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AEROELASTIC RESPONSE CALCULATION USING DUHAMEL INTEGRAL AND ITS APPLICATIONS

Lucas Bortolotto[1], Roberto G.A. da Silva[2] & Marcos H.J. Pedras[3]

[1]ITA – Instituto Tecnológico de Aeronautica
São José dos Campos, SP, Brazil
lucasb@ita.br

[2]ITA – Instituto Tecnológico de Aeronautica
São José dos Campos, Brazil
gil@ita.br

[3]Embraer S.A.
São José dos Campos, Brazil
marcos.pedras@embraer.com.br

Abstract

A method for calculation of dynamic response to gust and control surface excitations, which allows the inclusion of nonlinear terms such as nonlinear control laws, without the need to use rational function approximation, is presented. This method is based on characteristic responses obtained with frequency domain equations, which associated with Fourier transforms, can be used to calculate time domain dynamic responses via Duhamel Integrals. The Duhamel Integral method is demonstrated for a typical airfoil section with three degrees of freedom, making use of frequency domain equations and direct and inverse Fourier Transforms for the characteristic response computation. In addition, applications where this method presents advantages, such as in the application of non-linear control laws and analysis of Oscillatory Malfunction (OMF), are demonstrated.

Keywords: Aeroelasticity, Aeroservoelasticity, Duhamel Integral, Gust Response, Oscillatory Malfunction

1. Introduction

The calculation of the aircraft's dynamic responses with respect to gust, excitation of control surfaces, failure of these surfaces, failure of the laws that control such surfaces and many other load cases are fundamental for the certification and safe operation of an aircraft.

This process requires extensive time simulations based on the solution of the aeroelastic equations of motion and the sooner and faster this is done, the less need to adopt remedial measures, such as the application of load monitors, rework and resizing cycles.

There are several methods for calculating these dynamic responses. Probably, the most common are procedures based on frequency domain formulations, which are used in commercial software such as NASTRAN [1] and ZAERO [2]. To obtain time domain responses, Fourier and inverse Fourier transforms can be used [3]. The main advantage of the Frequency Domain approach is that the aerodynamic force coefficients (AFC) are well described for an oscillating airfoil and can be obtained by established commercial software. However, the Frequency Domain approach does not allow the inclusion of non-linear terms, such as non-linear control laws for load alleviation, which may have significant impacts on the resulting loads and are increasingly common in the aeronautical industry.

One option to include non-linear terms is a state-space formulation of the aeroelastic equations of motion in the time-domain. This approach has several advantages on fight mechanics and aeroservoelastic design and analysis. However, the conversion of the aeroelastic motion equations to the time-domain requires the approximation of the aerodynamic force coefficients (AFC) with rational functions of the Laplace variable. There are several rational function approximation (RFA) methods [4] to fit the tabulated AFC with the ones calculated via RFA such as the Roger's term-by-term approximation [5] and the Minimum-State method [6]. Still, these methods have disadvantages in accuracy and require careful adjustments because of the AFC approximation. In addition, they can add an excessive number of augmenting states due to the required lag terms, increasing the order of the state-space model substantially.

Another difficulty encountered with this method is the approximation of the aerodynamic terms referring to gust inputs, which is especially challenging as it can not be done in the reduced frequencies due to the penetration component of the gust. These terms are necessary to calculate gust responses and design load alleviation control laws. An option to circumvent this problem is the hybrid approach [7], where the rational approximations are only applied to the generalized aerodynamic forces (GAFs) due to structural modes and control surface and the time domain generalized gust forces are obtained via inverse Fourier transform, thus avoiding the RFA of the GAF due to gust.

As a result of these disadvantages, a recent trend is to return the aeroservoelastic analysis to the frequency axis [8], without the need to transform the unsteady aerodynamic models to the time-domain, thus non-linear aeroservoelasticity problems can be solved by separating their linear and non-linear parts. This was done in [9], where the computation of dynamic response to discrete gust excitation with nonlinear control system effects, using Fourier transforms and Convolution Integrals in a three-stage process with several linear and non-linear control terms, was demonstrated.

The approach presented in this work is based on characteristic responses obtained with a frequency domain model, which associated with Fourier transforms, can be used to calculate the dynamic responses via Duhamel Integrals.

The advantage of this approach is that the system's response to any excitation can be described using characteristic behaviors obtained once in a commercial software. This can reduce the number of cases analyzed and the time needed for analysis, optimizing the use of licenses. Furthermore, these characteristic behaviors are calculated via a Frequency Domain formulation, thus avoiding the disadvantages of RFA.

The control surface excitation can also have a dependence, linear on non-linear, on the system's response configuring a closed loop. In this way, the linear characteristic of the aeroelastic plant is used in its integration with non-linear control laws.

The objective of this work is to demonstrate this methodology based on the Duhamel Integral in a typical airfoil section with a trailing edge control surface, as well as the applications where this method has advantages, such as in the application of non-linear control laws and analysis of Oscillatory Malfunction (OMF).

2. Duhamel Integral Methodology

As stated previously, the method based on the Duhamel Integral is capable of optimizing the time needed for analysis and the use of commercial software licenses. In addition, this methodology allows to take advantage of the linear characteristic of the aeroelastic system in the integration with non-linear control laws.

This is made possible due to the Superposition Principle, which states that for all linear systems (which is the case of the aeroelastic plant), the net response caused by two or more stimuli is the sum of the responses that would have been caused by each stimulus individually.

The Duhamel Integral is the generalization of this principle and makes it possible to obtain the dynamic response of a system to any excitation through its characteristic behavior. This characteristic behavior could be the system’s impulse response or the step response for example. It can be demonstrated that if the characteristic behavior is specified in the form of a step response (S), the dynamic response to a given input (F) is described by Equation 1.

$$x(t) = \int_0^t F(\tau)S'(t-\tau)d\tau \quad (1)$$

Since both the input data and the characteristic response data are normally discretized over time, it is necessary to discretize the Equation 1, one of the discretized forms of Equation 1 is described in the Equation 2 [10].

$$x(ii) = F(1)S(ii) + \sum_{jj=1}^{ii-1}(F(jj+1) - F(jj))S(ii-(jj+1)) \quad (2)$$

In this way, using the Duhamel Integral, the system's response to any excitation can be described using a characteristic behavior obtained once in a commercial software. This excitation can also have a dependence, linear or non-linear, on the system's response, configuring a closed loop.

For a better representation, the flowchart presented on Figure 1 shows the methodology using the Duhamel Integral to calculate the dynamic response to gust and control surface excitation. Where δ is the control surface input, y is the system's response at a given time step, which consists in the sum between the responses to gust and control surface excitations, and w_G is a given gust velocity profile, e.g., '1-cos' gust profile.

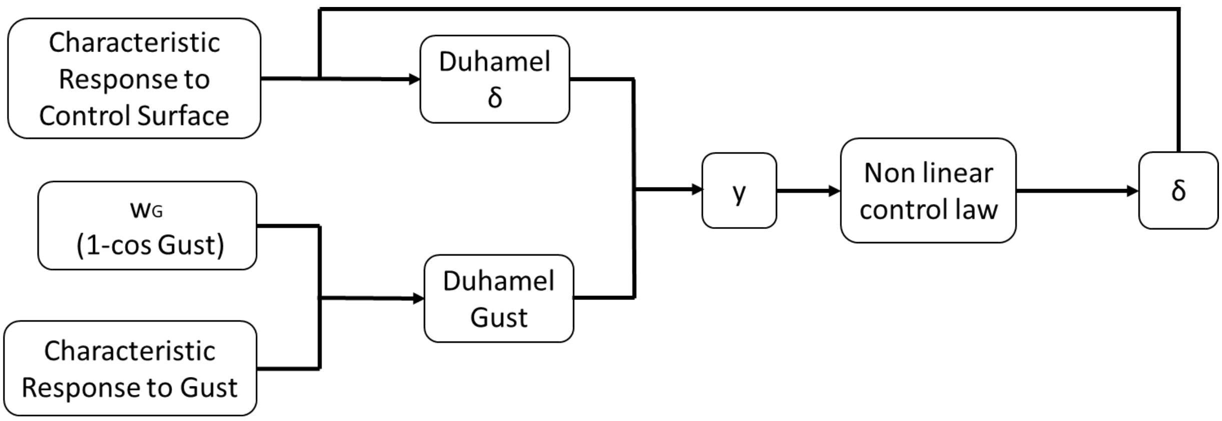

Figure 1 – Duhamel integral methodology flowchart.

3. Characteristic Responses for a Typical Airfoil Section

Several commercial software in the aeronautical industry are capable of calculating the characteristic responses to gust and control surface excitation shown in Figure 1, this makes the methodology very practical for these applications.

However, in this work, the objective is to demonstrate the methodology for the typical airfoil section with three degrees of freedom (DOFs) shown in Figure 2.

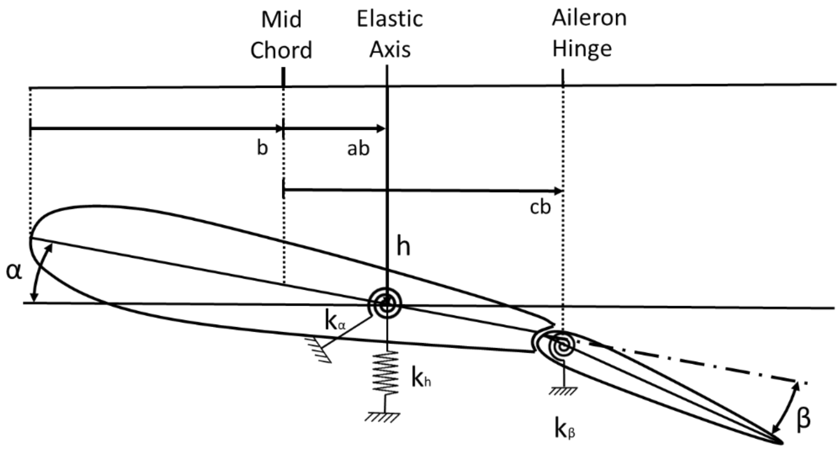

Figure 2 – Typical airfoil section with 3 DOFs.

The equation of motion of this typical airfoil section in the frequency domain is given by Equation 3.

$$-\omega^2[M_s]\{x_s\} + i\omega[B_s]\{x_s\} + [K_s]\{x_s\} - \omega^2[M_\delta]\{\delta\} =$$
$$= q_\infty[Q(V_\infty,\omega)]\{x_s\} + q_\infty[Q_\delta(V_\infty,\omega)]\{\delta\} + q_\infty[Q_G]\frac{w_G}{V_\infty} \quad (3)$$

Where $[M_s]$, $[B_s]$ and $[K_s]$ are the system's mass, damping and stiffness matrices, respectively. $[Q(V_\infty,\omega)]$ is the aerodynamic coefficient matrix, $[M_\delta]$ and $[Q_\delta]$ are the inertia and aerodynamic control matrices, $[Q_G]$ is the gust column, V_∞ and q_∞ are the free-stream airflow velocity and dynamic pressure and w_G represents the gust vertical velocity. The development of the system's matrices for a typical airfoil section with 3 DOFs can be found in [10] and [11].

In addition, the input $\{\delta\}$ is the control surface deflection and the column vector $\{x_s\}$ is the system's response, which contains the 3 DOFs shown in Figure 2: the plunge (*h*), the pitch angle (*α*) and the control surface angle (*β*).

The time domain characteristic responses are then calculated with the equations in the frequency domain, using Inverse and Direct Fourier Transforms. To check the accuracy of the Fourier Transforms, these responses are also validated with the ones obtained via time domain equations, which make use of rational function approximation (RFA). The process of obtaining these characteristic responses and the validation using the time domain model is described in the Flowchart of Figure 3.

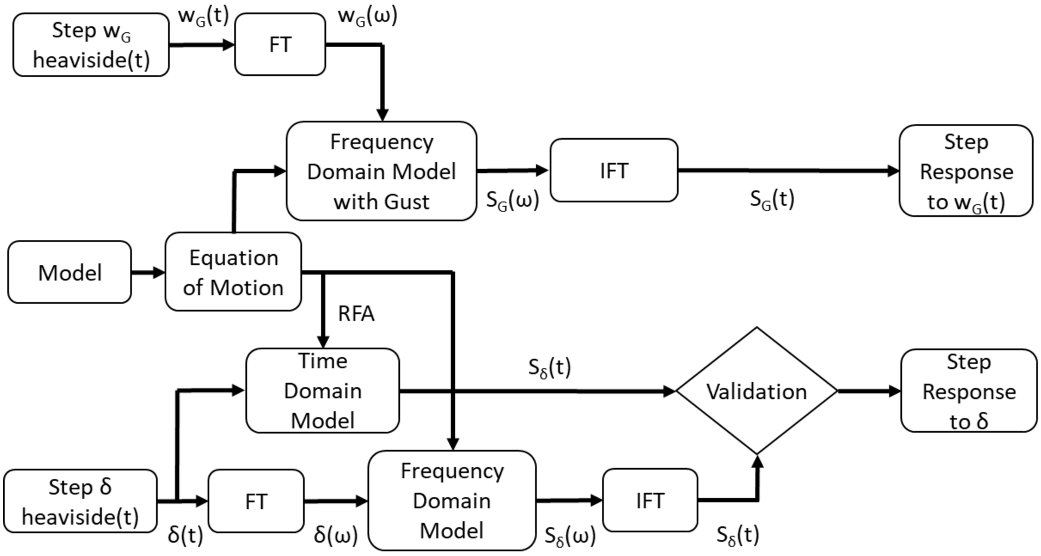

Figure 3 – Characteristic responses calculation for a typical airfoil section flowchart.

Starting from the bottom of the flowchart, a step input in the control surface (*δ(t)*) is transformed to the frequency domain using a Fourier Transform. This frequency domain input (*δ(ω)*) is applied to the frequency domain model and the result is transformed back to the time domain using Inverse Fourier Transform. The result of this transformation is the time domain response to a control surface step input ($S_\delta(t)$) which is validated with the time domain model. With the step response to *δ* in hands it is possible to calculate the typical airfoil section response to any input in the control surface, which can be arbitrary or given by a control law, using the Duhamel Integral.

As for the upper part of the flowchart, the same process can be done, but this time with a step gust velocity profile ($w_G(t)$), thus obtaining the characteristic response to gust ($S_G(t)$). With that, the typical airfoil section response to any gust profile can be calculated using the Duhamel Integral.

As shown in the flowchart, it is possible to rearrange the equation of motion (Equation 3) in order to obtain the state space Equation 4, used to calculate the response in the frequency domain.

$$i\omega\begin{bmatrix}\{x_s\}\\ i\omega\{x_s\}\end{bmatrix} = \begin{bmatrix}0 & I\\ [A_{21}] & [A_{22}]\end{bmatrix}\begin{bmatrix}\{x_s\}\\ i\omega\{x_s\}\end{bmatrix} + \begin{bmatrix}0 & 0\\ B_\delta & B_G\end{bmatrix}\begin{bmatrix}\delta\\ [Q_G]\frac{w_G}{V_\infty}\end{bmatrix} \quad (4)$$

As for the time domain, other procedures are necessary, such as rational function approximation, which will not be detailed here. In this way, the open loop state-space equation used to calculate the response in the time domain is Equation 5.

$$\{\dot{X}\} = [A]\{X\} + [B][u] \tag{5}$$

Where the $\{X\}$ column vector, contains not only the 3 coordinates of the system, but also *ns***nl* aerodynamic states generated in the rational function approximation. The variable *ns* being the number of structural modes, in this case 3, and *nl* is the number of lag terms used in the RFA. It is noteworthy that the number of structural modes grows a lot for practical applications, therefore the system order increases considerably using RFA.

4. Open Loop Results

4.1 Characteristic Responses

As shown in the Flowchart of Figure 1, characteristic responses such as step responses to gust and control surface excitations are fundamental in the application of Duhamel Integral methodology.

The characteristic response to a step control surface input is calculated in the frequency domain, using direct and inverse Fourier Transforms, and is validated with a time domain model, using the *step* MATLAB function as shown in the Figure 4.

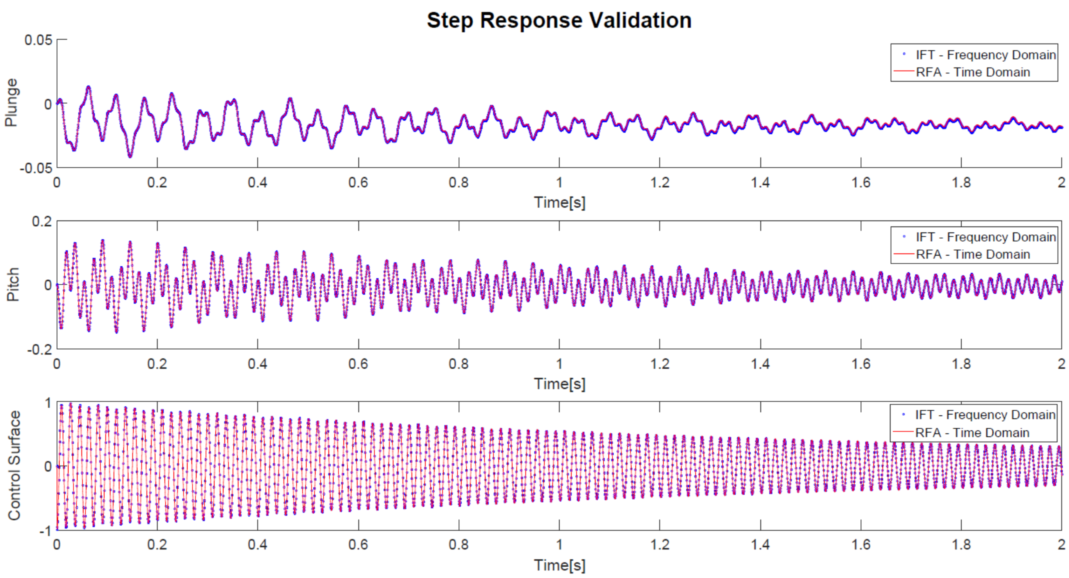

Figure 4 – Control surface step response validation.

This coherent result validates the model in the frequency domain using Fast Fourier Transform tools.

In addition, to calculate the response to an arbitrary gust velocity profile, it is also required the characteristic gust response. For this, the gust profile (w_G) used in the Equation 4 is obtained via a Fourier Transform of the Heaviside function and δ is assumed to be zero. In this way, the step response to gust, obtained using the inverse Fourier Transform on ($\{x_s\}$) of Equation 4, is presented on Figure 5.

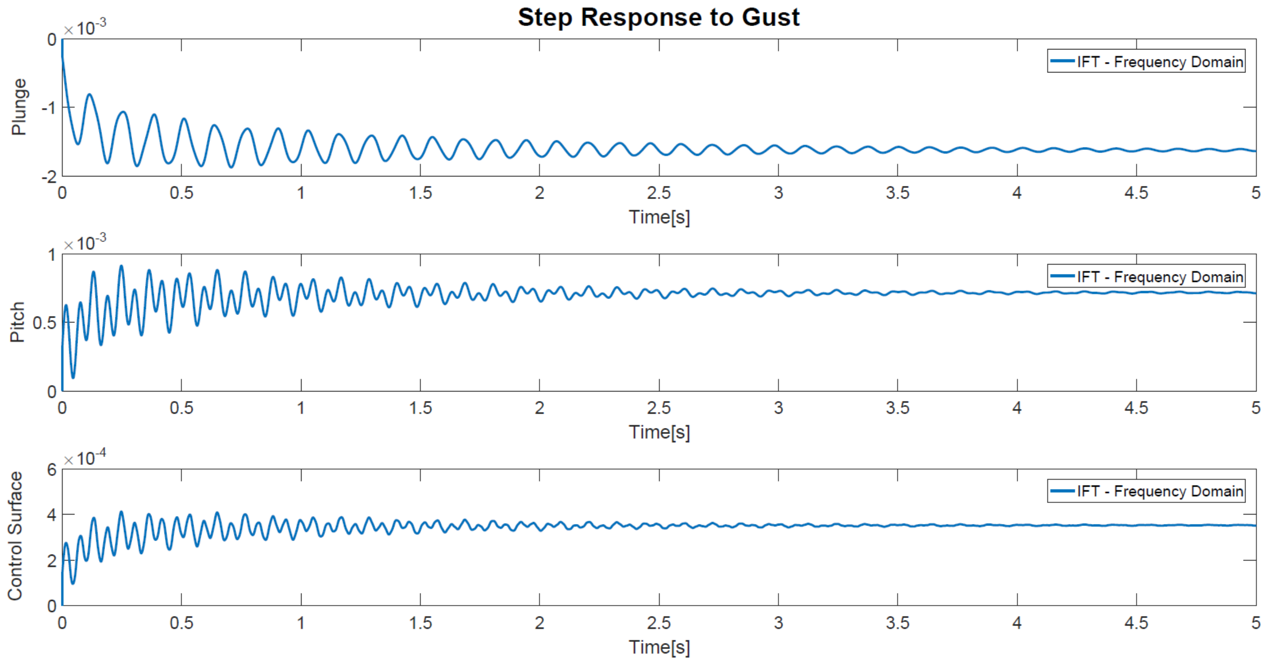

Figure 5 – Step response to gust.

4.2 Duhamel Integral Validation

With possession of the step response to a control surface input it is possible to calculate the response to an arbitrary control surface input using the Duhamel Integral. To test the Duhamel Integral implementation, it is used a sinusoidal control surface input, presented in Figure 6.

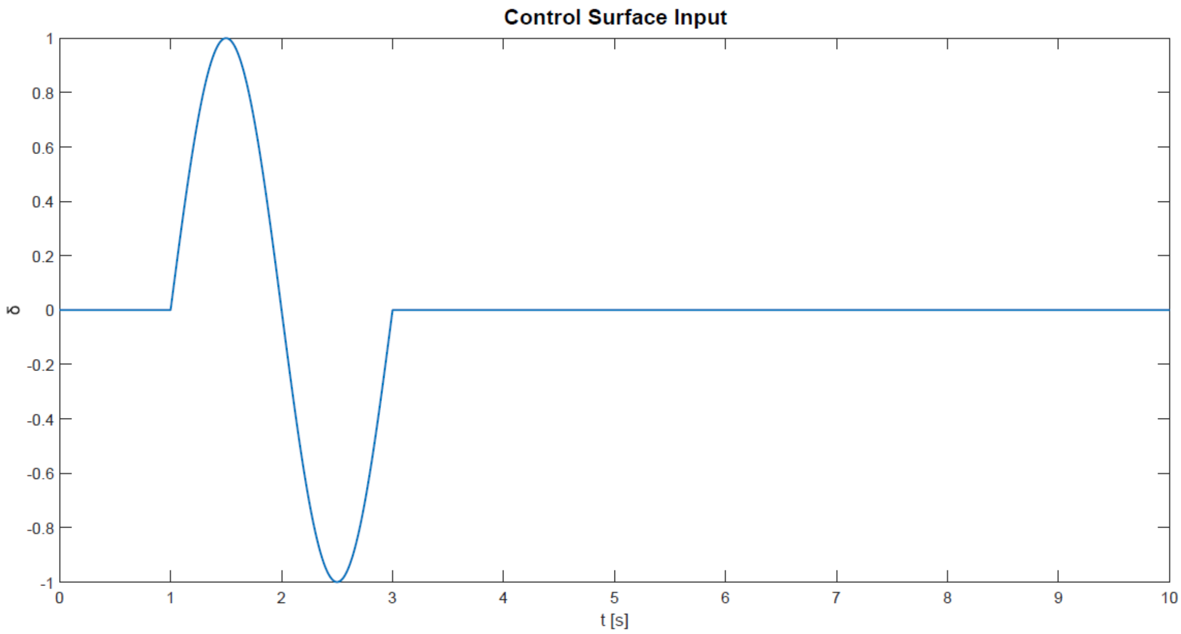

Figure 6 – Sinusoidal control surface input.

The response to this sinusoidal input obtained via Duhamel Integral is compared with the one obtained with a time domain model, using the *lsim* MATLAB function as shown in the Figure 7.

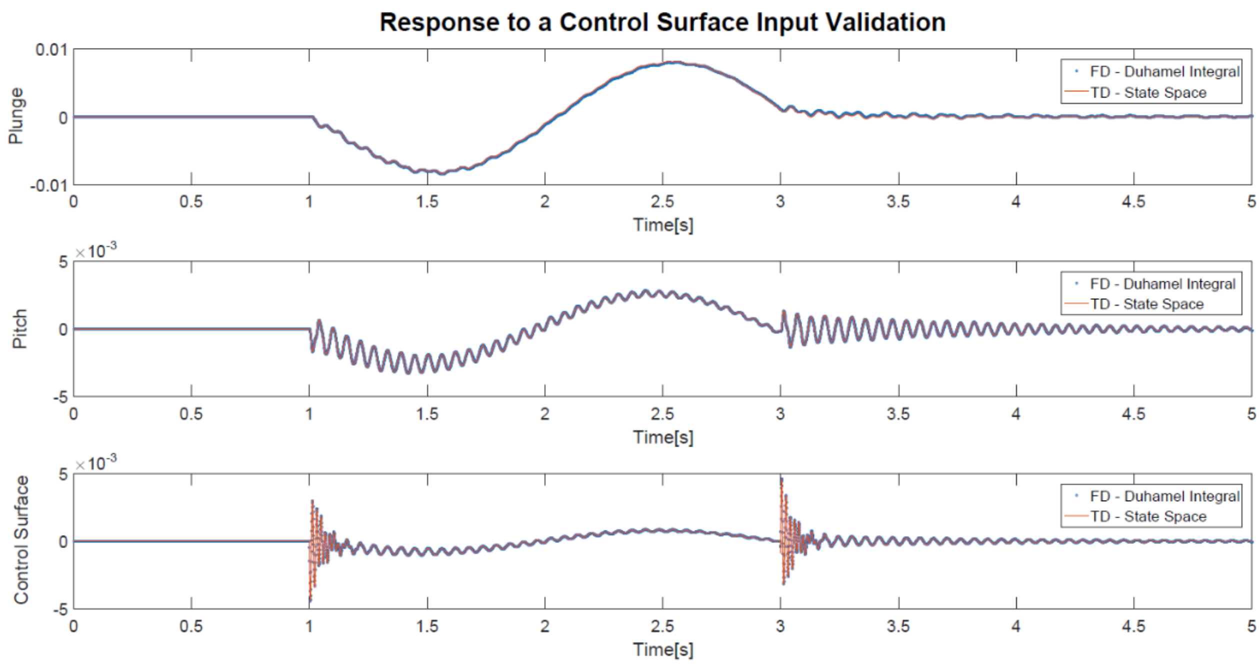

Figure 7 – Response to control surface angle via Duhamel Integral validation.

This coherent result validates the Duhamel Integral implementation. Therefore, one of the major objectives of the work, which is to calculate dynamic responses via the Duhamel Integral for a typical airfoil section with a trailing edge control surface, is achieved.

4.3 Gust and Combined Responses

With possession of the response to a step gust profile, shown in the Figure 5, it is also possible to calculate the response to a given gust profile, for example the '1-cos' profile shown in Figure 8, using the Duhamel Integral.

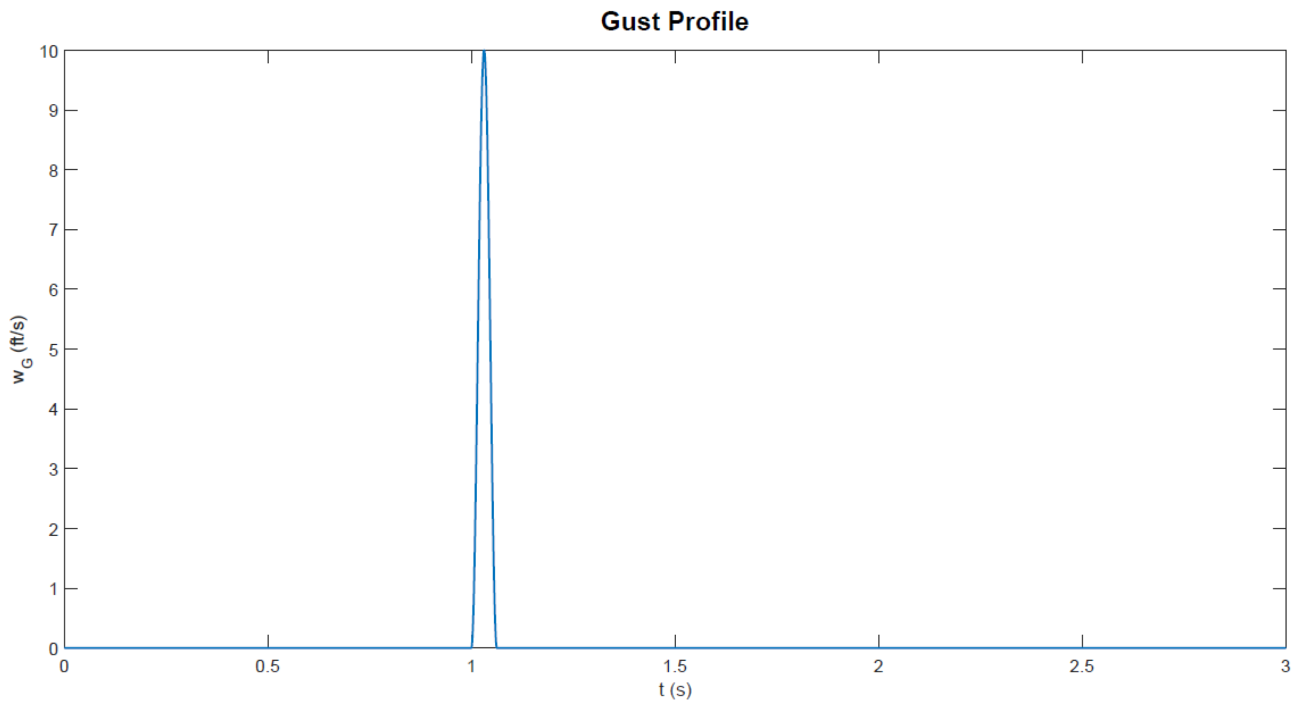

Figure 8 – Gust profile.

The response to this gust profile obtained with the Duhamel Integral is presented in Figure 9.

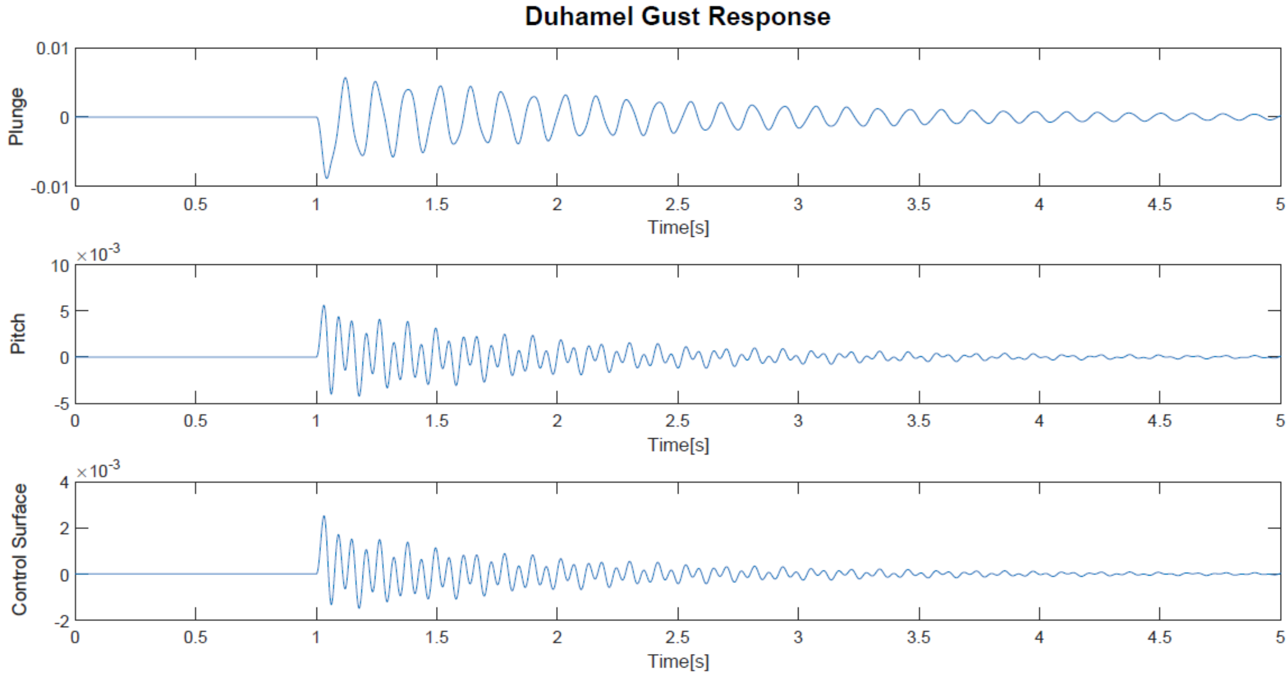

Figure 9 – Response to ‘1-cos’ gust profile.

The combined response consists of the sum of the gust and control surface responses presented on Figures 9 and 7. The combined response is shown in the Figure 10.

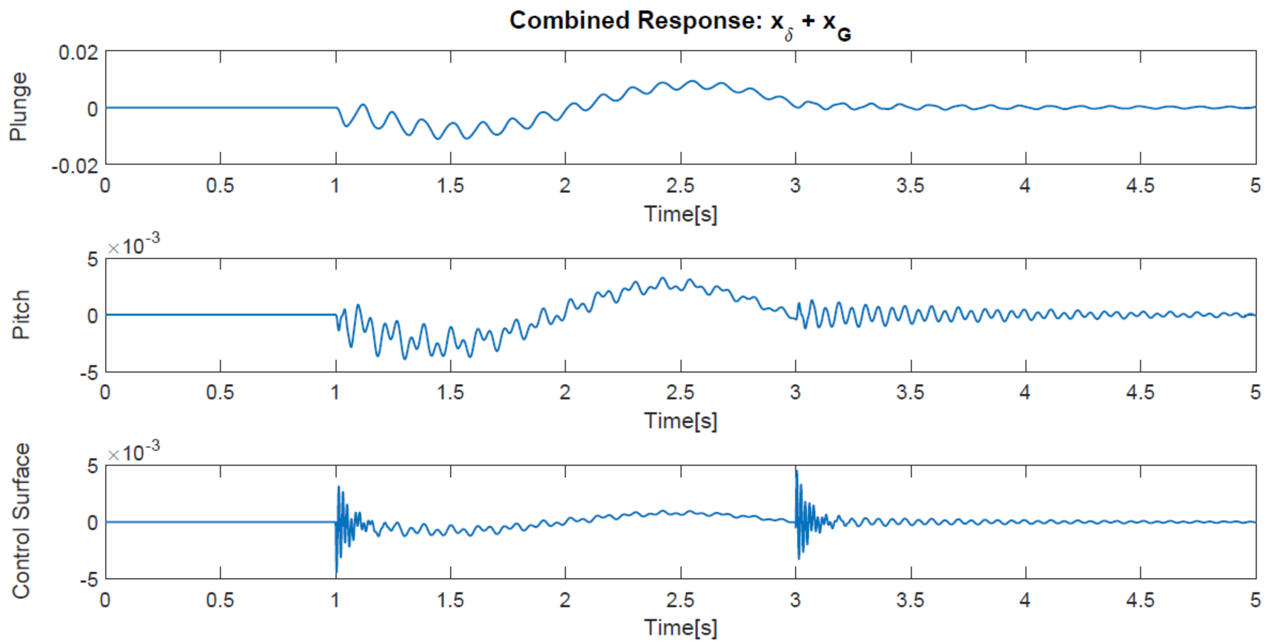

Figure 10 – Open Loop combined response.

The combined response is the one fed back into the control law, resulting in the control surface input used in the next time step.

5. Active Control and Oscillatory Malfunction (OMF)

Active control technologies such as gust load alleviation, flutter suppression, among others, are increasingly common and represent important tools in aircraft design. The effect of these tools must be taken into account in the analysis because they may have significant impacts on the resulting loads.

Because of that, the application of control techniques is demonstrated in the analysis via Duhamel Integral. As described in Figure 1, the control law receives the system response feedback as input. This feedback can be any measurable state of the system response or even a combination of them. As an output, the control law generates the deflection of the control surface to be applied in the next time step. The control law can be further optimized for any desired purpose, such as to minimize the load factor or maximize critical flutter speed, for an example.

In the test case presented in this work, the pitch speed ($\dot{\alpha}$) was chosen as the feedback variable and the control law parameters were optimized to reduce the plunge load factor.

An advantage of the Duhamel Integral methodology is that nonlinear control terms can be easily included in the analysis. This non-linearity can be found either in the control law itself, including quadratic or cubic factors on the feedback variables for example, or even non-linearities associated with control surface deflection and actuation rate limitations. In addition, through this methodology, it is possible to take into account the system's time history response in the control law.

When talking about control techniques and their consideration in the resulting loads, one must also consider the flaws in this system. One of them, known as oscillatory malfunction (OMF), is a failure that can occur in the actuator or in the control law itself that causes the failed control surface to oscillate, which can cause a significant impact on loads.

If this fault generates loads above the projected limit load of the structure, structural reinforcement is required, or deflection detectors must be installed to identify the failure and inform the pilots to disengage the failed surface. Both solutions can be very costly, and therefore, the earlier in the project and the faster these analyzes are made, the less the impact on the program and aircraft costs.

In addition, these deflection detectors can only identify the failure with oscillation amplitudes above a certain level depending on their sensitivity. Therefore, an analysis to determine if the loads generated by a non-identified failure are below the projected limit load of the aircraft structure is also necessary. In this case, OMF analysis can be also used to define requisites to the oscillatory failure detection system.

In the OMF analysis, the failure is modeled as a white noise in the deflection of the control surface that can be easily included in the analyzes via Duhamel Integral. Because of this, another great advantage of using the Duhamel integral methodology is the evaluation of the loads caused by OMF in a simple and fast way, as demonstrated in this work.

It is worth mentioning that there are two types of OMF, in one of them, the failed surface is no longer controlled and by the control law, this type of failure will be here called a solid failure, and in the other, the failed surface is still being controlled and will be here called a liquid failure.

6. Closed Loop Results

As stated previously, the pitch velocity ($\dot{\alpha}$) was chosen as the feedback variable and the control law was optimized to minimize the plunge load factor. Since we are using the Duahmel Integral, the control law can contain nonlinearities and, in this case, a cubic factor is applied to the feedback variable. The control law used in this example case with a proportional (linear) and a cubic (nonlinear) portion is demonstrated in the Equation 6.

$$\delta(ii+1) = K_1\dot{\alpha}(ii) + K_2\dot{\alpha}(ii)^3 \tag{6}$$

The parameters K_1 and K_2 can be optimized to minimize the plunge load factor. For this the MATLAB *fminsearch* function is used and the objective function is defined as the norm of the load factor vector. An optimization surface, in which the parameters K_1 and K_2 are displayed in the x and y axes and the objective function is displayed on the z axis is shown in Figure 11.

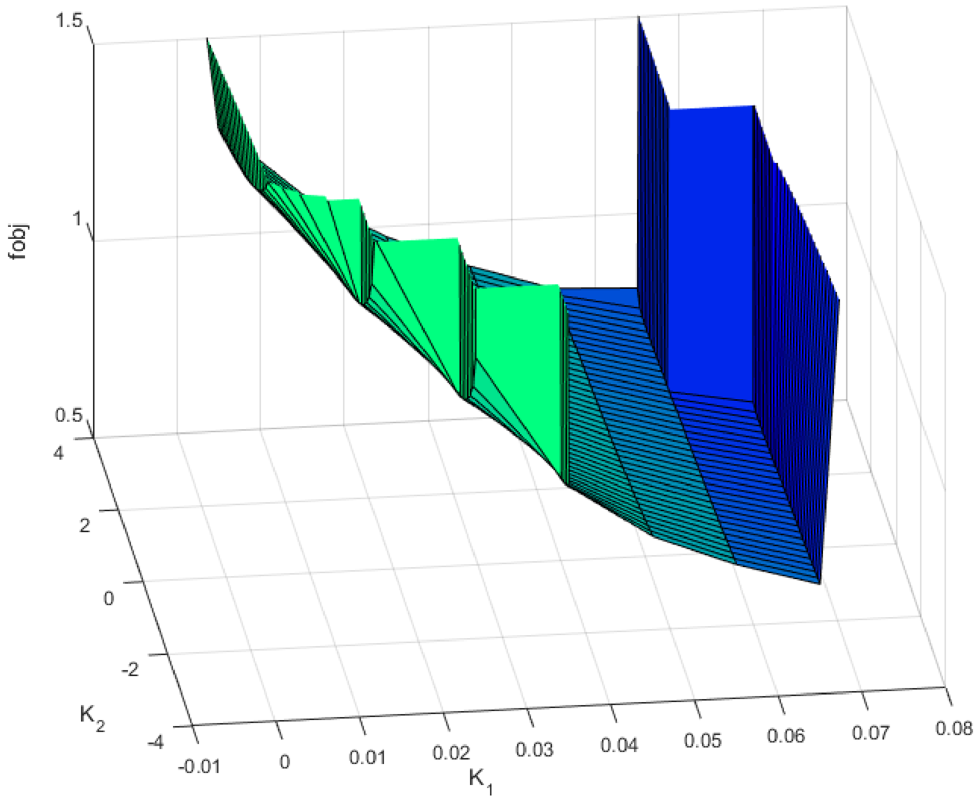

Figure 11 – Optimization surface.

Using the K_1 and K_2 values from the optimization, it is possible to compare the open loop to the closed loop response. This comparison made in terms of plunge load factor is represented in Figure 12, which shows an expressive reduction on the load factor with the closed loop.

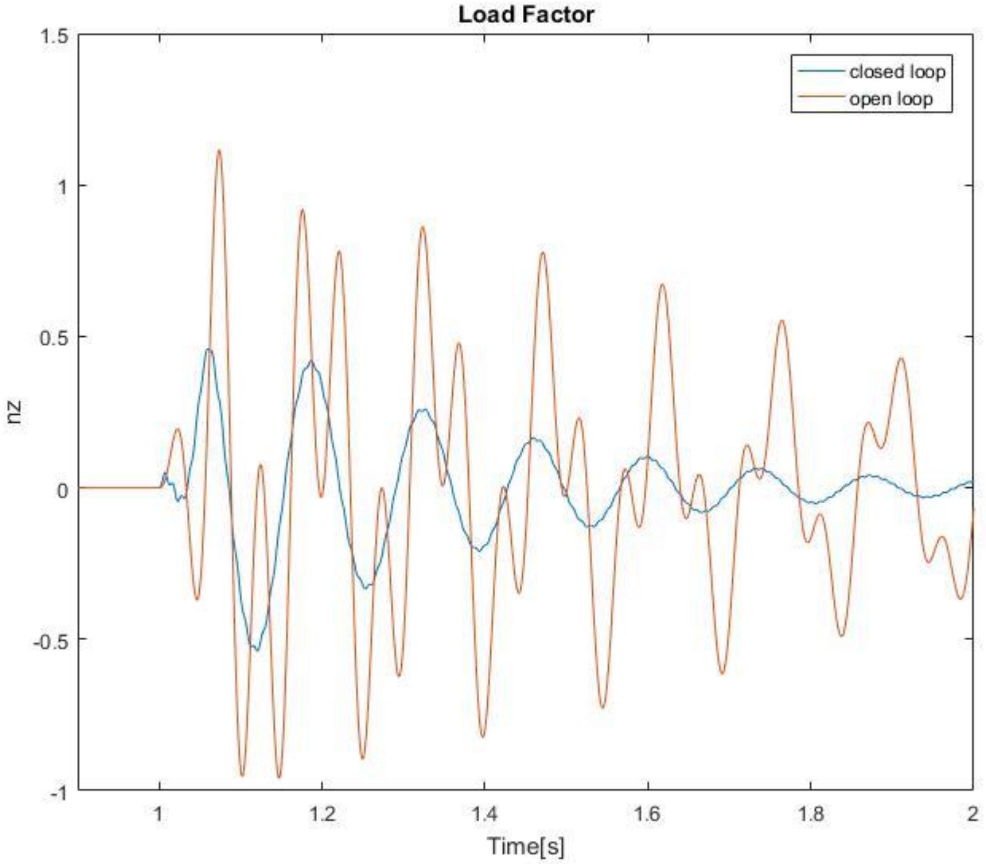

Figure 12 – Closed loop and open loop load factor comparison.

6.1 Oscilatory Malfunction (OMF)

Having presented the results with the integrated control law, it now remains to be evaluated the impacts to the load factor in case of a control system failure. The failure by OMF was modeled as a white noise (δ_{Fail}) and, in the case of a liquid failure, this noise is added to the command given by the control law, as shown in Equation 7.

$$\delta_{Liquid\_OMF}(ii+1) = K_1\dot{\alpha}(ii) + K_2\dot{\alpha}(ii)^3 + \delta_{Fail}(ii+1) \quad (7)$$

In the case of a solid failure, the input in the control surface is equal to the white noise used to model the OMF excitation, as shown in Equation 8.

$$\delta_{Solid\_OMF}(ii+1) = \delta_{Fail}(ii+1) \quad (8)$$

In practical cases, if it is desired to test whether a certain deflection detector is suitable, the white noise amplitude should be the minimum detectable amplitude by the failure detection system. Then, it should be evaluated whether the loads generated by this minimum detectable failure exceed the projected limit loads of the aircraft structure, if they do not exceed, the detection system is adequate.

In the example case presented here, the white noise amplitude was assumed to be thirty times lower than the maximum control law command. The comparison between the closed loop systems excited by '1-cos' gust with and without failure in terms of plunge load factor is presented in the Figure 13.

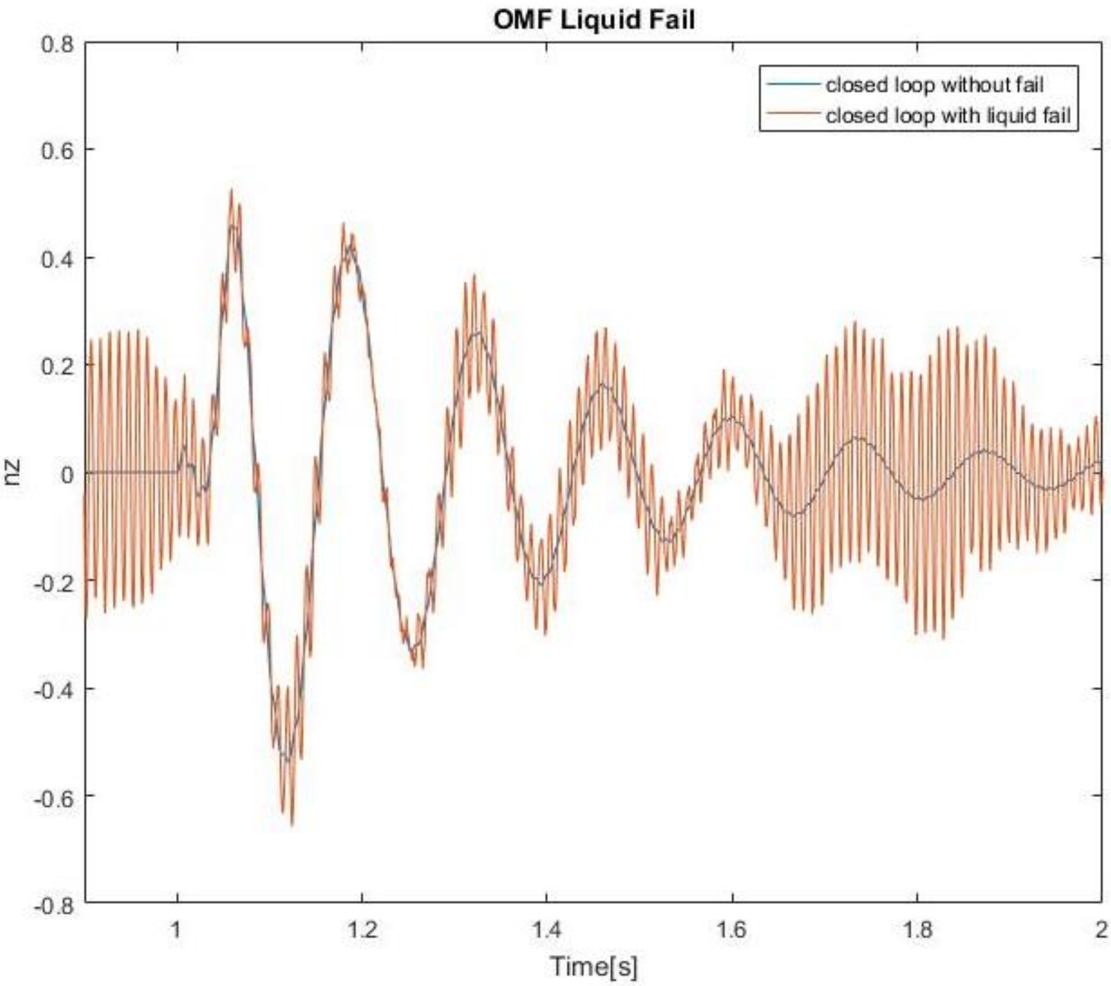

Figure 13 – Closed loop systems excited by gust with and without OMF.

In another example, it is compared the liquid and the solid failures in terms of plunge load factor without the effect of gust as shown in Figure 14.

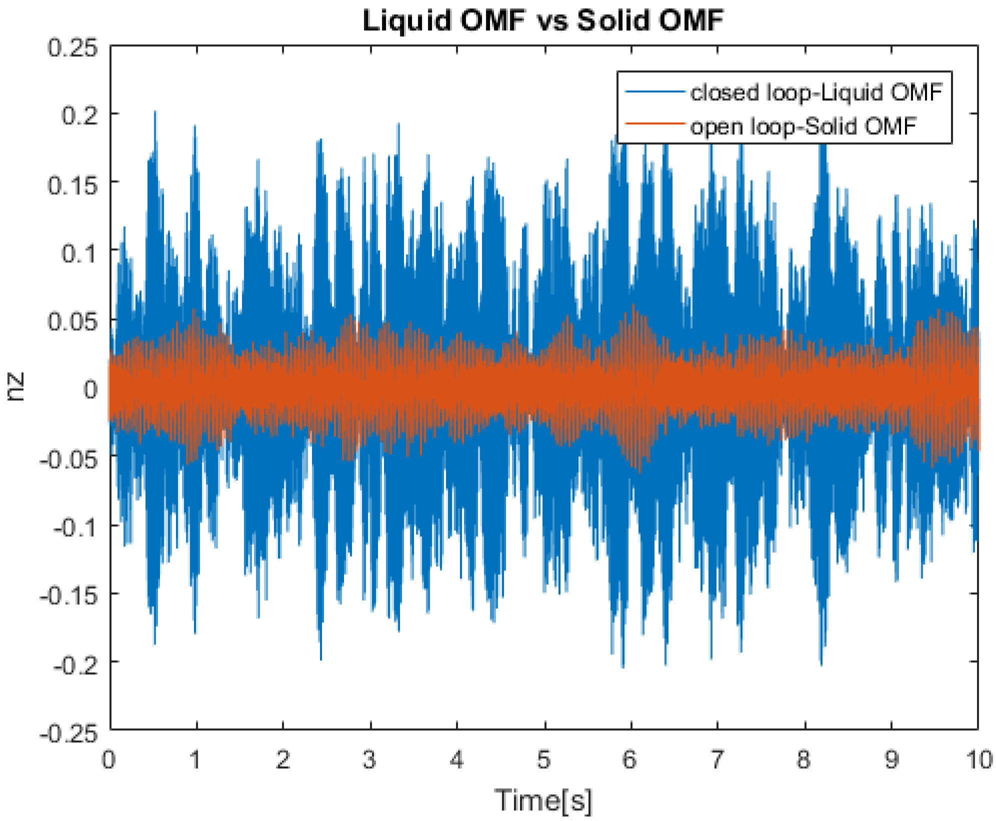

Figure 14 – Comparison between liquid and solid OMF.

One could assume that the loads generated by solid OMF would envelope the ones generated by the liquid OMF and perform only solid OMF analysis, that are simpler since they do not require to integrate the control law, thinking it is a conservative approach. In the case presented, that would be a terrible mistake as it can be seen that the liquid failure was more severe than the solid failure. This happened because the control law, which is still active in the case of the liquid failure, was optimized to alleviate gust loads and, in this case, it even amplifies the loads generated by the OMF.

To evaluate liquid OMF it is necessary integrate the control-law. If it has non-linear terms, it is fundamental to have a prepared tool for analysis. This example shows the importance to have analysis tools that are capable to integrate nonlinear control laws as is the case of the Duhamel Integral approach.

7. Conclusion

Using a typical airfoil section with a trailing edge control surface model, the characteristic responses to control surface and gust inputs are calculated in the frequency domain and transformed to the time domain using Fourier Transforms. Among them, the characteristic response to control surface input is compared with the one obtained via a time domain state space model, thus validating the Frequency Domain approach using direct and inverse Fourier Transforms.

Then, these characteristic responses are utilized to calculate the response to a sinusoidal control surface input and the response to a '1-cos' gust profile via Duhamel Integrals. The response to a sinusoidal control surface input is compared with the one obtained via a time domain state space model, thus validating the Duhamel Integral implementation.

In addition, a control law is integrated in order to minimize the load factor of the typical airfoil section. To do this, the system’s pitch velocity is chosen as the feedback variable, and the parameters of a non-linear control law are optimized to reduce the plunge load factor. In this way, comparing the open and closed loop dynamic responses, a significant reduction in the load factor is found.

Regarding the relationship between control techniques and loads, a failure in the control system known as Oscillatory Malfunction is presented. This failure can be modeled as a white noise input for the control surface deflection. An Oscillatory Malfunction analysis for the typical section is then carried out, paralleling it with practical applications of this analysis.

With that, the major objectives of the work, which are the calculation of dynamic responses via the Duhamel Integral and the demonstration of applications where this methodology presents advantages, such as the application of nonlinear control laws and OMF analysis, for a typical airfoil section, are achieved.

8. Contact Author Email Address

mailto: lucasb@ita.br

9. Copyright Statement

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