evolution rate, the information of all iteration steps is considered. The evolution rate change information factor d_i is introduced, which can be expressed as

$$d_i = \frac{d_2}{d_1} \tag{10}$$

Where

$$\begin{cases} m = \left[\dfrac{i}{2}\right] \\ d_2 = \dfrac{\sum_{j=1}^{m} \beta_j}{m} \quad (j = 1, 2, 3, ..., m) \\ d_1 = \dfrac{\sum_{l=m+1}^{i} \beta_l}{i - m} \quad (l = m+1, m+2, ..., i) \end{cases} \tag{11}$$

For the *i*th iterative step, it divides the performance index into two halves, the average value of the first half is d_2, the other half is d_1 ,and their ratio is d_i. d_i is the index to control the evolution rate. Finally, this paper uses the piecewise function to construct the self-control functions in the fast deletion phase and the smooth deletion phase respectively. When $ERR_{i-1} < 0.04$ and $\beta_i > \tilde{\beta}$, it is in the fast deletion phase, the element removal rate will be increased and its control function is:

$$ERR_i = ERR_{i-1} + \kappa \times \sin\left(\frac{\pi}{2} \times \frac{d_2}{d_1}\right) \tag{12}$$

Where κ is a relatively small number used to control the largest change of the element removal rate. When $ERR_{i-1} > 0.04$ or $\beta_i < \tilde{\beta}$, it will automatically enter the smooth deletion phase, and its element removal rate will decrease on the basis of the previous iteration ERR_{i-1}. Its control function is:

$$ERR_i = ERR_0 + \left(ERR_{i-1} - ERR_0\right) \times \sin\left(\frac{\pi}{2} \times \frac{d_2}{d_1}\right) \tag{13}$$

Where ERR_0 is the initial element removal rate. The value between 0.02 and 0.03 is more appropriate. In this paper it used $ERR_0 = 0.025$.

3.3 The procedure of the BESO method with a dynamic self-control ERR strategy

The evolutionary procedures for the improved BESO with the above-mentioned dynamic self-control ERR strategy are as follows:

Step 1: Define design domain, loads, constraints and boundary conditions;

Step 2: Discretize the design domain and perform finite element analysis;

Step3: Calculate the elemental sensitivity number and then solve its the relative standard deviation according to formula (9) as a performance index;

Step 4: Average the performance index number with its history information using Equation（10）and

(11), and then determine the element removal rate according to formulas (12) and (13);

Step 5: Filter and modify the elemental sensitivity number, then sort the new sensitivity from largest to smallest and determine the sensitivity threshold according to the structural target volume and the element removal rate of the *i*th iteration.

Step 6: Remove the solid elements with sensitivity less than α_{th} , and add empty elements with greater than α_{th} ;

Step 7: Repeat steps 2~ 6 until the volume constraint is achieved and convergence criterion is satisfied.

4. Example of the BESO method with a dynamic self-control ERR strategy

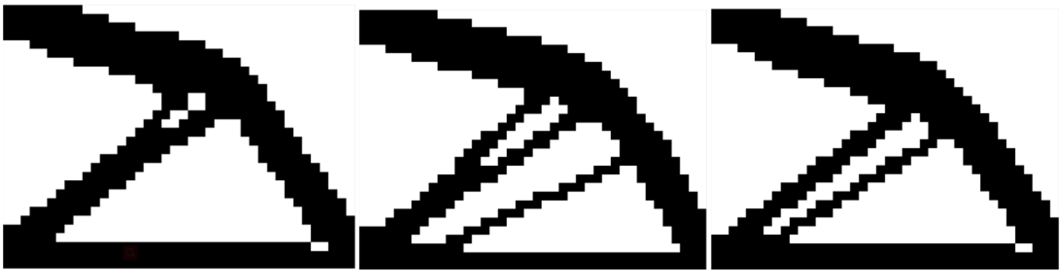
The calculation example in this paper compares the BESO method with the self-updating ERR method and the BESO method with a fixed removal rate (the ERR method). The correlation coefficient is unified as the elastic modulus of the material is 206GPa, Poisson's ratio is 0.3.

4.1 Topology Optimization of the cantilever beam

This example considers the stiffness optimization of a cantilever shown in Figure 4.1. The left side of the structure is fixed, and the lower right corner is subjected to a 1N concentrated force. The volume fraction of the final design will be 40%. The design domain is divided into 40×30 four node plane elements. The problem is solved using the penalty factor $p = 3$,filter radius $r = 1.2$ and the initial element removal rate $ERR_0 = 0.025$. The remaining parameters of the DSC-ERR method are: $\kappa = 0.00085$ and $\tilde{\beta} = 1.2$.

Figure 4.1– A cantilever beam

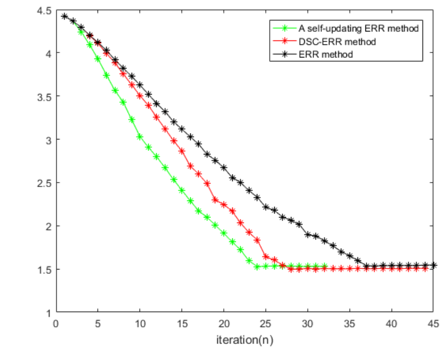
The final topology of three methods are shown in Figure 4.2. It is seen that the three topology optimization algorithms with different element removal rate adopted produce very similar optimized structure. The evolution histories of the structural performance indicators, the structural compliance, the maximum element sensitivity number and the element removal rate in the three optimization algorithms are shown in Figure 4.3. For structural performance indicators (Figure 4.2(a)), the three methods are steadily decreasing and tending to stabilize. Among them, the self-updating ERR method tends to stabilize the fastest, followed by the DSC-ERR method, and finally the ERR method with a fixed element removal rate. It is seen that the larger the element removal rate, the faster it tends to stabilize. Similar conclusions can also be obtained from Figure 4.2(b). The mean compliance of the three methods is improving and then converges to a stable value at the final stage. The self-updating ERR method has the fastest convergence rate for compliance, the DSC-ERR method is the second, and the ERR method is the slowest. However, table 4.3 shows that the final mean compliance of the self-updating ERR method is more than the DSC-ERR method, so that the topology obtained by the DSC-ERR method is slightly better than the self-updating ERR method.



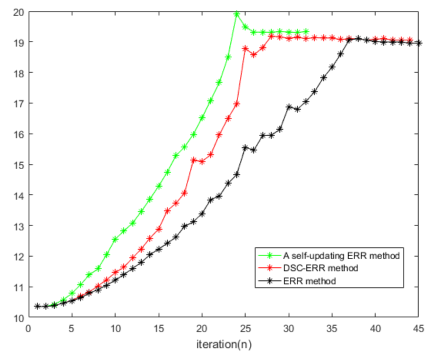
(a) DSC-ERR method (b) A self-updating ERR method (c) ERR method

Figure 4.2 – Topology optimization of a cantilever beam by different methods

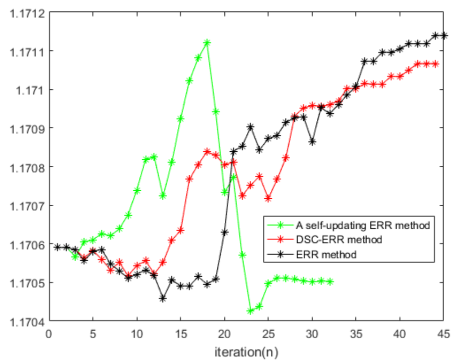
Among the three methods, the element removal rate of the self-updating ERR method adjusted as the structure changes, but it is generally maintained at the maximum value 0.04. The DSC-ERR method is based on a given initial element removal rate and continuously increases according to the changing structure. After reaching the maximum value 0.04, it starts to decrease, and realizes dynamic self-control. The stability of the algorithm can be reflected by the evolution histories of the maximum element sensitivity number. The maximum element sensitivity number of the DSC-ERR method and the ERR method rises relatively steadily, which shows that these two algorithms are generally stable. In contrast, the maximum sensitivity of the self-updating ERR method occurs volatility, low stability of the described algorithm. The reason may be that the greater the element removal rate, the lower its stability. In short, the DSC-ERR method improves the efficiency of the algorithm while considering the stability.



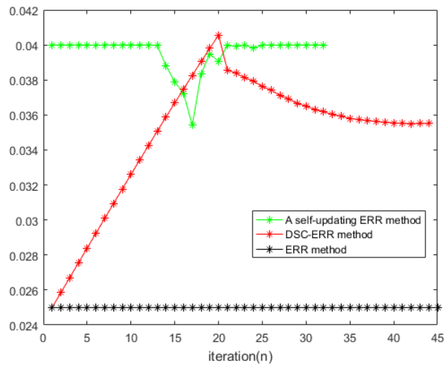
(a) Structural performance index of different methods



(b) The mean compliance of different methods



(c)The maximal element sensitivity number of different methods



(d) The element removal ratio of different methods

Figure 4.3 – Variation process of the performance indexes by three methods

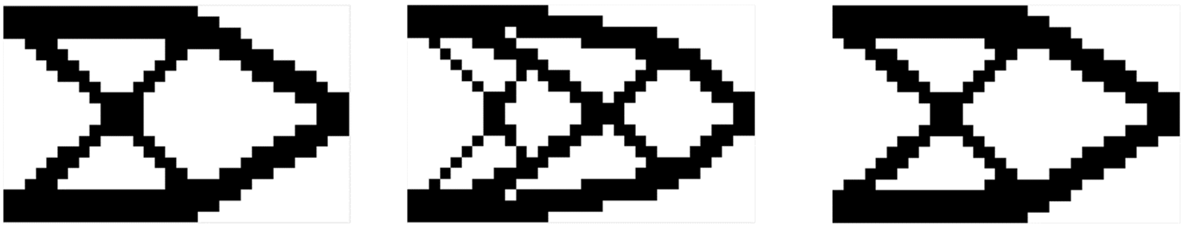
Table 4.3 – The index values of the final topology obtained by the three methods

The method	The mean compliance	Structural performance index
The DSC-ERR method	19.05763	1.508019
The self-updating ERR method	19.33963	1.528956
The ERR method	18.95451	1.545186

4.2 Topology Optimization of the second cantilever beam

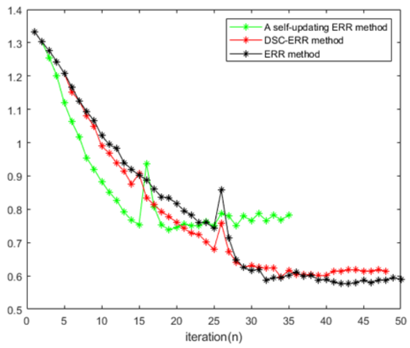
Consider the topology optimization problem of the beam structure with a vertical load of 1N applied at the middle of the free end shown in Figure 4.4. The design domain is divided into 32×20 four node elements. The following parameters are the same as the previous example: $v^{*}=0.4$, $ERR_0=0.025$, $\kappa=0.00085$ and $\tilde{\beta}=1.2$. The final topology of three methods are shown in Figure 4.5. It is seen that the final topology of the DSC-ERR method is very similar to the ERR method, while the structure of the self-updating ERR method is different. The structural inner part (Fig4.5(b)) has more branches and slower manufacturability. Figure 4.6 show the evolution histories of the structural performance indicators, the structural compliance, the maximum element sensitivity number and the element removal rate in the three optimization algorithms. Similar to the previous example, the mean compliance gradually increases and the structural performance indicators gradually decrease, and both will stabilize. The final mean compliance and performance indicator of the three method are 23.9492 and 0.614609 (the DSC-ERR method), 25.1044 and 0.787951 (the self-update ERR method) ,23.9097 and 0.589223 (the ERR method). Generally, a better structure has lower structural performance indicators and the mean compliance. Therefore, it can be considered that the structure obtained by the DSC-ERR method and the ERR method is slightly better than the self-update ERR method. For the element removed rate, the element removed rate of the DSC-ERR method increases first and then decreases, and its maximum element removed rate (the value of inflection point) is 0.028366571, which is less than the maximum value 0.04. The meaning of this inflection point is to switch the optimization process from the fast deletion phase to the smooth deletion phase. This proves that the DSC-ERR method uses the iterative information of the structure to achieve two-stage switching, thereby controlling the element removal rate.

Figure 4.4 – A cantilever beam

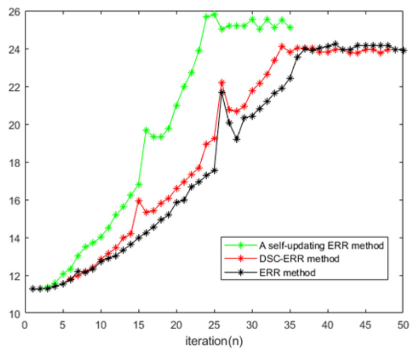


(a) DSC-ERR method (b) A self-updating ERR method (c) ERR method

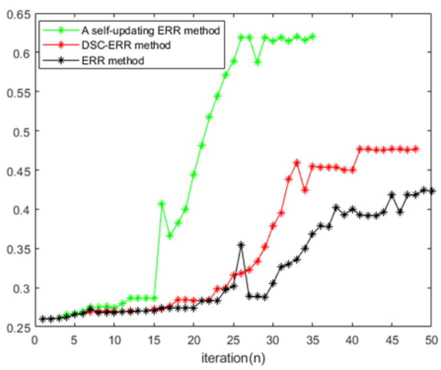
Figure 4.5 – Topology optimization of a cantilever beam by different methods



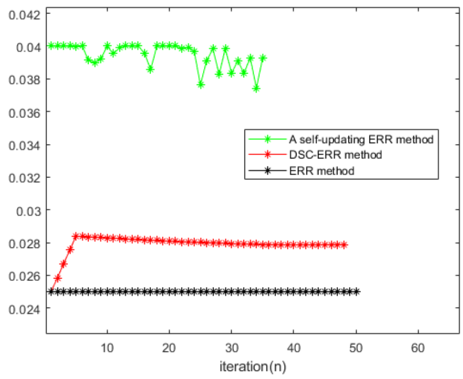
(a) Structural performance index of different methods



(b) The mean compliance of different methods



(c)The maximal element sensitivity number of different methods



(d) The element removal ratio of different methods

Figure 4.6 – Variation process of the performance indexes by three methods

Table 4.7 The index values of the final topology obtained by the three methods

The method	The mean compliance	Structural performance index
The DSC-ERR method	23.9492	0.614609
The self-updating ERR method	25.1044	0.787951
The ERR method	23.9097	0.589223

5. Conclusion

This paper has presented a improved BESO with a dynamic self-control ERR strategy. Its purpose is to improve the existing dynamic element removal rate strategy in the BESO method, which ignores the influence of the initial element removal rate, excessively pursues efficiency and ignores the stability of the algorithm. The DSC-ERR method divides the topology optimization process into two stages, the fast deletion phase and the smooth deletion phase, and gives the conditions for automatically switching from the fast deletion phase to the smooth deletion phase. According to the characteristics of these two stages, two self-control function of the element removal rate are constructed. The case of two cantilever beams shows that this method can realize the automatic switching of the two deletion stages and the self-control of element removal rate according to the degree of uniformity of the structural sensitivity number and the recommended element removal rate range. By comparing with the results of the self-updating ERR method and the ERR method, considering the stability, the DSC-ERR method appropriately improves the efficiency of the algorithm.

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