

# STUDY ON THE FATIGUE SCATTER FACTOR UNDER SEVERE LOAD SPECTRUM

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#### **Abstract**

Fatigue scatter factor is the life reliability index of aircraft structure under load spectrum, and it's used to describe the reliability of fatigue analysis and test results. The military aircraft structural strength specification (GJB67.6-2008) stipulates that when the average load spectrum is used for fatigue analysis, the scatter factor is generally 4, corresponding to the reliability level of 99.87%; when the severe load spectrum is used for fatigue analysis, the scatter factor is 2~4, but the specific value is not clear. This research refers to a large number of relevant data, assuming that the fatigue damage caused by the load spectrum and the critical damage value of the structure obey log-normal distribution, and the scatter factor is theoretically deduced from the perspective of probability statistics, and the fatigue scatter factors of structural life under different reliability levels are given. The differential fatigue scatter factors of typical structural details are further determined, which lay the foundation for military aircraft structural fatigue design and full-scale fatigue testing using severe load spectrum.

Keywords: Severe Load Spectrum; Fatigue Scatter Factor; Reliability; Fatigue life

### 1. Introduction

The military aircraft structure strength specification (GJB67A-2008) clearly stipulates that the design usage load spectrum for durability analysis and test should be compiled to reflect the severe usage of the aircraft, so that 90% of the aircraft in the fleet are expected to meet the design service life. Compared with the load spectrum compiled based on the average usage, the severe spectrum has a higher damage rate, and a lower fatigue scatter factor can be used in analysis and test. The fatigue scatter factor is the life reliability index of the aircraft structure under the load spectrum [1,2]. When the average load spectrum is used for fatigue analysis, if the material fatigue performance data is accurate and corresponds to the crack initiation, with appropriate analysis method, the scatter factor is generally 4, corresponding to the reliability requirements of 99.87%. When using severe spectrum for fatigue analysis, the scatter factor is between 2 and 4, but the specific value is not clear in the structural strength specification [3].

Aviation developed countries have been trying since the 1980s to compile load spectrum with different reliability levels for different aircrafts. For example, in 1998, the P-3C aircraft used an 85% reliability spectrum for life evaluation; F/A-18C/D used 3 $\sigma$  design spectrum (corresponding to a reliability of 99.87%). When evaluating life of F/A-18E/F, it was recommended to take a spectrum with a reliability of 90%. Subsequently, the United States added improvement of reliability of the load spectrum into JSSG-2006 and MIL-STD-1530C, which was successfully applied in the full-scale fatigue test of F/A-18-E/F and F-35. Among them, the fatigue scatter factor used in F-35 durability analysis was reduced to 2.67 [4], corresponding to durability test of 2 lifetimes [5]. The durability test began to expose structural fatigue problems in 2000 flight hours, which save the time to optimize the structure design. Meanwhile, damage caused by the potential risks of serving too many aircraft with fatigue problems was also avoided.

With the development and application of individual-aircraft tracking data and structural health monitoring technologies, the acquisition of flight parameters/loads used by aircraft in service has become more comprehensive and realistic [6-8]. Through flight parameter/load big data analysis,

not only the overall average statistics can be obtained, but also a more accurate distribution law can be deduced, making it possible to obtain load samples under different reliability levels, so as to compile high-reliability load spectrum. Based on the individual-aircraft load spectrum obtained by the individual-aircraft tracking, the stress spectrum of the typical structures are further obtained, and the damage distribution law of the individual-aircraft local load spectrum is studied, and the load spectrum dispersion of different structures can be obtained, and then the differentiated fatigue scatter factor is determined for the structure.

# 2. Life reliability and load spectrum reliability

Life reliability requirements are based on the safety considerations of the fleet. It is required to consider the inevitable differences in the usage and structure of the individual-aircraft of the fleet. The life of 99.87% or 99.9% airplanes can meet the design life requirements, i.e. the life reliability level is 99.87% or 99.9%.

Life reliability requirements are put forward by combining the load spectrum and structural differences, and there is no clear and independent requirement for the reliability of the load spectrum or the structure in the specification. Some researchers [9] proposed that the fatigue scatter factor is the product of the load-independent scatter factor and the structure-independent scatter factor. If the scatter factor is taken separately, it is necessary to independently provide the reliability requirement index for the load spectrum and the structure. Some researchers believe that the reliability of the load spectrum can be 90% (without consideration for 10% of the aircraft usage), and the reliability of the structure can be 99.9%, but no matter how comprehensive this situation is, it is impossible to achieve life of 99.9% reliability. This research is based on the comprehensive indicators of the load spectrum and structure, and do not separate them, but because the focus is on the load spectrum, this study adopts a recommended average value for the dispersion of the structure.

Load spectrum reliability means that the load spectrum used in life assessment is at the level of all possible load spectrum (or fleet usage). It is generally measured according to the load level and load frequency, which directly reflect the damage of the load spectrum. In order to study the dispersion of fleet usage, researchers have introduced various probability models, among which the lognormal distribution and Weibull distribution are the most commonly used. In this research, lognormal distribution model is used to describe the dispersion of load spectrum in the fleet. The positions of the load spectrum with different reliability levels relative to the damage distribution of the fleet are shown in Figure 1.

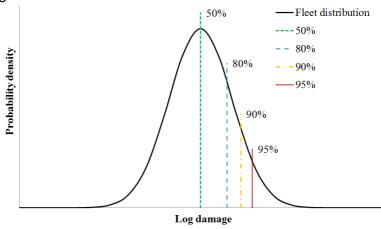


Figure 1 – Damage levels of load spectrum with different reliability levels

The log-normal distribution is expressed as:

$$f(x) = \frac{1}{\sigma x \sqrt{2\pi}} \exp\left\{-\frac{(\ln x - u)^2}{2\sigma^2}\right\}$$
 (1)

Where, x is the damage of individual-aircraft load spectrum,  $\mu$  is logarithm mean of the damage of fleet load spectrum, and  $\sigma$  is logarithmic standard deviation of the damage of fleet load spectrum.

## 3. Damage distribution of fleet load spectrum

There are certain differences in the load history of each aircraft structure in the fleet. From the perspective of possible damage to the structure, the differences in the usage of individual aircraft in the fleet are as follows: 1) the difference in load level; 2) the difference in load frequency; 3) the difference in order of maneuvers. To study the distribution law of the load in these aspects, the actual usage load history of the fleet is necessary, and then the reliability analysis method of probability statistics is adopted for analysis.

Based on the large-scale usage data of aircraft in service, this research calculates the structural component load according to the flight data based load equation, obtains the load history of various maneuvers, and counts the distribution law of the load level and frequency of different missions and maneuvers, and then calculate the structural damage under the individual aircraft load spectrum, and further obtain the damage distribution law through statistical analysis.

Based on the statistical test of skewness and peak degree, it is found that the fatigue damage of wing load spectrum under various maneuvers approximately obeys the log-normal distribution, as shown in Figure 2.

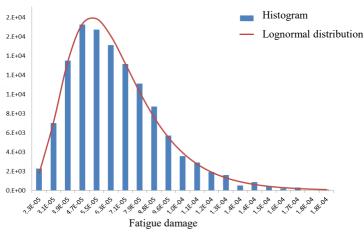


Figure 2 – Damage distribution of wing bending moment spectrum of typical maneuvers

## 4. Derivation of scatter factor model

The fatigue scatter factor is mainly affected by two factors, one is the dispersion caused by the difference in structural materials, assembly, and process; the other is the dispersion caused by the difference in the load spectrum of individual aircraft [10-14]. Through the fatigue test researches under average usage spectrum and severe usage spectrum, a large number of literatures found that the dispersion of the structure is not affected by the severity of the load spectrum [15]. A large number of fatigue tests have proved that the fatigue life of the structure under the specified spectrum obeys the lognormal distribution [16,17]. Therefore, studying the influence of load difference on structural fatigue life can be equivalently transformed into studying the difference of fatigue damage under different load spectrum. As mentioned above, based on the statistical analysis of the big data of the individual-aircraft tracking load, it has been concluded that the damage of the fleet load spectrum also obeys the log-normal distribution [18,19].

Let the critical fatigue damage S of the structure obeys  $LN(\mu_s, \sigma_s^2)$ , the damage L of the load spectrum in the fleet obeys  $LN(\mu_t, \sigma_t^2)$ , and the distribution density functions of S and L are:

$$f(s) = \frac{1}{s\sigma_{s}\sqrt{2\pi}}e^{-\frac{(\log s - \mu_{s})^{2}}{2\sigma_{s}^{2}}}$$

$$g(l) = \frac{1}{l\sigma_{l}\sqrt{2\pi}}e^{-\frac{(\log l - \mu_{l})^{2}}{2\sigma_{l}^{2}}}$$
(2)

Then life N of the structure under any load spectrum can be expressed as S divided by L, the unit is load block, and the formula after taking the logarithm is:

$$\log N = \log \frac{S}{L} = \log S - \log L \tag{3}$$

According to the characteristics of the normal distribution, the difference between two independent normal distributions is also a normal distribution. Therefore, under the premise that S and L are independent of each other, N obeys  $LN(\mu_s - \mu_l, \sigma_s^2 + \sigma_l^2)$ , and the distribution density function is:

$$h(n) = \frac{1}{n\sqrt{\sigma_l^2 + \sigma_s^2}} \sqrt{2\pi} e^{-\frac{(\log n - (\mu_s - \mu_l))^2}{2(\sigma_s^2 + \sigma_l^2)}}$$
(4)

The total standard deviation  $\sigma$  and life expectation  $\mu$  are given as:

$$\sigma = \sqrt{\sigma_s^2 + \sigma_l^2}$$

$$\mu = \mu_s - \mu_l$$
(5)

When the load spectrum is specified, the life distribution only contains structure dispersion, and the life distribution is reduced to  $LN(\mu_s-\mu_l,\sigma_s^2)$ . Introducing the severity p of the load spectrum (also known as the reliability of the load spectrum), the fatigue life  $N_p$  under the specified spectrum obeys  $LN(\mu_s-\mu_{l,p},\sigma_s^2)$ , where  $\mu_{l,p}$  is obtained from the damage distribution law of the load spectrum:

$$\mu_{l,p} = \mu_l + \Phi^{-1}(p)\sigma_l \tag{6}$$

In the formula,  $\Phi$  represents the standard normal distribution function, and p is the probability expressed as a percentage. According to engineering practice, the fatigue scatter factor is the ratio of the median life  $N_{50,p}$  of the structure under the specified spectrum to the safe life  $N_{99.87}$  of the fleet (i.e. life reliability is 99.87%). When the reliability is specified, the median life under the spectrum load can be obtained by the formula:

$$N_{50,p} = 10^{\mu_s - \mu_{l,p}} = 10^{\mu_s - \mu_l - \Phi^{-1}(p)\sigma_l}$$
 (7)

The safe life of an aircraft is the 99.87% quantile value of the life distribution of the fleet in consideration of all possible load histories and individual-aircraft structure states. From the lognormal distribution  $LN(\mu, \sigma^2)$  of fleet life, we can get:

$$N_{99.87} = 10^{\mu - 3\sigma} \tag{8}$$

In the equation,  $\sigma$  and  $\mu$  are calculated by Equ. (5).

Therefore, the following equation can be derived:

$$SF_p = \frac{N_{50,p}}{N_{99.87}} = 10^{(3\sqrt{\sigma_i^2 + \sigma_s^2} - \Phi^{-1}(p)\sigma_i)}$$
(9)

The scatter factor of the 90% reliability spectrum is:

$$SF_{90} = \frac{N_{50,90}}{N_{99,87}} = 10^{(3\sqrt{\sigma_i^2 + \sigma_s^2} - 1.28\sigma_i)}$$
 (10)

Based on the above derivation, the relationship between 50% average spectrum and 90% severe spectrum life distribution and scatter factor is shown in Figure 3.

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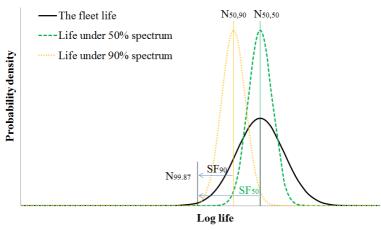


Figure 3 – Comparison of the fleet life distribution and life distribution under specified spectrum

This study collected coupon and element-level fatigue test data during the development of various aircrafts in the past ten years, including 149 groups of tests with various structural details (holes, notches, R-zones, nail connections, etc.). Calculate the standard deviation of the initiation life  $\sigma_s$ , and get  $\sigma_s$  =0.1, as shown in Figure 4.

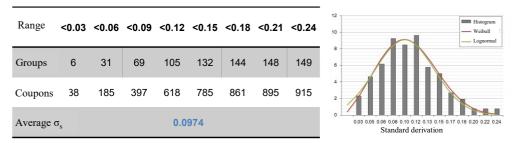


Figure 4 – Statistics of initiation life standard deviation based on coupon fatigue test

The study also collected the individual-aircraft load history of more than 200 aircrafts in service in recent years, calculated the equivalent damage based on normal acceleration  $N_z$ , and calculated the damage standard deviation  $\sigma_l$  of the individual-aircraft load spectrum, and obtained  $\sigma_l$  =0.18, as shown in Figure 5.

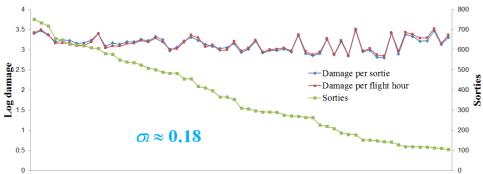


Figure 5 – Statistics of standard deviation of equivalent damage based on  $N_z$  spectrum

Substituting  $\sigma_s$  and  $\sigma_l$  into the above equations, the scatter factor of any severity load spectrum can be calculated. The relationship between reliability and scatter factor is shown as Figure 6.

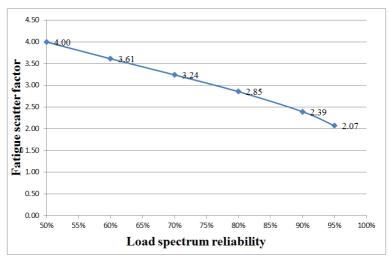


Figure 6 – Reliability and fatigue scatter factor ( $\sigma_s = 0.1$ )

Compared with the 50% average spectrum, the scatter factor of the 90% severe spectrum can be reduced by about 40%. Taking into account the large number of cycles of the severe spectrum, the use of 90% severe spectrum for the full-scale fatigue test can shorten the test duration by at least 30% and help expose the weak parts of the structure in advance.

## 5. Differential scatter factor of various structures

In traditional fatigue analysis and life monitoring of structures, uniform life reliability requirements (99.87%) and fatigue scatter factors are adopted for all critical structures. The advantage of this is that it is simple and easy to manage, but it will cause a lot of structural design waste, and the weight cannot be reduced, and a small part of the structure may be dangerous. Since the usage dispersion of aircraft structure has been concerned, the difference in  $N_z$  spectrum at the center of gravity or wing load spectrum between individual aircraft is mainly used to evaluate the dispersion of fleet usage. The fatigue design of the airframe structure of the new generation of aircraft should be based on refined design philosophies, with different reliability load spectrum and fatigue scatter factors for different structures, and verified by the full-scale fatigue test.

According to the importance of the structure, load characteristics, load transfer path, detectability and repair/replacement cost, the reliability requirements are graded, and the matched fatigue scatter factor is given to carry out more refined structural fatigue design and life monitoring. The main steps are:

- 1) Obtain the individual-aircraft load spectrum based on the flight data of aircraft in service;
- 2) Obtain the stress spectrum of each typical structure based on FEA;
- 3) Calculate the damage of each critical structure under stress spectrum;
- 4) Study the damage distribution law of each type of critical structure and obtain the damage standard deviation;
- 5) Study the fatigue scatter factor of each type of critical structure under different reliability levels.

There are obvious differences in the dispersion of the usage of different structures in the fleet, Table 1 shows the damage standard deviations of four different structural details based on the different components load spectrum. Comparing the 4 types of details, the dispersion of the fleet usage of different structures is obviously different. Therefore, it is unreliable for the fatigue scatter factor of various structures to take a uniform value in the structural durability analysis. In order to ensure the safety, economy and reliability of the structure, it is necessary to fully consider the differences in the usage of different structures in the fleet. Under high-reliability load spectrum, different structures also need to adopt different fatigue scatter factors and make a precise fatigue design to achieve lightweight and long-life design.

Table 1 – Damage standard deviation and scatter factors of different component load spectrum

Spectrum	Details	Standard deviation $\sigma_l$	scatter factors of different severe spectrum( $\sigma_s$ =0.1)		
			50%	80%	90%
spectrum A	Open hole	0.1679	3.86	2.79	2.35
spectrum B	Open hole	0.1468	3.41	2.57	2.21
spectrum C	Fillet	0.0742	2.36	2.05	1.9
spectrum D	Fillet	0.0802	2.42	2.08	1.91

## 6. Conclusion

In this study, the damage distribution law of the load spectrum of the fleet is studied, and the load spectrum dispersion (standard deviation) is determined, and the load spectrum dispersion and structure dispersion are considered comprehensively, a fatigue scatter factor model with the reliability index of the load spectrum is determined, and the damage severity of the load spectrum under different reliability level is given.

The stress/strain spectrum of typical structural details based on the individual-aircraft is derived. Based on analysis and tests, the load spectrum dispersion and damage distribution law of the structure are studied, and then the differential fatigue scatter factor is determined for the airframe.

The study of the scatter factor of severe spectrum lays the foundation for the application of severe spectrum in aircraft fatigue design and full-scale fatigue test.

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