EFFECTS OF ELASTICITY, VISCOSITY AND BOUNDARY LAYER TRANSITION ON AEROELASTICITY CHARACTERISTICS OF LAMINAR WINGS

S. Kuzmina[1], F. Ishmuratov[2], O. Karas[3]

[1]Leading research scientist
[2]Head of Division
[3]Senior research scientist
Central Aerohydrodynamic Institute (TsAGI), Russia, 140180

Abstract

The paper briefly describes the features of the developed mathematical model of aerodynamic forces in transonic flow. The model is based on the Euler equations in various approximations, taking into account the viscosity by using the boundary layer theory and the viscid - inviscid interaction. Elastic structure deformations are described by the equations of vibrations in modal coordinates. An algorithm for computation of dynamic aeroelasticity characteristics in transonic flow is developed based on the energy method for the mathematical model. The problem of transonic self-oscillations and their connection with the peculiarities of the pressure distribution and the shock wave movement is numerically investigated using the CAST wing model section as example. The dependence of aeroelastic stability on the aerodynamic work distribution along the wing chord is shown.

Keywords: aeroelasticity, transonic, aerodynamic, Euler equations

1. Introduction

Numerous computational and experimental studies of the flow around the swept high aspect ratio wing of a modern transport aircraft show that with the flight speed increase the flow becomes transonic and the shock waves appear. The question of how the moving shock waves that occur on the wing surface in transonic flow affect flutter and self-oscillations remains relevant at the present time. The flutter boundary can be determined by standard linear methods in the absence of shock waves on the wing surface. Naturally, in aerodynamic design, the wing airfoils are built in such a way that the flow is without shock waves in cruising flight to ensure maximum lift-to-drag ratio.

Limit cycle oscillations of lifting surfaces in transonic flow were studied on models in wind tunnels by a number of authors [1-3]. It was assumed that in a certain speed range close to the sound speed, there is a dynamic instability caused by negative aerodynamic damping. The study of the phenomenon was carried out on models of the CAST wing, the kinematic scheme of which approached a system with one degree of freedom – the pitch of the wing relative to the rotational axis, which is located at a quarter of the wing chord. The two-dimensional flow around the CAST model in the wind tunnel in the presence of a turbulent and laminar boundary layer, that is, with a fixed and free transition, was studied. The self-oscillations of the model that spontaneously occur in a certain range of the flow Mach numbers close to harmonic ones, were studied using modern experimental methods. It was found that with a mixed flow around the airfoil, the self-oscillations of the wing are accompanied by shock wave oscillation that ends the local supersonic zone. In this case, the shock wave moves in the phase with the airfoil displacement or with a certain phase shift, depending on the flow regime. In the above works, limit cycle oscillations (LCO) are associated with the appearance of a boundary layer separation induced by the shock wave. So, LCO are considered as the result of the shock - boundary layer interaction in transonic flow.

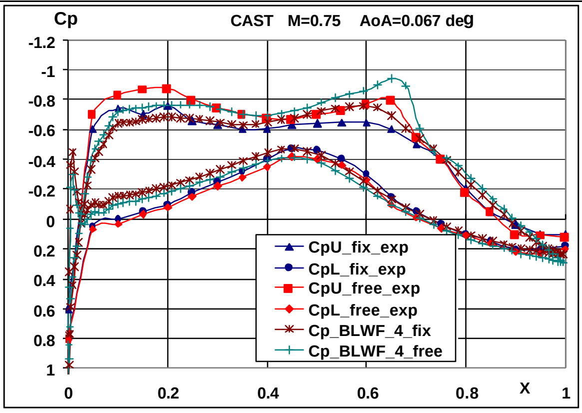
To study complex dynamic aeroelasticity phenomena various methods must be used: computation of the root trajectory and frequency characteristics considering different approaches to linearization, determination of time dependencies by marching methods, etc. The features and peculiarities of the structure behavior in the presence of non – linearities associated with the occurrence of shock waves in conditions of mixed subsonic-supersonic flow require the use of nonlinear aerodynamic theory to solve conjugate problems of the aircraft structure interaction with the flow. In many such cases the analysis of the work of aerodynamic forces contributes to a better understanding of the mechanism of the phenomenon. Such complex phenomena include flutter and self-oscillations in transonic flow with the movement of shock waves, which are considered in this paper.

2. Computational methods

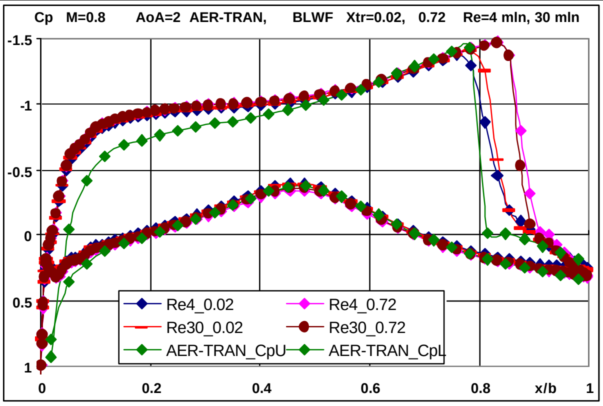
TsAGI has developed two approaches to solving aeroelasticity problems using different implementations of transonic aerodynamics based on the Euler equations [4-6], which complement each other. Based on the developed algorithms, aeroelasticity problems can be solved for different disciplines in transonic flow. The main features of such problems are non-linearities in the amplitude of vibrations, in the load (angle of attack) and the increased influence of viscosity. The computational method AER-TRAN for determining the dynamic reaction and flutter in frequency and time domain was developed using the Godunov finite-difference method for integrating nonlinear Euler equations for an ideal gas [5-6]. The method combines two algorithms: flutter computation according to linear theory and the determination of the non-stationary inviscid flow around an elastic wing. In the time domain, the method of joint numerical integration of equations describing the displacements of the lifting surface and aerodynamic equations of transonic flow is used to determine the dynamic response and parameters of nonlinear self-oscillations. It is important to note the main feature of the AER-TRAN method - the ability to study in transonic flow the dependence of the level of dynamic reaction, the flutter velocity and the LCO on the amplitude of vibrations and the magnitude of the angle of attack.

The computation of flow around a harmonically oscillating lifting surface of a complex configuration in transonic viscous flow is carried out using the BLWF program [4-5], taking into account the effects of viscosity, including the separation zones of the boundary layer. In this case, the flutter analysis in the frequency domain is supplemented by an effective procedure for computing the coefficients of the aerodynamic influence matrix.

In this paper, the possibility of self-oscillations with one degree of freedom, their connection with the peculiarities of the pressure distribution and the movement of the shock wave is considered on the example of the CAST wing model [3]. Figure 1 shows the pressure distributions along the CAST airfoil, obtained with the use of both the AER-TRAN and BLWF programs: 1) for the case of inviscid ideal gas flow around the wing and 2) in the presence of a turbulent or laminar boundary layer. It can be seen that the presented results agree quite well with each other. The position of the shock wave on the upper surface of the wing, for an ideal gas, agrees better with the result of the BLWF program for the case of a turbulent boundary layer than in the case of a free transition on a laminar wing. In addition, the position of the transition point significantly affects the flow around the wing. An increase in the Reynolds number Re from 4 million to 30 million did not significantly change the pressure distribution over the CAST wing ($z^*=0.3875$) both for the case of a fixed ($x_{tr}=0.02$) and for a free transition ($x_{tr}=0.72$) of the boundary layer.

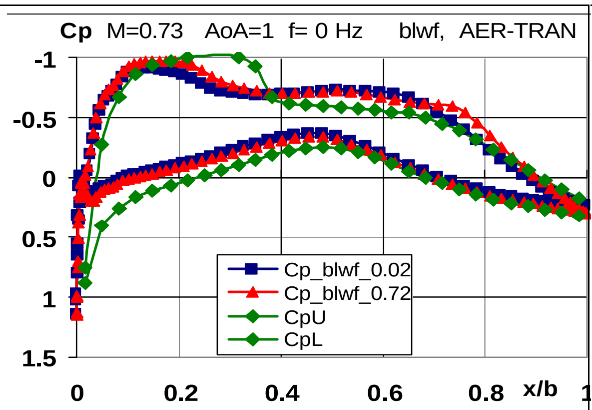


a. Comparison of computation (BLWF) and experiment [3]. AoA=0.067°, Re=4 mln.



b. Stationary pressure distribution in wing section z^*=0.3875, AoA=2°, AER-TRAN and BLWF. Re=4 mln and 30 mln.

Figure 1 – Comparison results of computation for the AIR-TRAN and BLWF programs with experimental data: M=0.75 (a), M=0.8 (b).



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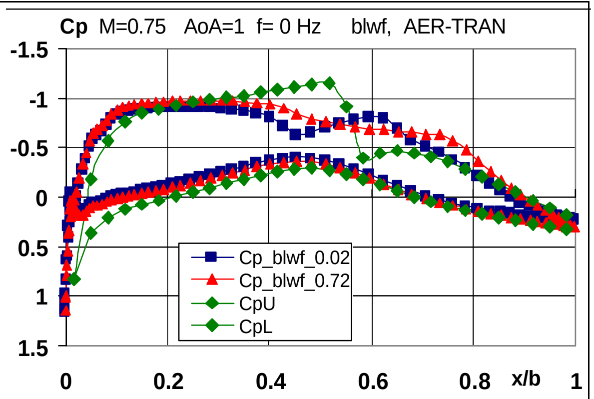

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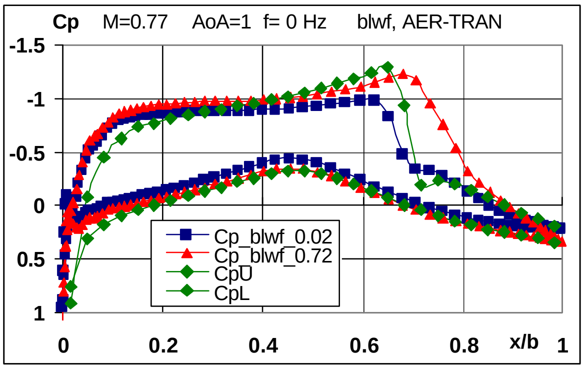

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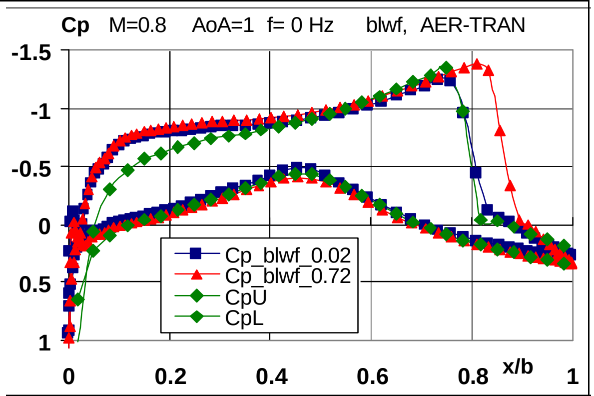


d.

Figure 2 – Comparison of computational results for AER-TRAN and BLWF programs. The CAST wing pressure distribution at four Mach numbers: M=0.73, M=0.75, M=0.77 and M=0.8 in wing section z=0.3875. The position of transition point along the chord: x/b=0.02 (fixed transition) and x/b=0.72 (free). Re=4mln

The distinctive pictures of the stationary pressure distribution along the chord are presented in Figure 2 (a, b, c, d), where the change in the position of the shock waves on the CAST wing surface in the transonic flow regime is shown with an increase of Mach number from M=0.73 to M=0.8. It should be noted that the results obtained under the assumption of inviscid ideal gas (AIR-TRAN) differ from turbulent and laminar flow (BLWF) mainly in cases of weak shock waves (M=0.73, M=0.75). With an increase of Mach number in regimes with sufficiently strong shocks, the agreement of the results is good (M=0.8).

Studies of non-stationary flow at transonic flight speeds were carried out for two variants of the CAST wing movement: forced wing vibrations with a given amplitude a_0 and frequency w: $\alpha(t)=\alpha_0\cos(\omega t)$, and free vibrations with one degree of freedom under the action of an aerodynamic moment and an elasticity of the structure. In this case, the behavior of the wing model in the air flow is determined by integrating equations in the time domain.

The stability investigation of wing vibrations in an ideal gas was carried out by solving nonlinear Euler equations using the Godunov finite-difference method (AER-TRAN). Distinctive of the transonic LCO regime is a significant displacement of the shock waves over the wing surface at a certain parameters combination such as Mach number, frequency and amplitude of vibrations. The shock movement corresponds to a sharp change in pressure at the points through which the shock passes. For the airfoil points lying away from the shock wave, the pressure change during the oscillation period is smooth and can be represented by a harmonic function. The dependences of the pressure coefficient distribution along the wing chord for different flow regimes are obtained. Figure 3 shows, for example, the pressure change over a period at three points along the wing chord: in the area of the shock wave movement (x/b=0.75) and at points ahead (x/b=0.65) and behind this area (x/b=0.85) for the case of the wing oscillations with frequency of f=20 Hz and amplitude of a_0 =A0=2° ($\omega=2pf$), M=0.8. Changes in the position of the shock waves along the chord at different wing oscillation frequencies (20 Hz, 35.8 Hz and 50 Hz) are shown in Fig. 4 for Mach number M=0.8. It can be seen from the presented results that the shock waves movements are harmonic functions with different phase shifts relative to the wing vibration. It is found that the magnitude of the phase shift significantly depends on Mach number and the oscillation frequency. In the same Figure 4, the red line shows the position of the shock wave in a stationary case with a sequential change in the angle of attack of the wing airfoil corresponding to the dynamic process of pitch vibrations of the wing.

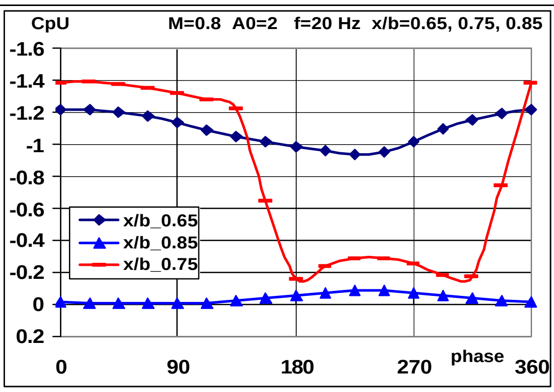


Figure 3 – Change in pressure distribution at three points on upper wing surface during oscillation period: $\alpha_0 = 2^0$, f=20 Hz, M=0.8

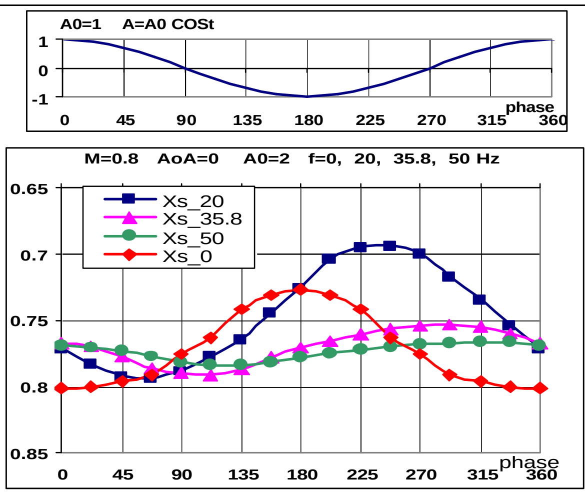


Figure 4 – Examples of shock waves movement on upper wing surface during one period of self-oscillation: frequencies f= 20, 35.8 and 50 Hz, M=0.8 α_0 =2^0

In the case of arbitrary dependence of aerodynamic characteristics on time, the criterion for the stability of the oscillatory process can be the work of the aerodynamic forces *W* during the oscillation period:

$$W = \int_0^b \int_t^{t+T} p(t,x)(x - x_0)\dot{\alpha}\,dx\,dt$$

The coefficient of aerodynamic damping corresponds to the dimensionless value of the aerodynamic work (*W*) for the period of oscillations and can be expressed by the formula:

$m_z^{\dot{\alpha}} = \dfrac{W}{\pi q \alpha_0^2 k b^2}$, where α_0 is the amplitude, $\omega = 2\pi f$ is circular frequency of the wing oscillation, b is chord of the CAST airfoil, x_0 is axis of rotation, $p(t, x)$ is pressure function, q is dynamic pressure, V is flow velocity, $k = \omega b / V$ is reduced frequency.

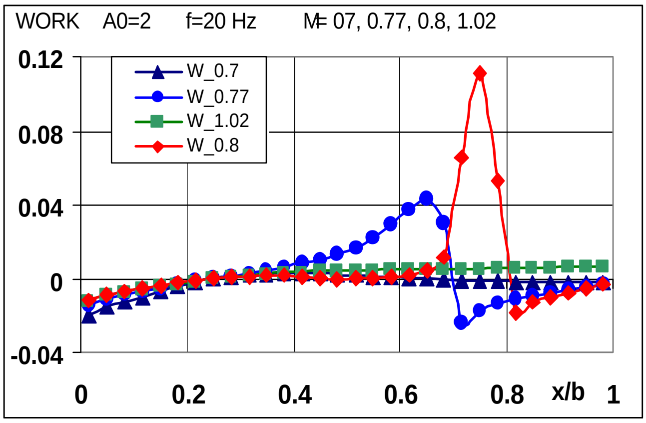
To determine the influence of various parts of the wing on the stability nature of the oscillatory process, it is convenient to consider the distribution of the aerodynamic work *W (x)* along chord:

$$W(x) = \int_0^T p(t,x)(x - x_0)\dot{\alpha}\,dt$$

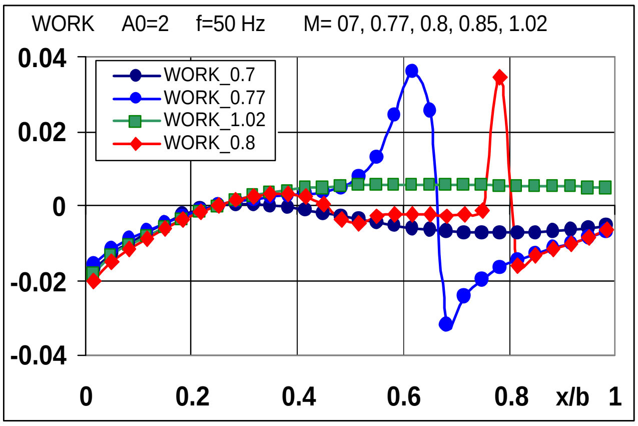
The estimation of both damping and anti-damping local unsteady aerodynamic forces arising on the oscillating wing and their relationship with the deflection angle is carried out.

3. Computational results

The work distribution *W (x)* along the chord of the two-dimensional CAST wing under forced pitch vibrations is shown in Fig. 5 (a, b). Computations were carried out in a wide range of Mach numbers, including both subsonic (M=0.7) and transonic (M=0.77 and M=0.8) and supersonic (M=1.02) flow regimes. The axis of rotation is located at the point $x_0 = 0.25b$.



a. Distribution of the aerodynamic work under forced wing vibrations with frequency f=20 Hz, α_0=A0=2^0.



b. Distribution of the aerodynamic work. f=50 Hz. α_0=A0=2^0.

Figure 5 – Distribution of the aerodynamic work under forced CAST wing vibrations.

In the case of steady-state vibrations, the wing surface is divided into areas of positive and negative values of the aerodynamic work (damping coefficient). In the region of negative aerodynamic damping, the aerodynamic work *W (x)*>0 (Fig. 5). In the region of positive aerodynamic damping, the work of the aerodynamic forces *W (x)<0*. Such distribution of the aerodynamic work along the chord is fundamentally different from its distribution, both in subsonic and supersonic unsteady flow

around the oscillating wing. Thus, in a fully subsonic flow (M=0.7), only negative values of the aerodynamic work are realized on the wing surface. At supersonic flow (M=1.02) relatively small positive values of work are realized (Fig. 5).
An essential feature of the transonic flow regime is that the wing vibrations cause the displacement of the shock waves, and the zones of interaction of shocks with the boundary layer on the wing surface. These movements can lead to the formation of additional non-stationary aerodynamic damping and anti-damping forces.
Suppose that for steady-state self-oscillations the motion of shocks can be described by a harmonic function:

$$x_s(t) = x_{0s} + \delta\cos(\omega t + \varphi),$$

where δ is the amplitude of the shock oscillation, φ is the phase shift of the shock movement relative to the wing movement, x_{0s} is the average position of the shock.

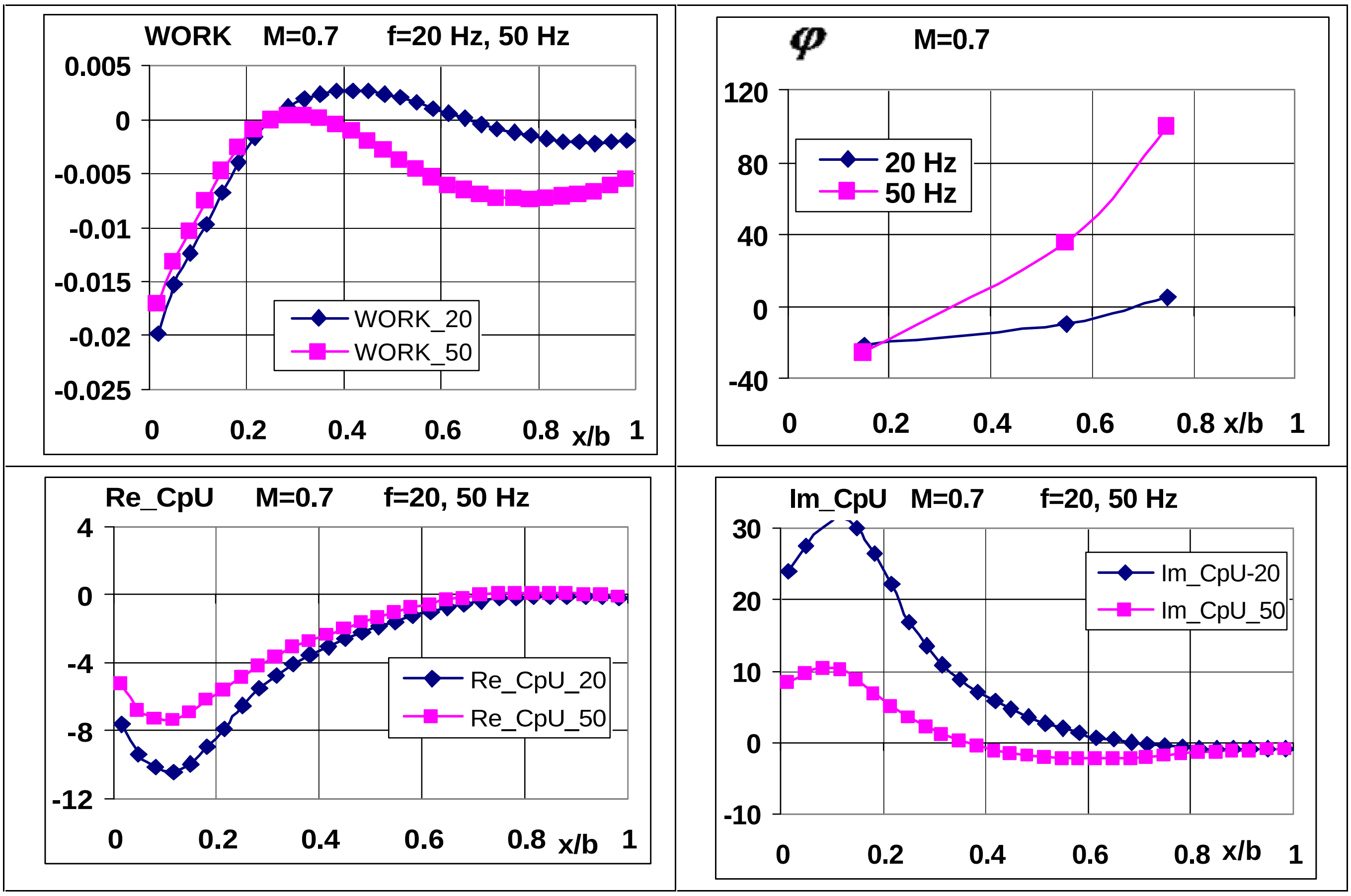


Figure 6 – Distribution of the aerodynamic work *W (x)*, the phase shift φ between the shock motion and the wing oscillation, the real and imaginary parts of pressure, M=0.7, f=20 Hz and 50 Hz.

It is well known that the amplitude, phase and average position of the shock depend on the Mach number and the parameters of the wing oscillation.
Investigations of the process of flow transformation with an increase of Mach number and evaluation of the role of shock waves in the occurrence of transonic LCO were carried out. As example, Figures 6-9 show the results related to subsonic, transonic and supersonic flow regimes, including four types of distribution along wing chord:

- *W(x)* - the work of aerodynamic forces,
- φ - phase shift between the shock motion and the wing oscillation,

– *Re_CpU* - the real and *Im_CpU* - the imaginary parts of the pressure coefficient on the upper wing surface.

In subsonic flow (Fig. 6, M=0.7), mainly positive phase shifts are shown on the wing surface, increasing towards the trailing edge. At supersonic flow (Fig. 9, M=1.02), relatively small phase delays are realized. The real and imaginary components of the pressure coefficient for these two regimes demonstrate a fairly smooth behavior for the frequency values shown in Figures 6 and 9.

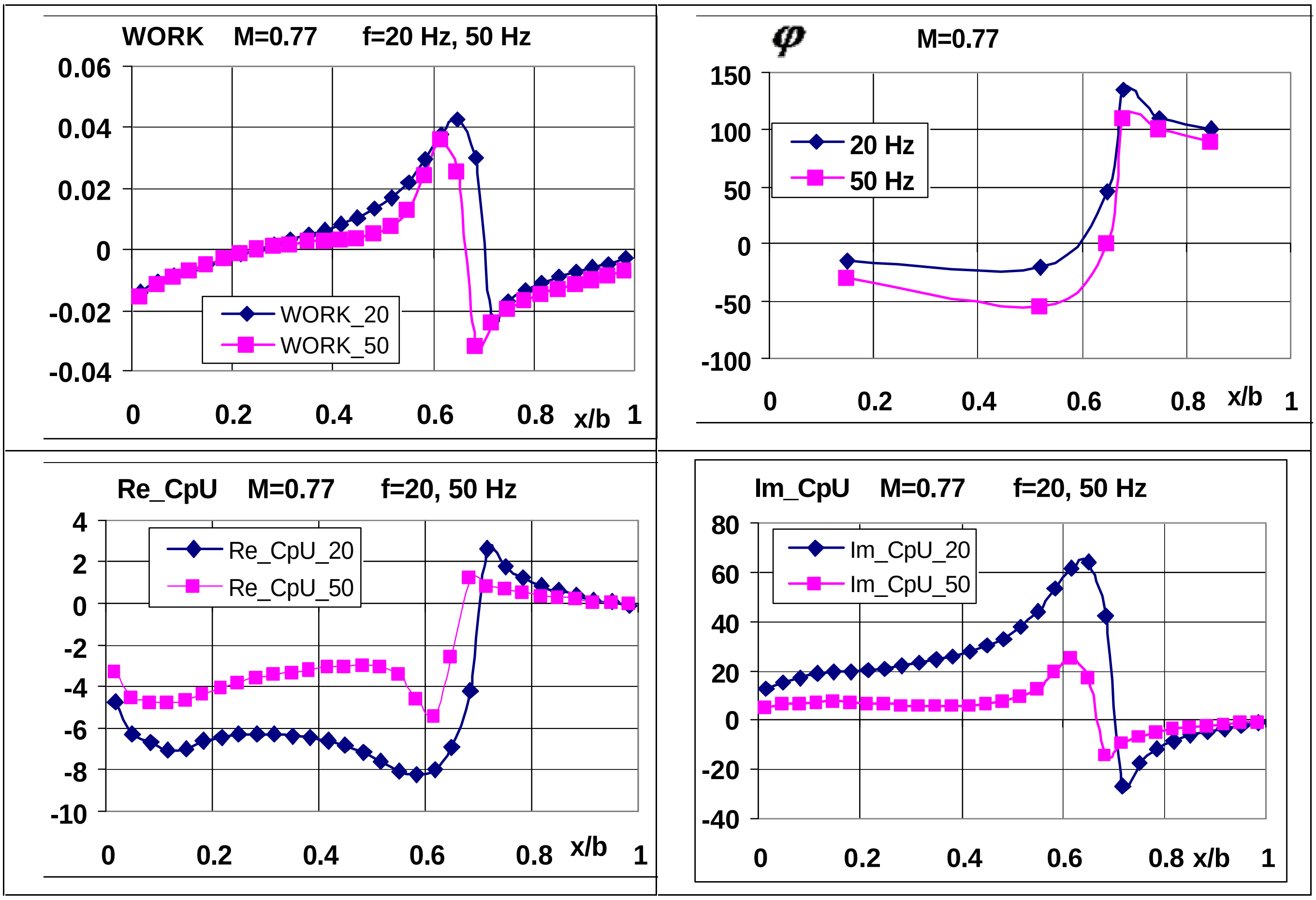


Figure 7 – Distribution of the aerodynamic work W (x), the phase shift φ between the shock motion and the wing oscillation, the real and imaginary parts of the pressure, M=0.77, f=20 Hz and 50 Hz.

In transonic flow behind the front of the oscillating shock a pressure component appears, which is ahead of the deflection angle: φ >0. This leads to aerodynamic damping in the rear part of the airfoil: *W (x)<0* (Fig. 7, 8). At those points of the wing that are ahead of the shock wave in the local supersonic zone, phase delays are realized (φ <0), which leads to negative damping. In this area of the wing, the aerodynamic forces perform positive work: *W (x)>0*. In this case, the interval of the shock movement along the chord can be considered as a kind of regulator of the amplitude of steady-state self-oscillations. Indeed, since an increase in the amplitude increases the oscillation interval of the shock, the damping areas are redistributed due to an increase of the positive damping area (Fig. 10). This will automatically lead to a corresponding decrease in the amplitude. The reverse process will occur when the amplitude decreases.

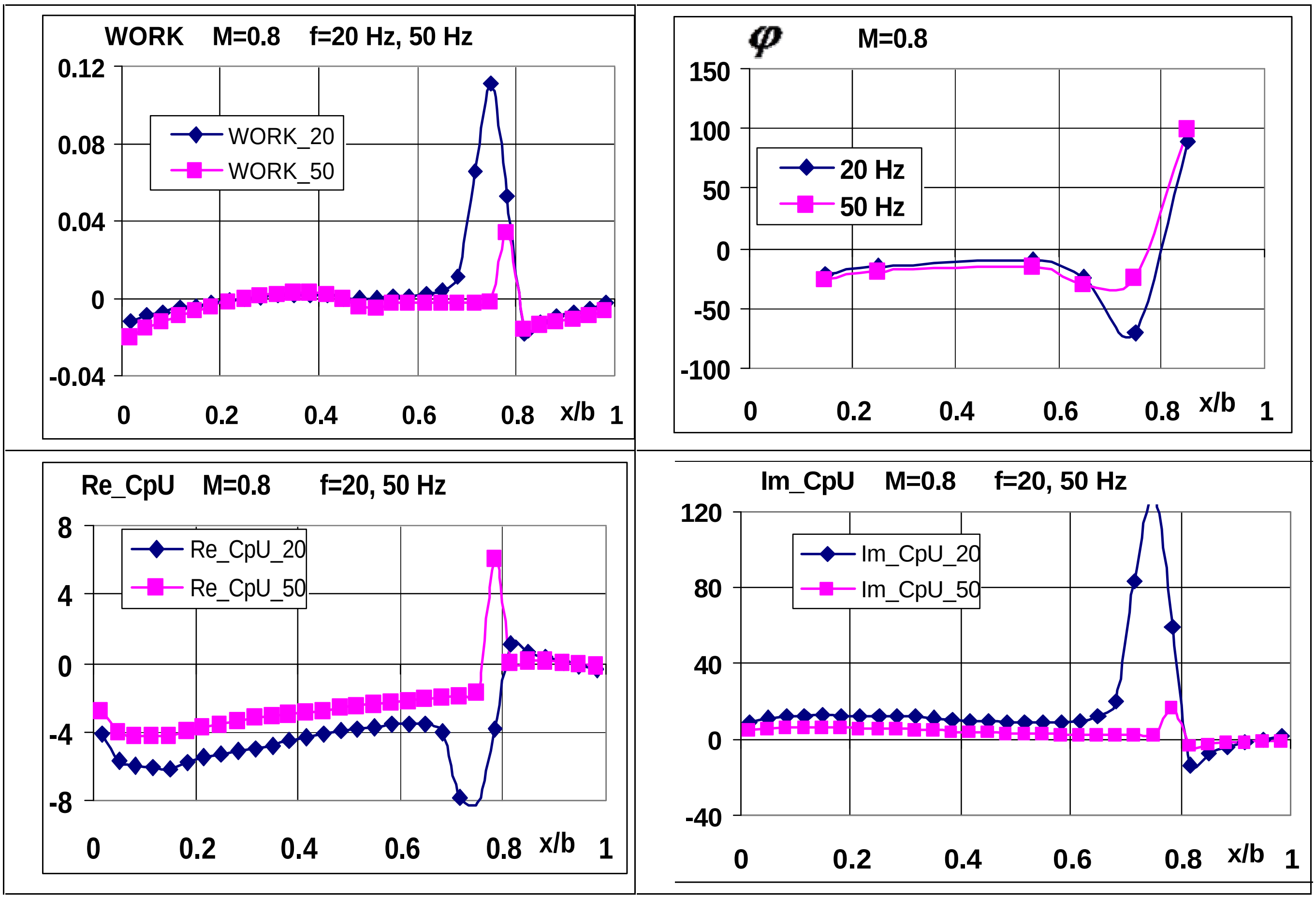
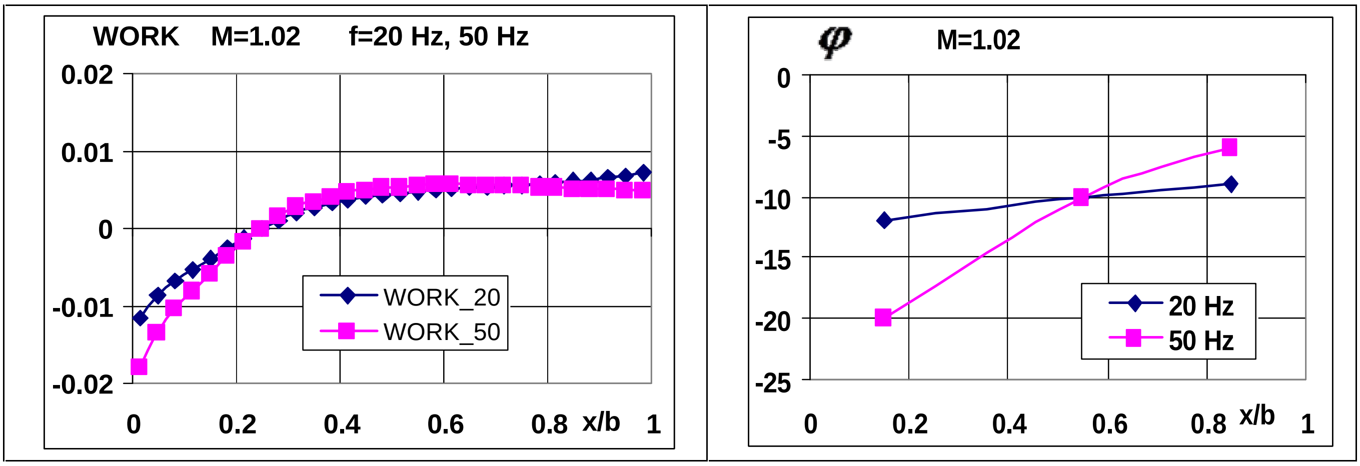


Figure 8 – Distribution of the aerodynamic work *W (x)*, the phase shift j between the shock motion and the wing oscillation, the real and imaginary parts of the pressure at M=0.8, f=20 Hz and 50 Hz.

The results presented in Fig. 8 clearly demonstrate the direct dependence of the aerodynamic work on the value of the pressure delay relative to the wing vibration. Comparing the flow regimes that differ in the oscillation frequencies, it can be seen that the interval of the shock movement increases with a decrease of frequency and the value of the pressure delay at the points of this interval also increases. As a result, the area of negative damping increases, while the area of positive damping behind the shock front does not change significantly.



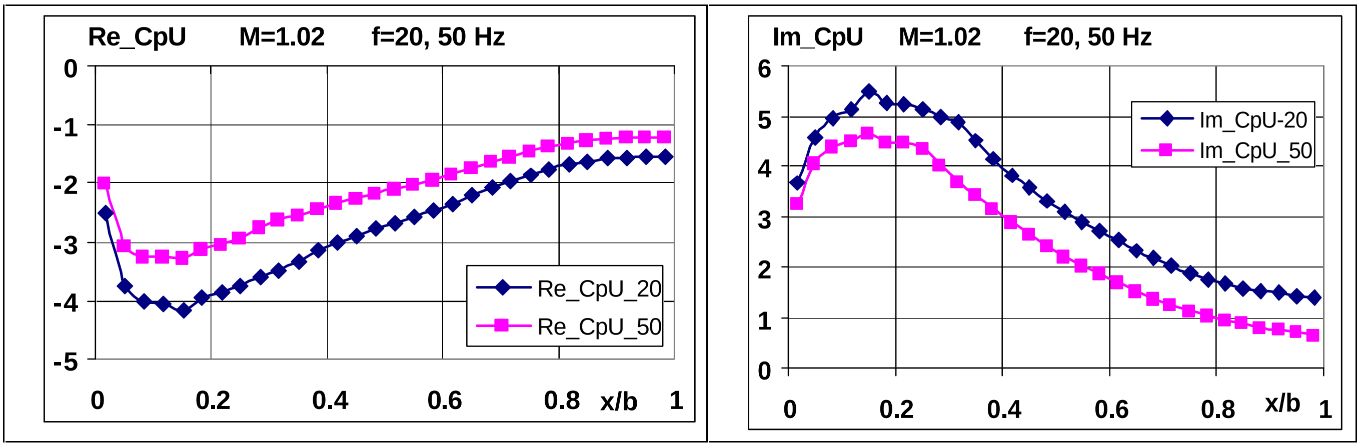
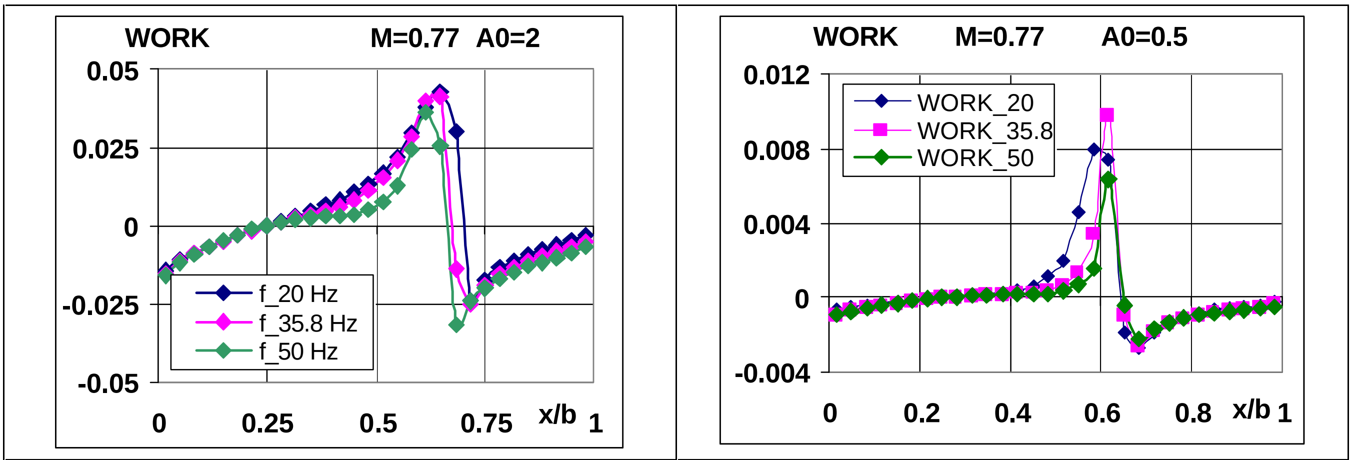


Figure 9 – Distribution of the aerodynamic work W (x), the phase shift j between the shock motion and the wing oscillation, the real and imaginary parts of the pressure at M=1.02, f=20 Hz and 50 Hz.

The influence of the frequency and amplitude of vibrations on the distribution of the aerodynamic work is shown, as example, in Figure 10 (a, b) at M=0.77. It can be seen that a change of the amplitude causes a change in the shock displacement interval and a redistribution of the areas of positive and negative damping. The nonlinear dependence of the damping coefficient on the oscillation amplitude, shown in Figure 11, demonstrates the possibility of LCO with an amplitude of about 1.5^0 in the flow regime: M=0.77, f=36 Hz. The dependence of the aerodynamic damping coefficient on the oscillation frequency obtained in computations is shown in Fig. 12.



a. b.

Figure 10 – Distribution of the aerodynamic work *W (x)*, M=0.77, f=20 Hz, 35.8 Hz and 50 Hz. Amplitude of oscillations: $À0=2^0$ (a), $À0= 0.5^0$ (b)

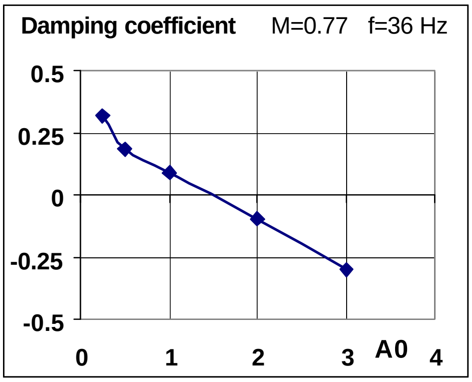


Figure 11 – Dependence of aerodynamic damping coefficient $m_z^{\dot{\alpha}}$ on amplitude of vibrations. f= 36 Hz. M=0.77

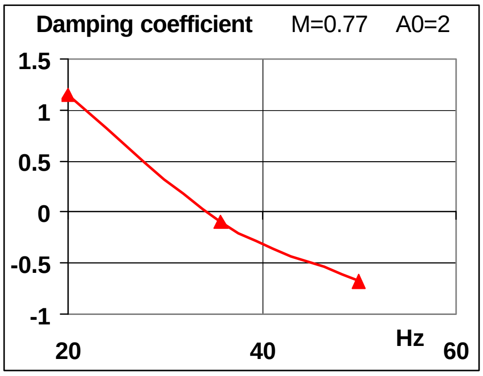


Figure 12 – Dependence of aerodynamic damping coefficient $m_z^{\dot{\alpha}}$ on vibration frequency. M=0.77, À0=2^0

To verify the proposed computational method, the obtained results were compared with the experimental data of the research model with supercritical CAST airfoil, which was tested in the transonic wind tunnel ADT TWG in Goettingen [3]. Figure 13 shows the experimental [3] (green line) and numerical results (red and blue lines) obtained in LCO of the CAST model.

The oscillation frequency is 50 Hz, the amplitude is 1^0, Mach numbers M=0.73 and 0.75. The position of the shock wave on the upper wing surface is in good agreement with the experiment for part of the oscillation period.

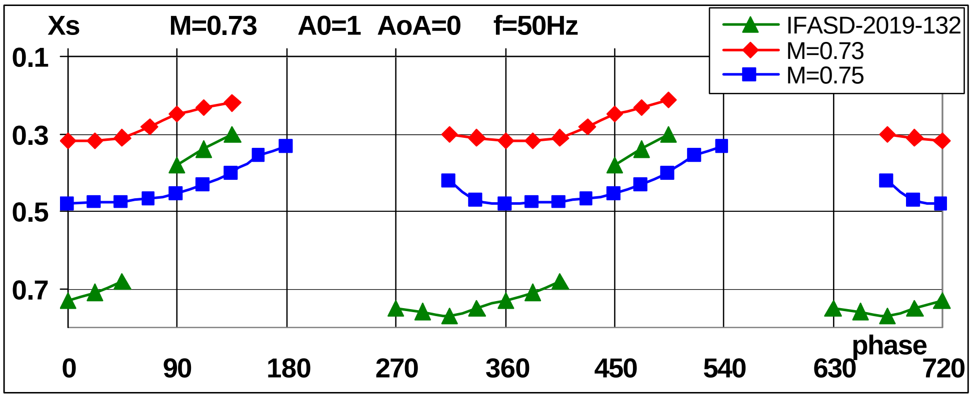


Figure 13 – Dependence of the shock position on time during LCO for two periods.

The process of shock waves formation and transformation during LCO with an increase of Mach number was studied. As example, the computation results are shown in Fig. 14 when Mach number changes from 0.73 to 0.8 for two frequencies of 20 Hz and 50 Hz. It can be seen that the shocks move with a certain phase shift, depending on the flow regime. We can assume that the shocks motion is periodic but interrupted at small transonic Mach numbers. Here, for comparison, the change of the wing deflection during its harmonic oscillations is presented. Figure 15 shows the values of phase shifts for the Mach numbers M=0.73 and M=0.77, depending on the frequency. Figure 16 demonstrates the dependence of the aerodynamic moment function on time, obtained at M=0.8, f=50 Hz. There is a noticeable delay of the aerodynamic moment relative to the wing vibration. Comparison of the delay angles of the aerodynamic moment and the shock wave is presented in Fig. 17. It is found that at low frequencies (20 Hz), both the moment and the shock are ahead of the wing deflection, while with an increase of frequency; both the moment and the shock are delayed.

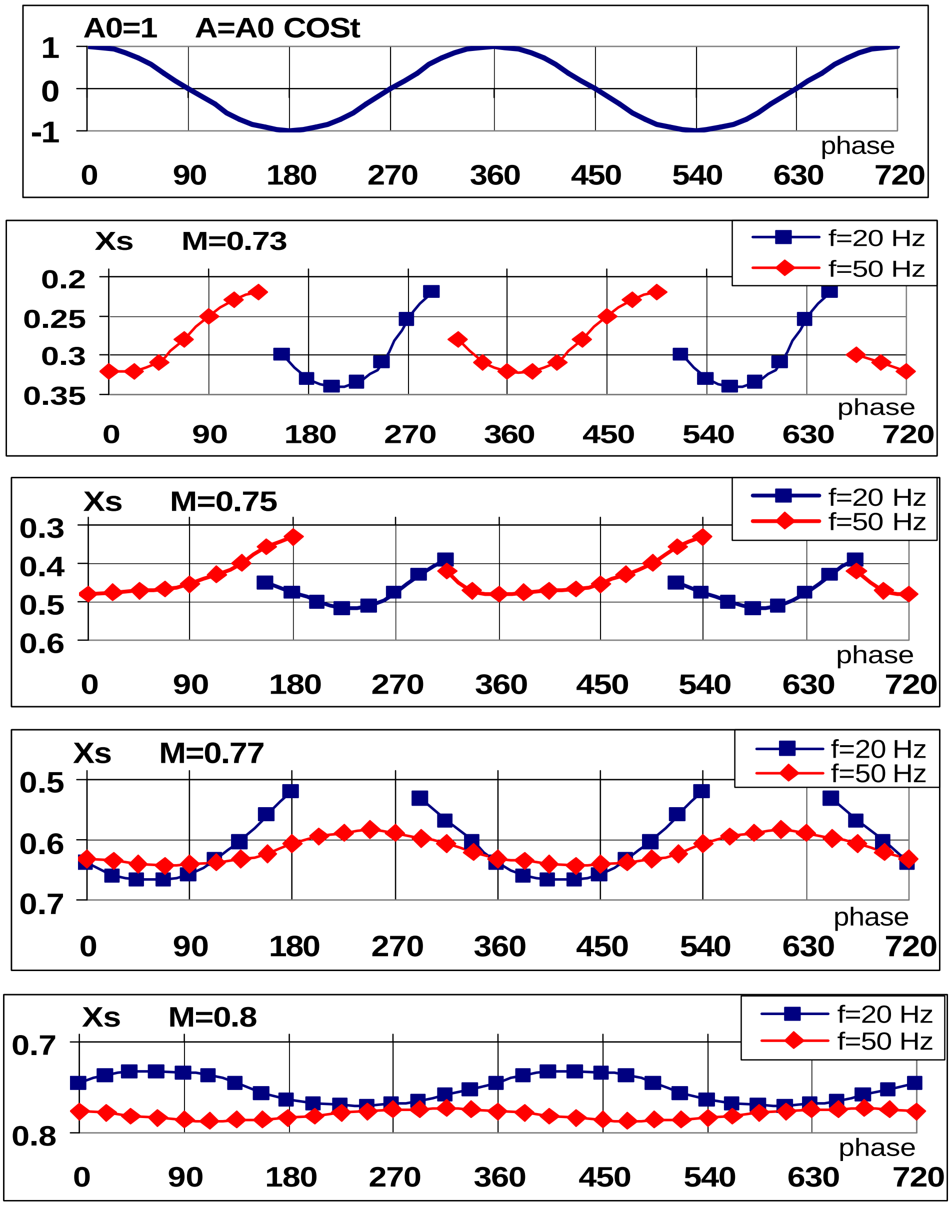


Figure 14 – Positions of the shocks along the wing chord during forced oscillations with frequencies of 20 and 50 Hz with the change in the Mach number from 0.73 to 0.8.

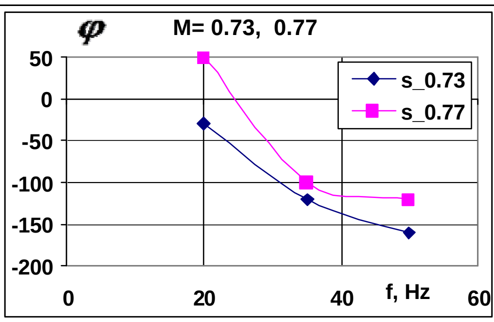


Figure 15 – Dependence of phase shift value of the shocks relative to wing deflection on oscillation frequency. M=0.73, 0.77.

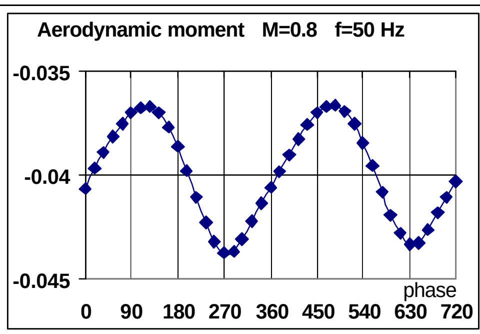


Figure 16 – Aerodynamic moment as function of time for harmonic vibrations of the wing. M=0.8.

The dependence of the aerodynamic damping coefficient on the vibrations frequency for transonic Mach numbers is shown in Fig. 18.

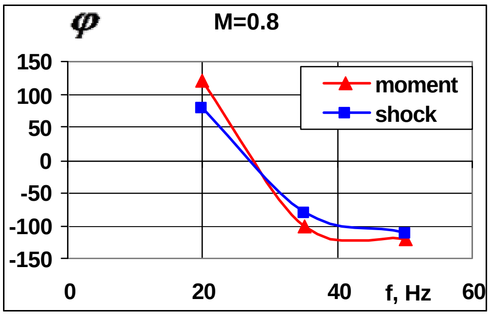


Figure 17 – Phase shifts computed from the moment function and from the harmonic displacements of the shock.

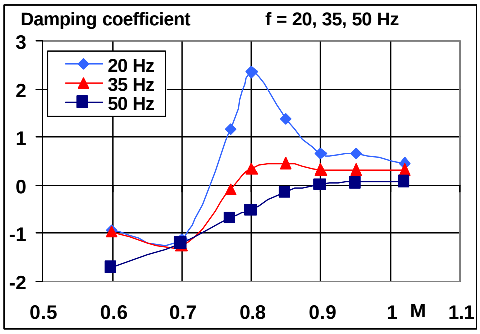


Figure 18 – Aerodynamic damping coefficient $m_z^{\dot{\alpha}}$ as function of Mach number and frequency of wing vibrations

4. Investigation of oscillations by direct modeling in the time domain

To confirm the results on the stability boundaries obtained in the frequency domain, the wing behavior was determined by integrating equations in the time domain. The solution of the related problem aerodynamics-elastic deformations in the time domain makes it necessary to simultaneously apply the methods of static and dynamic aeroelasticity. As it was found out above, the characteristics of aerodynamic damping significantly depend on the specified angle of wing deflection (angle of attack AoA_stat). In time modeling, the angle of attack is realized in given flow regime as the sum of the wing deflection angle AoA_jig_ that was set outside the flow and the additional angle AoA_flex due to elasticity of the attachment to rotation: AoA_stat= AoA_jig+ AoA_flex. Therefore, the determination of the specified angle of deflection AoA_jig (the so-called jig shape), which provides the required angle of attack in the flow, is performed iteratively. This procedure is performed for a number of oscillation frequencies (rotational stiffness). The method of forced oscillations with harmonic linearization used above made it possible to find out that the most critical from the view point of aeroelastic stability at Mach number M=0.8 are oscillation modes with an amplitude about 2 degrees around zero angle of attack. Therefore, the specified deflection angles and initial perturbations were selected iteratively for the implementation of such modes. Figure 19 shows the obtained dependence of the jig shape angle on the oscillation frequency. The time processes confirm the above results about the stability boundaries. At high frequencies, the vibrations are stable. When the frequency decreases to 36 Hz, self-oscillations with a limit cycle occur (fig. 20).

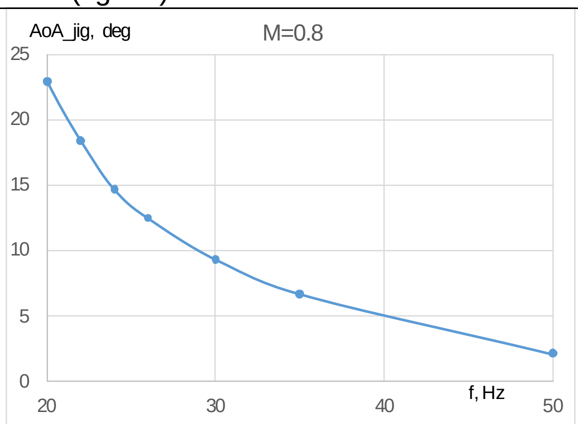


Figure 19 – Dependence of AoA_jig (jig deflection angle) on the vibration frequency.

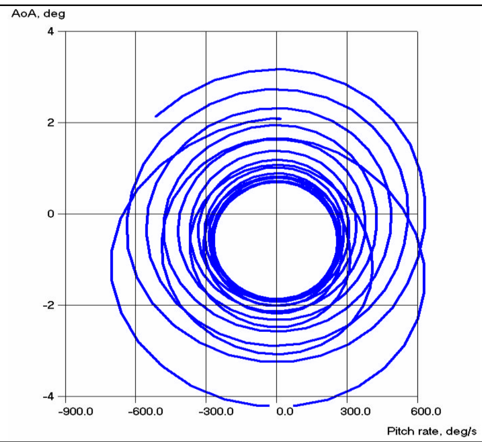


Figure 20 – Phase portrait of time process. f=36 Hz. M=0.8.

Phase portraits of processes in the plane "angle of attack – angular velocity" confirm the results obtained on the basis of the energy approach.

Conclusions

- The features of nonlinear mathematical model of aerodynamic forces in transonic flow based on Euler equations are considered, using as the example the CAST wing model.
- Approach to the creation of methodology for numerical modeling in the time domain of transonic self-oscillation processes has been developed.

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Contact Author Email Address

Mailto: kuzmina@tsagi.ru

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