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ESTIMATION OF WIND BY ESTIMATING DERIVATIVES OF A LATERAL-DIRECTIONAL MOTION MODEL

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Abstract

This paper considers estimating wind using only the data of an inertial measurement unit without any information from an air-data sensor by estimating stability derivatives of a lateral-directional motion model of an aircraft. Assuming that the wind is constant, we derive the relation between wind and stability derivatives, which allows us to calculate wind. An advantage of this method is that estimated wind is relatively insensitive to the estimation error of the derivatives. We illustrate the effectiveness of the proposed method by simulation using NASA's Generic Transport Model.

Keywords: Aircraft flight dynamics, Parameter estimation, Least squares method, Wind disturbance

1. Introduction

Obtaining wind information in the flight environment is vital to achieve a flight mission successfully and improve operational efficiency since wind can be a disturbance for flight control of aircraft and guidance and navigation. Wind information is also necessary for flight data analysis, such as parameter estimation of an aircraft's dynamic model. Although a weather report provides wind speed and direction in an area, they vary with time, location, and altitude, making it difficult to know precisely the wind in the actual flight environment.

Studies on wind estimation, such as [1] to [8], are grouped into three categories. The first category estimates wind or relative airspeed using aerodynamic models and static relations among variables expressing aircraft motion. For example, Reference [1] shows the results of estimating the vertical wind (downdraft) from the flight path angle computed with actual climb rate and airspeed. Reference [2] estimates the angle of attack and sideslip angle using aerodynamic forces obtained from accelerometer output and aerodynamic parameters such as a lift coefficient to calculate airspeed and then wind from the difference between the airspeed and the ground speed. The second category estimates the wind based on or using a dynamic model of an aircraft. Reference [3] shows an estimation method of steady wind by applying a nonlinear observer to a six-degree-of-freedom nonlinear motion model. Reference [4] estimates airspeed along with wind using a cascaded extended Kalman filter with only IMU (Inertial Measurement Unit) and GPS (Global Positioning System) measurement data but requiring an aircraft's dynamic model. The third category estimates wind using a Kalman filter designed for a simple kinematic model. In Reference [5], the wind is estimated simply as the difference between ground speed and airspeed. Reference [6] defines a linear system where the states are components of a wind vector, the inputs are airspeed, and the outputs are ground speed, estimating wind with a Kalman filter designed for this system. Reference [7] also estimates steady wind by applying a Kalman filter to a linear model with the state variables of wind and an airspeed sensor calibration factor. Reference [8] presents a method to estimate wind and bias of airspeed from ground speed measurements, attitude, and airspeed. The methods of the first and second categories require aerodynamic parameters such as lift and drag coefficients, on which preciseness of estimation depends. The methods of the third category use neither aerodynamic models nor equations of motion, which excludes modeling errors due to incorrect physical parameters. However, the methods require measurements of airspeed. Although air data systems provide an airspeed vector, they are expensive. In order to obtain an airspeed vector using an inexpensive Pitot-static probe, an aerodynamic or aircraft dynamics model is necessary [8].

This paper considers estimating steady wind by estimating stability derivatives of an aircraft's linear model without using measurements of air data sensors and known aerodynamic parameters but using IMU measurements alone. The underlying idea is that wind is an additive disturbance to the ground speed in a dynamic model of an aircraft, making wind speed part of stability derivatives in a linear model so that it is possible to calculate wind speed from stability derivatives. The wind is added to ground speed to obtain airspeed and dominantly affects aerodynamic forces and moments. In a linear model, this means that we can consider wind as an additive disturbance to the ground speed. Wind in the inertial frame can be expressed in the body-fixed frame with Euler angles by coordinate transformation. Hence, when linearizing the equations of motion, the derivatives for Euler angles become linear with respect to the wind. Thus, wind can be obtained by estimating stability derivatives and solving linear equations. A latent problem with this method is that errors in estimated derivatives can enlarge wind estimation error. However, we will show numerically and analytically that estimated wind is not so sensitive to errors in estimated derivatives. This paper deals with the lateral-directional motion since longitudinal motion provides an estimated wind in the flight direction but does not the north and east wind components in the inertial frame [4]. In addition, the estimation error of vertical wind obtained from the longitudinal motion tends to be large, although we do not describe it in detail in this paper. The effectiveness of the proposed method is demonstrated by simulation using NASA's Generic Transport Model (GTM) [9, 10].

This paper is organized as follows. Section 2 gives a general description of the problem to present how additive disturbances to states appear in parameters of a linear system under the assumption that the disturbance is a function of states. In Section 3 and 4, we apply this formulation to a lateral-directional motion model of an aircraft, showing how to obtain steady wind from estimated stability derivatives. Section 5 shows simulation and numerical analysis results. Finally, Section 6 gives conclusions.

2. Linearized systems with additive disturbances to state variables

Consider the nonlinear system described by the following equation:

$$\dot{x} = f(x + d(x), \delta) \tag{1}$$

where $x \in \mathrm{R}^n$ is the state vector, $d \in \mathrm{R}^n$ the disturbance vector being a known function of x with unknown parameters, and $\delta \in \mathrm{R}^m$ is the input vector. For a trim input δ^*, let x_0^* be the equilibrium state in the absence of disturbance. Let x^* be the trim state when adding a disturbance d and the disturbance at x^* be $d^* = d(x^*)$.

Applying Taylor series expansion to the right-hand side of (1) around the equilibrium point (x^*, δ^*), we have

$$\dot{x} = f(x^* + d(x^*), \delta^*) + A(\Delta x + \Delta d(x)) + B\Delta\delta + o(\Delta x, \Delta\delta) \tag{2}$$

where $\Delta x = x - x^*$, $\Delta\delta = \delta - \delta^*$,

$$A = \left. \frac{\partial f(x + d(x), \delta)}{\partial \left(x + d(x)\right)^T} \right|_{\substack{x=x^* \\ \delta=\delta^*}}, \quad B = \left. \frac{\partial f(x + d(x), \delta)}{\partial \delta^T} \right|_{\substack{x=x^* \\ \delta=\delta^*}} \tag{3}$$

$$\Delta d(x) = \left. \frac{\partial d(x)}{\partial x^T} \right|_{\substack{x=x^* \\ \delta=\delta^*}} \Delta x \tag{4}$$

and o(Δx, $\Delta\delta$) denotes the higher-order terms. Substituting (3) and (4) into (2) and neglecting the higher-order terms than the first order, (2) becomes

$$\dot{x} = A\left(I_n + \left. \frac{\partial d(x)}{\partial x^T} \right|_{\substack{x=x^* \\ \delta=\delta^*}} \right)\Delta x + B\Delta\delta =: A_d x + B\Delta\delta + x_{Ad}^* \tag{5}$$

where I_n is an n-by-n identify matrix and x_{Ad}^* is defined as

$$x_{Ad}^* = -A\left(I_n + \left.\frac{\partial d(x)}{\partial x^T}\right|_{\substack{x=x^* \\ \delta=\delta^*}} \right) x^* = -A_d x^* \tag{6}$$

Equation (5) is the estimation model used to estimate A, B, and x^*. Note, however, that x^* cannot be estimated directly. Instead, A_d, B, and x_{Ad}^* can be estimated, and then x^* is calculated as $x^* = -A_d^{-1} x_{Ad}^*$.

3. Lateral-directional motion model affected by wind

This section first represents wind defined in inertial coordinates $\sum_I = (X_I, Y_I, Z_I)$, in the body-fixed frame $\sum_B = (X_B, Y_B, Z_B)$, and then applies the representation of (5) to a lateral-directional model with the wind disturbance.

Let the wind in $\sum_I$ be $V_{wI} = [u_{wI}\ v_{wI}\ w_{wI}]^T$, and let us express V_{wI} in $\sum_B$ as

$$V_{wB} = T_{BI} V_I \tag{7}$$

where T_{BI} is the coordinate transformation matrix from $\sum_I$ to $\sum_B$ given by

$$T_{BI}(\Phi,\Theta,\Psi) = \begin{bmatrix} \cos\Theta\cos\Psi & \cos\Theta\sin\Psi & -\sin\Theta \\ \sin\Phi\sin\Theta\cos\Psi - \cos\Phi\sin\Psi & \sin\Phi\sin\Theta\sin\Psi + \cos\Phi\cos\Psi & \sin\Phi\cos\Theta \\ \cos\Phi\sin\Theta\cos\Psi + \sin\Phi\sin\Psi & \cos\Phi\sin\Theta\sin\Psi - \sin\Phi\cos\Psi & \cos\Phi\cos\Theta \end{bmatrix} \tag{8}$$

where Φ, Θ, and Ψ are the roll, pitch, and yaw angle (rad), respectively.

Generally, the forces act on the aircraft in flight are composed of aerodynamic force, gravitational force, inertial force, and thrust. Note that the airspeed affects the aerodynamic force and thrust and that the ground speed does not affect the gravitational force. As for the inertial force, although the ground speed affects the acceleration due to the rotation of the moving frame in nonlinear equations of motion, this is not the case in the linear equation of motion. It follows that by replacing the velocity on the right-hand side $V = [u\ v\ w]^T$ (ft/s) in $\sum_B$ with the airspeed $V_{aB} =: [u_{aB}\ v_{aB}\ w_{aB}]^T = V - V_{wB} = [u-u_{wB}\ v-v_{wB}\ w-w_{wB}]^T$, we obtain the equation of motion in the presence of wind. More specifically, we can express the dynamic model under the influence of steady wind as (1), except for the inertial term, which we should note does not include the ground speed as a variable in a linear model used in this study. We do not take into account here the effect of wind on thrust. The state variables are $x = [V^T\ p\ q\ r\ \Phi\ \Theta\ \Psi]^T$, the input $\delta = [\delta_e\ \delta_t\ \delta_a\ \delta_r]^T$, and the disturbance $d = [V_{wB}^T\ 0\ 0\ 0\ 0\ 0\ 0]^T$, where p, q, and r are the roll, pitch, and yaw angular velocities (rad/s), respectively; Φ, Θ, and Ψ are the roll, pitch, and yaw angles (rad), respectively; δ_e, δ_a, and δ_r are the deflection angles (deg) of the elevator, aileron, and rudder, respectively; and δ_t is the thrust (N). Its linear motion model is given by (5). In the following, we consider lateral-directional motion.

The lateral-directional motion is described by

$$\dot{x}_{lat} = A_{lat}\left(x_{lat} - d_{lat}\right) + B_{lat}\delta_{lat} \tag{9}$$

where $x_{lat} = [v\ p\ r\ \Phi]^T$, $\delta_{lat} = [\delta_a\ \delta_r]^T$, $d_{lat} = [v_{wB}\ 0\ 0\ 0]^T$ and the derivative matrices are given by

$$A_{lat} = \begin{bmatrix} Y_v & W^* + Y_p & -U^* + Y_r & g\cos\Theta^* \\ L_v' & L_p' & L_r' & 0 \\ N_v' & N_p' & N_r' & 0 \\ 0 & 1 & \tan\Theta^* & 0 \end{bmatrix}, \quad B_{lat} = \begin{bmatrix} Y_{\delta_a} & Y_{\delta_r} \\ L_{\delta_a}' & L_{\delta_r}' \\ N_{\delta_a}' & N_{\delta_r}' \\ 0 & 0 \end{bmatrix} \tag{10}$$

and from (7) and (8) v_{wB} in d_{lat} is given by

$$v_{wB} = u_{wI}\left(\Phi\sin\Theta^*\cos\Psi - \sin\Psi\right) + v_{wI}\left(\Phi\sin\Theta^*\sin\Psi + \cos\Psi\right) + w_{wI}\Phi\cos\Theta^* \tag{11}$$

where Θ^* is a trim pitch angle and the approximations, $\sin\Phi \cong \Phi$ and $\cos\Phi \cong 1$, are used. Substituting (11) into (9) yields

$$\begin{bmatrix}\dot{v}\\ \dot{p}\\ \dot{r}\\ \dot{\Phi}\end{bmatrix}=\begin{bmatrix}Y_v & W^*+Y_p & -U^*+Y_r & \left(g-Y_v w_{wI}\right)\cos\Theta^*\\ L_v' & L_p' & L_r' & -L_v' w_{wI}\cos\Theta^*\\ N_v' & N_p' & N_r' & -N_v' w_{wI}\cos\Theta^*\\ 0 & 1 & \tan\Theta^* & 0\end{bmatrix}\begin{bmatrix}v\\ p\\ r\\ \Phi\end{bmatrix}+\begin{bmatrix}Y_{\delta_a} & Y_{\delta_r}\\ L_{\delta_a}' & L_{\delta_r}'\\ N_{\delta_a}' & N_{\delta_r}'\\ 0 & 0\end{bmatrix}\begin{bmatrix}\delta_a\\ \delta_r\end{bmatrix}$$

$$-\begin{bmatrix}Y_v\\ L_v'\\ N_v'\\ 0\end{bmatrix}\left\{u_{wI}\left(\sin\Theta^*\Phi\cos\Psi-\sin\Psi\right)+v_{wI}\left(\sin\Theta^*\Phi\sin\Psi+\cos\Psi\right)\right\} \tag{12}$$

$$=: A_{dlat}\begin{bmatrix}v\\ p\\ r\\ \Phi\end{bmatrix}+B_{lat}\begin{bmatrix}\delta_a\\ \delta_r\end{bmatrix}+D_{lat}\begin{bmatrix}-\sin\Theta^*\Phi\cos\Psi+\sin\Psi\\ -\sin\Theta^*\Phi\sin\Psi-\cos\Psi\end{bmatrix}$$

where

$$D_{lat}=\begin{bmatrix}Y_v\\ L_v'\\ N_v'\\ 0\end{bmatrix}\begin{bmatrix}u_{wI} & v_{wI}\end{bmatrix}=\begin{bmatrix}Y_v u_{wI} & Y_v v_{wI}\\ L_v' u_{wI} & L_v' v_{wI}\\ N_v' u_{wI} & N_v' v_{wI}\\ 0 & 0\end{bmatrix}=:\begin{bmatrix}Y_{vu} & Y_{vv}\\ L_{vu}' & L_{vv}'\\ N_{vu}' & N_{vv}'\\ 0 & 0\end{bmatrix} \tag{13}$$

Ψ is given by

$$\dot{\Psi}=r\cos\Phi\sec\Theta^* \tag{14}$$

Note that (12) is nonlinear with respect to the yaw angle, which is not a state variable in this linear system and that the trim states for the lateral-directional motion are zero except for v.

Given a trim yaw angle Ψ^*, we can rewrite (12) into a linear state equation. Suppose that the aircraft is flying at a trim yaw angle Ψ^* such that the roll angle $\Phi^* = 0$ and $v_{aB}^* = 0$, that is, wing level and no sideslip for the airspeed vector. We then have

$$\begin{bmatrix}\dot{v}\\ \dot{p}\\ \dot{r}\\ \dot{\Phi}\end{bmatrix}=\begin{bmatrix}Y_v & W^*+Y_p & -U^*+Y_r & g\cos\Theta^*+Y_v V_{w\Phi}\\ L_v' & L_p' & L_r' & L_v' V_{w\Phi}\\ N_v' & N_p' & N_r' & N_v' V_{w\Phi}\\ 0 & 1 & \tan\Theta^* & 0\end{bmatrix}\begin{bmatrix}v\\ p\\ r\\ \Phi\end{bmatrix}+\begin{bmatrix}Y_{\delta_a} & Y_{\delta_r}\\ L_{\delta_a}' & L_{\delta_r}'\\ N_{\delta_a}' & N_{\delta_r}'\\ 0 & 0\end{bmatrix}\begin{bmatrix}\delta_a\\ \delta_r\end{bmatrix}-\begin{bmatrix}Y_v v_{wB}^*\\ L_v' v_{wB}^*\\ N_v' v_{wB}^*\\ 0\end{bmatrix} \tag{15}$$

where $V_{w\Phi} = (u_{wI}\cos\Psi^*+v_{wI}\sin\Psi^*)\sin\Theta^*+w_{wI}\cos\Theta^*$ and $v_{wB}^* = -u_{wI}\sin\Psi^*+v_{wI}\cos\Psi^*$. Note that $v_{wB}^* = v^*$ in the trim flight, which means $v_{aB}^* = v^*-v_{wB} = 0$, i.e., the airspeed vector lies in the X_BZ_B-plane.

In order to show the validity of the dynamic model (12), we compare the stability derivatives of (15) with those given by GTM, which provides linear models for a trim flight without sideslip for airspeed in the presence of wind. First, when there is no wind, GTM gives the stability derivative matrix in (12) as

$$A_{lat0}=\begin{bmatrix}\text{-0.6638} & 7.9085 & \text{-171.09} & 32.141\\ \text{-0.8173} & \text{-8.3752} & 1.8935 & 0\\ 0.2848 & \text{-0.5282} & \text{-1.8217} & 0\\ 0 & 1 & 0.04541 & 0\end{bmatrix}$$

for the trim angle of attack (AoA), 2.6 deg. The trim ground speed is V^* = [171.94 -0.02 7.65]^T (ft/s). In the presence of the wind V_{wI} = [-10 20 5]^T (ft/s), the stability derivative matrix becomes

$$A_{dlat1} = \begin{bmatrix} -0.6596 & 14.436 & -191.94 & 36.463 \\ -0.8118 & -8.3221 & 1.8731 & 5.3414 \\ 0.2831 & -0.5265 & -1.8103 & -1.8817 \\ 0 & 1 & 0.0739 & 0.0002 \end{bmatrix}$$

for the same AoA, 2.6 deg. The trim state vector is x_{lat}^* = [0.67 0.0 0.0 0.0]$^{\mathrm{T}}$ and the trim yaw angle Ψ^* = 114.78 deg in $\sum_I$ defined as the NED (North-East-Down) coordinates. The stability derivative matrix in (15) computed using the nominal derivatives for no wind, and U^*= 193.86 ft/s, W^*= 14.28 ft/s, and Θ^*=4.21 deg for the new trim condition in the wind becomes

$$A_{dlat2} = \begin{bmatrix} -0.6638 & 14.335 & -191.93 & 35.375 \\ -0.8173 & -8.3752 & 1.8935 & 4.0753 \\ 0.2848 & -0.5283 & -1.8217 & -1.4204 \\ 0 & 1 & 0.0737 & 0 \end{bmatrix}$$

A_{dlat1} given by GTM is close to A_{dlat2} computed from (14) except for (2, 4) elements, which justifies the linear model (14) and allows computing the wind from the derivatives.

4. Wind estimation and its sensitivity to estimation error of derivatives

We assume that all the variables in (12), including the yaw angle and control inputs, are measurable. Applying a parameter estimation algorithm such as the least-squares method to (12), we can obtain the estimates of A_{dlat}, B_{lat}, and D_{lat}. From these estimates, the wind components are calculated as

$$\begin{aligned} \begin{bmatrix} u_{wI} & v_{wI} \end{bmatrix} &= \frac{1}{Y_v}\begin{bmatrix} Y_{vu} & Y_{vv} \end{bmatrix} \\ w_{wI} &= \frac{-A_{14} + g\cos\Theta^*}{Y_v \cos\Theta^*} \end{aligned} \tag{16}$$

$$\begin{aligned} \begin{bmatrix} u_{wI} & v_{wI} \end{bmatrix} &= \frac{1}{L_v^{'}}\begin{bmatrix} L_{vu}^{'} & L_{vv}^{'} \end{bmatrix} \\ w_{wI} &= \frac{-A_{24}}{L_v^{'} \cos\Theta^*} \end{aligned} \tag{17}$$

$$\begin{aligned} \begin{bmatrix} u_{wI} & v_{wI} \end{bmatrix} &= \frac{1}{N_v^{'}}\begin{bmatrix} N_{vu}^{'} & N_{vv}^{'} \end{bmatrix} \\ w_{wI} &= \frac{-A_{34}}{N_v^{'} \cos\Theta^*} \end{aligned} \tag{18}$$

Alternatively, from the relation of (16) to (18), the wind can be calculated as

$$\begin{aligned} \begin{bmatrix} u_{wI} & v_{wI} \end{bmatrix} &= \begin{bmatrix} Y_v \\ L_v^{'} \\ N_v^{'} \end{bmatrix}^{\#} \begin{bmatrix} Y_{vu} & Y_{vv} \\ L_{vu}^{'} & L_{vv}^{'} \\ N_{vu}^{'} & N_{vv}^{'} \end{bmatrix} = \frac{1}{Y_v}\begin{bmatrix} Y_{vu} & Y_{vv} \end{bmatrix} = \frac{1}{L_v^{'}}\begin{bmatrix} L_{vu}^{'} & L_{vv}^{'} \end{bmatrix} = \frac{1}{N_v^{'}}\begin{bmatrix} N_{vu}^{'} & N_{vv}^{'} \end{bmatrix} \\ w_{wI} &= \frac{1}{\cos\Theta^*}\begin{bmatrix} Y_v \\ L_v^{'} \\ N_v^{'} \end{bmatrix}^{\#} \begin{bmatrix} -A_{14} + g\cos\Theta^* \\ -A_{24} \\ -A_{34} \end{bmatrix} = \frac{-A_{14} + g\cos\Theta^*}{Y_v \cos\Theta^*} = \frac{-A_{24}}{L_v^{'} \cos\Theta^*} = \frac{-A_{34}}{N_v^{'} \cos\Theta^*} \end{aligned} \tag{19}$$

where # denotes a pseudo-inverse, i.e., $X^{\#} = (X^{\mathrm{T}}X)^{-1}X^{\mathrm{T}}$.

Using estimated parameters θ_0 to compute other parameters θ_1 can enlarge estimation error in θ_0, resulting in a more significant estimation error in θ_1. However, this is not the case as long as the wind estimation by (16) to (19) is concerned, as shown below.

Let the first three columns of A_{dlat} be A_1, A_2, and A_3, respectively, and define $\theta_1 = \begin{bmatrix} A_1 & A_1 & A_1 & A_1 \end{bmatrix}$

$\in \mathrm{R}^{4\times4}$ and $\theta_2 = \begin{bmatrix} A_2 & A_3 & B_{lat} \end{bmatrix} \in \mathrm{R}^{4\times4}$. Further define a diagonal matrix T_4 with the diagonal elements: 1, u_{wI}, v_{wI}, and $w_{wI}\cos\Theta^*$, and define $\xi_1 = \begin{bmatrix} v & -\sin\Theta^*\Phi\cos\Psi + \sin\Psi & -\sin\Theta^*\Phi\sin\Psi - \cos\Psi & -\Phi \end{bmatrix}^T \in \mathrm{R}^4$ and $\xi_2 = \begin{bmatrix} p & r & \delta_a & \delta_r \end{bmatrix}^T \in \mathrm{R}^4$. We can then write (12) as

$$y = \begin{bmatrix} \theta_1 & \theta_2 \end{bmatrix} \begin{bmatrix} T_4 & 0 \\ 0 & I_4 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} =: \theta T \xi =: \theta_w \xi \tag{20}$$

where $y = \begin{bmatrix} \dot{v} - (g\cos\Theta^*)\Phi & \dot{p} & \dot{r} & \dot{\Theta} \end{bmatrix}^T \in \mathrm{R}^4$. Given N observations, $Y = [y_1\ y_2\ ...\ y_N] \in \mathrm{R}^{4\times N}$ and $\Xi = [\xi_1\ \xi_2\ ...\ \xi_N] \in \mathrm{R}^{8\times N}$ and using the LSM, we have the estimate:

$$\theta_w = Y\Xi^T\left(\Xi\Xi^T\right)^{-1} \tag{21}$$

Similarly, if the wind is known and constant, we obtain the estimate:

$$\theta = Y\left(T\Xi\right)^T\left(T\Xi\left(T\Xi\right)^T\right)^{-1} = Y\Xi^T\left(\Xi\Xi^T\right)^{-1}T^{-1} \tag{22}$$

Equations (21) and (22) yield the following relation:

$$\theta = \theta_w T^{-1} \tag{23}$$

The equations for the first 4 columns of (23) are $A_1 = \theta_{w1}$, $A_1 = \theta_{w2}/u_{wI}$, $A_1 = \theta_{w3}/v_{wI}$, and $A_1 = \theta_{w4}/(w_{wI}\cos\Theta^*)$, where $\theta_{wi} \in \mathrm{R}^4$ is the i-th column of θ_w. From these equations, we have $\theta_{w1} = \theta_{w2}/u_{wI} = \theta_{w3}/v_{wI} = \theta_{w4}/(w_{wI}\cos\Theta^*)$, which in turn yields (17) and (18). Note that θ_{w1} given by (21) is the same as that by (22), which means that we use θ_{w1} estimated for true wind, even if it has estimation error, to calculate wind. Thus, the wind calculated from the estimated parameters θ_w corresponds to the true wind. Although it is difficult to estimate true wind due to noise and nonlinearity, this property makes the estimates less sensitive to estimation error in θ_w. As for the calculation of w_{wI}, we used Θ^*, but the Θ is time-varying in motion. Therefore, assuming Θ is constant to estimate A_{14}, A_{24}, and A_{34} can cause a significant estimation error in the derivatives. We suppose that the estimation error and the assumption of constant Θ in calculating w_{wI} may result in a significant estimation error in w_{wI}.

5. Simulation results

We used GTM in the same flight condition and wind as in Section 4 for simulation, adding the noise with the standard deviation of [0.5 0.01 0.01 0.02 0.02]^T to the measurements of the variables $[v\ p\ r\ \Phi\ \Psi]^T$ (see [4] and [5]). The sampling period of data is 0.01s. No turbulence is assumed. Figure 1 shows the input's time history of the rudder and aileron angles. GTM generates the time response of the state variables for the input, as shown in Fig. 2. From the measurements of the variables and inputs, we can estimate the derivative matrices A_{dlat}, B_{lat}, and D_{lat} in (12) by the batch-type LSM and an extended Kalman filter (EKF) [11]. In the estimation by the LSM, a second-order Butterworth filter, $1/(s^2+1.41s+1)$, is applied in the form of a digital filter.

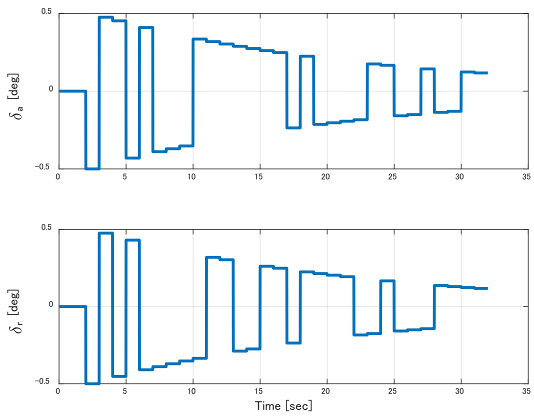

Fig. 1 Time history of inputs (aileron and rudder)

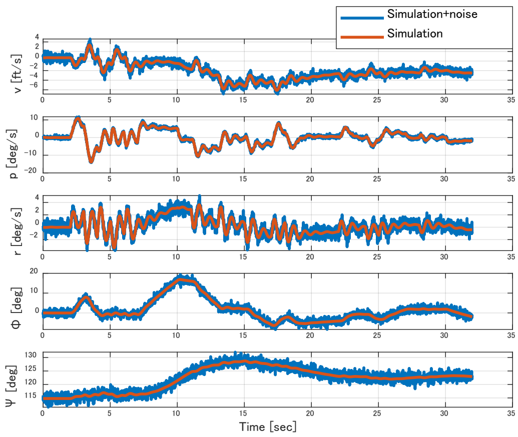

Fig. 2 Time history of measured outputs

Tables 1 and 2 show the mean of estimates of the derivatives Y_v, L_v', and N_v' and wind computed using (16), (17), (18), and (19) and the mean and standard deviation of the estimation error in percent, over 100 trials. The elapsed time for this computation is about 4 seconds. Although we stated that the wind estimate is insensitive to the estimation error of the derivatives, Tables 1 and 2 indicate that the minor estimation error of the derivatives is, the smaller the estimation error of wind, especially the downward component of the wind. For example, the mean of estimation errors for u_{wI}, v_{wI} and, w_{wI} is 0.3%, 0.2% and, 21.8% using (18), respectively. As comparing the estimation error of w_{wI} with that of u_{wI} and v_{wI}, the estimation error is minor for wind in the horizontal plane (u_{wI} and v_{wI}). Notably, the estimation error for u_{wI} and v_{wI} by (18) is smaller than the estimation error in N_v', which is as large as nearly 35%. Note that the means of estimation error in percent for X_v, X_{vu}, and X_{vv}, where $X = Y$, L', or N', are not small, they are similar. This result implies that the estimation error in X_v propagates to X_{vu} and X_{vv}, as stated below (23), which makes the estimation error of X_v less significantly deteriorate the accuracy of the wind estimation. However, we should also note that the larger the estimation error in the derivatives, the more error in wind results.

Table 1 Estimated parameters and estimation errors by LSM

Derivatives	True	Estimated (mean)	Error (%) (mean)	Error (%) (Stnd. dev.)
Y_v	-0.6596	-0.2386	-63.8	34.6
Y_{vu}	6.5956	2.3638	-64.2	34.8
Y_{vv}	-13.191	-4.7013	-64.4	34.7
L_v'	-0.8118	-0.4144	-49.0	1.7
L_{vu}'	8.1181	4.0575	-50.0	1.9
L_{vv}'	-16.236	-8.1529	-49.8	1.8
N_v'	0.2831	0.1851	-34.6	2.7
N_{vu}'	-2.8313	-1.8453	-34.8	2.8
N_{vv}'	5.6625	3.6938	-34.8	2.7
A_{dlat14}	36.463	30.8640	-15.4	4.6
A_{dlat24}	5.3414	0.8359	-84.4	2.6
A_{dlat34}	-1.8817	-0.7227	-61.6	3.7

Table 2 Estimated wind and estimation errors by LSM

Equation	wind	True	Estimated (mean)	Error (%) (mean)	Error (%) (Stnd. dev.)
(16) [Y]	u_{wI}	-10	-9.6784	-3.2	96.2
	v_{wI}	20	18.8300	-5.9	76.7
	w_{wI}	5	-22.0210	-540.4	2319.3
(17) [L']	u_{wI}	-10	-9.7917	-2.1	1.1
	v_{wI}	20	19.6750	-1.6	0.8
	w_{wI}	5	2.0161	-59.7	5.8
(18) [N']	u_{wI}	-10	-9.9684	-0.3	1.1
	v_{wI}	20	19.9540	-0.2	0.9
	w_{wI}	5	3.9087	-21.8	5.3
(19) [#]	u_{wI}	-10	-9.8589	-1.4	2.6
	v_{wI}	20	19.7570	-1.2	2.1
	w_{wI}	5	1.5249	-69.5	28.5

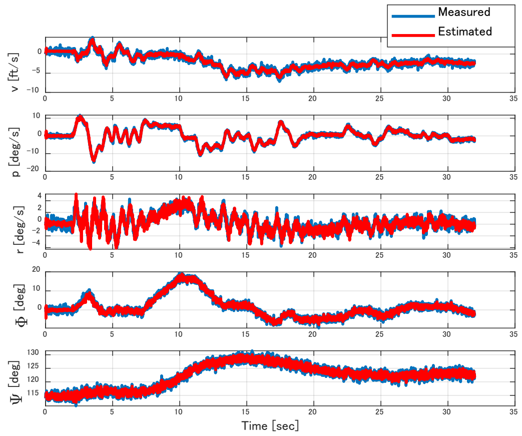

Fig. 3 Time history of the measured and estimated state variables

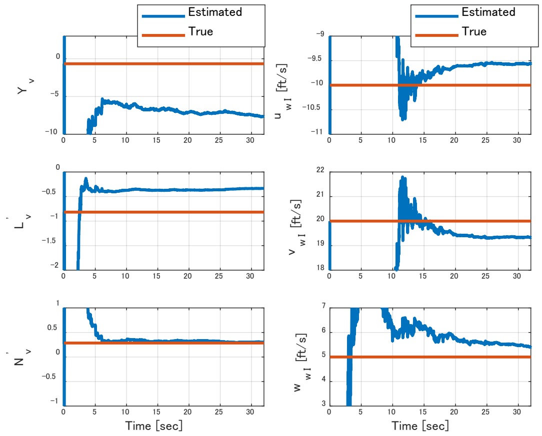

Fig. 4 Time histories of the estimated derivatives of Y_v, L_v', and N_v', and the wind of u_{wI}, v_{wI}, and w_{wI} by (19)

Table 3 Estimated parameters and wind and estimation errors by EKF

Derivatives	True	Estimated (mean)	Error (%) (mean)	Error (%) (Stnd. dev.)
Y_v	-0.6596	-5.9620	803.9	111.2
Y_{vu}	6.5956	57.6030	773.4	110.8
Y_{vv}	-13.191	-116.3100	781.8	110.8
L_v'	-0.8118	-0.3487	-57.0	4.3
L_{vu}'	8.1181	3.4329	-57.7	3.9
L_{vv}'	-16.236	-6.8777	-57.6	4.0
N_v'	0.2831	0.2613	-7.7	19.7
N_{vu}'	-2.8313	-2.4600	-13.1	18.6
N_{vv}'	5.6625	4.9873	-11.9	18.9
A_{dlat14}	36.463	61.1800	67.8	14.6
A_{dlat24}	5.3414	0.6411	-88.0	5.3
A_{dlat34}	-1.8817	-0.7074	-62.4	22.6

Table 4 Estimated wind and estimation errors by EKF

Equation	wind	True	Estimated (mean)	Error (%) (mean)	Error (%) (Stnd. dev.)
(16) [Y]	u_{wI}	-10	-9.6583	-3.4	1.4
	v_{wI}	20	19.5040	-2.5	1.1
	w_{wI}	5	4.8642	-2.7	7.2
(17) [L']	u_{wI}	-10	-9.8557	-1.4	1.9
	v_{wI}	20	19.7390	-1.3	1.4
	w_{wI}	5	1.7797	-64.4	13.7
(18) [N']	u_{wI}	-10	-9.4172	-5.8	2.5
	v_{wI}	20	19.0870	-4.6	1.8
	w_{wI}	5	2.4764	-50.5	25.5
(19) [#]	u_{wI}	-10	-9.6584	-3.4	1.4
	v_{wI}	20	19.5040	-2.5	1.1
	w_{wI}	5	4.8237	-3.5	7.2

For comparison, the EKF was designed and applied to estimate the parameters using the MATLAB® software provided by [11]. Equations (11) and (12) give the dynamic model for the time-update of the state variables, combined with the state equations for the parameters to be estimated. The selected covariance matrix of the measurements is R = diag([20 0.5 0.5 0.1 0.1]), where 'diag(x)' denotes a diagonal matrix with the diagonal elements x, and the covariance of the process noise is P_x = diag([1.0 0.1 0.1 0.1 0.1]) for the state variables and P_θ = diag(10^6) $\in \mathrm{R}^{32\times 32}$ for the parameters. Tables 3 and 4 show the estimates of the derivatives and wind and the mean and standard deviation of the estimation error in percent over 100 trials. The elapsed time for this computation is 1889 seconds (31 minutes). Figure 3 shows the time histories of the measured and estimated state variables in a trial. Figure 4 shows the time histories of the derivatives of Y_v, L_v', and N_v', and the wind of u_{wI}, v_{wI}, and w_{wI} estimated by (19). We chose a value at the terminal time of estimation as an estimate for each derivative.

As Table 3 shows, the estimation errors of the derivatives are not necessarily minor, especially for Y_v, Y_{vu}, and Y_{vv}. Despite this, the mean estimation errors of u_{wI} and v_{wI} are as small as 1.4% and 1.3% using (17), respectively. This result indicates that the estimation error of the derivatives does not significantly affect the accuracy of the estimation of the wind in the horizontal plane (u_{wI} and v_{wI}). On the other hand, the estimation error of w_{wI} is 2.7%, which is more significant than those for u_{wI} and v_{wI} but smaller than the LSM. Thus, wind estimates are less sensitive to the estimation error of the derivatives in EKF as well, except for w_{wI} by (18). This insensitivity is due to the invariance of state estimation to linear state transformation such as (23); if the linear state transformation holds between actual states, then the same transformation holds for their estimated states, which is valid for the derivatives since EKF treats them as state variables. A massive error in the estimate of w_{wI} by (18) may be due to a significant estimation error in A_{dlat34}. A problem with an EKF is that the computation time is considerably larger than that by LSM, but with a good computation capability, the real-time estimation will be possible by EKF, as shown in Fig. 4.

6. Conclusions

This paper has shown that steady wind is an additive disturbance to the states of an aircraft's dynamic model, making the wind appear as part of stability derivatives in the linearized model, which allows us to estimate wind by estimating stability derivatives of a lateral-directional motion model of an aircraft. We applied the proposed method to flight data generated by NASA's Generic Transport Model. The simulation results show that the estimation error of horizontal wind is less than 6% by the least-squared method (LSM) and 6% by an extended Kalman filter (EKF). In either method, the estimation error of wind is much smaller than the derivatives used to calculate the wind. We gave the reason for this concerning the estimation by the LSM and an EKF. On the other hand, both LSM and EKF result in a more significant estimation error in the vertical wind. We should investigate estimation performance for various winds and improve the precision for vertical wind. We should also consider biases of measurements and (slow) variations of wind as future work.

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