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AN INVERSE APPROACH BASED ON EUCLIDEAN SPACE FOR DETERMINING STRUCTURAL LOAD DISTRIBUTION FROM STRAIN MEASUREMENTS

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Abstract

Aiming at the inverse problem of static distributed load recovery from strain measurements, several existing influence coefficient methods are compared and the key problems affecting load inversion accuracy are discussed. To solve the selection of unit load cases, a stepwise method based on Schmidt's orthogonalization of the maximum vertical distance is proposed to select the basis vector from Euclidean space, which is used to sequentially select the basis cases from load space and strain space. Thus, a complete set of engineering feasible distributed load recovery approach is established. Meanwhile, the Tikhonov regularization method is introduced to solve the ill-conditioning of the inverse matrix. Finally, load rams and optical fiber sensor data of a certain aircraft wing fatigue test are used as a case study, with 0, 5% and 10% random strain measurement errors introduced, respectively. It is verified that the Euclidean space method has high prediction accuracy and robustness, and Tikhonov regularization method can effectively reduce the influence of strain measurement error.

Keywords: Distributed load recovery; Influence coefficient method, Euclidean space, Schmidt orthogonality, Tikhonov regularization

1. Introduction

Structural health monitoring is an important and challenging technology for high-performance aerospace vehicles [1]. The ability to identify flight load distribution has the potential to significantly improve fatigue monitoring capabilities, which cannot be accomplished by traditional load measurement program. It is unrealistic to instrument load sensors on the surface of the structure to directly measure the load distribution. The commonly used method is to determine the load using strain measurements. This is obviously an inverse problem of determining the input forces from structure response. Solving the inverse problem is very challenging because it tends to be highly ill-conditioned, i.e., even very small variations (noise) in strain measurement can cause large deviations in the load estimation.

The traditional load measurement program based on strain bridges is a typical inverse problem. It measures the overall load of the component, such as the bending moment, shear and torque at the root of the wing, by establishing a linear regression equation that is directly mapped from the strain bridge to the overall load. Traditional load calibration cannot obtain the global load distribution of the structure, and usually only the prediction of the bending moment is good, followed by the shear and torque is the worst. This is the inherent characteristic of this method.

Compared with the traditional load calibration, the distributed load inverse method studied in this paper needs to measure strain at more locations over the structure in order to obtain a sufficiently broad and reliable strain distribution data. Compared with the most commonly used strain gages, fiber optic sensors have the advantages of light weight, long life, resistance to electromagnetic interference and harsh environments, which will favorably promote the development of inverse approach for distributed load recovery from strain measurements.

Many authors have proposed different methods for load recovery from strain data. Among them, the most widely used method is influence coefficient method [2]-[6]. Most case studies in the literature are verified only by FEA or some simple physical tests such as cantilever beams. At present, there are no applications in the large and complex load-carrying structures (such as real wings) in the literature.

In this paper, the terms distributed load and load distribution are used interchangeably. Similarly, identification, recovery and estimation mean the same in the context.

2. Influence coefficient method

Under the assumption of linear elasticity in structural mechanics analysis, the linear combination of structural responses (such as displacement, stress, strain, etc.) at different load cases can be mapped to the linear combination of structural external loads.

The physical quantities at each load case can be represented by an *n*-dimensional column vector $\{\boldsymbol{x}\}=(x_1,x_2,...,x_n)^T$, where each element x_i represents the load value at different loading points or the response value at different locations. $\{\boldsymbol{x}\}$ determines the unique load case.

The influence coefficient method directly establishes the mathematical relations between known and unknown quantities, expressed as a linear matrix equation,

$$\{\boldsymbol{\varepsilon}_{g\times 1}\} = [\boldsymbol{A}_{g\times m}]\{\boldsymbol{\beta}_{m\times 1}\} \tag{1}$$

$$\{\boldsymbol{f}_{n\times 1}\} = [\boldsymbol{P}_{n\times m}]\{\boldsymbol{\beta}_{m\times 1}\} \tag{2}$$

where, {·} is a vector, [·] is a matrix, $\{\boldsymbol{\varepsilon}\}$ and $\{\boldsymbol{f}\}$ is the strain vector and the load vector at an unknown load case, $[\boldsymbol{A}]$ is influence coefficient matrix, $[\boldsymbol{P}]$ is unit load matrix, $\{\boldsymbol{\beta}\}$ is the linear coefficient, g is the number of strains, n is the number of loading points, and m is the number of unit load cases.

The matrix $[\boldsymbol{A}]$ are determined with a forward unit loading procedure by applying unit load matrix $[\boldsymbol{P}]$, in which the unit loadings can be point loads or a proper load distribution basis function, which will be detailed later.

2.1 Point loads method

Point loads method means the unit load is applied at one loading point once at a time. Then, $m=n$, and $[\boldsymbol{P}]$ is an unit matrix, $[\boldsymbol{P}]=\text{diag}(1,1,...,1)$. Thus, Eqn. (1) and (2) can be combined into Eqn. (3),

$$\{\boldsymbol{\varepsilon}_{g\times 1}\} = [\boldsymbol{A}_{g\times m}]\{\boldsymbol{f}_{m\times 1}\} \tag{3}$$

The influence coefficient a_{ij} in $[\boldsymbol{A}]$ represents the strain at location i due to a unit point load applied at the point j, $i=1,2,...,g$，$j=1,2,...,m$.

For the point loads method, the maximum load allowable at each loading point in the actual test of complex structures is limited due to the effect of concentrated loads on the local area, especially the loading points on the control surfaces and those near the wing tip. Also, the strain response in most structural regions is small at unit point load case, which will affect the accuracy of load identification. Therefore, the point load method is not suitable for ground test of complex structures.

2.2 Load basis function method

Load basis function method means that the load profile is in the form of a certain basis function. In general, $m \neq n$, $[\boldsymbol{P}]=(\{\boldsymbol{f}_1\},\{\boldsymbol{f}_2\},...,\{\boldsymbol{f}_m\})$. The influence coefficient a_{ij} in $[\boldsymbol{A}]$ represents the strain at location i due to the jth load basis function $\{\boldsymbol{f}_j\}$, $i=1,2,...,g$, $j=1,2,...,m$.

In Ref. [3], the basis function $\{\boldsymbol{f}_j\}$ is taken as a polynomial distribution function along the wing span (x) and chord (y), such as constant, linear x and y, quadratic x^2 and y^2, exponents e^x and e^y, and Sine function (*sin*), etc.

In Ref. [4], the basis function $\{\boldsymbol{f}_j\}$ is taken as the Fourier series terms along the wing span (x) and chord (y), such as constants, single Fourier terms $\cos(k\pi x/L)$ and $\cos(k\pi y/H)$, and the double Fourier term $\cos(k\pi x/L)\bullet\cos(k\pi y/H)$, where L is half of the span and H is the wing root chord length, usually $k=1,2,3$.

For the load basis function method, the difficulty is to select the appropriate type and number of load basis functions. In actual tests, the number of loading points is limited, and the load distribution proportion of the tension-compression load pads under one set of hydraulic jack and whiffletrees is fixed. Thus it is difficult to simulate the load basis functions. Therefore, for complex structures, the load basis function method is also not suitable for ground tests..

2.3 Discussions

The identification accuracy of the distributed load mainly depends on the following aspects,

- a) Selection of unit load cases (determines [***P***]). For the point loads method, unit load is directly applied at each loading point. For the load basis function method, the type and number of load basis functions directly affect the sufficiency and completeness of the load space.
- b) Selection of the number and location of strain sensors (determines [***A***]). The number and spread (variance) of the strain sensors should be increased as far as possible and the locations with large response to the loads should be selected. If the change of loads at some loading points cannot be reflected on the strain measurements, it is impossible to accurately predict the loads at these loading points.
- c) Reduction of ill-conditioning of the inverse matrix (determines {$\boldsymbol{\beta}$}). In practical applications, strain measurement errors are inevitable. It is necessary to introduce an effective regularization method to solve the linear matrix equation (Eqn. (1)) to reduce the influence of the measurement error on the load solution.

Researches of a) and c) are the focus in this paper.

3. Euclidean Space method

In order to solve the selection of unit load cases, a stepwise method based on Schmidt's orthogonalization of the maximum vertical distance to select the basis vectors from Euclidean space is proposed, which is called Euclidean space method. The general idea of Euclidean space method is to select the vector with the largest vertical distance from the current Euclidean space in turn to form a new Euclidean space until all vectors are in that Euclidean space and the selected vectors are the basis cases. Details of the process are provided in Section 3.1.

Compared with the point loads method and load basis function method, Euclidean space method proposed in this paper can select linearly independent basis cases from the design load case database. Thus, the basis case is a more realistic load distribution and it is easy to apply in the ground test.

The process of distributed load inverse approach based on Euclidean space method is shown in Figure 1. The key point is to first select load basis cases from the load space, and then further select strain basis cases from the strain space under the premise that the strain basis cases include the load basis cases, see section 3.2 for detailed steps. The candidate strain set in Figure 1 refers to the set of strain measurement points that can be arranged in the structure.

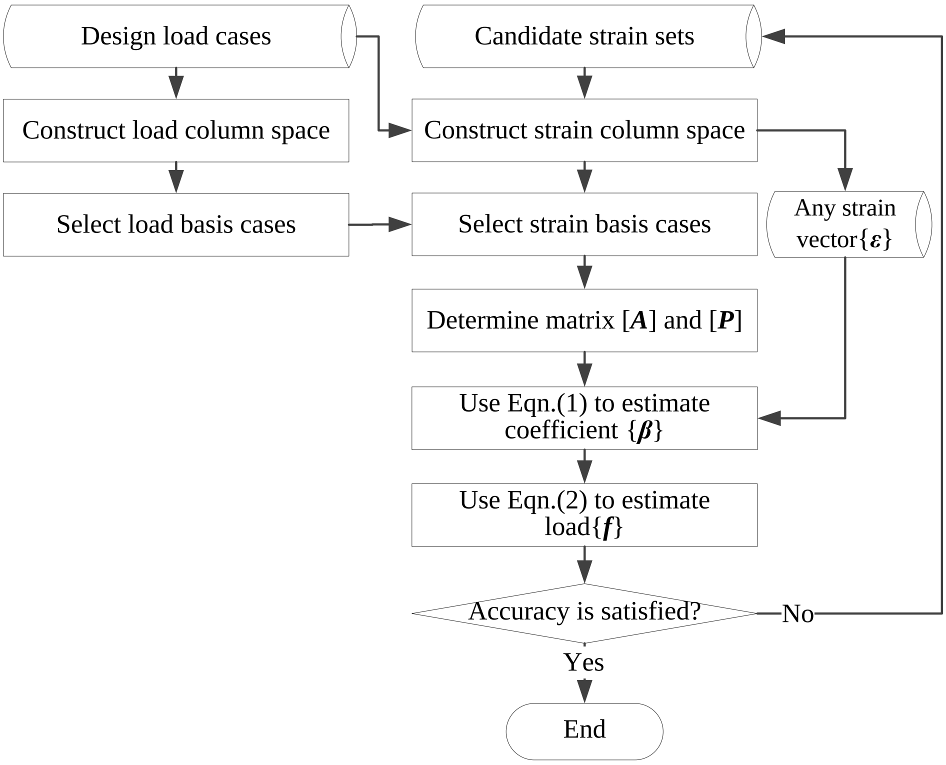


Figure 1 –The process of distributed load inverse approach based on Euclidean space method

3.1 Selection of load basis cases

The distributed load matrix of design load cases is expressed as $[\boldsymbol{P}_{n,M}]=(\{\boldsymbol{f}_1\},\{\boldsymbol{f}_2\},...,\{\boldsymbol{f}_M\}),\{\boldsymbol{f}_i\}$ represents a load case, M is the number of design load cases, n is the number of loading points. The load column space is a subspace of n-dimensional Euclidean space. As long as a set of linearly independent basis vectors, i.e. the basis cases, are determined, any load case can be linearly represented by this set of basis cases. The Euclidean space is referred as $\boldsymbol{E}_{n,m_f}$, m_f is the number of basis cases.

The steps [7] of stepwise method to select basis vectors based on Schmidt orthogonality of maximum vertical distance is as follows:

- Step1: Standardize all design load cases, and each unit load case is $\{\boldsymbol{f}_i^e\}=\{\boldsymbol{f}_i\}/|\{\boldsymbol{f}_i\}|$, i=1,2,...,M;
- Step2: Select a load case as the initial basis case, recorded as $\{\boldsymbol{f}_1\}$, and its standardized vector is $\{\boldsymbol{e}_1\}$. Then one-dimensional load column space $\boldsymbol{E}_{n,m_f}$ is obtained, m_f=1;
- Step3: Calculate the vertical components of all unit load cases $\{\boldsymbol{f}_i^e\}$ to the load column space $\boldsymbol{E}_{n,m_f}$;

$$\{\boldsymbol{f}_i^e\}^\perp=\{\boldsymbol{f}_i^e\}-\sum_{j=1}^{m_f}(\{\boldsymbol{f}_i^e\}^{\mathrm{T}}\{\boldsymbol{e}_j\})\{\boldsymbol{e}_j\} \tag{4}$$

- Step 4: Select the maximum $|\{\boldsymbol{f}_i^e\}^\perp|$ load case to enter the basis cases group. When $|\{\boldsymbol{f}_i^e\}^\perp|>\alpha$ (α is the preset error threshold), increase m_f by 1, and then $\{\boldsymbol{f}_i^e\}^\perp$ is standardized as $\{\boldsymbol{e}_{m_f}\}$, which constitutes the m_f-dimensional load column space $\boldsymbol{E}_{n,m_f}$;
- Step 5: Repeat steps 3 and 4 until $|\{\boldsymbol{f}_i^e\}^\perp|\leqslant\alpha$, then, all the basis cases are selected, $m_f\leqslant M$, $m_f\leqslant n$.

We can see from the above steps that the initial basis case $\{\boldsymbol{f}_1\}$ and error threshold α determine the final basis cases group. In practical application, the important case (such as the maximum load case) can be selected as the initial basis case. It's reasonable that error threshold α be taken as 1/10~1/15 of the minimum standard deviation after the unitization of all design load cases.

Generally, the larger the dimensionality (n) of the load vector is, the more basis cases (m_f) are required to construct a complete load column space. In structural FEA, the aerodynamic load and inertial load nodes are usually dense (*n* is very large), but in actual ground test, the distributed load can be expressed only by the independent hydraulic jack loads.

3.2 Selection of strain basis cases

The distributed strain matrix of design load cases is expressed as $[\boldsymbol{A}_{g,M}]=(\{\boldsymbol{\varepsilon}_1\},\{\boldsymbol{\varepsilon}_2\},\ldots,\{\boldsymbol{\varepsilon}_M\})$,$\{\boldsymbol{\varepsilon}_i\}$ represents a load case, *M* is the number of design load cases, *g* is the number of strain measurements.

Based on the load basis cases obtained above, the strain basis cases are further selected from the design load case database. Replace the load matrix $[\boldsymbol{P}_{n,M}]$ in Section 3.1 with the strain matrix $[\boldsymbol{A}_{g,M}]$, and modify the Step 2 as follows:

- Step 2: Select the selected m_f load basis cases as the initial strain basis cases, and convert them to the standard orthogonal vectors according to the Schmidt orthogonalization formula. Then the m_f-dimensional strain column space $\boldsymbol{E}_{g,m_f}$ is obtained;

The remaining steps remain the same, and m_ε strain basis cases can be selected in order, $m_\varepsilon \geqslant m_f$, $m_\varepsilon \leqslant M$, $m_\varepsilon \leqslant g$. Then, the influence coefficient matrix $[\boldsymbol{A}_{g,m_\varepsilon}]$ and the basis load matrix $[\boldsymbol{P}_{n,m_\varepsilon}]$ are determined.

Step 2 is the key step. The criterion that strain basis cases include load basis cases ensures the accuracy of distributed load recovery.

4. Regularization

The inverse problem nearly always leads to an ill-conditioned (ill-posed) equation. The conventional least square method cannot solve this problem. It is necessary to introduce a suitable regularization method to reduce the ill-conditioning of the inverse matrix equation, thereby improving the stability and robustness of load estimation.

Among the many regularization methods, the most widely used is Tikhonov's regularization method. Different from the conventional least square method, the key of the regularization method is to add a specific regular term to weight the condition of the minimal sum of squared errors, in order to overcome the ill-posedness.

For the linear matrix equation of Eqn. (1), Tikhonov's regularization criterion is

$$\arg\min\{\|[\boldsymbol{A}]\{\boldsymbol{\beta}\}-\{\boldsymbol{\varepsilon}\}\|_2^2+\lambda\|[\boldsymbol{H}]\{\boldsymbol{\beta}\}\|_2^2\} \tag{5}$$

Where, $\|\bullet\|_2$ is the Euclidean second norm, $[\boldsymbol{H}]$ is the regularization matrix, and λ is the regularization parameter. Generally, the L-curve method and generalized cross-validation (GCV) are used to optimize the parameter values. Which method is better depends on the specific structure in question. For the detailed algorithms of Tikhonov's regularization, please refer to reference [8].

5. Case study

Load rams and fiber-optic sensor data of a certain aircraft wing fatigue test are used as a case study to verify the Euclidean Space method proposed above. All the loads in the illustrations in this section have been standardized.

There are a series of 749 unique load cases in the fatigue test spectra, i.e. *M*=749. There are 24 load rams that have applied loads through whiffletrees to tension-compression load pads bonded to the lower wing surfaces, i.e. *n*=24. The 4 main beams are all equipped with optical fiber sensors along the span direction, and there are 116 effective strain measurements (excluding damaged/abnormal strains), which are used as the candidate strain set in this example (g=116). In order to eliminate the influence of strain measurement errors, the median of multiple strain values at all frequencies of each load case is taken to obtain the candidate strain matrix $[\boldsymbol{A}_{116,749}]$.

Following the implementation steps of Euclidean Space method, first based on the distributed load matrix $[\boldsymbol{P}_{24,749}]$, using the method described in Section 3.1, with the maximum wing bending moment case taken as the initial case, and the error threshold taken as 1/10 of the minimum standard deviation after the standardization of all load vectors, 23 basis cases in the load column space are stepwisely selected, m_f=23. Distributed loads at the basis cases are shown in Fig. 2.

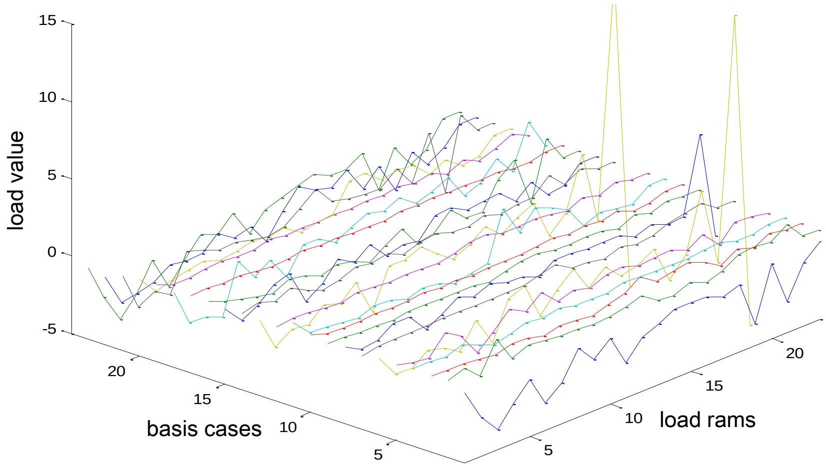


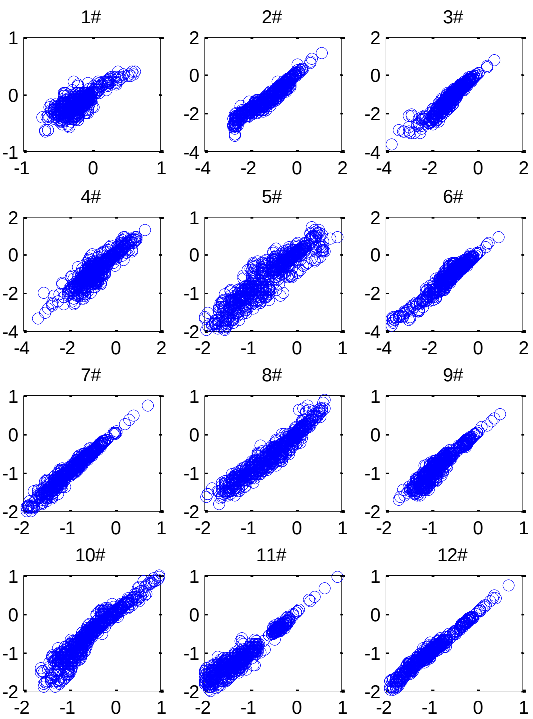
Figure 2 – Distributed loads at 23 load basis cases

Then, based on the candidate strain matrix [$\boldsymbol{A}_{116,749}$], using the method described in Section 3.2, 82 strain basis cases are stepwisely selected, m_ε=82, which included the 23 load basis cases. Then, the influence coefficient matrix [$\boldsymbol{A}_{116,82}$] and the basis load matrix [$\boldsymbol{P}_{24,82}$] are determined.

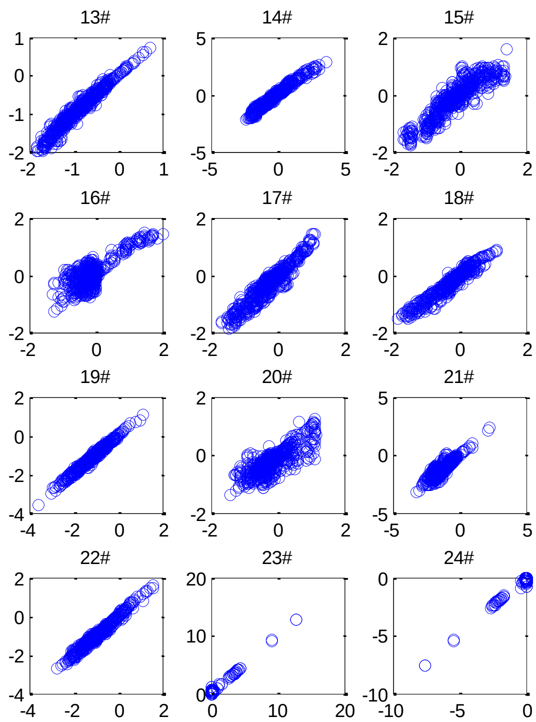
Assuming that the distributed loads of 749 test cases is unknown, only the strain distribution {$\boldsymbol{\varepsilon}_i$} at each case is known, and strain measurement errors are introduced according to the following three conditions: no error, 5% random error and 10% random error. According to Equations (1) and (2), conventional least squares method and Tikhonov regularization method are used to predict the load rams of 749 load cases, respectively. The overall load of sub-components (wing root shear/bending moment/torque, control surface shear/hinge moment) is further calculated according to position of the actuators, and the predicted load values are compared with the actual values to verify the prediction accuracy and robustness of Euclidean Space method.

5.1 No measurement errors

Assuming that there is no error in the strain measurements, the above steps are used to carry out the distributed load recovery. As shown in Fig. 3, the load prediction values of 24 load rams using the conventional least square method are compared with the actual values. Each data point represents a load case, with the x-axis being the actual value and the y-axis being the predicted value.



（a）1#~12#



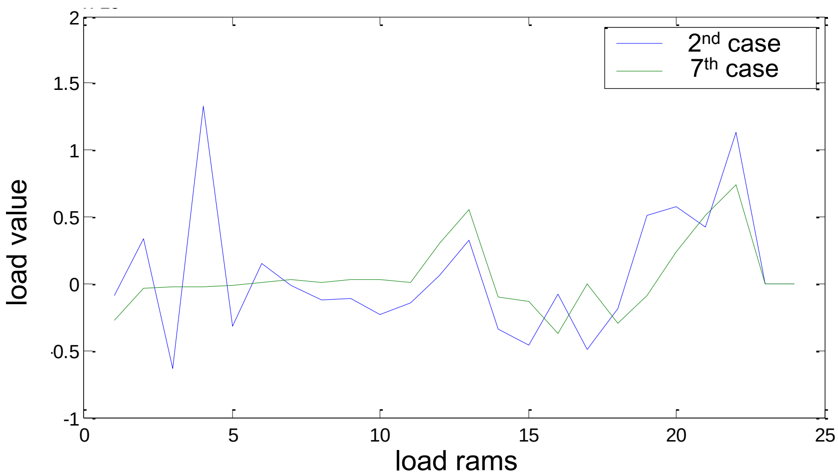
（b）13#~24#

Figure 3 – Load prediction comparisons of 24 load rams

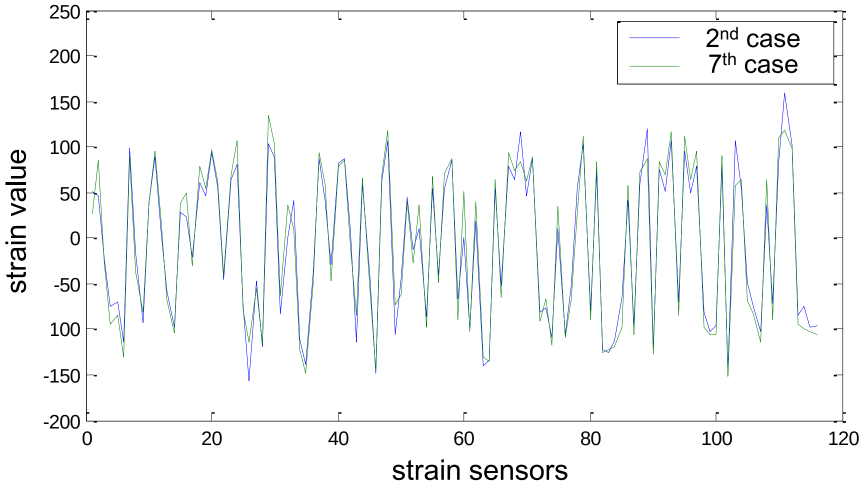
It can be seen that, except for 4#, 5#, 15#, 16#, and 20# load rams, the load prediction accuracy of most load rams are pretty high. Among them, 4# and 5# are close to the root of the wing, 15# and 16# are located on the aileron, and 20# is located on the leading edge flap.

The poor prediction accuracy of control surfaces load (15#, 16#, and 20#) is in line with expectations, because the optical fiber sensors are only arranged on the main beams, and these strain measurements are not responsive to the control surfaces load.

The poor prediction accuracy of 4# and 5# load rams may be due to that different distributed load will cause similar strain responses. As shown in Fig.4, the load distribution of the 2nd and 7th basis cases is obviously different, but the strain distribution is close. This situation is inevitable. Although the prediction accuracy of each loading points cannot be guaranteed, the overall component load (such as shear/bending moment/torque at the root of the wing) and overall strain distribution can be accurately reflected.



（a）load distribution



（b）strain distribution

Figure 4 – Different distributed loads cause similar strain responses

The overall loads of sub-components at 749 load cases are as shown in Fig. 5, with the x-axis being the actual value and the y-axis being the predicted value. It can be seen that the prediction accuracy of overall load is very high, although the prediction accuracy of some load rams is limited.

Since the strain space dimensionality is sufficient in this example, the accuracy improvement of regularization method is not obvious, so only the results of least square method are listed here.

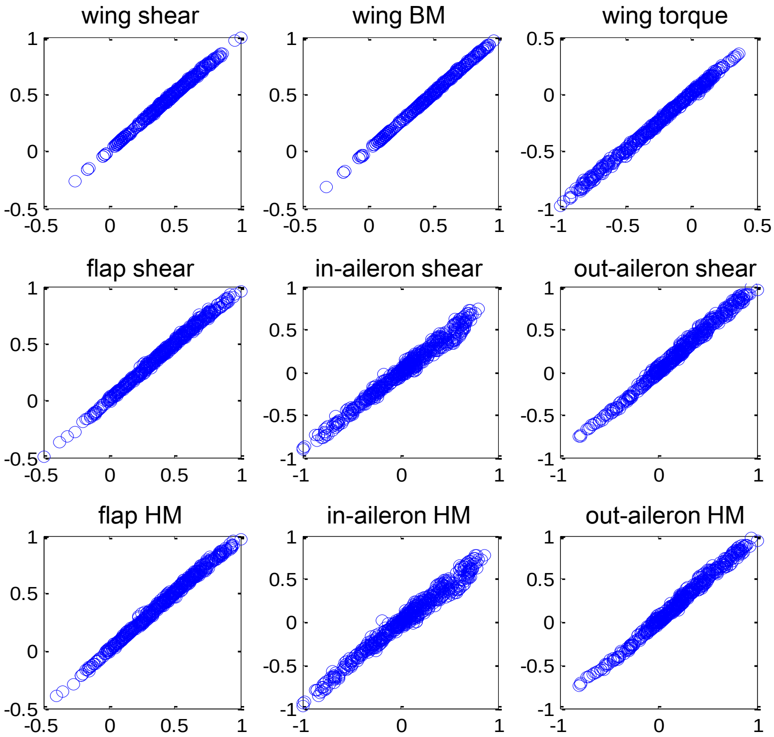


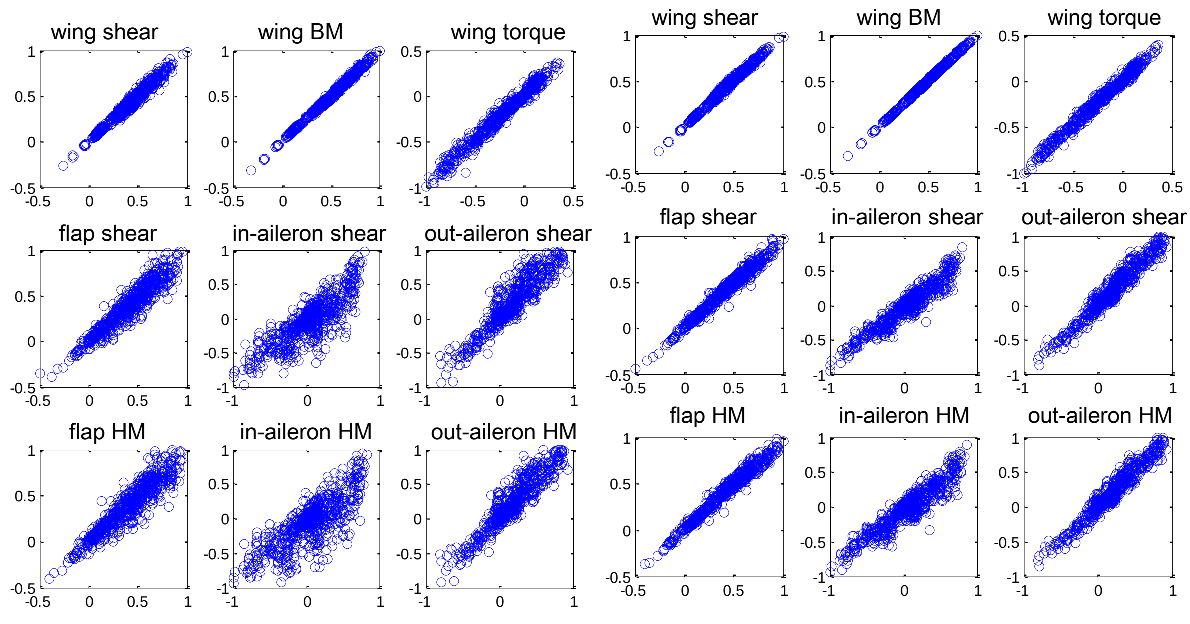
Figure 5 – Sub-component load prediction comparisons (least square method)

5.2 Random measurement errors

Assuming a random error of 5% and 10% is introduced in the strain measurements, Euclidean space method are used to carry out the distributed load recovery. Through comparison, the generalized cross-validation (GCV) method is used to determine the Tikhonov regularization parameters.

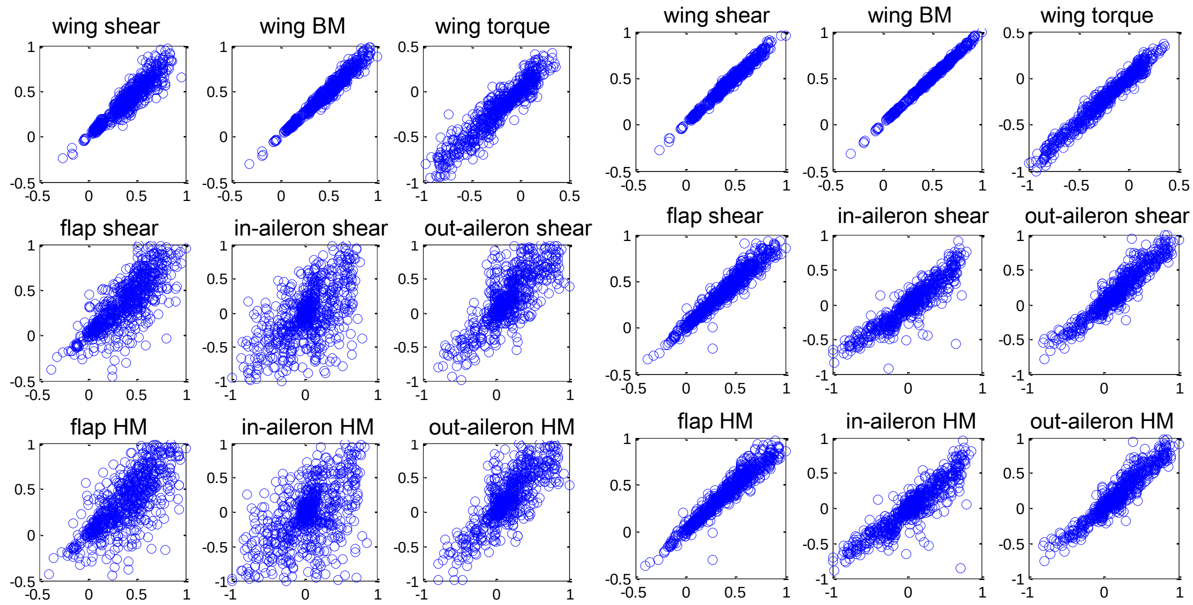
Given the limited space available, only the sub-component overall load comparison is drawn here, as shown in Figs. 6~7. Each data point represents a load case. It can be seen that

- For the conventional least square method, with the increase of measurement error, the load estimation accuracy becomes worse rapidly. When the random error is 10%, the accuracy is unacceptably poor.
- For the Tikhonov regularization method, although the estimation accuracy also become worse with the increase of measurement error, the accuracy reduction is slow, which indicates that the regularization method can effectively weaken the influence of strain measurement error and greatly improve the accuracy and robustness of load estimation.



(a)least square method (b)Tikhonov regularization method

Figure 6 – Sub-component load prediction comparisons with introduction of 5% random measurement error



(a)least square method (b)Tikhonov regularization method

Figure 7 – Sub-component load prediction comparisons with introduction of 10% random measurement error

6. Conclusions

Aiming at distributed load recovery from strain measurements, a stepwise method based on Schmidt's orthogonalization of the maximum vertical distance is proposed to select the basis cases from Euclidean space, and a complete set of engineering feasible distributed load recovery approach is established. The details are summarized as below:

- Compared with point loads method and load basis function method, the Euclidean space method proposed in this paper can select linearly independent basis cases from the design load cases. The basis case is a more realistic load distribution and it is easy to apply in the ground test.
- The criterion is proposed that strain basis cases selected from the strain space should include the load basis cases in the load space to ensure the load estimation accuracy.

- It is verified that Tikhonov regularization method can effectively alleviate the ill-conditioning of the inverse matrix equation and weaken the influence of the strain measurement error.
- It is proved that the inverse approach based on Euclidean space proposed in this paper is feasible and has high prediction accuracy and robustness, which provides a reliable approach for flight load identification of structure health monitoring.

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8. Copyright Statement

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