Study of Non-rotating microphone array for spinning duct mode detection

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Abstract

The traditional analytical model based rotational in duct microphone array for acoustic duct mode identification techniques are well developed and were widely used in fan duct mode analysis of aero engine, This paper try to find a way to see the induct mode shape of fan noise by the far field microphone array, so that we don't need to make many holes at the fan case and nacelle to install microphone array. This study provides a numerical model based tonal noise duct mode decomposition method, the in duct modes at noise source could be identified by the far field microphone array through the numerical propagation functions or transfer functions. The numerical case study shows that it is feasible to identify the induct discrete duct modes of tonal noise..

Keywords: far field microphone array; acoustic duct mode; numerical calibration; mode identification

1. General Introduction

The acoustic signal of turbo machine is rotating speed dependent especially for rotor stator interaction tones and combination tones. Spinning duct modes identification is one of the effective processes to analyze the mechanism of turbo machine noise generation and propagation, since all of tonal signal generated from turbo machine can be expressed as a superposition of a series of spinning duct modes, and the spinning duct modes are also the most important input information for acoustic liner design.

Traditional spinning duct modes detection method is based on the analytical modal of circular duct or annular duct with one dimension uniform potential flow. NASA Glenn Research Center developed a continuous rotating microphone rake system to detect the rotor-stator interaction tonal spinning modal structure of turbo fan duct acoustic from 1990s, the rotating rake system has been used in developing and evaluating a number of low noise fan concepts as well as in verification of several aero-acoustic codes[1][2][3][4]. DLR developed a rotating microphone rake optimization method based on duct acoustic analytical model and least-mean-squares fit method in 2006[5]. From 2009 to 2019,DLR kept developing analytical model based fan broad band noise modal decomposition technique by using rotating induct microphone array[6][7][8]. All of above provide many interesting techniques to see how it works on fan noise generation and propagation, the critical points for duct mode decomposition include: the microphone array position, the phase match among all the measurement channels, the accurate propagation function or transfer functions from noise source to receiver, the frequency and the flow parameters. But traditional induct microphone array method need to build up expensive rotating devices such as rotating duct or rotating rake to scan the sound map of measurement surface. Actually from point of view of industry, the next question is if we could find a way to see the induct mode shape of fan noise by the non-rotating far field microphone array, so that we don't need to make the rotating device or to drill many holes at the fan case and nacelle for microphone array installation. Thanks for the well developed numerical CAA techniques, CAA could help to get the accurate transfer functions or propagation functions from the noise source to the receiver microphone array. This paper try to find a numerical model based tonal noise duct mode identification method by using far field microphone array. The in duct modes at noise source could be decomposed by the far field microphone array through the numerical transfer functions or propagation functions. The numerical case study shows that this method is feasible to identify the induct discrete duct modes of tonal noise.

2. The mode decomposition model for far field microphone array technique

As described above, the quality of the duct mode decomposition are related to the accurate microphone array position, the phase match among all the measurement channels, the accurate propagation function or transfer functions from noise source to receiver, the frequency and the flow parameters. The far field microphone array technique leverage the concept of analytical model for duct mode decomposition, the only different point is to change the analytical mode shape function to numerical mode shape function. The following describes the model derivation.

2.1 A brief review of analytical model for mode decomposition with rotating microphone array

The analytical in duct solution for one dimension uniform flow annular duct of the convective Helmholtz equation is a linear superposition of spinning duct modes as follows:

$$p(x,r,\theta,t)=\sum_{\omega}\sum_{m=-\infty}^{\infty}\sum_{n=0}^{\infty}A_{mn}^{\pm}\psi_{mn}(r)e^{jk_{mn}^{\pm}x}e^{jm\theta}e^{j\omega t} \tag{1}$$

Here ω is angular frequency. $A_{mn}^{\pm}$ denotes the complex amplitudes of the mode (*m*,*n*) with the circumferential order *m* and the radial order *n* for propagation in and against flow direction, and $k_{mn}^{\pm}$ is the axial wave number in and against flow direction of the corresponded (*m,n*) mode, the real $k_{mn}^{\pm}$ means the mode is propagation cut on mode, the image $k_{mn}^{\pm}$ means the mode is a damping cut off mode. $\psi_{mn}(r)$ is the radial modal shape function of the (*m,n*) mode. For the hard wall boundary condition, the spinning radial modes form an orthogonal eigen system, the radial modal shape function are given by[5]:

$$\psi_{mn}(r)=\frac{1}{[N_n^m]}\left(J_m(\sigma_{mn}\frac{r}{R_o})+Q_{mn}(\sigma_{mn}\frac{r}{R_o})\right) \tag{2}$$

Here R_o is the outer duct radius. J_m and Y_m being the Bessel functions of first and second kind and order m with associated hard wall cylindrical eigenvalues σ_{mn} . The eigenvalues depend on the hub-to-tip ratio *η*. Q_{mn} is zero for non-annular cylinders. The definition of the normalization factor $[N_n^m]$ is the norm of the radial modal shape function, given by:

$$[N_n^m]^2=\frac{1}{R_o^2}\int_{R_i}^{R_o}r\left(J_m(\sigma_{mn}\frac{r}{R})+Q_{mn}(\sigma_{mn}\frac{r}{R})\right)^2dr \tag{3}$$

Here R_i is the inner duct radius. For a single (*m,n*) mode at a certain frequency, the normalized modal shape function is given by:

$$P_{mn}^{\pm}(x,r,\theta)=\psi_{mn}(r)e^{jk_{mn}^{\pm}x}e^{jm\theta} \tag{4}$$

The purpose of mode decomposition is to calculate the complex amplitudes or mode factor $A_{mn}^{\pm}$ based on the measured pressures of microphone array $p(x,r,\theta,t)$ and the normalized duct modal shape function $\psi_{mn}(r)e^{jk_{mn}^{\pm}x}e^{jm\theta}$ in equation(1). The general process of mode decomposition include several round Fourier Transform to equation(1). The first round time series Fourier Transform to equation(1) can get the measured sound map in duct at frequency domain:

$$P(x,r,\theta)=\sum_{m=-\infty}^{\infty}\sum_{n=0}^{\infty}A_{mn}^{\pm}\psi_{mn}(r)e^{jk_{mn}^{\pm}x}e^{jm\theta} \tag{5}$$

The second round circumferential series Fourier Transform to equation(5) at a specified frequency(eg. 1BPF) can get the measured sound map induct for each circumferential mode.

$$P_m(x,r,\theta)=\sum_{n=0}^{\infty}A_{mn}^{\pm}\psi_{mn}(r)e^{jk_{mn}^{\pm}x}e^{jm\theta} \quad (6)$$

The final round two dimension Fourier Transform(Finite Hankel transform[9]) to equation (6) at the x section of the duct can get the mode factor $A_{mn}^{\pm}$. For the engineering application, the least-mean-squares fit method is normally applied to final round transform in consideration of the non-perfect orthogonal problems or low signal to noise ratio problems.

2.2 The numerical model for mode decomposition with far field microphone array

As described above, the analytical model based method for duct mode decomposition could help to get mode factors for one dimension uniform flow in annular duct. In this paper, we try to investigate if we could get the mode factors of fan noise by the non-rotating far field microphone array, so that we don't need to make the rotating device or to drill many holes on the fan case or nacelle for microphone array installation. We try to leverage the far field microphone array for aero engine ground static noise measurement, the general far field microphone array are positioned as Figure 1. The microphone azimuthal interval is about 5 degree.

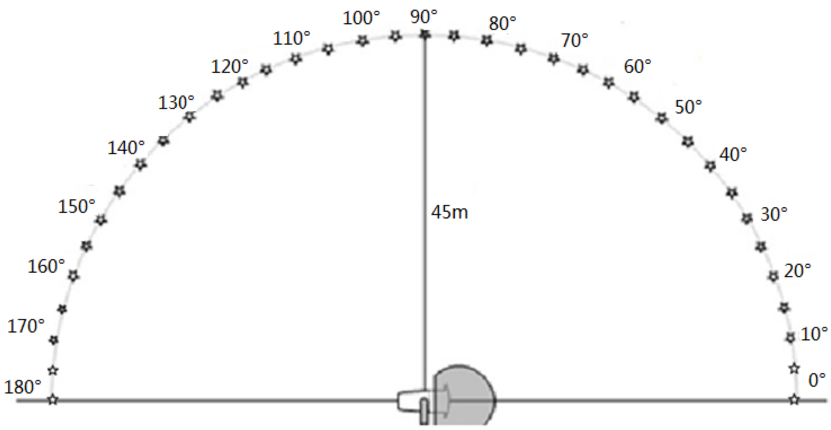


Figure 1 – Far field microphone array for engine static noise measurement.

Since the governing equation for the acoustic propagation from fan duct to the far field array is still the convective Helmholtz equation, the response of far field microphone array to the spinning duct modes of fan noise source is also linear superposition of spinning modes. The difference between the far field microphone array and induct microphone array is the propagation function or transfer function. The propagation function of induct microphone array to each normalized (*m,n*) mode could be expressed by equation(4), and for the propagation function of each normalized (*m,n*) mode to the far field microphone array could be solved by numerical CAA model. Then the measured pressure of far field microphone array corresponded to the spinning duct modes of fan noise source is given by:

$$p(x,y,z,t)=\sum_{\omega}\sum_{m=-\infty}^{\infty}\sum_{n=0}^{\infty}A_{mn}^{\pm}P_{mn}^{\pm}(x,y,z)e^{jm\theta}e^{j\omega t} \quad (7)$$

Where $P_{mn}^{\pm}(x,y,z)$ denotes the propagation function from the normalized fan duct modes (*m,n*) to far field microphone array at (*x,y,z*). Following the same procedure for mode decomposition above, we can get the following equation after the time series Fourier Transform and the circumferential series Fourier Transform:

$$P_m(x,y,z)=\sum_{n=0}^{\infty}A_{mn}^{\pm}P_{mn}^{\pm}(x,y,z) \quad (8)$$

Normally the $P_{mn}^{\pm}(x,y,z)$ cannot meet the orthogonal condition for the propagation function from the normalized fan duct modes (*m,n*) to far field microphone array, so the least-mean-squares fit method is applied to get the complex amplitude $A_{mn}^{\pm}$. For a far field microphone array with *i* microphones, if the cut on radial mode number of the *m* order mode is $N_n^{(m)}$, the equation(8) could be expressed as following：

$$P_m(x_i,y_i,z_i)=\sum_{n=0}^{N_n^{(n)}} A_{mn}^{\pm} P_{mn}^{\pm}(x_i,y_i,z_i) \quad (9)$$

Where $P_m(x_i,y_i,z_i)$ denotes the measured pressure of the m order mode at the No.*i* microphone, $P_{mn}^{\pm}(x_i,y_i,z_i)$ denotes the propagation functions from the normalized (*m,n*) mode to the No.*i* microphone. Hence, a linear matrix equation system can be established for each circumferential mode order m based on equation (9):

$$P_m = E_m A \quad (10)$$

Where P_m is the measured pressures vector of the m order mode at the far field microphone locations dimensioned $i\times 1$, E_m is the propagation function matrix of the normalized (*m,n*) mode to far field microphone array dimensioned $i\times N_n^m$, and *A* is the mode factors vector of cut on radial mode in m order dimensioned $N_n^m\times 1$.

Applying least-squares analysis, inverting and solving for the modal amplitudes:

$$A=[E_m^H E_m]^{-1}E_m^H P_m = E_m^+ P_m \quad (11)$$

Where the E_m^+ is the pseudo-inverse of matrix E_m. The condition number of the matrix E_m is normally applied to evaluate how sensitivity of the solution for a system of linear equations to errors in the measured data. It gives an indication of the accuracy of the results from matrix inversion and the linear equation solution. The lower condition number means the better accuracy. Hence, the critical issues to complete the duct mode decomposition are to get the propagation function matrix and the measured pressure vector.

3. Numerical case study for mode decomposition with far field microphone array

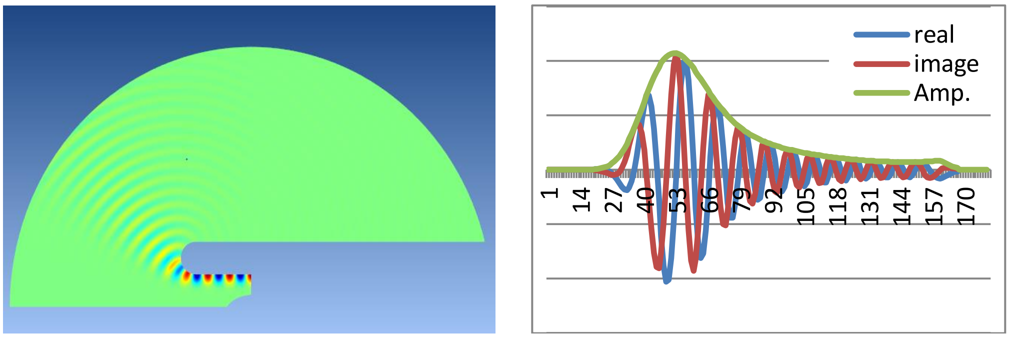
In order to verify the feasibility, we would like to build up a numerical model to conduct a virtue test for fan forward noise duct mode decomposition with far field microphone. In this paper, the Actran TM of FFT company is applied to build up the CAA numerical modeling. The JT15D[10] hard wall intake is applied to conduct the numerical virtue test, the frequency of fan tonal noise source is 3150Hz, the in flow Mach number is zero, the cut on modes includes(13,0), (12,0), (11,0), (10,0), (9,1) and (8,1). The radii of the far field microphone array is 45 meters, there are totally 181 microphones uniformly installed in the semicircle as Figure 1.The numerical investigation process are as following:

- The step 1 is to build up the propagation functions of the cut on normalized modes to far field microphone array;
- The step 2 is to optimize the microphone azimuthal interval based on the condition number analysis, in order to trade off the decomposition accuracy and microphone number ;
- The step 3 is to verify the optimized far field microphone azimuthal interval is suitable for the cut on mode decomposition.

Figure 2 and Figure 3 are the predicted propagation map(a) and propagation function(b) from the normalized (11,0) and (9,1) modes to far field microphone array by numerical CAA model.

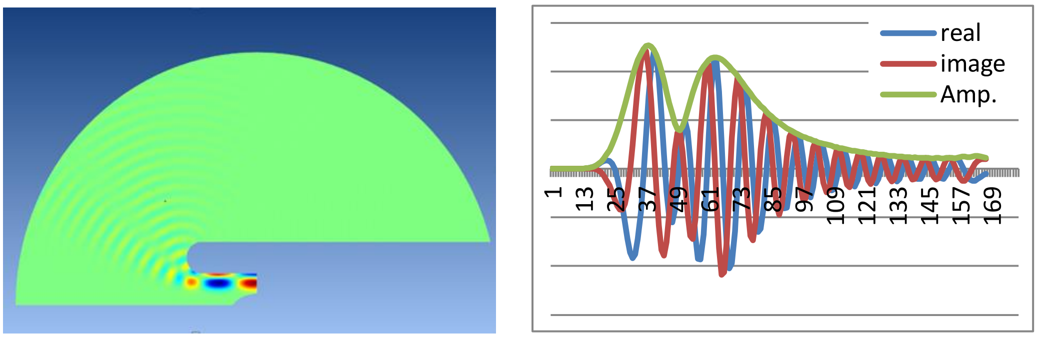
In order to select a optimized microphone azimuthal interval and microphone number, a condition number comparison study of propagation matrix for different microphone azimuthal interval was conducted. Figure 4 shows the far field microphone azimuthal interval of 5 degree is enough for this case, and the corresponded number of far field microphone is 34. The condition number will be unstable and divergent for the greater azimuthal interval than 5 degree.

A virtual case study results for a series of cut on mode with different complex amplitudes as listed in table 1. The far field microphone azimuthal interval is the optimized 5 degree, the table 1 compares the complex amplitudes of loaded cut on modes and the decomposed complex amplitudes. The potential decomposition error of the mode (11,0) may comes from the numerical error, the non-orthogonal condition of the propagation function, the energy aliasing from strong mode to weak mode, and the lower signal to noise ratio of the weak mode etc.. Even so, this method still could get a proper amplitudes of the stronger modes as listed in table 1, the stronger mode is the most important issue for fan noise control after all . It should be noted that for different fan design and different fan rotating speed, the cut on modes are also different for different tonal noise, the detailed cut on mode propagation numerical study is necessary to evaluate the proper far field microphone azimuthal interval and microphone number for different fan noise measurement.



(a) The sound map (b) The propagation function

Figure 2 – The predicted far field microphone array response to normalized (11,0) mode.



(a) The sound map (b) The propagation function

Figure 3 – The predicted far field microphone array response to normalized (9,1) mode.

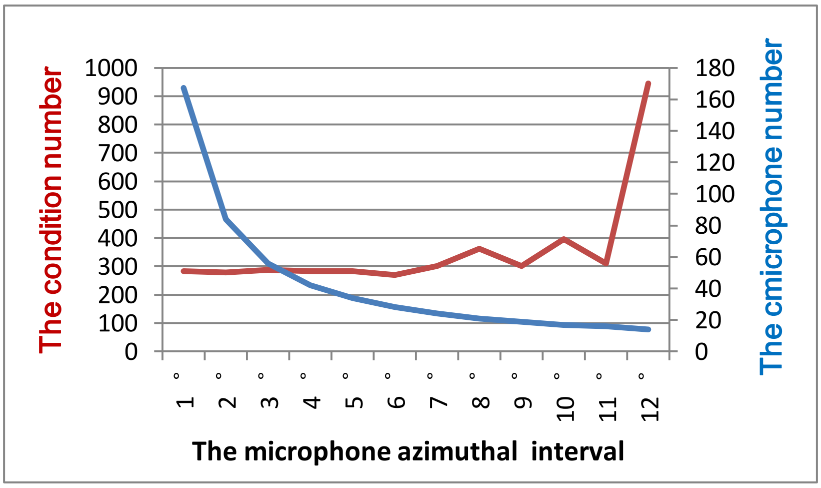


Figure4 – The microphone azimuthal interval optimization results

Table 1 the numerical case study results

cut on modes	loaded Amp.	loaded Phase	decomposed Amp.	decomposed Phase
(13,0)	1	0 °	0.991434133	-0.939549649
(12,0)	2	30 °	1.970482564	31.32831793
(11,0)	0.5	60 °	0.471817309	53.00694894
(10,0)	1.2	120 °	1.200448585	118.8737689
(9,1)	0.7	-60 °	0.6865902	-58.38204563
(8,1)	10	90 °	10.01498651	90.00676728

4. Summary

This study provides a numerical model based tonal noise duct mode decomposition method by using non-rotating far filed microphone. A numerical model case study demonstrates it is feasible to use the far field microphone array for aero engine ground static measurement to get the cut on modes factors induct which closed to fan section. The method applied to evaluate the proper microphone azimuthal interval could help engineer to trade off the decomposition accuracy and microphone number. This method study provide another option for duct mode decomposition which may not be necessary to make a rotating device or to drill many holes at the fan case and nacelle for microphone array installation.

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