32nd Congress of the International Council of the Aeronautical Sciences
September 6-10, 2021
Pudong Shangri-La, Shanghai, China

Research on Trajectory Optimization Technology Considering Meteorological Information Prediction

Wei Guo [1], Ruiyang Zhang [1]

[1] China National Aeronautical Radio Electronics Research Institute

Abstract

In order to cope with the airspace capacity brought by the growth of air transport, the 4D trajectory-based operation will be gradually implemented in accordance with the Aviation System Block Upgrade Plan (ASBU) issued by ICAO. Accurate and efficient aircraft trajectory optimization is an important basis for conducting the above research. At present, relevant researches are mainly aimed at performance optimization in a single flight phase, mostly concentrated in cruise segments, and lack of research on trajectory optimization during full flight phase considering weather information prediction. In this paper, a trajectory optimization algorithm considering weather information prediction is proposed, which mainly includes a meteorological information fusion algorithm and a flight profile optimization algorithm based on energy state method.

Keywords: trajectory based operation; weather information prediction; trajectory optimization

1. General Introduction

The rapid development of the economy has made the global air transport industry more and more prosperous, that is, the number of civil aviation aircraft and the flight flow are gradually increasing. The problems of traffic order maintenance and flight safety caused by flight delays, air traffic jams, etc. are also becoming more frequent. For this reason, flight path conflict detection, arrival and departure sequencing, performance-based operation, and aircraft air traffic control intention have become hot topics in air traffic management research, and accurate and efficient four-dimensional flight path prediction and optimization is the basis of the above research [1].

In short, the Four-dimensional (4D) trajectory refers to the time factor based on the three-dimensional trajectory, that is, the geographical coordinates of the whole process of the flight of the aircraft described by the latitude and longitude combined with the time in the position. Aircraft 4D trajectory prediction and optimization techniques are the basis for managing airspace in high-density aircraft operating environments. It can not only improve the automation of ATC, reduce the physical and mental pressure of ATC controllers, but also improve the safety of aircraft operation, thus ensuring the efficient operation of air traffic flight management. In summary, 4D trajectory prediction and optimization technology research is extremely important. In the 4D trajectory, the horizontal trajectory is generally pre-set by the flight plan to planned and optimized. Vertical optimal trajectory refers to the vertical reference trajectory designed when the given target reaches the optimal value under the combination of one or more performance indicators.

During the 1950s to the 1970s, the main aspect of the study was the calculation based on mathematical theory for the optimization of aircraft vertical section trajectorys. In the 1980s, optimization algorithms based on optimal control theory began to develop and gradually applied to various fields. The optimal control method is developed on the basis of the flight performance optimization theory. It applies the variational theory and the minimum value principle in the optimal control theory, and optimizes a certain performance index by optimizing to obtain the flight time. Required longitudinal flight path.

The basic principle of the energy state method [2-4] is to express the flight performance of the aircraft as the form of energy, and then express the motion of the aircraft according to the principle of mutual conversion between basic potential energy and kinetic energy, and consider the potential energy and kinetic energy during the flight of the aircraft. It can be converted instantly [5]. The energy state method was proposed as early as the 1950s. In the early 1970s, Zagalsky and Schultz used the fuel approximation as the optimization index, and used the energy approximation method to give the trajectory under a given voyage. Firstly, they assume that the thrust is limited in the climbing and descending stages, and use the minimum principle to obtain the corresponding velocity function.

At the same time, in order to meet the comprehensive issues of economy and environmental protection, countries have successively launched Optimized Profile Descents (OPDs) [5-9] and Continuous Descent Approach (CDA) [10] Research to change and optimize the flight path in the aircraft's descent. When the OPDs mode is lowered, the aircraft can continuously descend from the cruising altitude to the end of the descent or the initial approach point without the need to level the flight. In order not to affect the throughput of the airport, RTA requirements are usually added at a measurable waypoint to ensure security at the time of entry. Due to the slow speed/height profile achieved by the slow train in the down process of the OPDs, it is more fuel efficient and cleaner than the conventional step down mode, and can also reduce engine noise. The research on the CDA method in Europe and the United States has developed rapidly. By 2016, more than 100 airports in Europe have successfully implemented continuous descent approach, i.e. the continuous smooth gradient method is adopted when the aircraft descends, which improves the navigation procedure of terminal area approach process, increases the available space above the airport, reduces the frequency of calls between pilots and controllers, and at the same time, reduces the frequency of calls between pilots and controllers. It lays a foundation for flight path design of continuous descent approach mode [11]. Although the domestic research on CDA is relatively lagging behind, in recent years through the continuous exploration of actual simulation and algorithm evaluation, the CDA flight path is applied to different airports in the terminal area operation.

In recent years, the performance of single flight stage or multiple flight stages has been studied mostly, and most of them focus on the cruise section where the performance is easy to optimize. Wu Shufan et al. gave the height, and then used the singular optimal control method to optimize the performance. Finally, the optimization results of cruising with the maximum range and the minimum fuel were given [12]. In order to optimize the straight continuous descent approach (CDA) flight path with constant descent angle,Gong Fengxun et al. transformed the Bolza optimal control problem (OCP) into a non-linear programming problem under different flaps by using the Gauss pseudospectral method, and obtained the time-optimal descent approach flight path [13].

To sum up, the existing vertical profile trajectory optimization research in China is mainly based on the optimal control theory, and then combined with numerical solution to achieve, and there are few studies of vertical profile optimization in full flight process. Therefore, it is of great engineering significance to divide the flight stages, construct vertical profiles by different climbing/descending modes and compare the performance during climbing and descending, and optimize the important stage of performance optimization, that is, the optimization of cruise section using four-dimensional trajectory optimization based on energy state method.

In this paper, a trajectory optimization algorithm considering weather information prediction is proposed, which includes weather information fusion algorithm and flight profile optimization algorithm based on energy state method. Four-dimensional trajectory is predicted based on the flight performance model. The prediction information includes the height, speed, time and remaining fuel at a certain flight path. Based on this prediction information, the vertical profiles with different optimization performance are generated by combining the selected optimization index.

2. Models for 4D Trajectory Prediction and Optimization

The premise of predicting and optimizing the four-dimensional trajectory and analyzing the influencing factors in the whole flight process is to establish all kinds of required models. These basic models mainly include the non-linear full-volume aerodynamic model describing aircraft motion, namely aircraft dynamics and kinematics model and engine model, the basic flight segment model of the whole flight process, and the meteorological model reflecting environmental factors. This chapter mainly elaborates on the description and establishment of these basic models.

2.1 Aircraft dynamics model

2.1.1 Linear Motion of Aircraft Under Combined External Forces

$$\sum F = m\frac{dV}{dt} \tag{1}$$

In the upper model, F is the combined external force of the aircraft, m is the mass of the aircraft, V is the centroid velocity vector of the aircraft.

In the moving coordinate system, right side of the equal sign in Formula (2) is:

$$\frac{dV}{dt} = 1_V\frac{\delta V}{\delta t} + \Omega \times V \tag{2}$$

Where 1_V is unit vector of velocity direction, $\delta V/\delta t$ is relative derivative, Ω is total angular velocity vector.

Resolution of V and Ω in the body axis system is:

$$V = iu + jv + kw \tag{3}$$

$$\Omega = ip + jq + kr \tag{4}$$

Where i, j, k are the unit vectors of three axis of airframe. Resolution of net force $\sum F$ in coordinate axes of airframe is:

$$\sum F = i\bar{X} + j\bar{Y} + k\bar{Z} \tag{5}$$

Force equations can be obtained from (2-3) to (2-6):

$$\begin{aligned} X &= m(\dot{u} + qw - rv) \\ Y &= m(\dot{v} + ru - pw) \\ Z &= m(\dot{w} + pv - qu) \end{aligned} \tag{6}$$

If the total aerodynamic force R_Σ and engine thrust T are decomposed into airframe coordinate axes (F_x, F_y, F_z), combined with decomposition formula of gravity in airframe coordinate axis, force equations can be obtained from (7) as in (8).

$$\begin{cases} \dot{u} = vr - wq - g\sin\theta + \dfrac{F_x}{m} \\ \dot{v} = -ur + wp + g\cos\theta\sin\phi + \dfrac{F_y}{m} \\ \dot{w} = uq - vp + g\cos\theta\cos\phi + \dfrac{F_z}{m} \end{cases} \tag{7}$$

2.1.2 Angular Motion of Aircraft under External Torque

The angular motion of an aircraft subjected to external moments is expressed as follows:

$$\sum M = \frac{dL}{dt} \tag{8}$$

In the formula, M represents the external moment on the body and L represents the momentum moment on the body.

There are the following relationships in the airframe system:

$$\frac{dL}{dt}=1_L\frac{\delta_L}{\delta_t}+\Omega\times L \tag{9}$$

Where 1_L is the unit vector along the direction of momentum L, δ_L/δ_t is relative derivative, Ω is relative total angular velocity , $\times$ is the sign of cross product.

The theorem of momentum moment in mechanical system is as follows：

$$L=\int dL=\int(r\times V)\delta_m \tag{10}$$

Where r is radius vector, V is velocity vector.

Since the plane is symmetric about the plane Oxz ($I_{xy}=I_{zy}\equiv 0$), substitute $r=ix+jy+kz$, $\Omega=ip+jq+kr$ and $V=\Omega\times r$ into the above formulas, it is obtained that the component of the momentum moment L in the three axes of the body is:

$$\begin{bmatrix}L_x\\L_y\\L_z\end{bmatrix}=\begin{bmatrix}pI_x-rI_{xz}\\qI_y\\rI_z-pI_{xz}\end{bmatrix} \tag{11}$$

And because:

$$\begin{aligned}1_L\frac{\delta_L}{\delta_t}&=i\frac{\delta_{Lx}}{\delta_t}+j\frac{\delta_{Ly}}{\delta_t}+k\frac{\delta_{Lz}}{\delta_t}\\&=i(\dot{p}I_x+p\dot{I}_x-\dot{r}I_{xz}-r\dot{I}_{xz})+j(\dot{q}I_y+q\dot{I}_y)+k(\dot{r}I_z+r\dot{I}_z-\dot{p}I_{xz}-p\dot{I}_{xz})\end{aligned} \tag{12}$$

Assume that the aircraft is a rigid body of constant quality, so:

$$1_L\frac{\delta_L}{\delta_t}=i\frac{\delta_{Lx}}{\delta_t}+j\frac{\delta_{Ly}}{\delta_t}+k\frac{\delta_{Lz}}{\delta_t}=i(\dot{p}I_x-\dot{r}I_{xz})+j(\dot{q}I_y)+k(\dot{r}I_z-\dot{p}I_{xz}) \tag{13}$$

$$\Omega\times L=\begin{vmatrix}i&j&k\\p&q&r\\L_x&L_y&L_z\end{vmatrix}=i(qL_z-rL_y)+j(rL_x-pL_z)+k(pL_y-qL_x) \tag{14}$$

Decomposing the external torque in the three axes of the Airframe system:

$$\sum M=i\bar{L}+jM+kN \tag{15}$$

From the above equations, the angular motion equations of the aircraft under the external moment are obtained in the body coordinate system as:

$$\begin{cases}\bar{L}=\dot{p}I_x-\dot{r}I_{xz}+qr(I_z-I_y)-pqI_{xz}\\M=\dot{q}I_y+pr(I_x-I_z)+(p^2-r^2)I_{xz}\\N=\dot{r}I_z-\dot{p}I_{xz}+pq(I_y-I_x)+qrI_{xz}\end{cases} \tag{16}$$

The following torque equations can be obtained from the above formula:

$$\begin{aligned}\dot{p}&=(c_1r+c_2p)q+c_3\bar{L}+c_4N\\\dot{q}&=c_5pr-c_6(p^2-r^2)+c_7M\\\dot{r}&=(c_8p-c_2r)q+c_4\bar{L}+c_9N\end{aligned} \tag{17}$$

where $c_1=\frac{(I_y-I_z)I_z-I_{xz}{}^2}{\Sigma}$, $c_2=\frac{(I_x-I_y+I_z)I_{xz}}{\Sigma}$, $c_3=\frac{I_z}{\Sigma}$, $c_4=\frac{I_{xz}}{\Sigma}$, $c_5=\frac{I_z-I_x}{I_y}$, $c_6=\frac{I_{xz}}{I_y}$, $c_7=\frac{1}{I_y}$, $c_8=\frac{(I_x-I_y)I_x-I_{xz}{}^2}{\Sigma}$, $c_9=\frac{I_x}{\Sigma}$, $\Sigma=I_xI_z-I_{xz}{}^2$.

2.2 Meteorological Forecast Model

When aircraft performance model is used for trajectory prediction, the overall performance of data can effectively improve the accuracy of prediction, and meteorological data is an important factor. Meteorological model mainly includes wind and temperature modeling. If the influence of wind and temperature can be considered in the prediction, the accuracy of prediction information will be greatly improved.

In the aircraft dynamics model, wind speed and temperature are projected to the body coordinate system by direction consine matrix, which reflects the changes of aircraft force, moment and attitude caused by the changes of external environment (wind speed and temperature). Therefore, the actual meteorological data during the flight will have a certain deviation from the grid prediction value, and the aircraft itself has weather radar can detect meteorological information within a certain distance in the real time. Therefore, the simulation system needs to be able to simulate the actual situation, that is, to predict and update the meteorological information of a certain distance in the remaining segment, and to improve the accuracy of 4D trajectory prediction.

2.2.1 Acquisition and use of real weather data

The meteorological data used in this paper are from the database of the European Meteorological Center. The ERA-Interim (Jan 1979-present) database is the latest data developed by ERA. It provides global meteorological data from January 1979 to January 2017. The advantages of this database are high accuracy, complete variables, fast updating speed and capability. Provide the required wind speed and temperature information. The corresponding meteorological information image can be viewed by opening the data file under matlab, as follows:

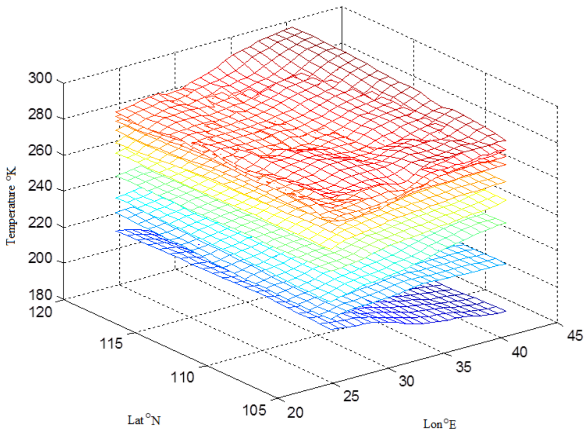

Figure 1 – Temperature Information

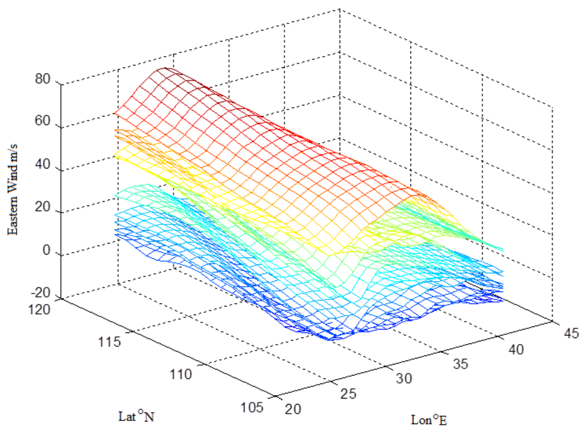

Figure 2 – East Wind Information

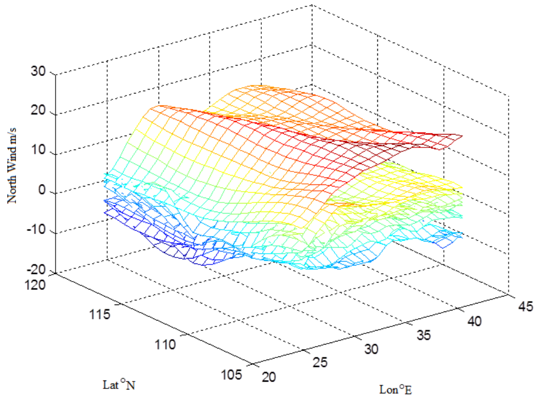

Figure 3 – North Wind Information

The above are the distribution and value of the temperature, east wind and north wind with the pressure plane change in the range of 0:00 on December 31, 2016. The size of the grid is 10 air pressure planes at equal intervals within the range.

After the mesh data is obtained, the wind speed value and the temperature value at the point related to the trajectory prediction and the aircraft flight are calculated by spatial three-dimensional linear interpolation. The principle of the three-dimensional linear interpolation look-up table is shown in the figure below:

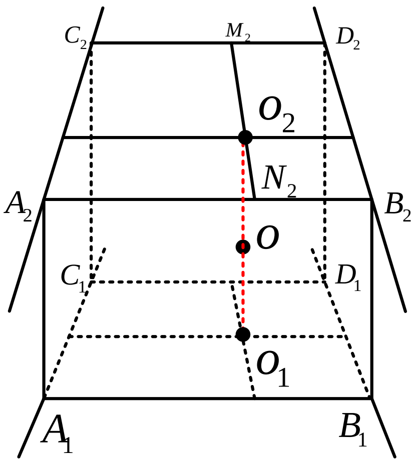

Figure 4 – Three-dimensional Interpolation Table Principle

Now we need the meteorological data of the O point. First, find the grid where the requested point is located in the grid, that is, find the latitude and longitude coordinates and corresponding data values of the eight boundary points of the grid where it is located. This part is realized by looking up the table. Where O_1 and O_2 are the projection points of the O points on the upper and lower surfaces of the grid, respectively, the latitude and longitude coordinates are the same as O, and then the two-dimensional interpolation of the latitude and longitude coordinates is performed on the upper and lower surfaces respectively. The process is as follows: the latitude, longitude and meteorological data of the C_2 and D_2 points are known, you can use the one-dimensional interpolation function to find the meteorological data at point O:

$$u_{M_2} = u_{C_2} + \left(u_{D_2} - u_{C_2}\right)\frac{x_{M_2} - x_{C_2}}{x_{D_2} - x_{C_2}} \tag{18}$$

The variable u is meteorological data (temperature, east wind, north wind), and the variable x is the spatial position coordinate (latitude and longitude). Similarly, the meteorological data of the N_2

point can be obtained, and the one-dimensional linear interpolation between the N_2 and M_2 points can be used to obtain the meteorological data of the O_2 point. The wind data of the point O_1 can be obtained by the same method. Finally, the one-dimensional linear interpolation of height between O_1 point and O_2 point can be used to obtain the meteorological data of O point based on grid. There are two main parts in the interpolation process: first, find the grid boundary of the location, and then do one-dimensional linear interpolation based on different coordinates seven times. The size and direction of the position wind are obtained by using the following formula:

$$V_{wind} = \sqrt{u^2 + v^2} \tag{19}$$

$$\varphi_{wind} = \begin{cases} \arctan(u/v) & u>0, v>0 \\ 180 - \arctan(u/(-v)) & u>0, v<0 \\ 180 + \arctan(u/v) & u<0, v<0 \\ 360 + \arctan(u/v) & u<0, v>0 \end{cases} \tag{20}$$

Where V_{wind} is wind speed, φ_{wind} is wind direction. In the North-South direction, the south wind (from south to north) is positive; in the East-West direction, the west wind is positive (from West to east).

2.2.2 Meteorological Information Prediction

In the research of flight path prediction based on aircraft dynamics and kinematics, accurate data support can improve the validity of prediction results, and meteorological data is one of the important data. The influence of wind data in meteorological data on flight path prediction is particularly important. If there is no accurate wind speed and direction, the accuracy of four-dimensional flight path prediction will be reduced. In the simulation process, because the simulation system has a meteorological perception input window, users can input actual wind speed and direction which deviate greatly from the predicted value, so there will be some deviation between the actual flight meteorological data and the grid forecast value. Without correction, the accuracy of ETA calculation and four-dimensional trajectory prediction will be reduced. It is necessary to have a practical method to predict the meteorological data of the remaining flight segments according to the real meteorological data of the flight segments, so as to reduce the adverse effects of meteorological errors on the trajectory prediction in a certain range.

Because the available data for meteorological prediction are only the grid forecast value and the real value of the flight segment, the real meteorological value is a dependent variable and the grid data is an independent variable. The online sample updating multiple linear regression prediction method is used to predict the real meteorological data of a certain distance in the remaining flight segment.

Taking temperature prediction as an example, if the real temperature value is y, the forecast data t、u、v is x_1、x_2、x_3. $(y_i, x_{1i}, x_{2i}, x_{3i})$, $i=1,2,3\cdots n$ is a set of samples composed of the forecast data and the real temperature value, then the regression model can be expressed as follows:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & x_{11} & x_{21} & x_{31} \\ 1 & x_{12} & x_{22} & x_{32} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_{1n} & x_{2n} & x_{3n} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_n \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix} \tag{21}$$

Set $Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$, $X = \begin{bmatrix} 1 & x_{11} & x_{21} & x_{31} \\ 1 & x_{12} & x_{22} & x_{32} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_{1n} & x_{2n} & x_{3n} \end{bmatrix}$, $\beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_n \end{bmatrix}$, $\varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$, Then the least squares estimate of regression parameter B is： $\hat{\beta}=\left(X^T X\right)^{-1} X^T Y$ ， if a point outside the sample is given $x_f=\left(1, x_{1f}, x_{2f}, x_{3f}\right)$， The predicted value of dependent variable y_f is： $\hat{y}_f=x_f\hat{\beta}$.

Generally, the traditional multivariate regression analysis prediction relies on large sample data and adopts a fixed regression prediction model. However, the data of each sample point in the flight of an airplane changes rapidly, and the data differs greatly. Moreover, the computer calculation speed is limited and it is impossible to use large sample for real-time calculation. Therefore, it is proposed to use five sample points to form the whole sample. On-line updating of the prediction model with the change of aircraft position can ensure the accuracy of prediction within a certain distance, without a large amount of data and computation, and can timely respond to the new changes in atmospheric image data.

The simulation conditions are as follows: the grid data at 0:00 Universal Time on December 31, 2016 is the real value, and the grid data at 18:00 is the forecast value. The position difference between each sample point is 0.75 longitude. The first five bats accumulate the sample number. After reaching the fifth bat, the analytic prediction model is used to solve the prediction value of the forward position point at this time, and the temperature, temperature and temperature.

From the simulation results, it can be seen that the regression model of five sample points has a good prediction effect for the location points within 2-3 sample distances from the current position, and the predicted value tends to approach the real value, especially when the real value is different from the predicted value. When the prediction point is far away from the current position, the predicted value becomes unreliable due to the decrease of data correlation and tends to diverge away from the real value. This is the inherent defect of the model, and the effective prediction distance is limited when the number of samples is limited. It can be seen that the regression prediction model established by this method can reduce the prediction error within a certain range, make the weather forecast more accurate for the airway points closer to the current location, reflect the reasonable response of the meteorological model to the user's input meteorological data, and simulate the actual situation of the meteorological management system to the sudden gas. Response and treatment of image changes to minimize meteorological errors.

A point about the prediction of multiple linear regression: For the general problem of multiple regression prediction, first of all, we need to judge the correlation and linear relationship between the selected independent variables and dependent variables, and determine that the corresponding prediction model can be established; after obtaining the least square estimation of the parameters, we also need to test and evaluate it. Price, to determine whether the model can be applied. Under the research background of this topic, the prediction of meteorological data is not the focus of attention, but is used to simulate the treatment and response of meteorological management system to external meteorological abrupt changes, to ensure that the simulation system has a certain coordination and response ability under the changing meteorological conditions, reflecting the authenticity of the simulation process. Therefore, the test and evaluation of the multiple linear regression prediction model are omitted and not reflected in the simulation system.

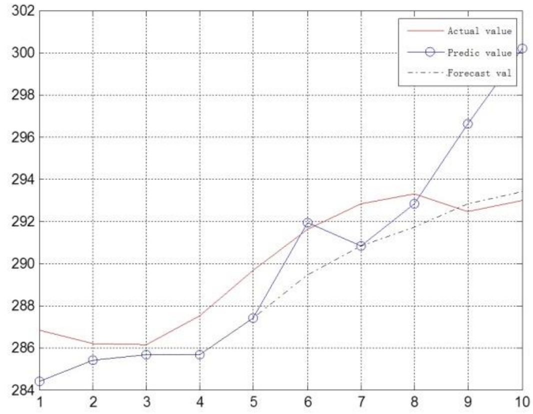

Figure 5 – Temperature prediction value (in k)

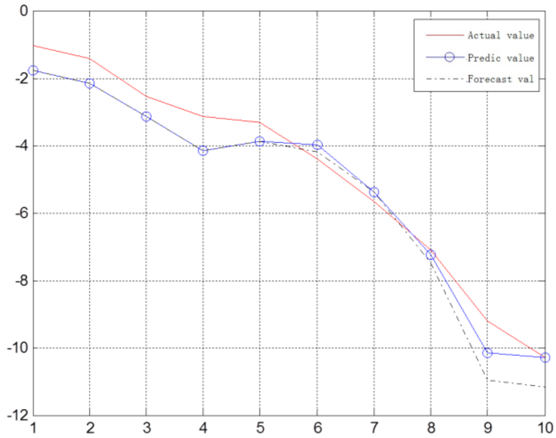

Figure 6 – East Wind Prediction Value(m/s)

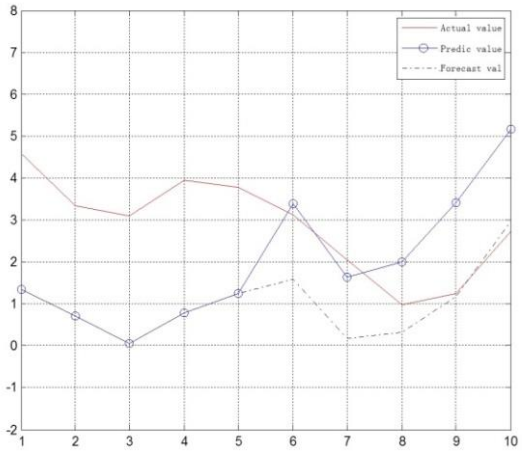

Figure 7 – North Wind Prediction Value

3. 4D trajectory Optimization Based on Energy State Method

4D trajectory is generated by adding time requirement on the basis of 3D trajectory. It can be divided into horizontal trajectory and vertical trajectory. Horizontal trajectory is usually connected by a given series of coordinates of route points. Vertical trajectory is generally composed of velocity/altitude profile. Because horizontal trajectory follows the route given in flight plan. Latitude determination, therefore, this paper focuses on the optimization algorithm of vertical trajectory. If there is no requirement, the 4D trajectory is generated according to the unconstrained optimal trajectory, the cost function of the aircraft is defined, and the velocity and height profiles are generated by the method of dynamic programming in the optimal control. If there is a requirement, the relative optimal trajectory is generated under the condition of satisfying the constraint.

3.1 Theoretical Basis

The optimization problem of aircraft flight profile can be attributed to the optimization problem of continuous dynamic system with fixed end time but no end constraint.

Known systems can be described as the following non-linear differential equations

$$\dot{x} = f(x,u,t) \quad t_0 \le t \le t_f \tag{22}$$

Where state vector x is in N-dimensional, and control vector u is in N-dimensional. Thus the cost function to be minimized is

$$J = \phi\left(x\left(t_f\right), t_f\right) + \int_{t_0}^{t_f} L(x,u,t)\,dt \tag{23}$$

Where ϕ is terminal cost function, L is cost per unit time on the route,

Firstly, the system equation (22) is substituted for the cost function J, and the multiplication factor $\lambda(t)$ is introduced to obtain:

$$J = \phi\left(x\left(t_f\right), t_f\right) + \int_{t_0}^{t_f} \left\{L(x,u,t) + \lambda^T(t)\left\{f(x,u,t) - \dot{x}\right\}\right\} dt \tag{24}$$

Then the Hamiltonian function can be defined as

$$H(x,u,t) = L(x,u,t) + \lambda^T(t) f(x,u,t) \tag{25}$$

By substituting Hamiltonian function $H(x,u,t)$ into equation (24), we can get:

$$\begin{aligned} J = {} & \phi\left(x\left(t_f\right), t_f\right) - \lambda^T\left(t_f\right)x\left(t_f\right) + \lambda^T\left(t_0\right)x\left(t_0\right) \\ & + \int_{t_0}^{t_f} \left\{H(x,u,t) + \dot{\lambda}^T(t)x(t)\right\} dt \end{aligned} \tag{26}$$

Secondly, for fixed t_0 and t_f, the increment of $u(t)$ and $x(t)$ causes the change of J is:

$$\begin{aligned} \delta J = {} & \left\{\left(\frac{\partial \phi}{\partial x} - \lambda^T\right)\delta x\right\}_{t=t_f} + \left(\lambda^T \delta x\right)_{t=t_0} \\ & + \int_{t_0}^{t_f} \left\{\left(\frac{\partial H}{\partial x} + \dot{\lambda}^T\right)\delta x + \frac{\partial H}{\partial u}\delta u\right\} dt \end{aligned} \tag{27}$$

If the proper $\lambda(t)$ is chosen so that the δx term in the integral term of formula (27) disappears at the time of t_f, then there are

$$\dot{\lambda}^T = -\frac{\partial H}{\partial x} = -\frac{\partial L}{\partial x} - \lambda^T \frac{\partial f}{\partial x}; \lambda^T\left(t_f\right) = \frac{\partial \phi}{\partial x} \tag{28}$$

According to (28), (27) transferred to:

$$\delta J = \lambda^T \delta x\left(t_0\right) + \int_{t_0}^{t_f} \frac{\partial H}{\partial u}\delta u\,dt \tag{29}$$

To minimize J, for arbitrary $u(t)$, δJ should be equal to 0, that is to say

$$\frac{\partial H}{\partial u} = 0 \qquad t_0 \le t \le t_f \tag{30}$$

This means that the range of u is not required in optimizing the trajectory. If the control variable constraint is:

$$C(u,t) \le 0 \tag{31}$$

For minimized $u(t)$, there must be $\delta J \ge 0$ within its allowable range, then equation (31) under all the t and all permissible $u(t)$ can be expressed as :

$$\frac{\partial H}{\partial u}\delta u \triangleq \delta H \ge 0 \tag{32}$$

That is to say, for all of the possible values of u , H is always the smallest, which is the minimization principle.

In conclusion, in order to get the minimum J , the differential equation (22) to (28) must be satisfied at the same time, where $u(t)$ can be determined by equation (30) or (32). $x(t_0)$ and $\lambda(t_f)$ are known, so this is a two-point boundary value problem.

If L and f is not the explicit function of t, then

$$\begin{aligned}\dot{H} &= \frac{\partial H}{\partial t} + \frac{\partial H}{\partial x}\dot{x} + \frac{\partial H}{\partial u}\dot{u} + \dot{\lambda}^T f \\ &= \frac{\partial H}{\partial t} + \frac{\partial H}{\partial u}\dot{u} + \left(\frac{\partial H}{\partial x} + \dot{\lambda}^T\right) f\end{aligned} \tag{33}$$

each item of (33) will be 0 in the optimal path, and we can see that H is a constant on the optimal trajectory.

3.2 Optimal vertical section based on energy state method

The above theory is applied to the optimization process of aircraft vertical profile in order to select the appropriate thrust and speed to control the aircraft from the starting point to the end point.

Firstly, the cost function in the process of aircraft control is defined as

$$J = \int_0^{t_f} \left(C_f \dot{w} + C_t\right) dt \triangleq \int_0^{t_f} C_d dt \tag{34}$$

Where C_f is the fuel cost coefficient(yuan/pound), $\dot{w}$ is the time cost(yuan/hour), C_d is the direct manipulation cost, t_f is the time to fly over specified distance d_f .

Assume that the typical flight profile is shown in Figure 9, which consists of three parts: ascent, flat cruise and descent. Then there are constraints

$$d_{up} + d_{dn} \le d_f \tag{35}$$

Where d_{up} is the distance from the starting point to the starting point of the cruise phase, $d_{up} = x(t_{ci})$, and d_{dn} is the distance from the end of the cruise phase to the end of the descending phase.

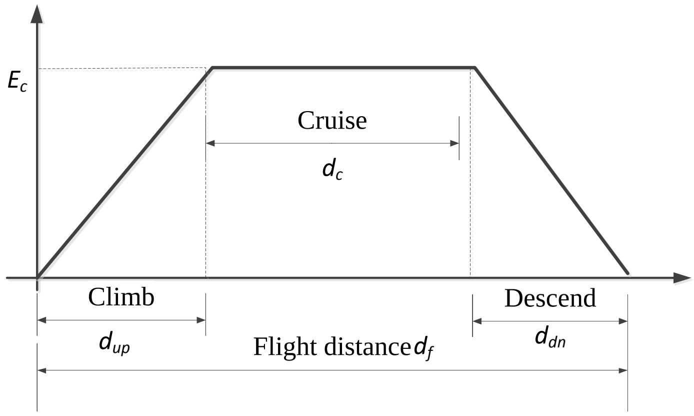

Figure 8 – Flight Profile

The cost function can be expressed as

$$\begin{aligned} J &= \int_0^{t_{ci}} C_d dt + \left(d_f - d_{up} - d_{dn}\right)\psi + \int_{t_{cf}}^{t_f} C_d dt \\ &= \int_0^{t_{ci}} C_d dt + \left\{d_f - x\left(t_{ci}\right) - \left[d_f - x\left(t_{cf}\right)\right]\right\}\psi + \int_{t_{cf}}^{t_f} C_d dt \end{aligned} \tag{36}$$

Where ψ is the unit cruise cost.

Energy is defined as

$$E = h + V^2/2g \tag{37}$$

The physical meaning of this formula is the sum of kinetic energy and potential energy per unit mass.

$$\dot{E} = V\left(T - D\right)/mg \tag{38}$$

Assuming that the aircraft energy monotonously increases when climbing and decreases when descending, the time differential can be expressed as an expression of energy, $dt = dE/\dot{E}$, in this way, the above cost function can be transformed into an expression of energy.

The definition variable x is the flight distance of the aircraft in the climbing and descending phases, and the distance of the cruise phase $d_f - x$, and the increment dx of the distance variable x is equivalent to the incremental change on x_{up} and x_{dn}, so

$$dx = d\left(x_{up} + x_{dn}\right) \tag{39}$$

In this way, the cost function can be rewritten as

$$J = \int_{0_i}^{t_{ci}} C_d dt + \left(d_f - x_{up}\left(t_{ci}\right) - x_{dn}\left(\tau_{cf}\right)\right) + \int_{0_f}^{\tau_{cf}} \left|C_d\right| d\tau \tag{40}$$

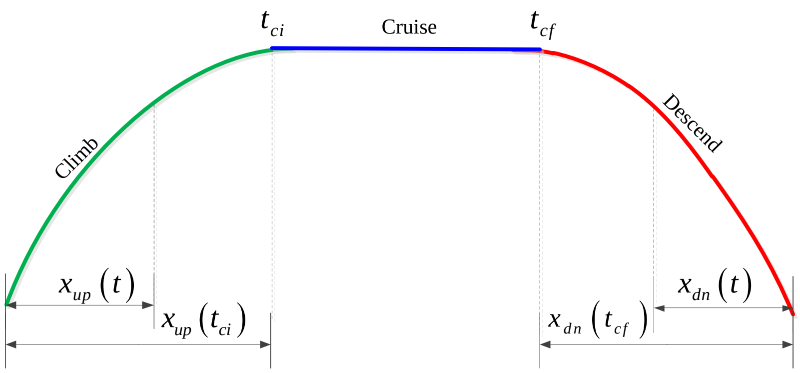

Figure 9 – Definition of flight distance

substituting $dt = \frac{dE}{\dot{E}}$ into (40)，it can be obtained that:

$$J = \int_{E_i}^{E_{ci}} \left(\frac{C_d}{\dot{E}} dE \right)_{\dot{E}>0} + \left(d_f - x_{up}\left(E_{ci}\right) - x_{dn}\left(E_{cf}\right) \right) + \int_{E_f}^{E_{cf}} \left(\frac{dE}{\left|\dot{E}\right|} dE \right)_{\dot{E}<0} \tag{41}$$

Where E_i , E_{ci} , E_{cf} and E_f are the energy value at time 0，t_{ci} ， t_{cf} and t_f .

(41) is the sum of the distance between the climbing phase and the descending phase.

$$\frac{dx}{dE} = \frac{d\left(x_{up} + x_{dn}\right)}{dE} = \left(\frac{\left(V_{up} + V_{wup}\right)}{\dot{E}} \right)_{\dot{E}>0} + \left(\frac{\left(V_{dn} + V_{wdn}\right)}{\dot{E}} \right)_{\dot{E}<0} \tag{41}$$

Where V_{wup} and V_{wdn} are the wind velocity of the climbing phase and the descending phase.

According to (25), Hamilton function is rewritten to:

$$H = \left\{ \left(\frac{C_d}{\dot{E}} dE \right)_{\dot{E}>0} + \left(\frac{C_d}{\dot{E}} dE \right)_{\dot{E}<0} + \lambda \left[\left(\frac{\left(V_{up} + V_{wup}\right)}{\dot{E}} \right)_{\dot{E}>0} + \left(\frac{\left(V_{dn} + V_{wdn}\right)}{\dot{E}} \right)_{\dot{E}<0} \right] \right\} \tag{42}$$

It can be divided into two parts:

$$H = \left(\frac{C_d + \lambda\left(V_{up} + V_{wup}\right)}{\dot{E}} \right)_{\dot{E}>0} + \left(\frac{C_d + \lambda\left(V_{dn} + V_{wdn}\right)}{\dot{E}} \right)_{\dot{E}<0} \tag{43}$$

The co-state equation is:

$$\frac{\partial \lambda}{\partial E} = -\frac{\partial H}{\partial x} = -\frac{\partial H}{d\left(x_{up} + x_{dn}\right)} = 0 \tag{44}$$

Therefore:

$$\lambda\left(E_{ci}\right) = \lambda\left(E_{cf}\right) = \frac{\partial \Phi}{\partial\left(x_{up} + x_{dn}\right)} = \frac{\partial\left[\left(d_f - x_{up} - x_{dn}\right)\psi\right]}{\partial\left(x_{up} + x_{dn}\right)} = -\psi \tag{45}$$

For convenience and processing, the whole process is divided into two stages: climbing + cruising phase half and descending + cruising phase half.

$$\frac{\partial \lambda}{\partial E} = -\frac{\partial H}{\partial x_{up}} = 0 \tag{46}$$

$$\lambda\left(E_{ci}\right) = \frac{\partial\left[\left(\frac{d_f}{2} - x_{up}\right)\psi\left(E_{ci}\right)\right]}{\partial x_{up}} = -\psi\left(E_{ci}\right) \tag{47}$$

Similarly, the descent phase can be expressed as

$$\frac{\partial \lambda}{\partial E} = -\frac{\partial H}{\partial x_{dn}} = 0 \tag{48}$$

$$\lambda\left(E_{cf}\right) = \frac{\partial\left[\left(\frac{d_f}{2} - x_{dn}\right)\psi\left(E_{cf}\right)\right]}{\partial x_{dn}} = -\psi\left(E_{cf}\right) \tag{49}$$

In this way, the trajectory optimization problem is transformed into the minimum value problem of piecewise algebraic functions represented by the sum of equations (48) and (49).

$$H_{up} = \min_{V_{up}} \left[\frac{C_d}{\dot{E}} - \psi\left(E_{ci}\right) \left(\frac{\left(V_{up} + V_{wup}\right)}{\dot{E}} \right) \right]_{\dot{E}>0} \tag{50}$$

$$H_{dn} = \min_{V_{dn}} \left[\frac{C_d}{\dot{E}} - \psi\left(E_{cf}\right) \left(\frac{\left(V_{dn} + V_{wdn}\right)}{\left|\dot{E}\right|} \right) \right]_{\dot{E}<0} \tag{51}$$

When using the optimal control theory based on total energy method to solve the optimal trajectory, because there is no accurate analytical differential expression, this solution can only be solved by trial-and-error method, that is, numerical solution.

The cruise cost $\psi\left(E_c\right)$ in formulas (50) and (51) is solved under the condition that the aircraft is in a steady straight-line equilibrium state. Satisfaction

$$\psi\left(E_c\right) = \min_{V_c} \frac{C_d}{\left(V_c + V_w\right)} \tag{52}$$

V_c is cruising speed and V_w is the wind speed during cruising. In other words, for any cruise altitude, there is an optimal thrust and airspeed to minimize the unit distance cost $\psi\left(E_c\right)$. Since the altitude, speed and total weight of the aircraft are known, the cruise energy (i.e. the sum of kinetic energy and potential energy of the aircraft) can be calculated. $\psi\left(E_c\right)$ can be regarded as a function of cruise energy, so there must be a cruise energy E_{copt} that minimizes the cost per unit distance $\psi\left(E_c\right)$.

In calculating the optimal profile, the following known conditions must be given:

a) The initial takeoff altitude, speed and total weight of the aircraft;
b) The final expected height and speed;
c) Distance from take-off to destination;
d) Wind speed and temperature varying with altitude;
e) Fuel cost coefficient C_f and time cost coefficient C_t in cost function.

Because the method of equal speed climbing and descending is adopted in this paper, there is no optimization in this process, so the optimization calculation of cruise section is only carried out here.

Firstly, the optimal cruise cost ψ and its derivative $\partial\psi/\partial E$ should be determined. This calculation process needs to use the cruise performance table of the aircraft. The cruise performance table is related to the total weight, wind speed, lift, drag, thrust and fuel consumption rate of the aircraft. The optimization process is to get the best flight altitude and airspeed according to the total re-look-up table of the aircraft.

Assuming that the aircraft is in equilibrium during cruising, the unit cruising cost under given weight can be obtained by formula (53)

$$\psi(w_c) = \min_{V} \left[\frac{c_f \dot{w} + c_t}{V + V_w} \right] \tag{53}$$

w_c is the total weight of the aircraft.

The optimization idea of cruise section can be described as follows: firstly, the altitude is discretized, and the maximum and minimum velocities at all altitudes are obtained according to the increment of altitude interval from sea level to aircraft lift. Secondly, the velocity interval calculated at that altitude is obtained by using formula (53). Finally, since the minimum cruise cost is a function of height, the Fibonacci search method can be used to search the cruise table.

The values of w_i are $45t$, $50t$, $55t$, $60t$ and $65t$, respectively. Then, the cruise performance interpolation data table with weight as input and the best speed and height as output can be obtained.

4. SIMULATION AND VERIFICATION

According to the optimization calculation process of cruise section given in the preceding section, the fuel consumption rates at different altitudes, masses and velocities are obtained firstly, then the maximum and minimum flight velocities at different altitudes are given, and the index functions are optimized by using Patternsearch function to obtain unit cruising cost under different fuel cost coefficients, time cost coefficients, height and mass.

Based on this, the flight altitude, i.e. the optimal cruise altitude, can be obtained by using the data table search method under the minimum unit cruise cost, and the optimal cruise speed can be obtained based on this. According to the optimal cruise altitude and cruise speed obtained, the optimal vertical profile with the minimum cost can be determined by combining the longitude and latitude coordinates of the given starting point and terminal point and the course of flight.

Thus, given different fuel cost coefficients, time cost coefficients and aircraft quality during cruising, the optimal cruising altitude and corresponding optimal cruising speed can be obtained according to the calculation function of cruising cost per unit, as shown in tables 1, 2 and 3, respectively.

Table 1 – Optimum cruise altitude and speed for different mass and CT when Cf = 2

Ct	mass (kg) Optimum cruising altitude (m)/optimum cruising speed (m/s)				
	45000	50000	55000	60000	65000
0	4000/144.1	3600/141.68	3400/140/7	4000144.3	4000/144.2
20	5680/162.5	5160/161.0	4960/160.5	5680/161.4	5680/160.9
40	7360/177.8	6720/176.9	6520/175.9	7360/175.2	7360/174.2
60	8200/184	7500/183	7300/182	8200/181	8200/180

Table 2 – Optimum cruise altitude and speed for different mass and CT when Cf = 4

Ct	mass (kg) Optimum cruising altitude (m)/optimum cruising speed (m/s)				
	45000	50000	55000	60000	65000
0	8000/173.7	7200/172.9	6800/172.0	8000/171.3	8000/170.5
20	8560/177.8	7720/176.9	7320/176.1	8560/175.2	8560/174.3
40	9120/181.9	8240/180.9	7840/180.0	9120/179.1	9120/178.1
60	9400/184	8500/183	8100/182	9400/181	9400/180

Table 3 – Optimum cruise altitude and speed for different mass and CT when Cf = 6/8/10/12

Ct	mass (kg) Optimum cruising altitude (m)/optimum cruising speed (m/s)				
	45000	50000	55000	60000	65000
0	10000/184	9000/183	8500/182	10000/181	10000/180
20	10000/184	9000/183	8500/182	10000/181	10000/180
40	10000/184	9000/183	8500/182	10000/181	10000/180
60	10000/184	9000/183	8500/182	10000/181	10000/180

From Table 1, Table 2 and Table 3, it can be seen that the effect of mass on the optimal cruise altitude and speed is obvious, that is, when the flight mass increases, the optimal altitude and speed decrease first and then increase. When the fuel cost coefficient is equal to 2 and 4, and the time cost coefficient is less than 20, that is, when the fuel cost is low and the time cost coefficient is small, the cruise altitude of the aircraft can be minimized at the cost of fuel consumption, and cruise at the most suitable speed at that altitude to reduce the total cost; but when the fuel cost coefficient and the time are successful, the cruise altitude can be reduced. When the coefficients are large, the aircraft cruises at higher altitudes and speeds as far as possible to reduce fuel consumption and flight time in flight, so as to comprehensively improve economic benefits.

5. Conclusion

In this paper, a trajectory optimization algorithm considering weather information prediction is proposed, which mainly includes weather information fusion algorithm and flight profile optimization algorithm based on energy state method. It can be seen from the experiments that the trajectory optimization algorithm proposed in this paper can reduce fuel consumption and flight time in flight, thus improving the flight economic benefits. The simulation results show that the algorithm is feasible.

6. Contact Author Email Address

Wei Guo: gwadams@126.com.

7. Copyright Statement

References

[1] Zhang Junfeng, Suidong, Tang Xinmin. Track Prediction Based on Hybrid Estimation of State-Dependent Modal Switching [J]. System Engineering Theory and Practice,2014,11:2955-2964.

[2] Sorensen J A. Concepts for generating optimum vertical flight profiles[J]. 1979.

[3] Sorensen J A, Waters M H, Patmore L C. Computer programs for generation and evaluation of near-optimum vertical flight provides (Contractor Report 3688)[J]. Hampton VA: NASA, 1983.

[4] IRONS, JR R P, Schultz R L, Zagalsky N R. Energy state approximation and minimum-fuel fixed-range trajectories[J]. Journal of Aircraft, 1971, 8(6): 488-490.

[5] Gong Fengxun, Yuan Kejian, Ma Yanqiu. Gauss pseudospectral optimization method for continuous descent approach (CDA) track [J].Traffic information and safety,2016,04:15-21.

[6] Young S Y R, Jerome K, Collins R. Fuel-efficient optimized profile descents with time control[C]//2015 Integrated Communication, Navigation and Surveillance Conference (ICNS). IEEE, 2015: I1-1-I1-11.

[7] Young S Y, Jerome K. RTA-compliant optimized profile descents with 4-D FMS[C]//2014 Integrated Communications, Navigation and Surveillance Conference (ICNS) Conference Proceedings. IEEE, 2014: N1-1-N1-15.

[8] Grabow C. A method to design a tie-point-based optimized profile descent (OPD) solution[C]//2013 IEEE/AIAA 32nd Digital Avionics Systems Conference (DASC). IEEE, 2013: 1D3-1-1D3-9.

[9] Herndon A, Cramer M, Miller S, et al. Analysis of advanced flight management systems (FMSs) optimized profile descent (OPD)[C]//2013 IEEE/AIAA 32nd Digital Avionics Systems Conference (DASC). IEEE, 2013: 1-26.

[10]Grabow C. A method to design a tie-point-based optimized profile descent (OPD) solution[C]//2013 IEEE/AIAA 32nd Digital Avionics Systems Conference (DASC). IEEE, 2013: 1D3-1-1D3-9.

[11]Stibor J, Nyberg A. Implementation of continuous descent approaches at Stockholm Arlanda airport, Sweden[C]//2009 IEEE/AIAA 28th Digital Avionics Systems Conference. IEEE, 2009: 2. C. 1-1-2. C. 1-13.

[12]Wu Shufan, Shen Yongzhang. Concept and main technology of "Future Air Navigation System" [R]. Research report of Nanjing University of Aeronautics and Astronautics.1994.

[13]Introduction to this issue [J].Traffic Information and Safety,2016,04:3.