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A NEW METHOD FOR ESTIMATING THE THERMOS-PHYSICAL PARAMETERS OF THERMAL PROTECTION MATERIALS BY ULTRASONIC MEASUREMENTS

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Abstract

A new method for measuring material thermos-physical parameters is presented in this paper by the ultrasonic thermometry. The principle for estimating material thermal properties is based on internal temperature distribution dependence of the velocity of the ultrasonic wave propagating through the heated material. This method combines the advantages of the ultrasonic measurement and heat conduction inverse problem analysis. A numerical model of sensing interior temperature field is obtained by ultrasound. Then, an inverse analysis by the adjoint equation method is developed to estimate material thermal properties such as thermal conductivity and specific heat. The influences of different number of segments of material properties, number of measuring points and measurement error on the prediction results are also studied in detail. The numerical example shows that the presented method can measure the material thermo-physical parameters with temperature non destructively and accurately.

Keywords: ultrasonic measurement, thermos-physical parameters, inverse heat conduction problem, piece-wise model, transient measurement

1. General Introduction

Thermal protection systems (TPS) that are usually made of different materials are required to protect a vehicle re-turning from space or entering an atmosphere [1]. In order to apply reliably and efficiently, the thermal protection materials are well understood and have sufficient test data under the appropriate conditions to provide confidence in their performance [2, 3]. Thermo-physical parameters are important characteristics of materials. They are not only the key parameters to assess whether materials can meet the needs of specific thermal processes, but also the critical parameters for basic research and engineering design of TPS [3]. Thus, rapid and accurate measurement of material thermos-physical parameters is of great significance in the design, evaluation and maintenance of TPS of an aircraft.

Numerous investigations on measuring material thermos-physical parameters are available in open literatures [5-8]. For example, there are so many method proposed to meet the requirements of various working conditions [9-12], such as the plate method, heat flow method, hot wire method, thermal probe method and so on. However, the assumption that the material parameters are constant or in the form of a known function with temperature is introduced in most methods. Besides, the samples are heated to a given temperature or a certain temperature gradient, which would lead to a long experimental time, or large environmental disturbance or very complex control measures. Furthermore, some methods are only suitable for measuring single thermos-physical parameters; at the same time, due to the sensors usually need to contact with the tested part, the measurement range will be limited by the high temperature resistances of the sensors [13-15].

The measurement technology of thermos-physical parameters based on ultrasound has been extensively studied in the prediction of elastic modulus and other mechanic parameters [16-18]. In this paper, the ultrasonic method is introduced into the estimation of thermos-physical parameters of materials such as thermal conductivity or specific heat capacity, which can effectively avoid the above mentioned problems.

2. Principle of ultrasonic thermometry

2.1 Theoretical model

The basic principle of the presented method is that the velocity of ultrasound propagation in the medium is affected by the material temperature, and the temperature distribution and its change with time are closely related to the thermos-physical parameters of the material. Assuming a single side of the structure of a rectangular parallelepiped-shape is uniformly heated, one-dimensional temperature distribution in interior of this structure is shown in Figure 1. By sending an ultrasonic pulse through a medium of known length, the propagation time of ultrasonic wave t_{tof} between the initial pulse and the reflection of the pulse from the opposite end of the medium can be given by [16]

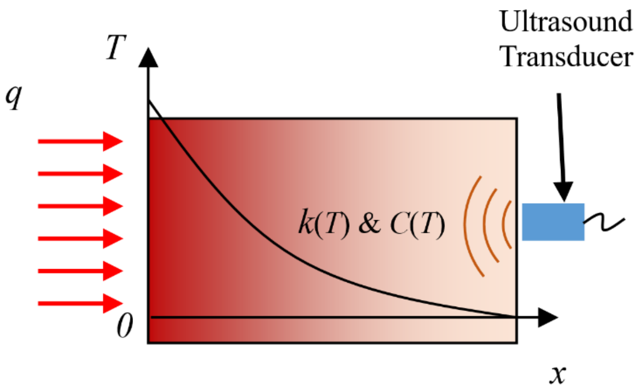


Figure 1-Ultrasonic detection model for thermos-physical parameters of materials.

$$t_{tof} = 2\int_0^L \frac{1}{V(T)} \mathrm{d}x \qquad (1)$$

where x is the direction of the temperature distribution of the heated structure, L is the single propagation distance and $V(T)$ is the acoustic velocity, which is a function of temperature T.

The forward problem of one-dimensional heat conduction problem without internal heat source can be expressed as:

$$C(x,t)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k(x,t)\frac{\partial T}{\partial x}\right) \qquad x \in [0,L], C = \rho c \qquad (2)$$

$$-k(x,t)\frac{\partial T}{\partial x}\Big|_{x=0} = q, \quad -k(x,t)\frac{\partial T}{\partial x}\Big|_{x=L} = 0 \qquad t > 0 \qquad (3)$$

$$T\big|_{t=0} = T_0 \qquad (4)$$

$$t_{tof,m} = t_{tof,ex} + \sigma_{tof,m} \qquad (5)$$

where k is coefficient of heat conduction, and C is thermal capacity of the heated material; ρ is density of medium; q is the thermal flux of heating side; $x = L$ is the other side of a medium, and this side usually is unheated. It is noted that the temperatures of these two side can be determined by the infrared or thermocouple technique. T_0 is initial temperature of a medium. $\sigma_{tof,m} = t_{tof,m} - t_{tof,ex}$ is the error between real values $t_{tof,ex}$ (or namely actual values) and experimental results of propagation time $t_{tof,m}$.

2.2 Optimization model of estimating thermo-physical parameters

Thermal properties of materials at high temperatures can be described as: propagation times of ultrasonic wave and heating conditions of materials are known, the coefficient of heat conduction *k* (*x*, *t*) and thermal capacity *c* (*x*, *t*) are estimated parameters. According to the observation equation, the objective function and constraint conditions are established as

$$\text{Minimize:} \quad J(k,C)=\sum_{i=1}^{n}\{t_{tof.i,c}-t_{tof,i,m}\}^2=\sum_{i=1}^{n}\{2\int_0^L \frac{1}{V[T(t_i)]}\mathrm{d}x-t_{tof,i,m}\}^2 \tag{6}$$

where k,C is the estimated thermo-physical parameters. The real values $t_{tof,ex}$ is replaced as the numerical values $t_{tof,c}$, i and n are ith measurement data and the total number of measurement values.

For a heat conduction problem with no heat source, when the cost function $J(k,C)$ is established, estimating thermo-physical parameters can be considered as solving the following mathematical programming problem

$$\text{Minimize:} \quad J(k,C) \tag{7a}$$

$$\text{Subject to:} \quad \frac{\partial}{\partial x}\left[k(T)\frac{\partial T}{\partial x}\right]=\rho C\frac{\partial T}{\partial t} \qquad x\in(0,L), t>0 \tag{7b}$$

$$T(x,t), t>0, x\in[0,L] \tag{7c}$$

2.3 The conjugate gradient method

The conjugate gradient method (CCM) is one of the most effective algorithm for solving large sparse symmetric positive-definite systems. There are many advantages of this method, such as less storage, fast convergence, and high stability and so on [19-21]. CGM is often implemented as an iterative solution process and is described fully in reference [21].

In order to make the objective function $J(k,C)$ regular, the Lagrange multiplier λ(*x*, *t*) is introduced. Combined with the heat conduction constraint equation, the objective function is obtained as follows

$$J=\sum_{i=1}^{n}[t_{tof,c,i}(k(x,t))-t_{tof,m.i}]^2+\int_0^{t_{\max}}\int_0^L \lambda(x,t)(C(x,t)\frac{\partial T}{\partial t}-\frac{\partial}{\partial x}\left(k(x,t)\frac{\partial T}{\partial x}\right))\mathrm{d}x\mathrm{d}t \tag{8}$$

where $t_{\max}$ is the maximal time of measurement.

According to the variational principle, the adjoint equation is shown as

$$\frac{\partial C(x,t)\lambda}{\partial t}+\frac{\partial}{\partial x}(k(x,t)\frac{\partial\lambda}{\partial x})=(t_{tof,c,i}-t_{tof,m,i})\frac{-4}{V^2}\frac{\partial V}{\partial T}\delta(t-t_i) \tag{9}$$

where *δ* is the Kronecker symbol.

The boundary conditions can be expressed as

$$\left.\partial\lambda/\partial x\right|_{x=0}=0, \quad \left.\partial\lambda/\partial x\right|_{x=L}=0 \tag{10}$$

Thus, the gradient of objective function can be obtained through the equations (8)-(10)

$$\frac{\partial J}{\partial k}\delta k=\int_0^{t_{\max}}\int_0^L \delta k(x,t)\frac{\partial\lambda}{\partial x}\frac{\partial T}{\partial x}\mathrm{d}x\mathrm{d}t \tag{11}$$

$$\frac{\partial J}{\partial C}\delta C=\int_0^{t_{\max}}\int_0^L \delta C(x,t)\lambda\frac{\partial T}{\partial x}\mathrm{d}x\mathrm{d}t \tag{12}$$

Usually, material thermos-physical parameters are the function of temperature, but the form of the function is unknown. $k(T)$ and $C(T)$ are written in the form of piecewise function, that is, the temperature range is divided into n segments, in which k and C are constant

$$k(T)=\sum_{i=1}^{N}k_i\varphi_i(T) \quad , \quad C(T)=\sum_{i=1}^{N}C_i\varphi_i(T) \ , \quad \varphi_i=\begin{cases} 1 \ ; & T\in[T_i,T_{i+1}] \\ 0 \ ; & other \end{cases} \tag{13}$$

Substituting equation (11) into equations (9) and (10), the gradient of the objective function with respect to k_i and C_i are：

$$\frac{\partial J}{\partial k_i}=\int_0^{t_{\max}}\int_0^L \varphi_i(T)\frac{\partial\lambda}{\partial x}\frac{\partial T}{\partial x}\mathrm{d}x\mathrm{d}t \quad , \quad \frac{\partial J}{\partial C_i}=\int_0^{t_{\max}}\int_0^L \varphi_i(T)\lambda\frac{\partial T}{\partial t}\mathrm{d}x\mathrm{d}t \tag{14}$$

The reconstruction algorithm of temperature field based on conjugate gradient method is described as follows

$$k_i^{l+1} = k_i^l - \beta\left(\partial J / \partial k_i\right) \quad , \quad c_i^{l+1} = c_i^l - \beta\left(\partial J / \partial c_i\right) \tag{15}$$

where the subscripts i and j respectively indicate the discreteness of space and time of heat flux , the superscripts l and $l + 1$ denote the iteration steps, and β is the iterative step length. The concrete steps of estimating equivalent thermos-physical parameters based on conjugate gradient method are shown in Fig. 2.

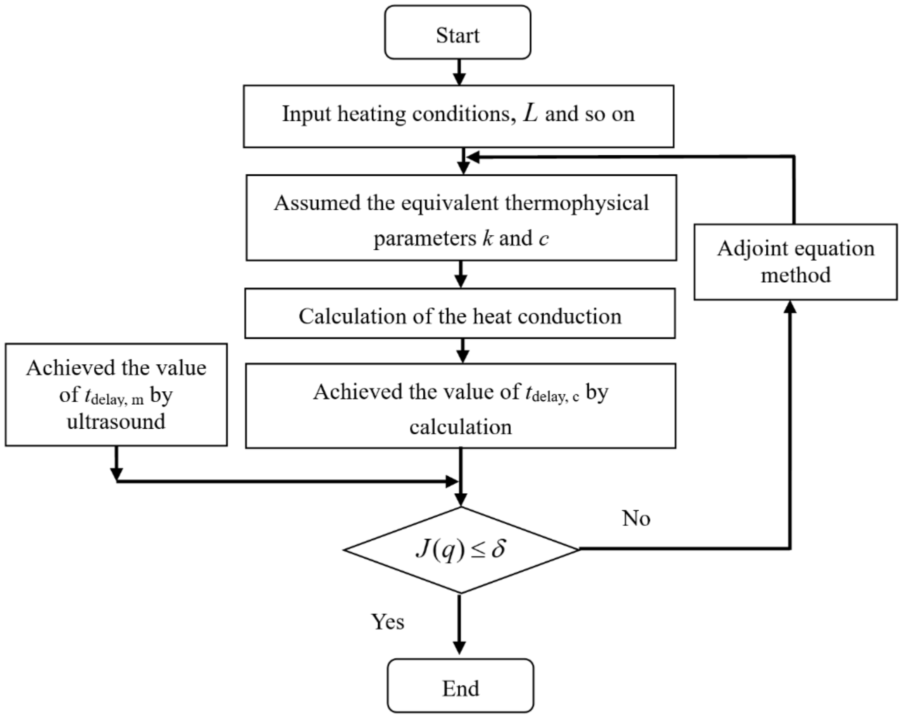


Figure 2-Flowchart for searching the optimum of equivalent thermos-physical parameters.

3. Numerical simulation and discussion

Since there is usually no prior knowledge about the thermos-physical parameters of materials in advance, the thermos-physical parameters can be expressed as piecewise functions varying with position and time in heat transfer model. Thus, this presented approach can provide separate or simultaneous estimations of the functions of thermal conductivity, specific heat and thermal diffusivity, regardless of the waveform of those functions even without a priori information on the kind of dependence concerned.

3.1 Prediction of single thermos-physical parameter

The thermos-physical parameters of a tested material are C = 7800×(654 T + 423.3) (J/℃) and k = 54.0-0.033T (W/(m*℃)) in the range of room temperature to 410, where T is the temperature. The above thermos-physical parameters of materials are obtained by experiments in advance.

Based on the measurement of the variation of bottom echo propagation time (as shown in Figure 1), the inverse problem of thermo-acoustic coupling is solved and relevant equivalent thermos-physical parameters are derived. Figure 3 and Figure 4 show the identification results of *k* and *c* with temperature change, respectively, when 6 subsection functions, 1e-9s accuracy of acoustic time measurement and 100 sampling points in 300 s measurement time range are used. The prediction errors are all less than 1%.

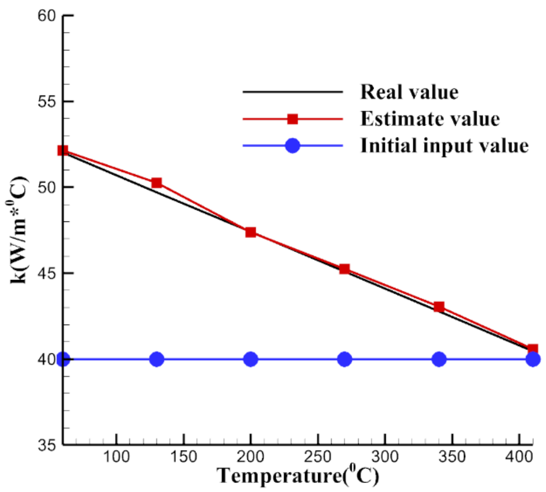


Figure 3-Estimation of coefficient of heat conduction k.

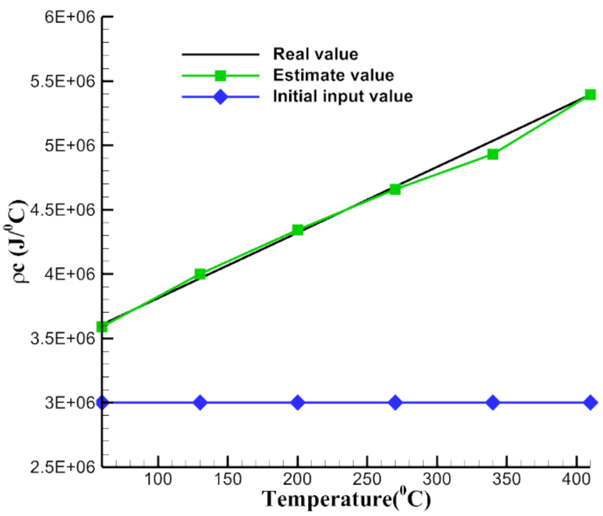


Figure 4-Estimation of thermal capacity C.

Based on the identification of thermal conductivity k, the influences of different number of segments, acoustic time measurement accuracy and the number of measurement points on the prediction accuracy is analyzed.

Figure 5 presents the prediction results of the piecewise function changing with position and time when the number of segments is 4, 5, 6, 7 and 8. Min is the starting point of the piecewise function, Max is the end point, and i is the length of the piecewise function. The results show that the errors of other models are less than 1% except for 4 segments, which indicates that the number of segments has little effect on the prediction accuracy.

In order to obtain better prediction accuracy of thermos-physical parameters, corresponding acoustic time measurement accuracy and sampling points are needed. The results show that when the accuracy of acoustic time measurement is more than 1e-8s and the sampling point N is more than 50, better prediction results of thermo-physical parameters can be obtained, as shown in Figure 6.

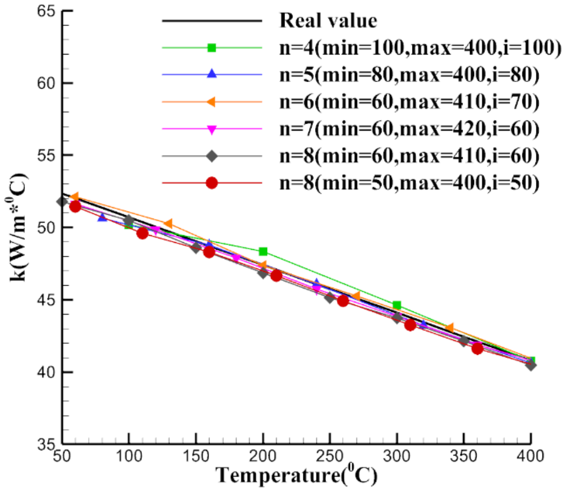
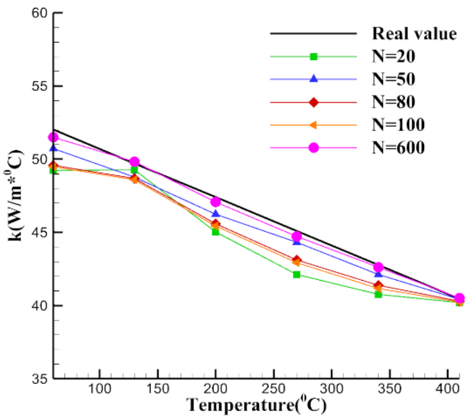
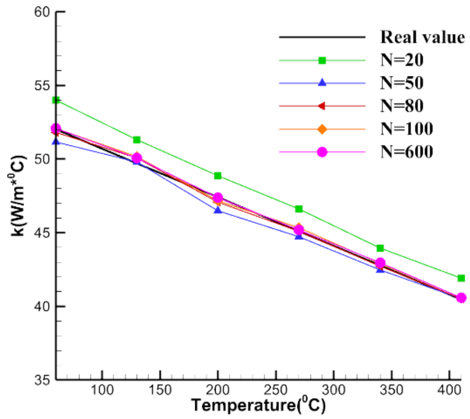


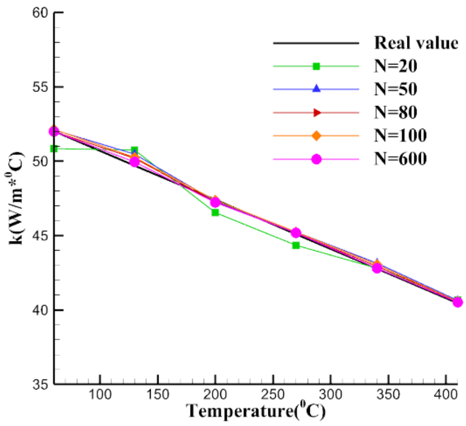
Figure 5-Effects of different piecewise function models on prediction accuracy.



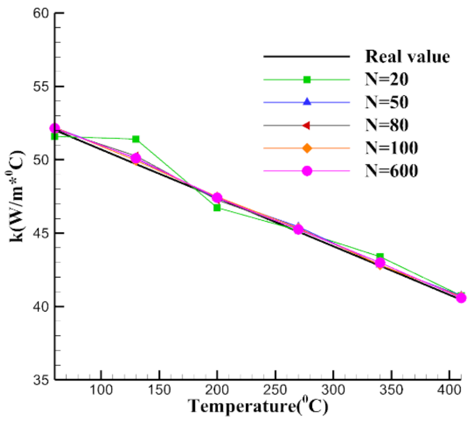
(a) Prediction accuracy of different sampling points under 1e-7s



(b) Prediction accuracy of different sampling points under 1e-8s



(c) Prediction accuracy of different sampling points under 1e-9s



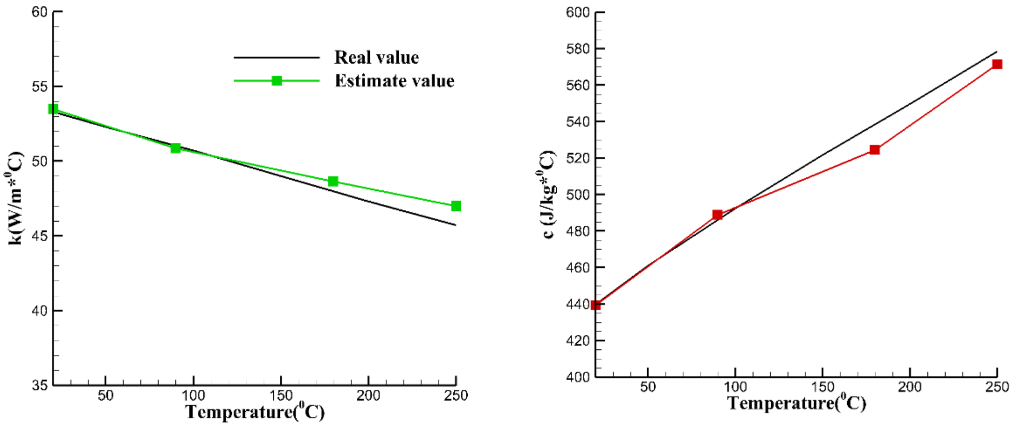
(d) Prediction accuracy of different sampling points under 1e-10s

Figure 6-Effects of different measurement data on prediction accuracy.

3.2 Simultaneous prediction of thermal conductivity and specific heat

Heat conductivity is usually fixed to estimate specific heat in most studies, or specific heat is assumed to determine heat conductivity. However, these two are usually unknown in engineering practice. Thus, it is more practical to carry out the algorithm research of simultaneous prediction of heat conductivity and specific heat. The difficulties of multi-parameter simultaneous identification include: firstly, the sensitivity of each parameter to the objective function is different, and there may be mutual coupling between the parameters, so the calculation process is complicated; secondly, with the increase of inversion parameters, the robustness and efficiency of the algorithm are required to be increased.

Figure 7 shows the results of simultaneously predicting k and C as a function of temperature. 4 piecewise functions, the measurement accuracy of acoustic time is 1e-9s and 100 sampling points are sampled in the prediction model. The prediction errors of k and C are 1.19% and 1.43% respectively. It can be found that the presented method can accurately and effectively predict the thermos-physical parameters simultaneously.



(a) Identification of k (b) Identification of C

Figure 7-Results of simultaneous prediction of thermo-physical parameters.

4. Conclusion

(1) Based on ultrasonic pulse echo measurement and inverse analysis of heat conduction problem, a new method of material thermo-physical parameters is proposed in this paper. The principle is that the velocity of ultrasonic propagation in the medium is affected by the temperature of the material, and the temperature distribution and its change with time are closely related to the thermo-physical parameters. It shows that the advantage of the presented method is that the thermo-physical parameters varying with temperature can be estimated with non-destructive, non-contact and accurate.

(2)Material thermo-physical parameters can be expressed as piecewise functions of coordinates and time. The advantage is that the thermal conductivity and specific heat capacity can be identified individually or simultaneously regardless of whether the functional relationship or prior information of thermo-physical parameters with temperature is known in advance, which has important engineering practical value.

(3) The parameter analysis shows that the number of piecewise functions has little effect on the prediction accuracy. However, in order to obtain better prediction accuracy of thermo-physical parameters, the corresponding acoustic time measurement accuracy and sampling points are demanded.

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7. Copyright Statement

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