

FAST METHOD AND AN INTEGRATED CODE FOR SONIC BOOM PREDICTION OF SUPERSONIC COMMERCIAL AIRCRAFT

Yu-Lin Ding, Zhong-Hua Han* Jian-Ling Qiao, Wen-Ping Song & Bi-Feng Song

Institute of Aerodynamic and Multidisciplinary Design Optimization, National Key Laboratory of Science and Technology on Aerodynamic Design and Research, School of Aeronautics, Northwestern Polytechnical University, Xi'an 710072, P. R. China

Abstract

Sonic boom prediction plays a significant role in the preliminary design of next-generation supersonic commercial aircraft. There is an enormous demand of sonic boom evaluations in a preliminary design stage, during which a suitable configuration needs to be determined through extensive modifications and evolutions. Therefore, it is of great interest to develop a robust, efficient and reasonably accurate sonic boom prediction method. A modified linear method is developed for sonic boom prediction to address the enormous computational cost associated with preliminary design. Aiming at improving prediction accuracy, a more reasonable method for calculating equivalent area distribution is applied. To be specific, calculation method of equivalent area distribution due to volume and lift is presented, which can improve the accuracy of the prediction. An integrated code named FA-Boom for fast sonic boom prediction is developed and its accuracy and efficiency is validated by comparison with high-fidelity result as well as experimental data.

Keywords: supersonic commercial aircraft, sonic boom, fast prediction method, FA-Boom

1. Introduction

The next-generation supersonic commercial aircraft has become one of the most promising aircrafts of the future. According to the development schedules and research progress of research institutions such as NASA, JAXA and so on, the next-generation supersonic commercial aircraft is expected to come into service in about 2030. However, the re-introduction of commercial supersonic aircraft still confronts a primary obstacle that the supersonic overland flight is prohibited due to sonic boom, which has taken effect from the era of Concorde. Obviously, reducing sonic boom to community acceptable level is the most crucial work to realize supersonic commercial operating[1].

Due to low boom concern, sonic boom evaluations are enormously demanded in conceptual design stage of a supersonic commercial aircraft. Generally, large number of configurations need to be modified and selected to obtain a well-performing baseline at this stage[2]. Therefore, sonic boom prediction method should have low computational cost and reasonable accuracy.

The available sonic boom prediction methods can be classified into two categories: high-fidelity and low-fidelity methods, according to precision, physical complexity and computational expense. Generally, high-fidelity prediction method incorporates computational fluid dynamics (CFD) simulation and nonlinear propagation method. CFD simulation is able to capture near-field overpressure signals accurately. Nonlinear propagation method that takes into account the effects of distortion, relaxation, absorption of atmosphere can obtain ground waveform. Although high-fidelity prediction method can present accurate far-field sonic boom, complex pre-processing and highly expensive calculation make high-fidelity method be not suitable for the preliminary design. To reduce complexity and computational cost, a low-fidelity and cheap prediction method is usually preferable by engineers. Method based on modified linear theory is the most common used approach. The modified linear theory introduces F-function to represent the disturbance intensity of the aircraft to the air. F-function is determined by equivalent area A_e . A_e approximates the aircraft as an equivalent body, according to a superposition of volume and lift disturbances generated by aircraft

along longitudinal axis. Obviously, precision of sonic boom prediction by modified linear theory is mainly depended on calculation of A_e . To improve fidelity of the modified linear method, it's of great significance to develop a more reasonable method to compute A_e .

Aiming at sonic boom evaluation in preliminary design of a supersonic commercial aircraft, this paper develops a fast prediction method based on modified linear theory. Effect of angle of attack (AOA) is considered in calculation of equivalent area distribution due to volume. Effect of AOA and geometry coordinates are considered in calculation of equivalent area distribution due to lift. With the two improvements above, an integrated code named FA-Boom is developed for purpose of efficient analysis. The precision of results by FA-Boom is validated by comparison with high-fidelity results or experimental data.

2. Methodology

2.1 Modified Linear Theory for Sonic Boom Prediction

The modified linear theory for sonic boom prediction is developed based on the Whitham's theory, which is originally presented for axisymmetric flow fields. The Whitham's theory[3] describes the supersonic slender body as an equivalent axisymmetric body with a source function named F-function, which is of the form

$$F(y) = \frac{1}{2\pi} \int_0^y \frac{A_e''(\xi)}{\sqrt{y-\xi}} d\xi \quad (1)$$

where A_e is the equivalent area, given by cross-section cuts by Mach planes. The theory works well for slender and non-lifting configurations. However, for a supersonic aircraft, lift has a significant impact on the sonic boom intensity. Walkden[4] extended Whitham's theory to wing-body combinations, which consider lift contribution effect to sonic boom. Walkden decomposed the A_e in equation (1) into volume and lift components. Thus, equation (1) is evolved into

$$F(y) = \frac{1}{2\pi} \int_0^y \frac{A_{eV}''(\xi) + A_{eL}''(\xi)}{\sqrt{y-\xi}} d\xi \quad (2)$$

where A_{eV} , A_{eL} are the equivalent areas due to volume and lift, respectively. A_{eL} is formulated by

$$A_{eL}(x) = \frac{\beta}{\rho U^2} \int_0^x L(x) dx \quad (3)$$

where $\beta = \sqrt{Ma^2 - 1}$, ρ is density at cruise altitude, U is velocity of aircraft. Once F-function is obtained, near-field overpressure can be calculated by Whitham's theory, expressed by equation (4). Crucially, nonlinear effect of shock wave propagation is simulated. The characteristic line becomes nonparallel, which is entirely different from that of supersonic linear theory. Therefore, the axial coordinate is given by equation (5).

$$\frac{dp}{p_0}(\chi) = \frac{\gamma Ma^2}{\sqrt{2\beta r}} F(y) \quad (4)$$

$$\chi = y - kF(y)\sqrt{r} \quad (5)$$

where $\gamma=1.4$, $\chi = x - Br$, $k = 2^{-1/2}(\gamma+1)Ma^4 B^{-3/2}$. Obviously, overpressure signals calculated with nonlinear distortion might lead to nonphysical waveform. To correct the near-field signal to a real pattern with shock, area-balanced method must be applied, and the principle sketch of balance operation is shown in Figure 1. The position of shock should satisfy area-balanced condition. For example, at the nose shock locations, the area of shadow zone A is equal to that of B, and it is same for C and D for rear shock.

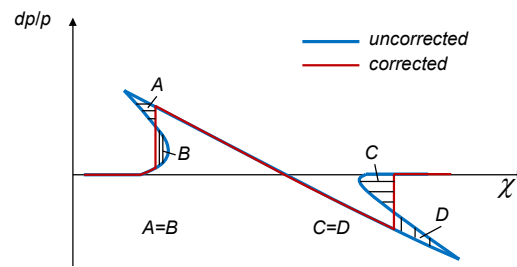


Figure 1 - Sketch of area-balanced method

In the conceptual design stage, complex configurations need to be evaluated. Aiming at improving the precision of sonic boom prediction to the greatest extent, the calculation of equivalent area distribution is crucial.

2.2 Calculation of Equivalent Area Due to Volume

Equivalent area due to volume (A_{eV}) comes from disturbances caused by volume of an aircraft. Taking an airplane as a reference, disturbance generated forms a backward conical surface, namely Mach cone. There is no disturbance before the cone, which is shown in Figure 2. For sonic boom analysis, reference point should be located at observer. The signal observer can receive must be generated on the surface of fore Mach cone which originates at observer. When the aircraft moves through the fore Mach cone, the disturbance is generated due to the cross-section of aircraft and fore cone. Thus, at the given moment, equivalent area due to volume is calculated from the cross-section area.

In a practical situation, aircrafts cruise far enough away from a observer, so the fore cone surface in the vicinity of the airplane can be approximated by a planar surface tangent to the cone. It means that cross section cut by the Mach cone can be considered as Mach plane, which is proved reasonable[5]. Two cut methods are shown in Figure 3. Based on the cross-section, equivalent area due to volume is normal projection of cuts along Mach planes. AOA has a significant impact on cross section area, which is sketched in Figure 4. Effect of AOA should be taken into account in Mach plane slice, otherwise an evident error of equivalent area will be introduced.

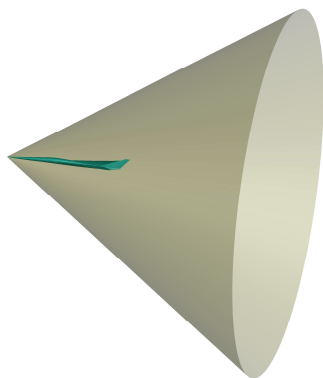


Figure 2 - Sketch of influence zone of supersonic aircraft

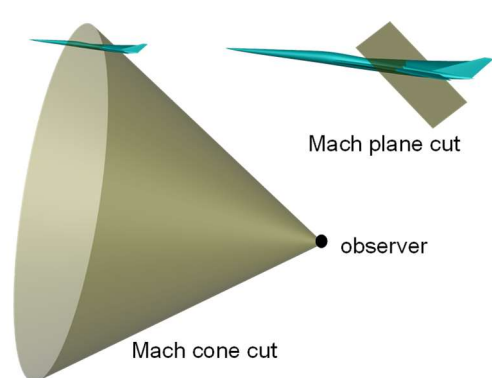


Figure 3 - Mach cone cut and plane cut for sonic boom analysis

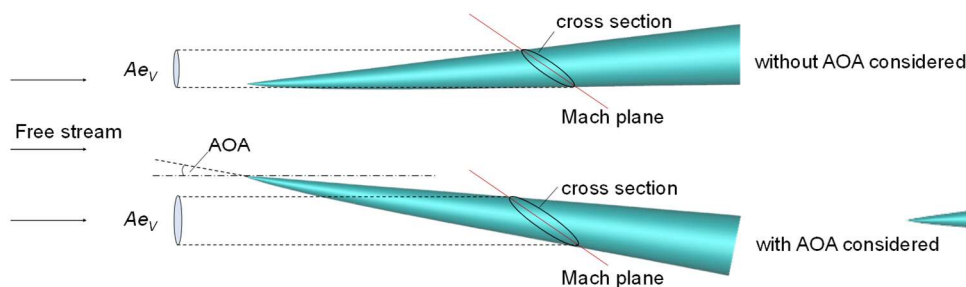
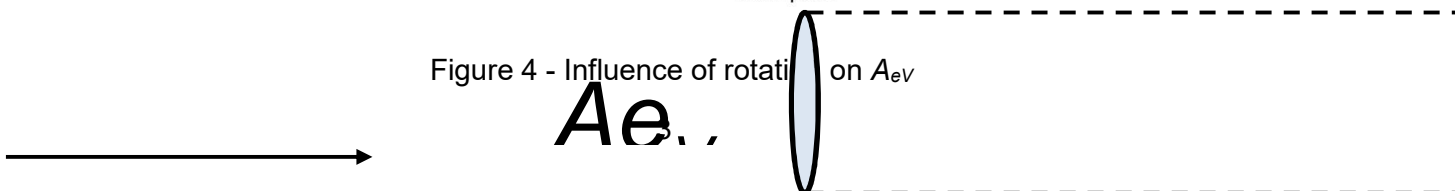


Figure 4 - Influence of rotation on A_{eV}



2.3 Calculation of Equivalent Area Due to Lift

For fast sonic boom prediction, low-fidelity aerodynamic calculating methods such as vortices lattice method and panel method are widely applied to compute lift characteristics. While pressure distribution is obtained, axial lift distribution can be computed. However, distinct from volume effect, lift effect is computed as the superposition of the lift along the axis before current slice, rather than the lift on the slice. Therefore, the lift contribution is a function of accumulated lift on each surface element, shown in Figure 5.

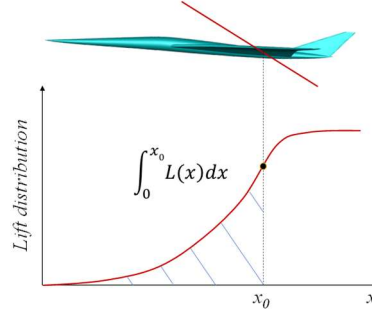


Figure 5 - Sketch of lift distribution calculation

Apparently, lift on surface element also has to be translated along Mach planes. Meanwhile, effect of AOA should also be considered as we have explained before. To establish correct axial coordinates of lift distribution, coordinate transformation is necessary. The relationship between original coordinates and transformed coordinates is shown in Figure 6. For simplicity, assume that coordinate of nose point is $(0,0,0)$. To operate coordinate transformation, a reference height h is set below the aircraft. All surface elements should be projected to reference height along Mach plane. Thus, for a surface element of aircraft whose coordinate is (x_0, y_0, z_0) , the axial coordinate of projected point is

$$x_1 = x_0 + (z_0 - h)\beta \quad (6)$$

So the axial coordinate of this element for lift distribution is

$$x_1 = x_0 + (z_0 - h)\beta + h\beta = x_0 + z_0\beta \quad (7)$$

After axial coordinates of all surface elements are calculated, lift distribution is obtained by accumulation, and equivalent area due to lift is converted by equation (3).

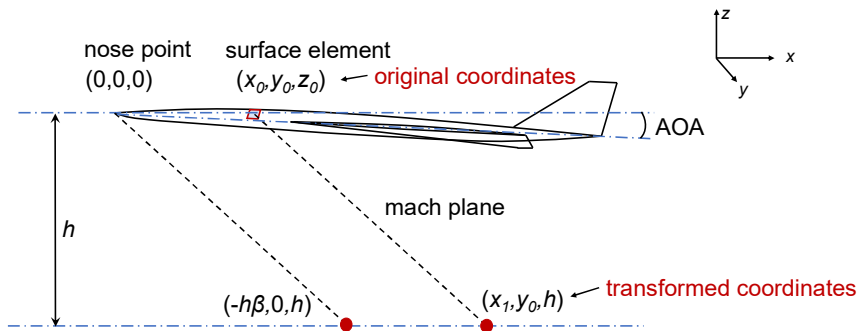


Figure 6 - relationship between original coordinates and transformed coordinates

2.4 Main Process of Fast Prediction Method

As presented above, the main process of fast prediction of near-field pressure is shown in Figure 7. The digital model of geometry and state parameters of computation are input. Then, by Mach plane slice and fast aerodynamic computation, equivalent area due to volume and lift can be calculated respectively. The total equivalent area distribution is the sum of that due to volume and lift directly, and then, F-function is computed from second derivative of equivalent area distribution as mentioned in section 2.1. Finally, with area balanced method applied to the distorted overpressure signal, real shocks in signal are found and near-field signal of sonic boom is predicted.

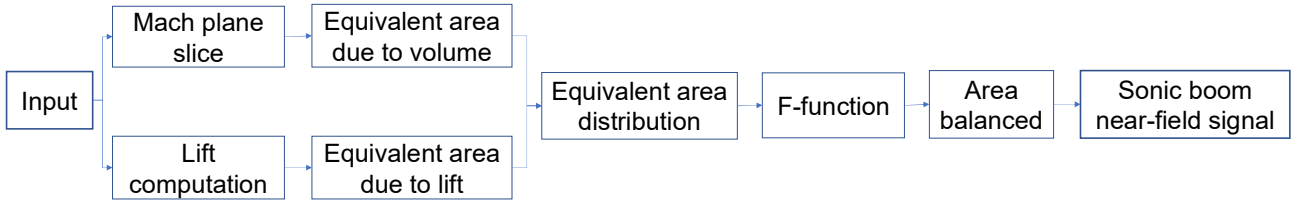


Figure 7 - Main process of fast prediction of near-field pressure

3. Integrated Code FA-Boom for Fast Prediction

This paper developed an integrated code named FA-Boom for fast sonic boom prediction, based on the method presented above. The code consists of five main modules of geometry parameterization, volume slice, lift prediction, F-function calculation and area balanced method. The details of modules are explained briefly in the following paragraph.

Geometry parameterization transforms input geometry to digital model, which can be used to calculate by volume slice and lift prediction. The form of digital model is chosen to be structured mesh, being convenient for definition and modification. Also, the structured mesh is required by lift prediction code PANAIR.

Volume slice is able to cut aircraft along axial direction and record the coordinates of slices. The code searches intersections of slices and mesh successively. Then the module computes the area of intersection line so that cross section area is obtained.

Lift prediction module integrates an open sourced code named PANAIR developed by Boeing corporation[6]. PANAIR is an aerodynamic analysis program which can predict inviscid subsonic and supersonic flows of arbitrary configurations. High-order panel method is applicated to solve linearized partial differential equation. Lift prediction module extracts pressure coefficient obtained by PANAIR and computes equivalent area distribution due to lift.

F-function calculation module mainly based on equivalent area distribution, computes F-function and distorted pressure signal by equations in section 2.1. What needs to be explained, linear extrapolation is applicated in the numerical integration of equation (1) at the upper limit, because the value will be infinity at that point, which is nonphysical.

Area balanced module can solve the problem of complex distorted pressure signal with large number of multivalued points shown in Figure 1 to find the position of real shocks.

Combined of modules above, FA-Boom is able to predict sonic boom near-field signal of arbitrary configuration with very cheap cost of analysis.

4. Validation of the Developed Method

To validate the fast prediction method of sonic boom near-field signal, several models are chosen for analysis, whose high-fidelity results or experimental measurements are used to be reference. To assess the applicability of FA-Boom, near-field signals got from FA-Boom are used as input to propagate to ground. Far-field propagation is conducted by an in-house code named bBoom based on augmented burgers equation, which has been described and validated in reference[7][8]. Also, the computation cost of FA-Boom and Euler method is investigated.

4.1 Axisymmetric Equivalent Area (AXIE)

Axisymmetric Equivalent Area (AXIE) is an axisymmetric body which has the same equivalent area distribution with C25F on the centerline. AXIE is one of standard model chosen for the Second AIAA Sonic Boom Workshop (SBPW-2). Reference[9] describes the detail of AXIE, meanwhile high-fidelity results from CFD analysis are provided. The geometry of AXIE is shown in Figure 8, The reference length of the model is 32.92 m. The AXIE is calculated at $Ma=1.6$, and near-field signals are extracted from 1, 3, 5 times reference body lengths away from the body. The high-fidelity results chosen for contrast is Euler simulation by Cart3D [10].

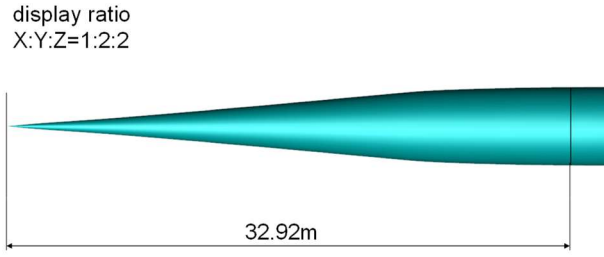


Figure 8 - Geometry of AXIE

Figure 9 presents comparison of near-field signals predicted by FA-Boom and simulation results by Cart3D at $H/L=1, 3, 5$. The waveforms calculated by FA-Boom shows good agreement with results of Cart3D at three positions, and main details of shocks profile are captured. Signals obtained by FA-Boom and Cart3D are propagated to ground from 15760m altitude by bBoom, meanwhile their perceived noise level are calculated. Comparisons of ground waveforms are shown in Figure 10. The waveforms propagated from FA-Boom are similar to that from Cart3D, with slight differences in details. Difference of perceived noise level between FA-Boom and Cart3D is less than 1 PLdB. Therefore, it can be concluded that FA-Boom is accurate enough for prediction of AXIE.

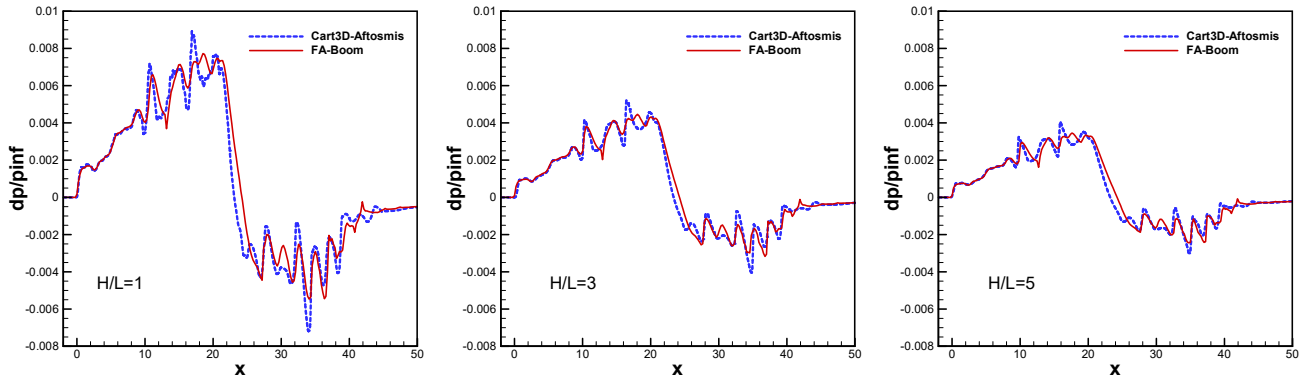


Figure 9 - Comparisons of signals predicted by FA-Boom and Cart3D[10] at three positions

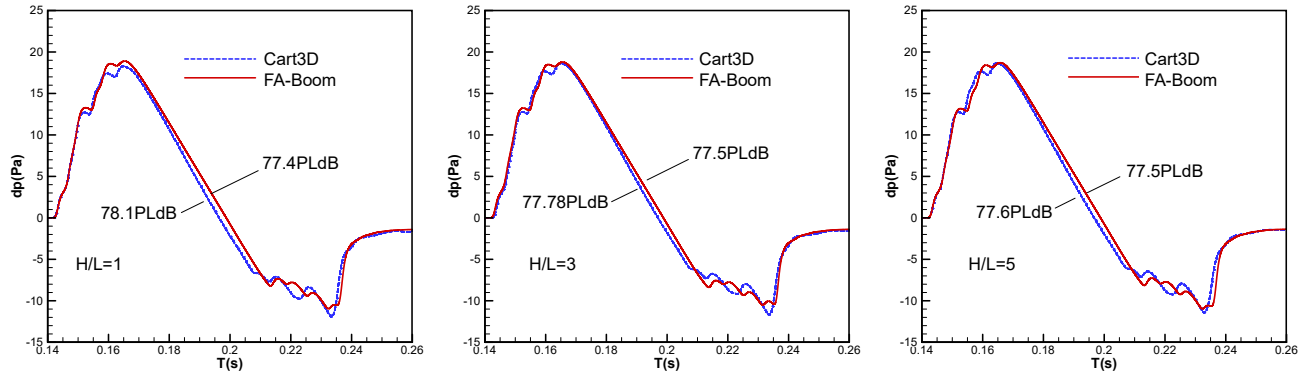


Figure 10 - Comparisons of ground waveforms by FA-Boom and reference at three positions

4.2 Delta Wing Body (DWB)

Delta wing body (DWB) case is from the First AIAA Sonic Boom Workshop (SBPW-1) because it has a simple axisymmetric fuselage and diamond airfoil wing with available wind tunnel data[11]. The platform of DWB and geometry input for FA-Boom are shown in Figure 11. The model has a reference length of $L = 0.1752\text{m}$. The sting of wind tunnel model is replaced by cylinder with constant diameter. The model is calculated at $Ma=1.7$ with no incidence. The near-field signal at 3.6 times reference length below on the centerline is analyzed, and wind tunnel experiment data is chosen as reference, including data of uncertainty effect[12].

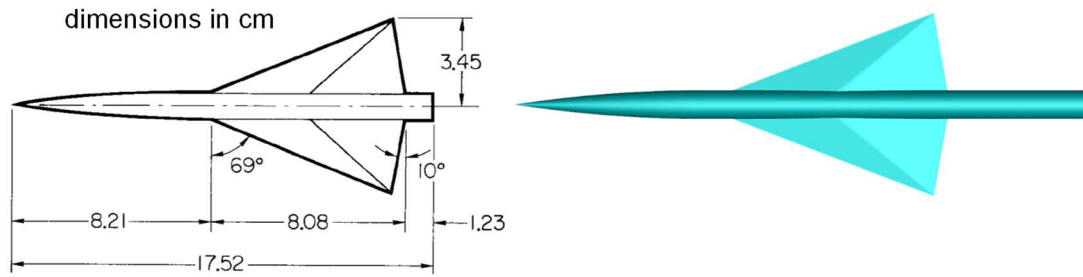


Figure 11 - The platform of DWB and geometry input for FA-Boom

Figure 1 shows comparison of near-field signal predicted by FA-Boom and wind tunnel data. Obviously, the amplitude and location of prediction are in good agreement with experiment at the shocks. The precision is acceptable from the point of view of fast prediction, even if the strength and location of rear shock is slightly different from experimental value. There is no altitude data of DWB for far-field propagation, so contrast of ground waveforms is omitted. From result presented above, FA-Boom achieves good accuracy for non-lift wind-body like DWB.

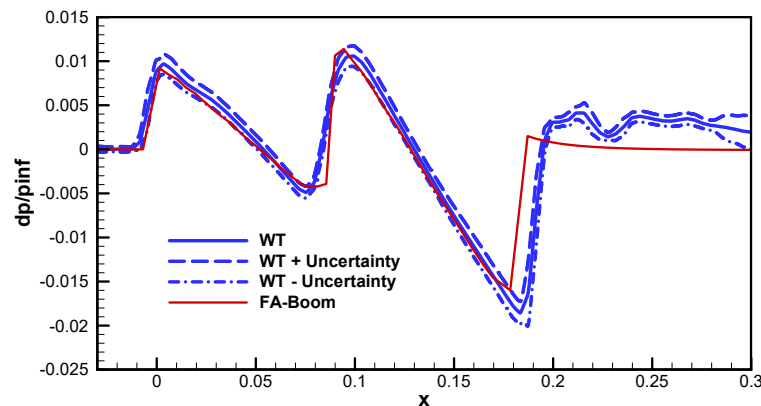


Figure 12 – Comparison of near-field signal predicted by FA-Boom and wind tunnel data

4.3 A simplified LM-1021 Model

LM-1021 is a concept of supersonic transport designed by Lockheed Martin, whose scaled (1/125) wind tunnel model is chosen as one of cases for near-field prediction in the SBPW-1[11]. This paper selects LM-1021 as validation case since it is a quite advanced low-boom configuration released in the workshop currently. However, for the sake of simplicity, the nacelles and V-tail are removed, the geometry for computation is shown in Figure 13. The angle of attack is specified as 2.1° which is required in SBPW-1, and the Mach number is 1.6. Inviscid simulation results using hybrid grids are high-fidelity reference for this case.

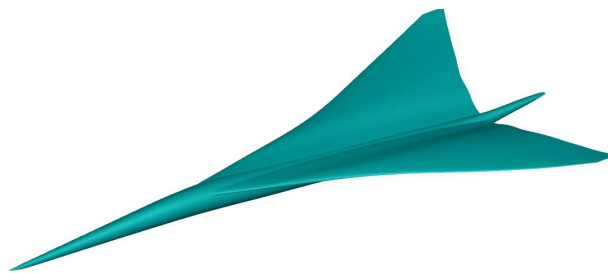


Figure 13 - Geometry of simplified LM-1021

Near-field overpressure predicted by FA-Boom is compared with result from Euler method, the comparison is shown in Figure 14. The waveforms predicted by FA-Boom match well with reference at three signal extraction positions. Especially, in the fore-body region of 0~55m, the predictions are almost identical to the reference waveforms. Although small oscillations occur along the waveform, the signals got by FA-Boom still show good agreement with that by Euler method. The results indicate that FA-Boom has satisfactory accuracy for sonic boom prediction in preliminary design process. Besides, it is much more efficient than the Euler method, which will be demonstrated later. Small difference can be observed in the aft-body region. The reason for this phenomenon is that modified linear theory counts volume and lift effects on sonic boom, corresponding to the monopole

and dipole effects on sonic boom in multipole theory. Namely, higher order pole effect which has a significant impact on sonic boom signal of aft-body is ignored in modified linear theory. This is required to improve in further work.

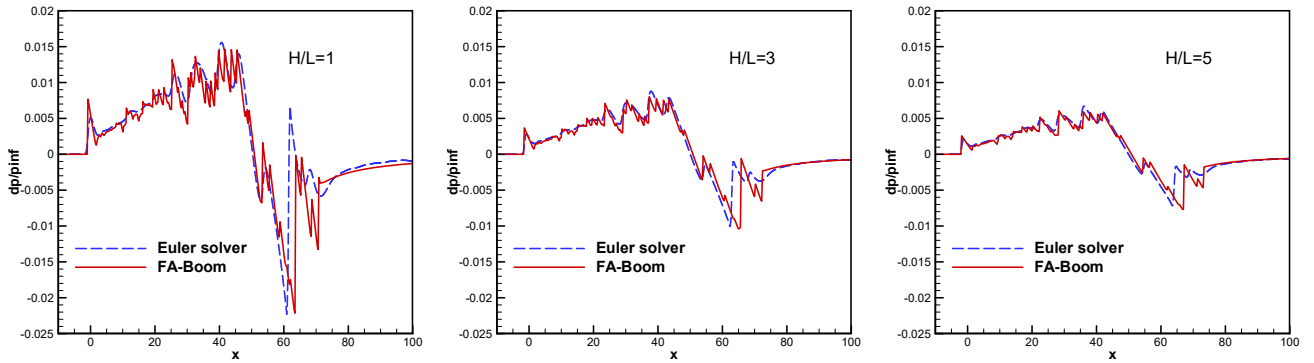


Figure 14 - Comparison of prediction and high-fidelity result by solving Euler equations for simplified LM-1021

The ground waveforms are calculated from 15240m altitude, which is designed cruising altitude for LM-1021 concept. The comparison of ground waveform is presented in Figure 15. The strength of overpressure from FA-Boom prediction is slightly weaker than that from Euler method in the fore-body region, and shape of signal tends to be similar. In the aft-body region, waveform from FA-Boom appears difference with Euler method, which results in larger PLdB. The results of simplified LM-1021 indicates that prediction of modified linear theory has acceptable precision for complex low-boom configurations.

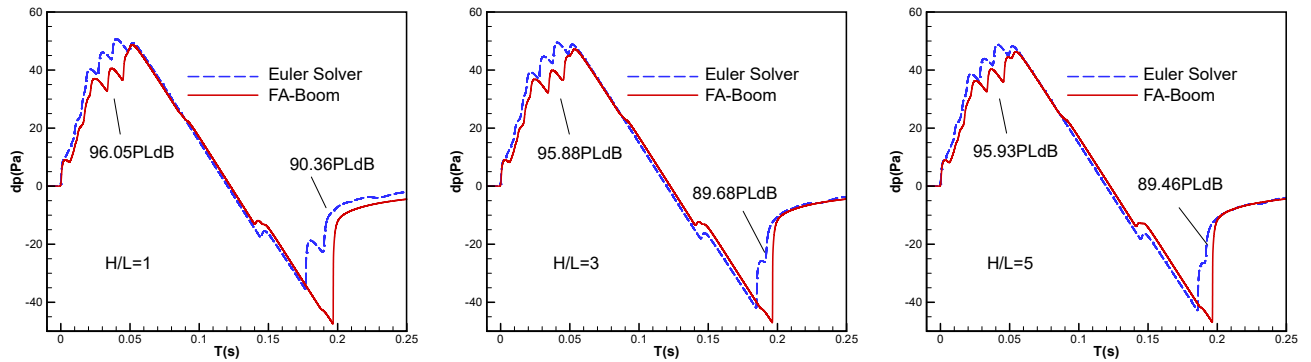


Figure 15 - Comparison of ground waveform of simplified LM-1021

4.4 Tupolev Tu-144

Tupolev Tu-144 is known as one of the first-generation supersonic transport which had been in operation. The paper obtains geometry of Tu-144 from reference[13], and nacelles are removed for simplification. The geometry for input is shown in Figure 16. The case is calculated at $Ma=2.0$, and angle of attack is 6.389° at which the lift coefficient is 0.126. Near-field signal simulated by Euler solver is high-fidelity reference for this case. The near-field overpressure is extracted at H/L=1, 3, 5 for contrast.



Figure 16 - Geometry of Tu-144

Figure 17 presents the comparison of near-field signals calculated by FA-Boom and Euler solver. The strength of nose shock, wing shock and rear shock are close to reference. In this case, the fact can be found that the wing shock of FA-Boom appears earlier than Euler result, and rear shock appears much later.

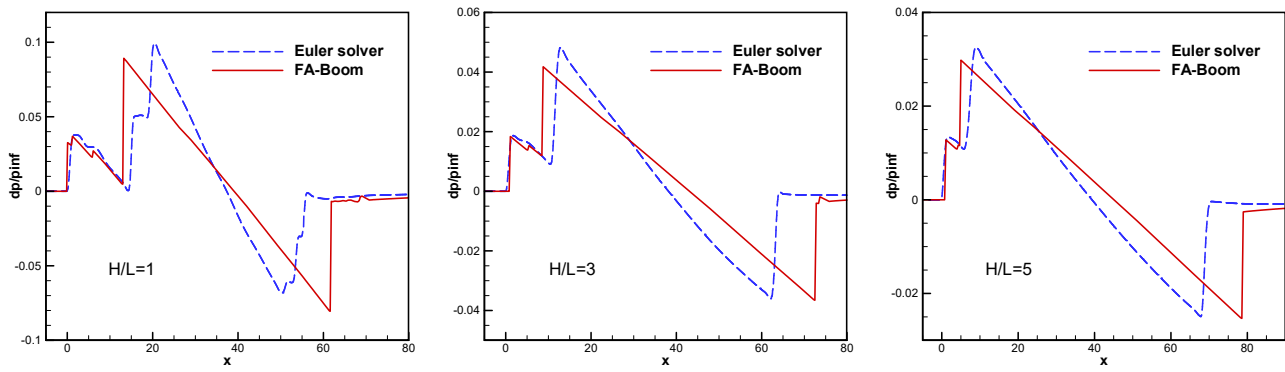


Figure 17 - Comparison of near-field signals calculated by FA-Boom and Euler solver

To take a deeper insight of the reason, surface pressure coefficient computed by FA-Boom and Euler solver are compared. Comparison of pressure coefficient contour are shown in Figure 18 and Figure 19. From Figure 18, the fact that pressure coefficient contour of lower surface computed by FA-Boom is similar to Euler solver. However, contour of upper surface is distinctly different between FA-Boom and Euler solver, primarily on the wing. Streamlines of above wing is extracted from flow field of high-fidelity simulation, presented in Figure 20. Due to sharp leading edge and large cruise AOA of Tu-144, strong vortices are generated along the leading edge of inner wing, which introduce strip-shaped area of low pressure. Because of the assumption that flow is irrotational of panel method, the vortex was not successfully calculated by FA-Boom. According to the reason, the surface pressure and the equivalent area distribution due to lift might be inconsistent with reality.

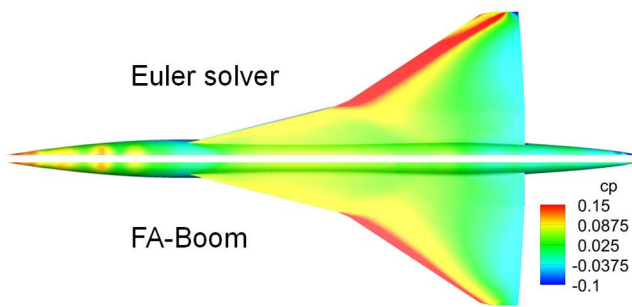


Figure 18 - Comparison of pressure coefficient contour at lower surface

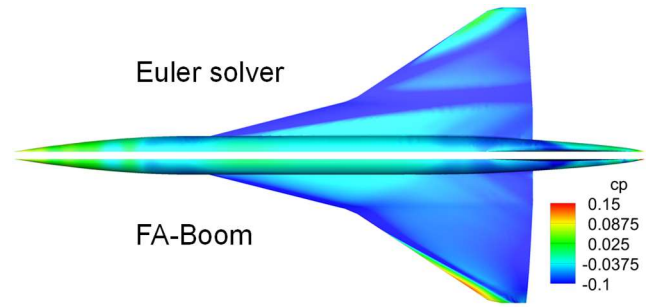


Figure 19 - Comparison of pressure coefficient contour at upper surface

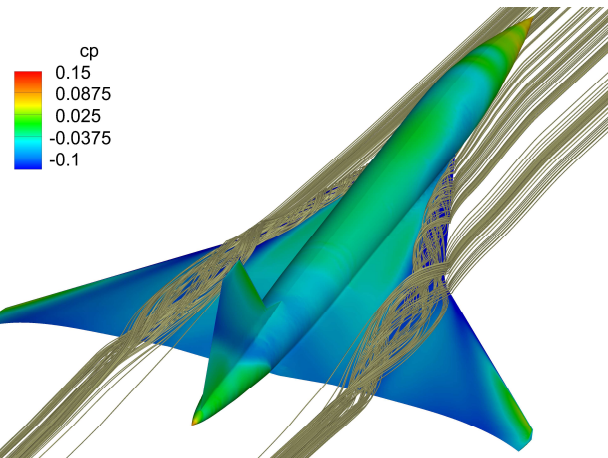


Figure 20 - Streamlines of vortex generated along leading edge of inner wing

Far-field propagation is carried out by bBoom and ground waveforms are shown in Figure 21. The ground waveforms of FA-Boom and Euler method show good agreement of amplitude and shape. Particularly, front shock has almost identical strength, only phase of rear shock appears small distinction. Difference of PLdB between prediction and high-fidelity result are less than 1 PLdB. The comparison indicates that precision of FA-Boom is reliable enough for large supersonic transport. Nevertheless, configurations with vortex are still challenge to FA-Boom. Aiming at improving precision, panel method can be replaced by Euler simulation.

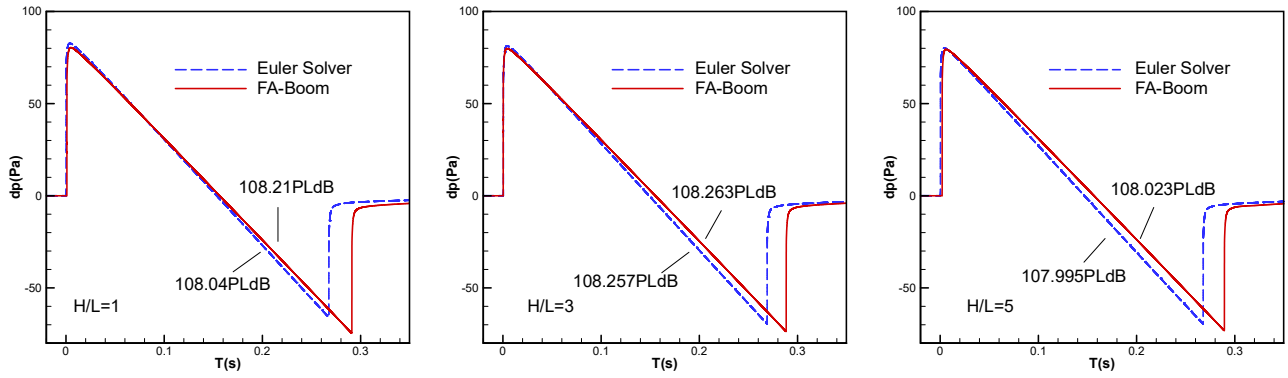


Figure 21 - Comparisons of ground waveform propagated from prediction and reference

4.5 Discussion of Computational Cost

The computational cost of FA-Boom method is compared with that of Euler method for the simplified LM-1021 configuration. For Euler simulation, a coarse grid and a fine grid are adopted, respectively. The symmetry plane mesh is shown in Figure 22. The quantity of coarse grid cells is 1.417 million, and 13.619 million of fine grid. The Euler simulation and the FA-Boom prediction are both operated on an Intel i9-9900K CPU. When convergence achieved, 278 minutes are taken by coarse grid and 21 hours by fine grid. CPU time taken by FA-Boom prediction is 61 seconds for the whole run. Obviously, the cost of FA-Boom is much less than Euler method. Table 1 summaries the information and cost of three calculations above.

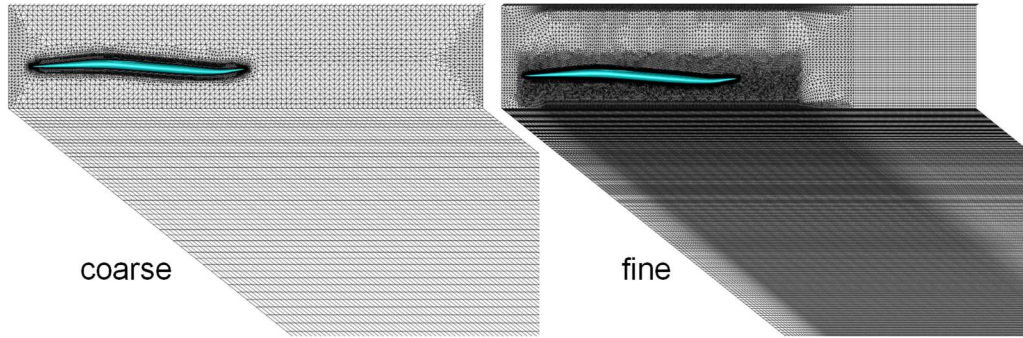


Figure 22 - Symmetry plane mesh of the coarse and fine grid

Table 1 the information and cost of three calculation

	Cells number	CPU Time
Fine grid	13.619 million	21 hours (8 cores)
Coarse grid	1.417 million	278 minutes (single core)
FA-Boom	\	61 seconds (single core)

Figure 23 shows the comparisons of results obtained by FA-Boom and Euler solution on two grids. Fine grid can capture shocks well but the cost is very expensive for conceptual design. Although high performance parallel computing can reduce computation time, the cost is still difficult to accept in conceptual design. The main waveform details are preserved by FA-Boom, while much dissipation occurs by Euler solution on coarse grids, which leads to the loss of details. Thus, accuracy of FA-Boom is slightly better than coarse grid in this case with much less CPU time. From the fact presented above, the conclusion can be obtained that FA-Boom has prominently less computational cost and better availability than Euler simulation for conceptual design.

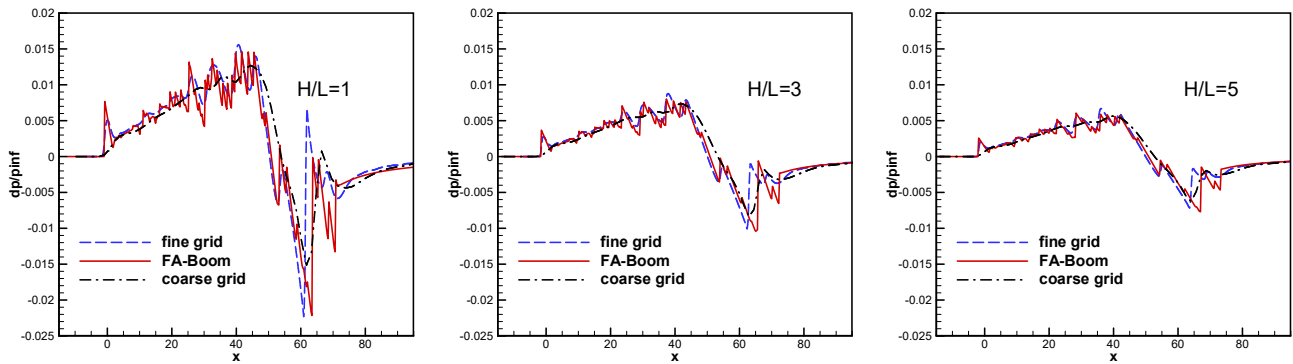


Figure 23 - Comparisons of results obtained by FA-Boom and Euler solution on coarse grid

5. Conclusion

Aiming at the significant demand of efficient and reasonably accurate sonic boom evaluation in preliminary design of a supersonic commercial aircraft, this paper develops a modified linear method for sonic boom prediction. The calculation methods of equivalent area distribution due to volume and lift are presented and applied to improve the precision of the prediction. Five main modules of geometry parameterization, volume slice, lift prediction, F-function calculation and area-balanced method are integrated into a fast prediction code named FA-Boom. Precision of FA-Boom is validated by calculating near-field overpressure signals of four cases, by comparison with results obtained from high-fidelity simulation or wind tunnel data. Research of validation indicates that FA-Boom has desirable accuracy for non-lifting slender body and wing-body configuration. For configurations with lift but without vortex above the wing, prediction by FA-Boom is accurate enough for the fore-body, while appears small deviation for the aft-body. Due to the ignorance of high-order pole effect in modified linear theory, the absolute value of negative pressure in aft-body region is calculated larger. For those configurations with vortex, FA-Boom has difficulty in lift calculation because of the irrotationality hypothesis of panel method. Replacing panel method with inviscid simulation is a potential and feasible approach to improve the fidelity of prediction. Besides, the FA-Boom is proved to have much higher efficiency than the Euler method in sonic boom prediction.

In summary, fast prediction method and the integrated code named FA-Boom are developed by this paper. The code is proved to have acceptable accuracy of sonic boom evaluation for supersonic commercial aircraft configurations, meanwhile the efficiency of analysis is much higher than Euler method. The code is able to satisfy requirements in preliminary design of supersonic commercial aircraft.

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7. Contact Author Email Address

Zhong-Hua Han, Professor, hanzh@nwpu.edu.cn, Corresponding author.

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