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# ACCURATE BUCKLING AND POST-BUCKLING ANALYSIS OF COMPOSITE STIFFENED PANELS

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### Abstract

The design and certification process of reinforced composite panels in aerospace applications require advanced tools able to accurately describe the three-dimensional stress state. Nevertheless, the complexity due to the anisotropy of the material and the large number of the adopted plies can severely increase the computational costs of finite element models. This work proposes a refined 1D beam model that consists of an enrichment of the popular equivalent single-layer approach for the description of accurate buckling and post-buckling of multi-plies aerospace composite stiffened panels.

**Keywords:** Composite stiffened panel; CUF 1D model; ESL and LW; Geometrical nonlinearities; Post-buckling.

## 1. Introduction

Fiber-reinforced composites are increasingly popular in many engineering fields to provide superior performances as compared to metals [1, 2]. The increasing usage of such sophisticated structures is due to their outstanding performances, their light weightiness [3] and the high specific strength and stiffness [4]. As a result, composite materials are adopted in various applications. Among the other, reinforced composite structures have encountered a wide adoption in the aerospace industry. [5, 6].
The adoption of such structures is made possible nowadays because of the improved predictive capabilities of numerical tools, able to simulate their behavior during working services, ensuring a proper design process. However, some particular phenomena which may occur to those components, such as in the case of damage and thermal effects, require an accurate evaluation of the three-dimensional (3D) complex internal stress states [7, 8]. This aspect results to be an issue for the modern commercial tools since they usually rely either on simplified hypotheses, based on classical models. For one-dimensional (1D) beam structures, classical theories are based on Euler-Boernoulli's [9] and Timoshenko's [10] assumptions, which considers a null (the former) and a constant (the latter) distribution of the shear strain and stress components over the cross-sectional thickness of the beam. For two-dimensional (2D) plate and shell structures, classical models are based on the Classical Lamination Theory (CLT) [11], which can be considered as the extension to laminates of the Kirchoff-Love theory [12, 13]. CLT neglects the effect of out-of-plane strains. An evolution of CLT is represented by the First Shear Deformation Theory (FSDT), which accounts for the shear deformation effects by linear variation of in-plane displacements. FSDT was widely developed in the framework of Finite Element Method (FEM) by Pryor and Barker [14], Noor [15], and it still plays a fundamental role in commercial codes. In order to overcome the limitations of 1D and 2D classical models, one can rely on 3D mathematical models. However, these models result to be heavy and they require a huge computational effort. For this reason, the development of high-order models able to capture the structural condition of composite laminates represents a challenge for engineers and scientific researchers.
The literature about higher-order theories for the analysis of composite structures is vast, see, for instance, the higher-order theories developed by Reddy [16], the so-called zig-zag theories [17] and the theories based on the Reissner's Mixed Variational Theorem (RMVT) [18]. One of the approach to analyze composite structures is represented by the Layer-Wise (LW) method [19], which ensures

own kinematics for each layer of the composites. Another popular tenchique is the Equivalent Single Layer (ESL), where the variables are independent of the number of layers.
Although accurate, LW models may require the use of high computational efforts. On the other hand, ESL models may lack the ability to accurately describe the 3D stress distribution over the plies of the composite structures. In fact, ESL models often rely on classical theories. In this paper, we adopt a refined ESL model based on Lagrange polynomials. Consequently, the static and buckling behavior of the reinforced composite structure can be evaluated with great accuracy compared with the LW model since a refined model is used, but the computational cost is drastically cut down since an ESL is adopted.
The models proposed in this study make use of the modeling features of CUF, which was proposed by Carrera more than one decade ago [20, 21] and allows for the automatic and eventually hierarchical generation of the theory of structures by using an extensive index notation and low- to higher-order generalized expansions of the primary mechanical variables. Thanks to CUF, both ESL [22] and LW [23] theories can be formulated with ease (and eventually combined) as in the case of the present beam model for the reinforced composite panel. The dynamic characteristics of the reinforced composite panel were evaluated in [24], and here the investigation is further extended for the buckling and post-buckling analysis.

## 2. The proposed refined ESL model

In the present work, LW and ESL models are built by using 1D refined CUF models. According to CUF and FEM, a LW displacement field of a composite beam is written as:

$$\mathbf{u}^k(x,y,z) = F_\tau(x,z)N_i(y)\mathbf{q}^k_{\tau i} \qquad \tau = 1,2,....,M \qquad i = 1,2,\dots,N_n \tag{1}$$

where $y$ is the direction of the axis of the refined 1D model; $(x,\ z)$ are the coordinates over the cross-section; $\mathbf{u}(x,y,z)$ is the 3D displacement field; $\mathbf{q}^k_{\tau i}$ is the displacement vector evaluated at each of the $N_n$ node at the $k$-th layer level; $N_i(y)$ are the shape functions in the $y$ direction; and $F_\tau(x,z)$ are the expansion functions of the cross-sectional are. The order of expansion functions are arbitrary, being M the maximum number of expansions, which is a parameter defined by the user. Repeating indexes denote summation. The ESL model can be derived from Eq. (1) without the index $k$ since the variables are independent of the number of layers. Figure 1 reports a schematic representation of the 1D CUF model.

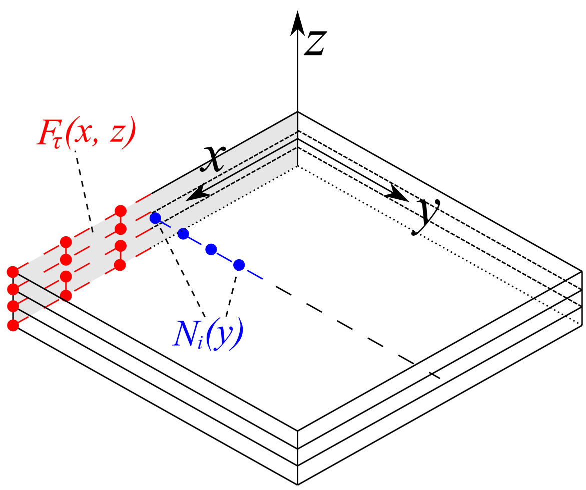



Figure 1 – Mathematical 1D CUF model.

In this work, refined models are employed by using Lagrange Expansions. This expansion function, introduced in [23], makes use of opportune interpolation of the variables evaluated at the Lagrange point. The interpolation functions are based on Lagrange polynomials, and they denote the order of the expansion. These Lagrange points can be used to define any geometric shape and, as in this work, to denote the domain of each layer in the LW approach and of the whole thickness in the refined ESL approach. In Fig. 2 the different LW and ESL approaches are reported.

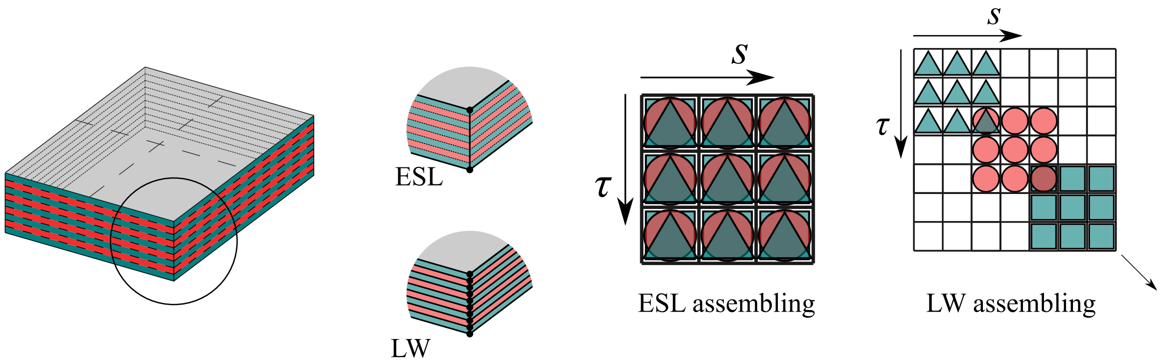


Figure 2 – ESl and LW approaches for composites.

The refined ESL approach ensures a great level of accuracy of the behavior of the composite structure at a macroscale level compared to the heavy LW models.

## 3. Numerical results

The numerical results report the linearized buckling analysis of the stiffened panel using both refined ESL and LW models. Finally, the main post-buckling results, using the refined ESL model, are given.

### 3.1 Linearized buckling

The reinforced composite panels were manufactured and tested at the Delft Aerospace Structures and Materials Laboratory [24]. The AS4 unidirectional prepreg employed has the following material properties: $E_1$ = 119 GPa, $E_2$ = 119 GPa, $E_3$ = 119 GPa, $\nu_{12}$= 0.316, $\nu_{13}$= 0.26, $\nu_{23}$= 0.33, $G_{12}$= 4.7 GPa, $G_{13}$ = $G_{23}$ = 1.76 GPa and $\rho$ = 1580 kg/m$^3$. Note that the properties in the out-of-plane directions (13 and 23) are assumed and not available from the manufacturer. Figure 3 shows the main geometrical properties and the boundary conditions used for the mathematical model. The two-stringer reinforced panel is 690mm long and the width is $b=$ 270 mm. The dimensions of the

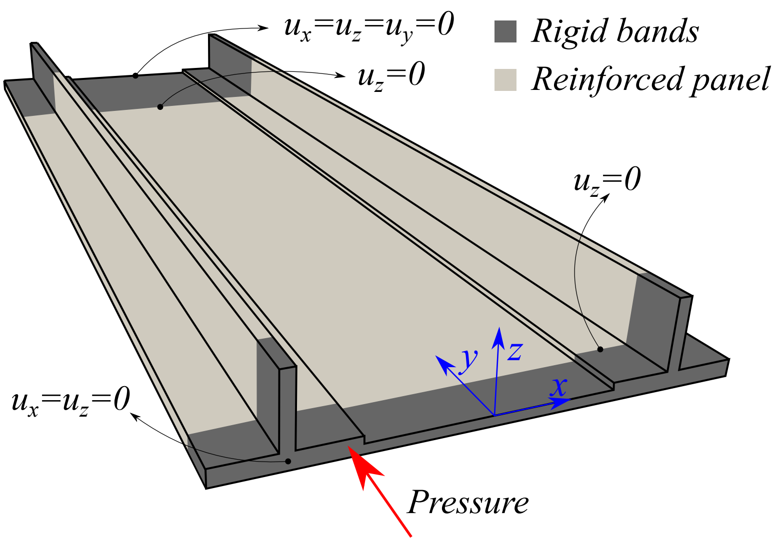


Figure 3 – Main geometrical features, loading and boundary conditions for the simulation of the reinforced composite panel.

cross-section are reported in Fig. 4. The stringer has height and thickness equal to $h=$ 39.3mm and $t=$ 7.3 mm, respectively, whereas $h_1=$ 9.52 mm and $h_2=$ 3.66 mm. Boundary conditions are imposed on displacement components on planes perpendicular to the $y$-axis, as shown in Figure 3. Two rigid bands of 50mm each are modeled at the panel ends to simulate the experimental setup. Note that at $y=$ 0 the cross-section is free to translate, whereas a pressure is applied on the entire plane. Figure 5 introduces the lamination sequences of the composite panels and the reinforcements. The first buckling loada evaluated with experimental data, LW and refined ESL models is reported in

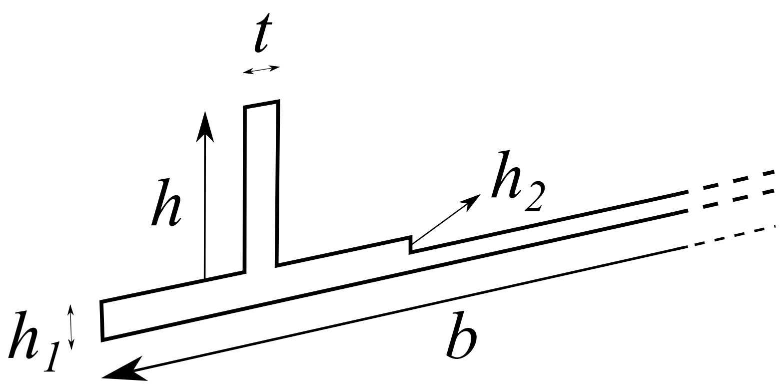



Figure 4 – Detail of the panel's cross-section.

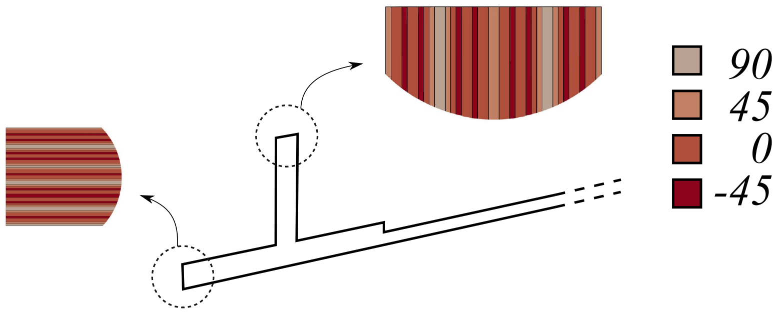



Figure 5 – Stacking sequence.

Table 1.

| Model | Buckling load, kN<br>Test 1 - Test 2 - Test 3 | DOF | Error % |
|---|---|---|---|
| Experimental | 739.90 - 740.30 - 738.00 | — | — |
| LW | 739.17 | 715365 | 0.03% |
| Refined ESL | 744.12 | 10521 | 0.64% |

Table 1 – Measured buckling load and comparison between experimental results and numerical simulation. Experimental results taken from Delft Aerospace Structures and Materials Laboratory tests [24].

The buckling load is perfectly evaluated by the proposed model, compared to the experimental results, and with a significant gain on the computational cost, compared to the LW model. Moreover, Table 2 reports the first four buckling modes numerically evaluated with LW and refined ESL.
It can be concluded that the proposed refined ESL model can evaluate the buckling behavior of the structure with a reliable accuracy while increasing the computational cost.

## 3.2 Nonlinear post-buckling

Post-buckling results are discussed hereafter. The geometrical nonlinear equations are included within the model with the same procedure and mathematical steps described in [25]. The stiffened panel was designed to have the first buckling modes in a small range of critical loads. For this reason, it is possible, from a numerical point of view, to simulate the post-buckling static behavior of the structures of various buckling modes. Figures 6 and 7 show the first and the third post-buckling equilibrium curves. Clearly, the post-buckling mode is equal to the correspondent linearized buckling one, proving the accuracy of the simulations. Moreover, in the figures the linearized buckling loads are reported, and they correspond to the collapse zone in the post-buckling curves.

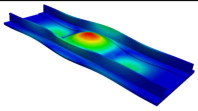
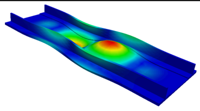
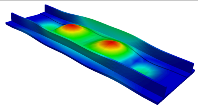
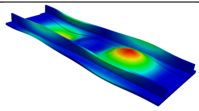
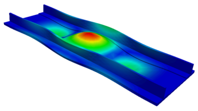
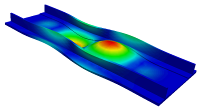
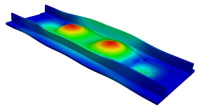
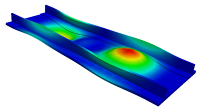
| Model | Buckling mode 1 | Buckling mode 2 | Buckling mode 3 | Buckling mode 4 |
|---|---|---|---|---|
| LW | <br>739.17, kN | <br>744.86, kN | <br>809.42, kN | <br>829.87, kN |
| Refined ESL | <br>744.12, kN | <br>749.96, kN | <br>815.78, kN | <br>836.96 kN |
| Diff % | 0.67% | 0.68% | 0.79% | 0.85% |

Table 2 – Measured buckling load and comparison between LW and refined ESL models.

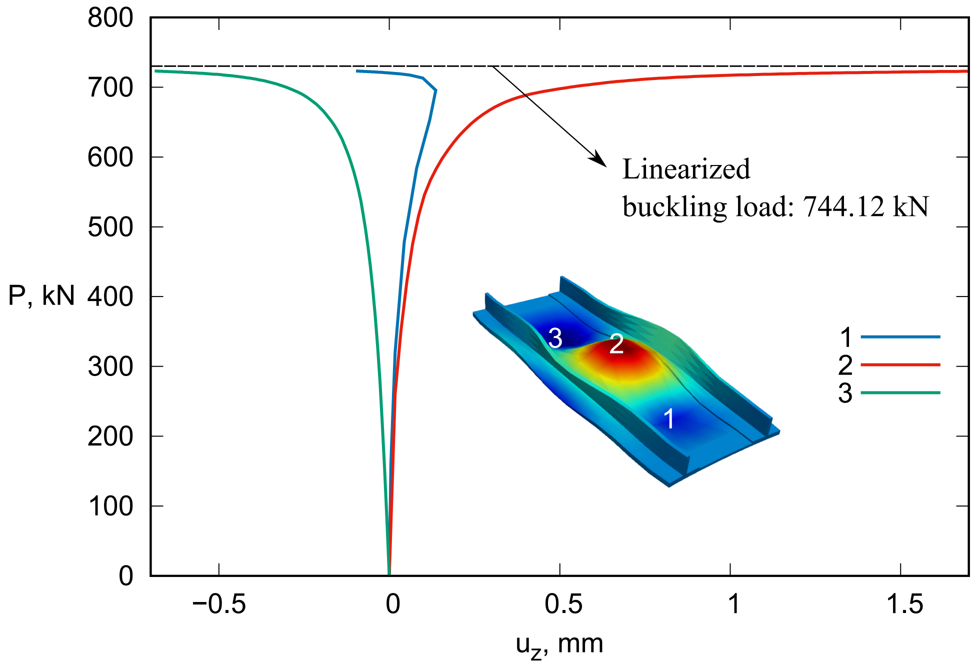



Figure 6 – Post-buckling equilibrium curve of the stiffened composite panel ($1^{st}$ buckling mode).

## 4. Conclusions

This paper has discussed the buckling and post-buckling analyses of composite reinforced panels for aerospace applications. The proposed model is based on the Carrera Unified Formulation (CUF), which allows generating refined finite elements with Equivalent Single-Layer (ESL) approach. In this way, the computational cost is definitely cut down compared to the LW models (more than 65 times less DOFs), whereas a great level of accuracy is still guaranteed. As a matter of fact, it is possible to analyze the post-buckling behavior of such structures, which numerical simulation is cumbersome due to geometrical nonlinearities. Several conclusions can be drawn from the results:

- Compared to the experimental results, the linearized buckling loads are evaluated with reliable accuracy;
- Looking at the linearized buckling modes and the post-buckling equilibrium curve, it is clear that **the stiffeners have a predominant role** on the behavior of the reinforced composite panel. For this reason, **a model which is able to describe the deformation of those is mandatory**;
- With this refined ESL model, the global behavior of the structure at a macroscale level is accurately caught. If one wants to investigate local phenomena (interlaminar strains, debonding or

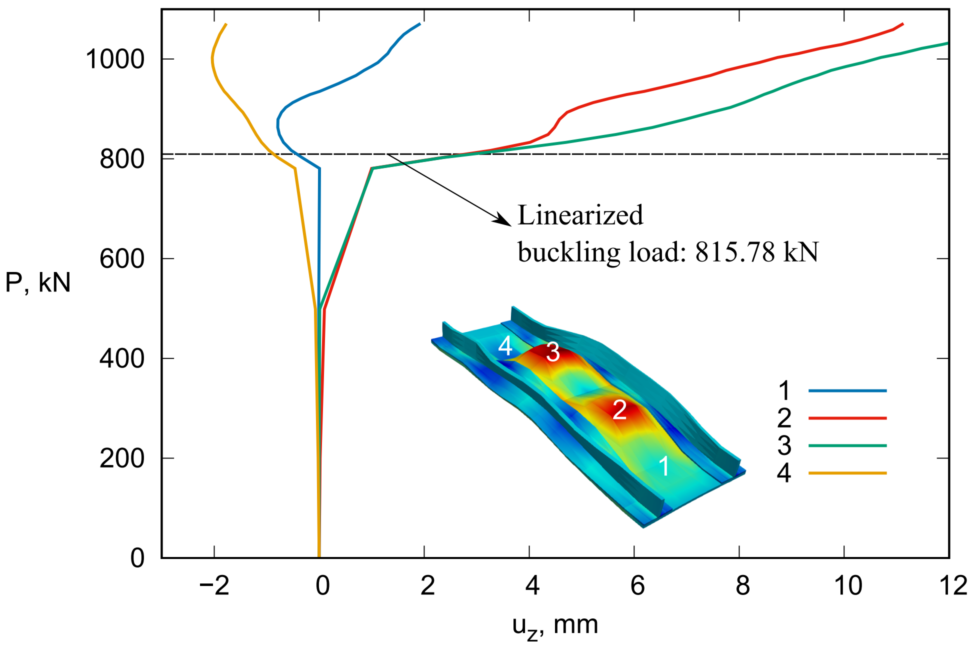



Figure 7 – Post-buckling equilibrium curve of the stiffened composite panel (1$^{st}$ buckling mode).

delamination), a Layer-Wise (LW) description is required. However, CUF allows for varying the kinematics of the three-dimensional domain, adding degrees of freedom only in the domains where it is necessary, in a global/local sense (see [26]), so future works in this direction do not represent an issue.

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## 6. Copyright Statement



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