METHODS FOR CALCULATING THE FATIGUE DAMAGE OF A STRUCTURE FOR VARIOUS SPECTRA OF VIBRO-ACOUSTIC LOADS

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Abstract

A computational technique is proposed for recalculating of the experimental fatigue life obtained under one loading spectrum to another one, that is, determining the equivalence of different loading spectra in terms of fatigue damage. The features of determining fatigue damage for metal structures in the frequency and time domains are highlighted. The real experimental vibro-acoustic time realizations were processed and analyzed. A possible application of the method under consideration is described and is offered some suggestions for its further improvement.

Keywords: fatigue damage, vibro-acoustic loads, fatigue life, spectral density.

1. Introduction

The assessment of the fatigue life of aircraft structures under acoustic and vibration-acoustic loading for the main load-bearing elements should be based on experimental data. Due to the fact that experiment requires significant material costs, there is a need to reduce the volume of required experimental research. The main purpose of this work is to develop a computational technique for recalculating the experimental fatigue life obtained under one loading spectrum for another, that is, determining the equivalence of different loading spectra in terms of fatigue damage. As an example of such tasks, one can mention aircraft remotorization or determination of the effect of static loading on spacecraft such as Buran or Shuttle. In the last case, the experimental determination of the fatigue life at various levels of the static load caused by the internal pressure in the cabin requires both significant material costs and the complication of the necessary experimental base. Therefore, the the experimental data for these vessels is usually obtained without reproducing the stresses caused by internal pressure. At the same time, at present, the calculation of changes in the spectrum of acting stresses at various values of static loading is possible at significantly lower costs.

The proposed approach was based on the method for determining equivalence in terms of fatigue damage, which can be used in case loading data in the time domain and fatigue curves of the considered structural elements are available. Since fatigue curves are mainly obtained under cyclic loading with a constant stress amplitude, a random process in the time domain is usually schematized using the rainflow method, which allows to form a table of completed cycles, which serves as the basis for calculating the fatigue life [1] and, accordingly, the fatigue damageability. This type of technique most accurately takes into account the main factors affecting fatigue resistance under irregular loading.

The direct application of this approach for acoustic and vibro-acoustic loading is inapplicable due to the fact that the fatigue life properties in this case are obtained not in the form of ordinary S-N curves or the loading program cycles until structural failure, but as dependence of time to failure on property of loading spectrum, usually root-mean-square value of load, that is, load data is derived from the frequency domain.

A number of methods have been developed to translate the load data in the frequency domain into a one-dimensional table of completed cycles (by amplitudes), which does not include data on the asymmetry of the cycle (average or maximum stress). These methods are based either on the introduction of additional assumptions about the form of a random process (Rice's method [2], based on the assumption of a narrow-band form of a random process), or on obtaining empirical dependencies between the one-dimensional distribution of completed cycles and the statistical characteristics of a random process (Dirlik's method [3] and etc.). An overview of these methods is presented in [4]. From Fig. 1 one can see that the approximate methods have rather strong deviations from the rainflow method in various areas. To prove the applicability of a particular method for a specific task, you need to have at disposal large number of such approaches and conduct a comparative analysis each time. In general, we can say that these methods are applicable only for implementations similar to those from which the empirical dependencies were obtained. As a consequence, the use in this case of the results of tests under random loading does not make it possible to determine the equivalence in terms of fatigue damage at very different spectral densities of the process.

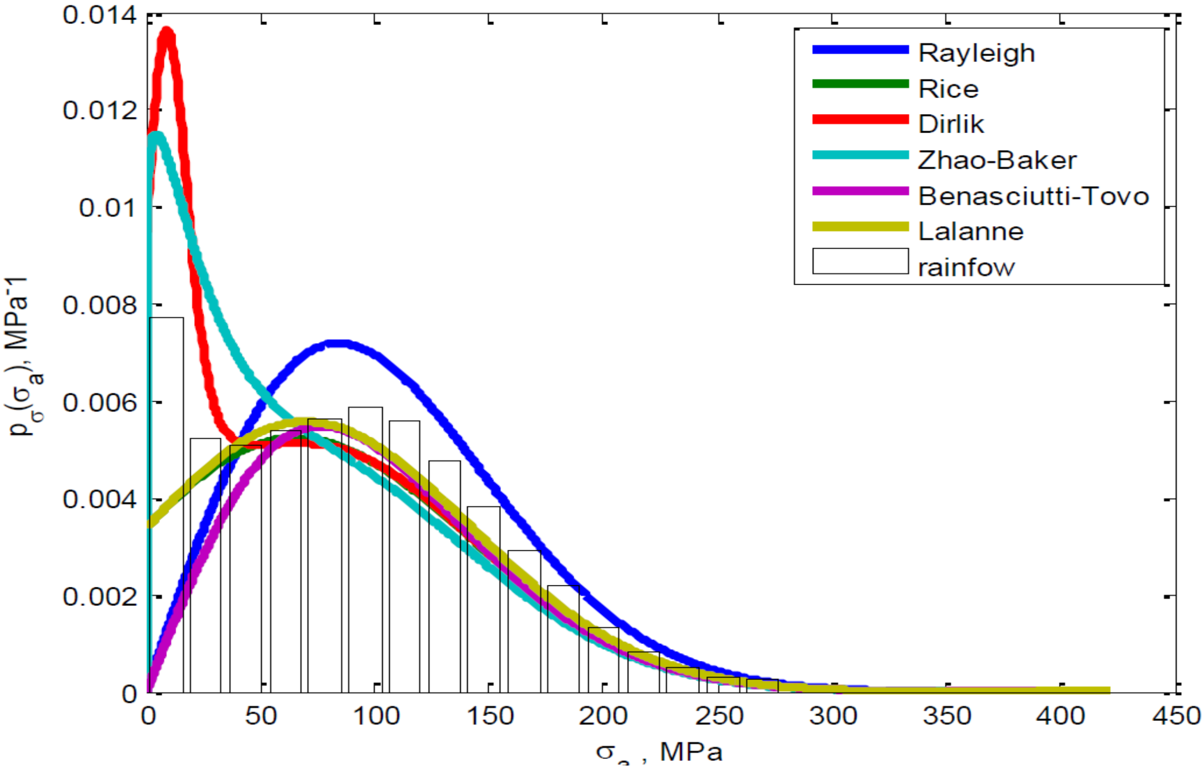

Figure 1 - [4] Comparison of the distribution functions of completed cycles obtained by different methods with the distribution obtained by the rainflow method

In the present work, a more universal approach for determining the equivalence in terms of fatigue damage at different power spectral densities (PSD) of the process is proposed. The paper also shows the application of this technique by the example of processing real experimental vibro-acoustic time realizations.

2. Loading process simulation

To analyze the effect of the assumptions made on the fatigue life estimation accuracy when using the characteristics of a random process in the frequency domain, the following approach was applied. A time realization (original time realization), recorded during vibration-acoustic tests of the representative specimen was chosen. This implementation has a duration of 10 seconds and consists of 240,000 samples, and its graph is shown in Fig. 2. Using the Fast Discrete Fourier Transform (FDFT) for this time history, the power spectral density (PSD), as well as the phase spectrum, were generated. The obtained PSD and the phase spectrum are shown in Fig. 3 and Fig. 4 respectively. The frequency axes in these graphs are cut off to a value of 2000 Hz, due to the fact that the PSD values at frequencies above 2000 Hz tend to zero.

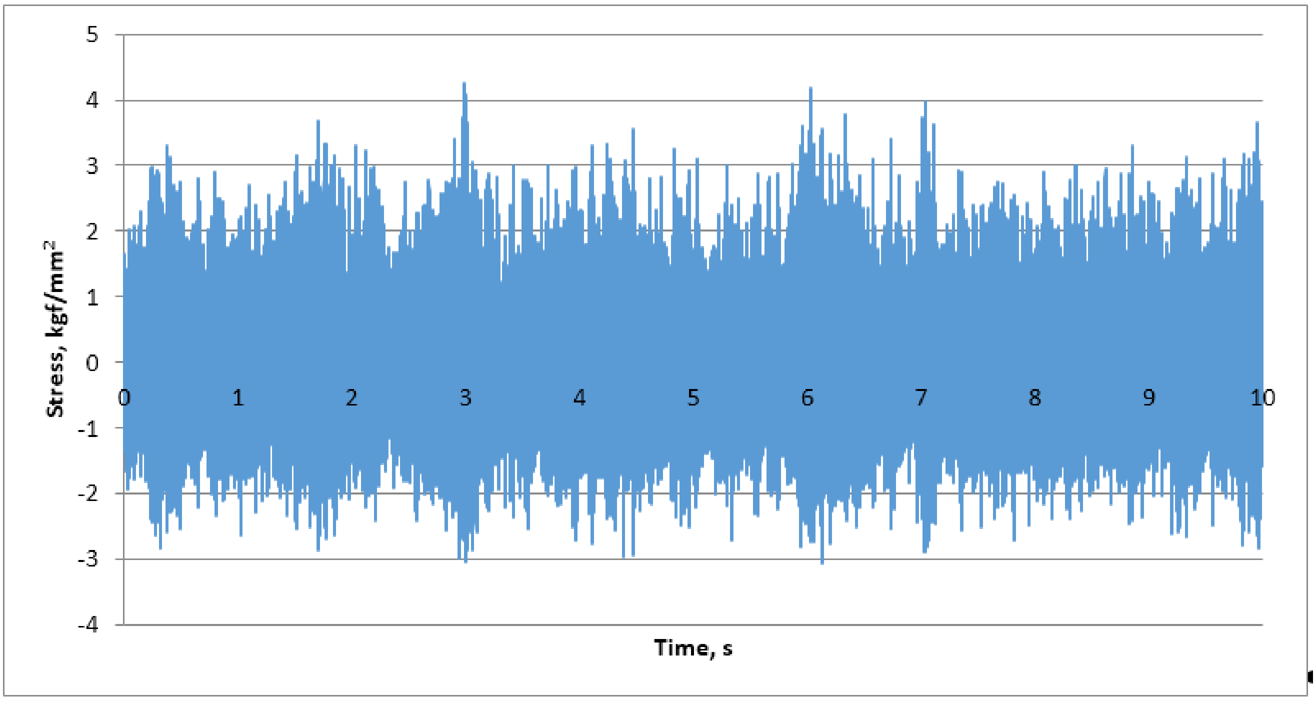

Figure 2 – Original time realization

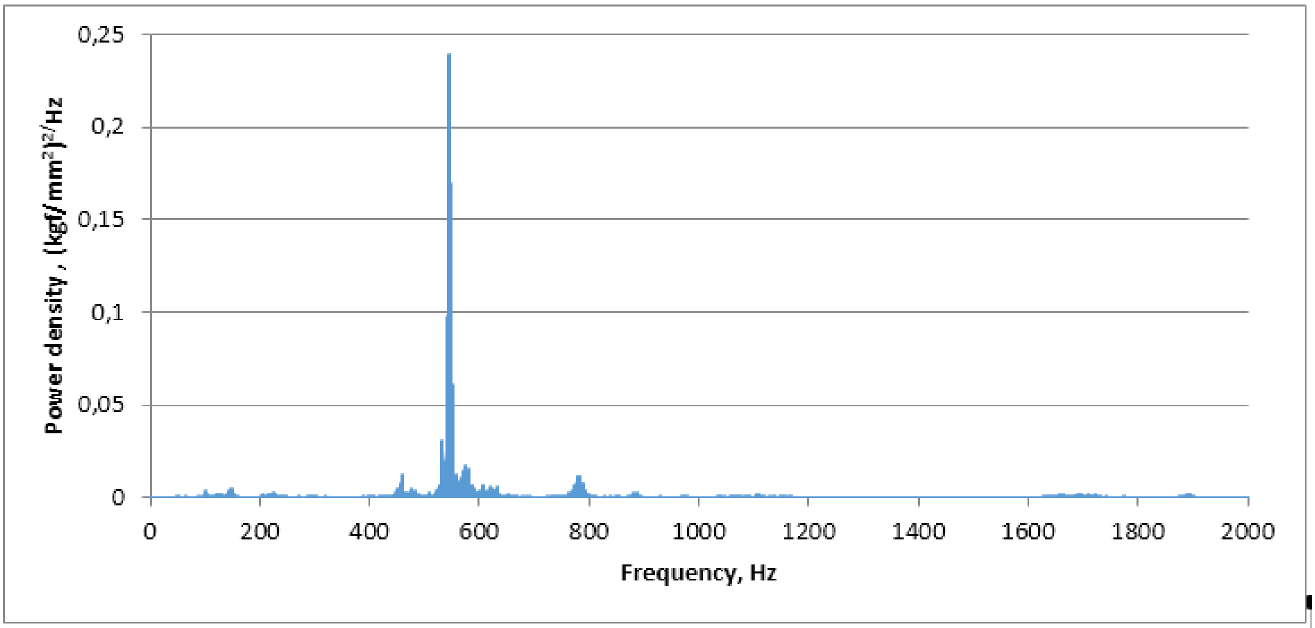

Figure 3 – PSD of original time realization

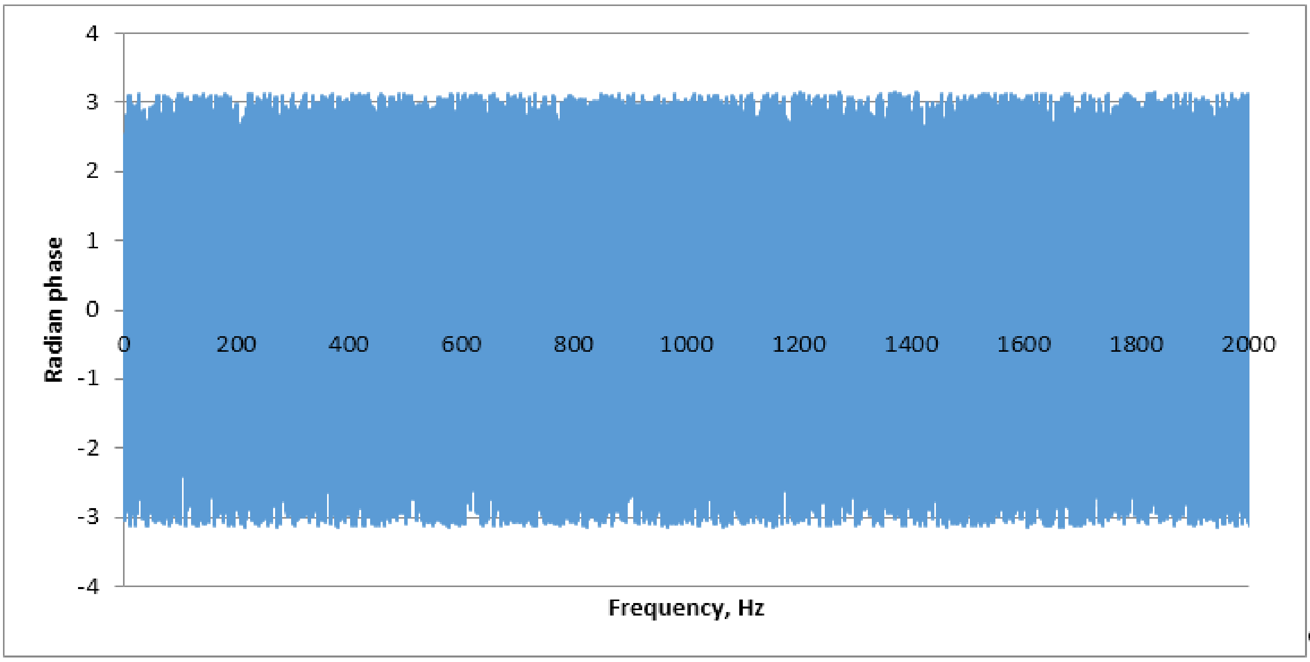

Figure 4 – Phase spectrum of original time realization

It should be noted that there are a number of important requirements for the time realization used in this methodology for modeling the loading process:

1. Its minimum duration should be chosen so that the accumulated amount of fatigue damage per unit of time $\overline{D}$ (see below (5)) has enough time to stabilize. In this work, the following stabilization criterion was taken: the deviations $\overline{D}$ from the average value of the relative damage, calculated for the right half of the original time realization should become a steady sinusoid with a constant amplitude. The original temporal realization considered in this work was stabilized for a duration of 4 seconds, the deviation for such a duration became less than ±5%. The resulting graph of deviations $\overline{D}$ is shown in Fig. 5.

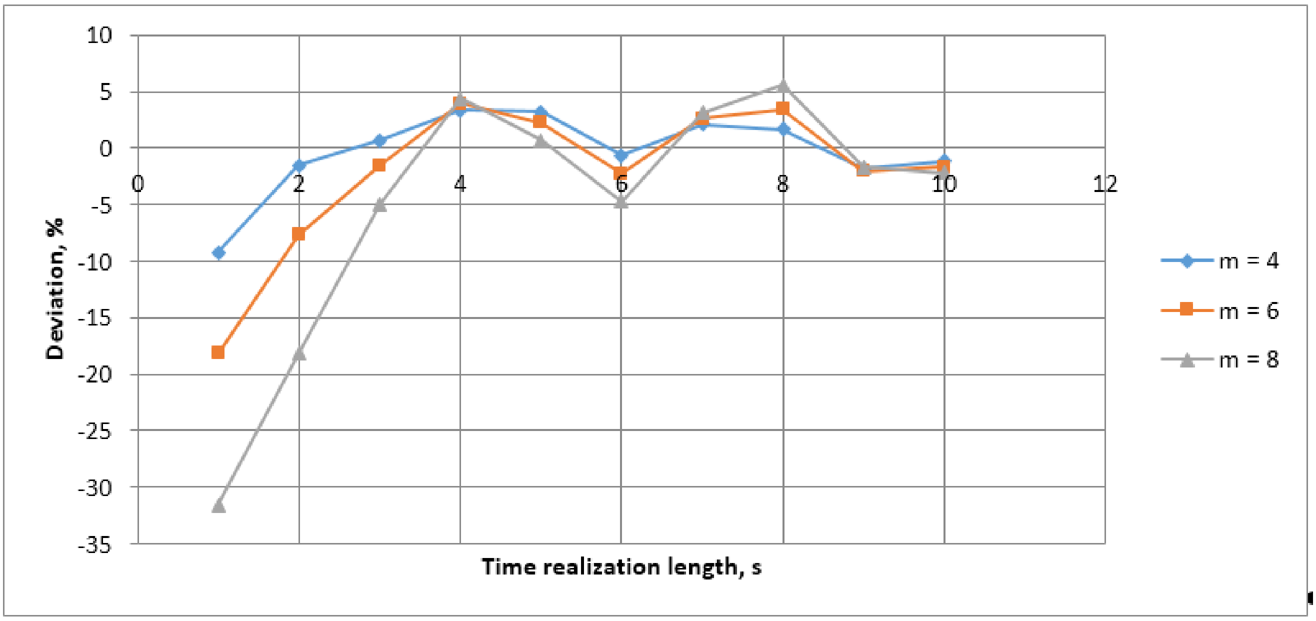

Figure 5 - Stabilization diagrams of the deviation of the relative equivalent in terms of damage

2. The sampling frequency of the time realization should be chosen at least twice as high as the highest loading frequency of the considered structure. In other words, the sampling rate should satisfy Kotelnikov (Nyquist-Shannon) sampling theorem [5]. The recommended oversampling should be at least 10. The sampling frequency of the temporal implementation investigated in this work is 24 kHz, and the maximum frequency of a spectrum that is substantially different from zero is 1.9 kHz.

3. The average value of the load in the considered time realization should be a constant value. If the average value changes, then the time realization should be divided into sections with a constant average and analyzed independently of each other. At the same time, requirements 1 and 2 must be true for each allocated area.

Further modeling of the loading process is based on the assumption that the obtained PSD is generated for a random loading process. As it was shown in [6], the phase spectrum of such a process does not carry any information due to its zeroing when the statistical characteristics of the loading process are averaged. Based on the above assumption, the loading process was simulated using the inverse FDFT, the input data for which were the original PSD (Fig. 3) and the phase spectrum filled with random numbers in the range [-π, π]. After the formation of the time realization, its was scaled to equate its root-mean-square value (RMS) with the RMS of the original PSD.

As it can be seen from Fig.4, the values of the phase spectrum are uniformly distributed in the range [-π, π]. Therefore, the Mersenne twister was chosen as a pseudorandom number generator (PRNG) for the phase spectrum. This PRNG provides one of the most uniform distributions of generated pseudo-random numbers among all existing algorithms. In addition, the Mersenne twister is free of many of the disadvantages inherent to other pseudo-random number generators, such as a small period, predictability, and easily identifiable statistical patterns.

The purpose of the simulation was to numerically assess the accuracy of methods used for determination of loads by characteristics of random processes.

3. Analysis procedure

The analysis considered 16 time realizations: 1 original time realization and 15 time realizations generated in accordance with the methodology described in the previous section.

For all 16 considered time realizations the extrema were identifiedand processed using the rainflow method [6-7] in order to obtain a table of completed half-cycles. Undeer completed half-cycle a half-cycle represented by the rainflow method is meant. Representing by the rainflow method was carried out according to the upgraded algorithm [8]. In contrast to the algorithm described in [8], in current algorithm the table of completed half-cycles has better accuracy. The increase in accuracy is achieved due to the fact that having the required amount of free disk space, the upgraded algorithm allows to record "raw" data on the determined half-cycles (without merging half-cycles with similar characteristics into one line of the table of completed half-cycles).

According to the proposed method, a table consisting of a sequence of lines and containing completed half-cycles was formed. Each line consists of cells, in which the parameters of a half-cycle were written, in this case, these are the maximum stress, stress range (double stress amplitude) and the number of half-cycles with these parameters.

The resulting tables were processed using the linear damage hypothesis and the traditional method for metal aircraft structures used to determine the equivalent Q in terms of damage

$$Q = \frac{D}{D_{\text{init}}} \quad (1)$$

$$D = \sum_{i=1}^{M} n_i \, D_i \tag{2}$$

$$D_i = \sigma_{\mathrm{eqv}i}^{m} \tag{3}$$

$$\sigma_{\mathrm{eqv}i} = \sigma_{\max i}^{\chi} (2\,\sigma_{a\,i})^{1-\chi} \tag{4}$$

$$\bar{D} = \frac{D}{L} \tag{5}$$

$$Q_m = \frac{1}{N} \sum_{j=1}^{N} Q_j \tag{6}$$

where D is the fatigue damageability of the analyzed time realization;

D_{init}- fatigue damageability of the original time realization;

D_i– fatigue damage of one full half-cycle, determined with an accuracy up to a constant;

m - the parameter of the fatigue curve;

M - the number of loading modes identified in this implementation;

n_i - the number of half-cycles in the i-th loading mode;

σ_{eqvi} - is the equivalent stress of a full half-cycle in the i-th mode;

$\sigma_{\max i}$ - Maximum stress of a full half-cycle in the i-th mode;

σ_{ai} - amplitude of a full half-cycle in the i -th mode;

χ - coefficient of the generalized Oding (Walker) formula, taken in this case equal to 0.5;

$\bar{D}$ - relative fatigue damage per unit time;

L - the length of the implementation in seconds.

Q_m - the average value of the equivalent damageability, calculated for N generated time realizations;

j -the serial number of the generated time realization;

Three values representative for metal alloys were taken as the constant m: m = 4, m = 6 and m = 8. Determination of Q_m was carried out for each of these values.

4. Analysis results

The values of Q obtained for all investigated time realizations are shown in comparative Table 1.

Table 1 - The obtained values of damage equivalents

Time realization	Q **(m = 4)**	Q **(m = 6)**	Q **(m = 8)**
Original	1	1	1
№1	0,944	0,894	0,825
№2	0,950	0,896	0,844
№3	0,985	1,003	1,042
№4	0,967	0,976	1,018
№5	0,938	0,910	0,875
№6	0,978	0,996	1,036
№7	0,964	0,964	1,007
№8	1,003	1,057	1,159
№9	0,950	0,907	0,866
№10	0,959	0,931	0,903
№11	0,953	0,942	0,959
№12	0,968	0,989	1,049
№13	0,991	1,035	1,142
№14	0,964	0,961	0,963
№15	0,949	0,916	0,881

As it can be seen from the Table 1, the average Q_m for the 15 generated time realizations is 0.964 at m = 4; 0.959 for m = 6 and 0.971 for m = 8. The charts of Q_m variation from unity depending on the number of generated time realizations (N) are shown on Fig. 5. It follows that when Q_m is averaged over more than 7 time realizations, the variation of Q_m becomes less than 5%, and with a further increase in the number of generated time realizations, it does not change much. Along with that the resulting variation Q_m doesn’t go beyond the variation of $\bar{D}$ when it stabilizes (Fig. 5).

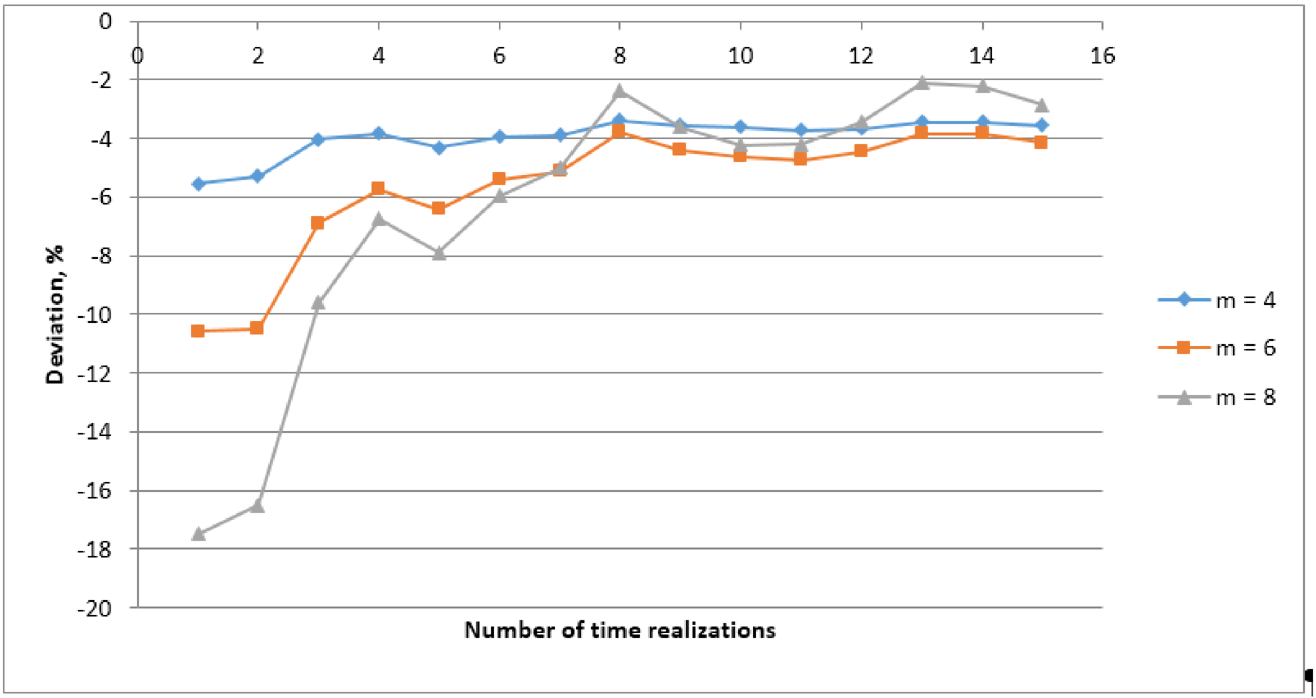

Figure 6 - Variation of Q_m, calculated for multiple time realizations

In addition, for the original and two generated time realizations, differential distributions of equivalent stresses and ranges of completed cycles were identified (Fig. 7 and Fig. 8, respectively). It can be seen from the charts that the distributions of the generated realizations quite well repeat the distributions obtained from the original time realization.

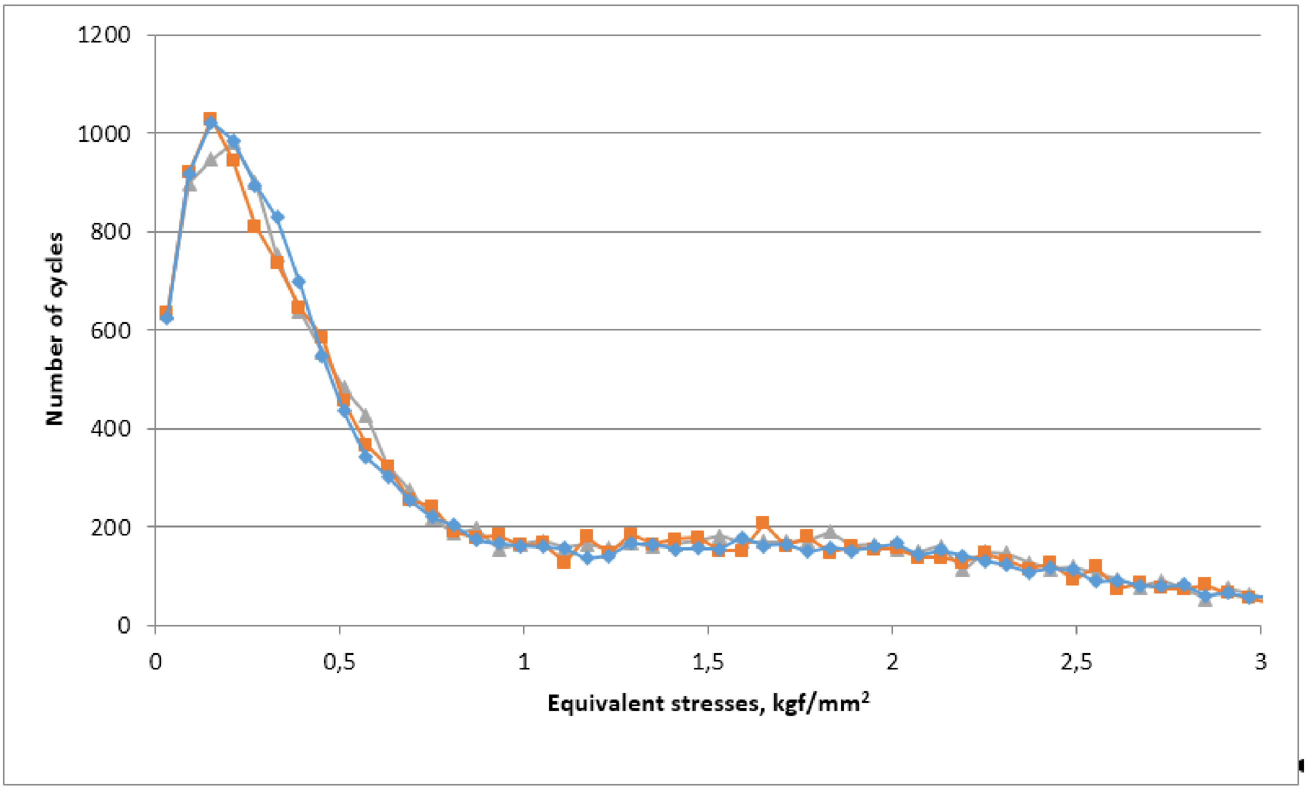

Figure 7- Differential distribution of equivalent stresses of completed cycles

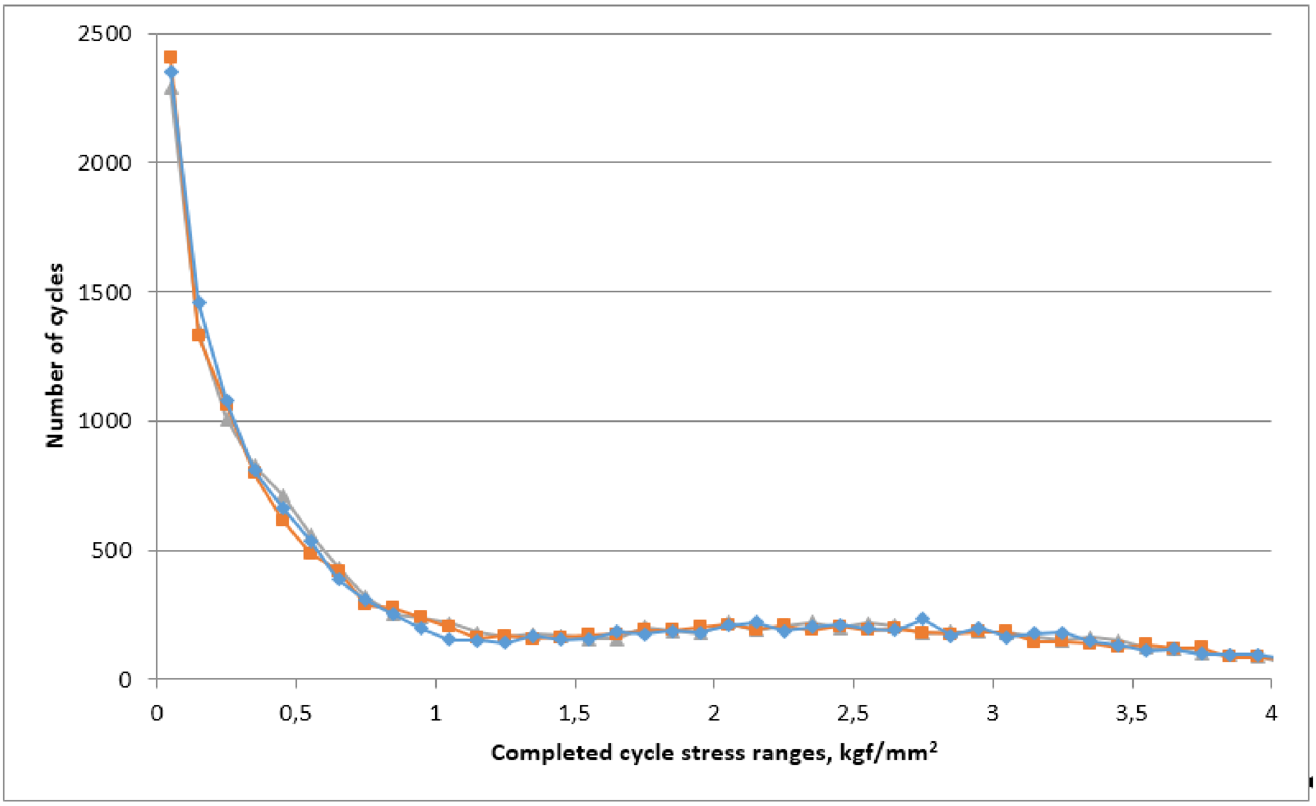

Figure 8 - Differential distribution of completed cycle stress ranges

However, it should be noted that the described method doesn't allow to record and save , the RMS and the average value of the original PSD in generated time realizations. In this relationit is proposed to scale the time realizations to achieve equality between the RMS of the obtained time realization and the RMS of the original PSD. For the further development of the technique it is recommended to implement the procedure of generating the original loading consistent with those used in obtained experimental data.

5. Possible application of the technique

The described method for calculating fatigue life can be used to solve a wide range of problems. One of such tasks may be the recalculation of the structural damage from one PSD of loads to another. The need for such a recalculation may arise, for example, if it is impossible to directly obtain data on fatigue damage caused by vibration-acoustic loading of the aircraft engine. Having the loads PSD and fatigue damageability caused by vibro-acoustic loading for aircraft configuration with "Engine 1" using this technique the fatigue damageability for aircraft configuration with "Engine 2" can be determined, provided that the loads PSD for "Engine 2" is known. Such a recalculation of the damage rates can be carried out during the remotorization of aircraft.

6. Conclusion

In the presented work, a universal approach is considered for determining the equivalence in terms of fatigue damage at various spectral densities of the process. The given processing of real experimental vibro-acoustic time realizations showed quite acceptable variations (<5%) of the obtained values of the equivalent in damage (Q_m) from 1.

Also, the paper describes a possible application of the method under consideration and offers suggestions for its further improvement.

7. Copyright Statement

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