

Evaluation and Validation of Aeroelastic Analysis Program for the “AeroStruct” Wing Optimization Platform

Sheng-Bo Ling¹, Ke-Shi Zhang¹, Zhong-hua Han¹, Zhong-Jian Gao²

¹School of Aeronautics, Northwestern Polytechnical University, Xi'an, 710072, P.R.China 1

²Xiantianjiahongtong Technology CO. Ltd., Xi'an, 710086, P.R.China 2

Abstract

Wing aero/structural optimization driven by higher-fidelity simulations plays an important role in the design of high-aspect-ratio transonic wings. Surrogate-based optimization (SBO) is one of the most popular optimization techniques, due to its capability of global search and high efficiency. Based on the framework of SBO, we are developing a wing aero/structural optimization platform, which is named as AeroStruct. Here we are focusing on validation and evaluation the aeroelastic analysis module of the AeroStruct. Navier-Stokes equation, Euler equation and full potential equation are optional for aerodynamic analysis, while ANSYS and NASTRAN are available for structural analysis. A radial basis function interpolation is used for data coupling between different meshes of aerodynamic and structural analysis. The benchmark wing models for aeroelastic analysis, CRM and HIRENASD, are used for validation and evaluation of the codes. It is shown that the computational results are in good agreement with the experimental data or other researchers' results. The developed codes are applied to the aeroelastic analysis of the uCRM-9 model that is the jig shape of CRM, which shows that the codes are efficient and robust.

Keywords: wing optimization; static aeroelastic analysis; surrogate-based optimization

1. Introduction

Aircraft design is no longer satisfied with traditional sequential design methods due to the increasing economic and environmental requirements of civil aircraft in the future. Collaborative multidisciplinary design optimization (MDO) is becoming the trend of aircraft design, as it helps to largely improve the overall performance by considering coupling between disciplines during optimization [1, 2]. Therefore advanced wing aero/structural design optimization which apply high-fidelity numerical simulation and consider strong coupling between aerodynamic and structure disciplines, is a basic technology for the development of advanced transport aircraft, and it is also a significant technology to improve the comprehensive performance of future transport aircraft.

The initial development and application of wing aero/structural design optimization can be traced back to the mid-19th century, Schmit and Haftka et al. introduced the influence of other disciplines in structural design [3-6]. Since the development of computer technology and numerical simulation technology in the 1990s, wing aero/structural optimization which is an important research area in MDO has played an increasingly significant role in aircraft design [7]. After over thirty years development, researchers had already realized wing aero/structural optimization based on high-fidelity analysis codes and Finite Element Analysis (FEA) [8]. Wan performed aero/structural design optimization for composite wing with high aspect ratio through genetic/sensitivity hybrid optimization algorithm and finite element model (FEM) which introduced the influence of aerodynamics [9]. Heinrich proposed a coupling strategy between computational fluid dynamics (CFD) and computational solid dynamics (CSD), and applied it to the design of complex high-lift configurations, rocket nozzle and maneuvering aircraft [10]. Piperni proposed an optimization methodology which is based on the integration of aerodynamic and structural analysis codes that combine computational,

analytical, and semi-empirical methods, and applied it to the preliminary design of a large business jet [11].

Furthermore, Yu proposed a two-level optimization method based on surrogate model to deal with wing aero/structural design optimization, including method of wing shape and structure parameterization, automatic updating of aerodynamic shape and finite element model by CATIA and PCL respectively, and a force loading method from aerodynamic to structure [12].

In recent years, Martins developed a mature adjoint-based aero/structural design optimization approach, which is a gradient based optimization framework for complex configurations. Reynolds average N-S equation (RANS) and the Analysis of Composite Structure (TACS) based on adjoint method were used in aerodynamic analysis and structural analysis respectively [13-14]. Long proposed an efficient aero/structural design optimization strategy named multi-model fusion method (MMF), in which they perform aero/structural analysis through CFD and FEA, and these two disciplines are coupled by spline interpolation method. At the process of optimization, they integrate the limited high-fidelity (HF) simulations with a large number of low-fidelity (LF) simulations to create a surrogate model with preferable accuracy [15]. Beyond that, Zhang [16], Brezillon [17], Bai [18], et al. also made corresponding research work in aero/structural design optimization.

The purpose of this paper is to introduce AeroStruct wing optimization platform, evaluate and validate the program of aero/structural analysis for the transonic aircraft wing, which is a part of AeroStruct. The framework and numerical calculation method for each part of the aero/structural analysis program will be introduced, and we set wind tunnel experimental model of CRM and HIRENASD model as examples to validate the program. Then apply the aero/structural analysis program to uCRM-9.

2. AeroStruct: Surrogate-based aero/structural design optimization platform

2.1 Optimization framework of AeroStruct

The surrogate-based optimization strategy is used to perform aero/structural design optimization. "SurroOpt" is developed by Professor Han's team which is based on Kriging model, it can solve problems with multi-objective and multi-constraint efficiently [19]. As shown in Figure 1, the process of surrogate-based optimization can be divided into the following steps:

- Step 1: Set the suitable design space firstly, and generate initial samples by design of experiments (DoE). Latin hypercube sampling (LHS), uniform design (UD) and Monte Carlo design (MC) can be selected as the way of DoE [20].
- Step 2: Secondly the initial samples should be simulated by aero/structural analysis code and get the initial responses.
- Step 3: Thirdly the surrogate model about object function or constraint function could be constructed. Kriging model, response surface model (RSM), gradient-enhanced Kriging model (GEK), hierarchical Kriging model (HK) can be selected as the surrogate model [19].
- Step 4: After constructing the initial surrogate model, new samples should be added by infill criteria. Minimum surrogate prediction (MSP), maximizing expected improvement (EI), minimizing lower confidence bounding (LCB), maximizing probability of improvement (PI) and maximizing mean squared error (MSE) can be selected as the way of adding new samples. These infill criteria can be used alone or in combination, so that we can add new samples in parallel [19].
- Step 5: After getting new samples, aero/structural analysis code is called for getting responses. Repeat steps 3 and 4 until we get a surrogate model that meets the requirements.
- Step 6: Finally, find the global optimum of the aero/structural design optimization problem.

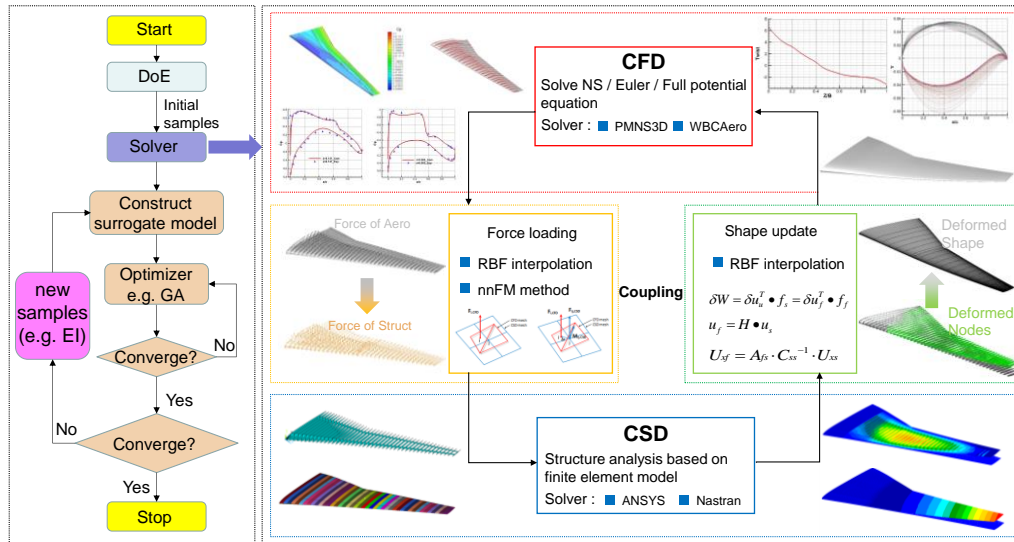


Figure 1 – Framework of surrogate-based aero/structural design optimization platform.

2.2 Aero/structural Analysis

As an important part of aero/structural design optimization, accurate aero/structural coupling analysis is necessary. The aero/structural analysis program which is based on weak coupled framework, focus on transmitting data between disciplines. The framework is shown on Figure 1, and the analysis process can be divided into four parts. 1) Firstly, compute to get the aerodynamic force by CFD. 2) Secondly, transmit the force of wing from aerodynamic to structure by nearest-neighbor Force and Moment method (nnFM) or Radius Based Function interpolation method (RBF). 3) Thirdly, Nastran or ANSYS is used to solve the finite element model of wing structure. 4) And finally, we extract the displacement of nodes to compute the deformation of wing through RBF interpolation and regenerate the mesh for CFD.

2.2.1 Aerodynamic Analysis

In order to predict aerodynamic forces on the wing surface, accurate aerodynamic solver is necessary. 1) WBCAero code is a CFD solver based on full potential equation which can predict lift of wing accurately. And viscosity of boundary layer is considered into the equation to predict drag accurately. The WBCAero code is capable of mesh automatically according to airfoil of each section along the transonic transport aircraft wing span. 2) PMNS3D code is another CFD solver which can solve steady and unsteady compressible Euler equation or Navier-Stokes equation. Where the 3D RANS solver applies finite volume method, and BL zero-equation model, SA equation model and $k-\omega$ equation model can be selected as the turbulence model. When we select PMNS3D code as the flow solver, a in-house code based on RBF is used for grid deformation. Usually, WBCAero is used to predict aerodynamic forces quickly, and PMNS3D is used to predict aerodynamic forces accurately.

2.2.2 Structural Analysis

In order to predict the deformation of wing, finite element analysis method was applied to the wing structure. Considering the maturity and versatility of commercial software, Nastran and ANSYS was packaged to solve finite element model of wing, and the solver is free to choose Nastran or ANSYS. We perform all processes of finite element analysis including pre-processing, solving and post-processing through ANSYS Parametric Design Language (APDL). The structure model of wing with high aspect ratio was conducted by skin, spars, ribs and stringers, their property are shell element and rod element. The material of wing is aluminum currently, however we consider using composite materials in future. As for research object with given FEM, it can be solved by Nastran or ANSYS directly. Therefore the structural solver is capable of handle complex bending and torsion problem of wings with complex aerodynamic force and displacement constraint.

2.2.3 Data Coupling

Because of the weak coupled framework and mesh difference between CFD and CSD, a high-fidelity code was used to transmit information between these two disciplines. As for transmitting of aerodynamic force, nnFM and RBF are available. The first channel is nnFM which is based on conservation of energy, therefore the force system loading on the wing is conserved. By automatically comparing the coordinate information of aerodynamic points and structural nodes, nnFM code searches for two nearest points. Aerodynamic forces loaded on grid of CFD are moved to grid nodes of CSD, and attach corresponding moments to these nodes.

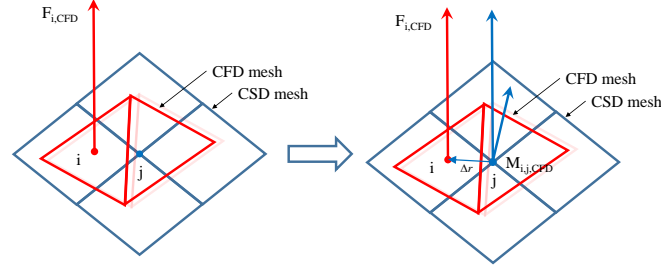


Figure 2 – Principle of nnFM.

The second channel is RBF interpolation which is an accurate global method based on conservation of energy. It works whether the data points are regular or not, therefore that RBF interpolation method is suitable for weak coupled framework. The transmitting matrix H is constructed by the principle of virtual work firstly, that the matrix H is related to nodes' information of both CFD grid and FEM grid. RBF interpolation method is capable of transmit displacements and forces of nodes. So as to update the shape of deformed wing, RBF interpolation method is used to transmit the nodes' displacements. And add the displacements to undeformed wing to get the deformed wing. Then wing grid is updated and perform CFD calculation once more.

$$f_s = H \bullet f_f \quad u_f = H \bullet u_s \quad (1)$$

3. Validation and application of aero/structural analysis codes

In order to validate the aero/structural analysis codes, the aeroelastic process of wind tunnel model of CRM and HIRENASD are simulate by AeroStruct, and we compare the results with experimental data. Then we apply the codes to predict the deformation of uCRM-9, and prepare for aero/structural design optimization.

3.1 Common Research Model (CRM)

The wind tunnel model of Common Research Model (CRM)[21] is the first example to validate the aero/structural codes. CRM is a standard research model published by the fourth Drag Prediction Workshop (DPW). CRM configuration was designed and developed by NASA's Subsonic Fixed Wing (SFW) aerodynamic technology studio and DPW, its detailed aerodynamic design was completed by Boeing. This configuration consists of fuselage, wing, horizontal tail and nacelle, and contains detailed wind tunnel experimental data. The configuration was initially used for aerodynamic analysis, and has since been gradually expanded by researchers into aero/structural validation. Figure 3 shows the wind tunnel model of CRM.

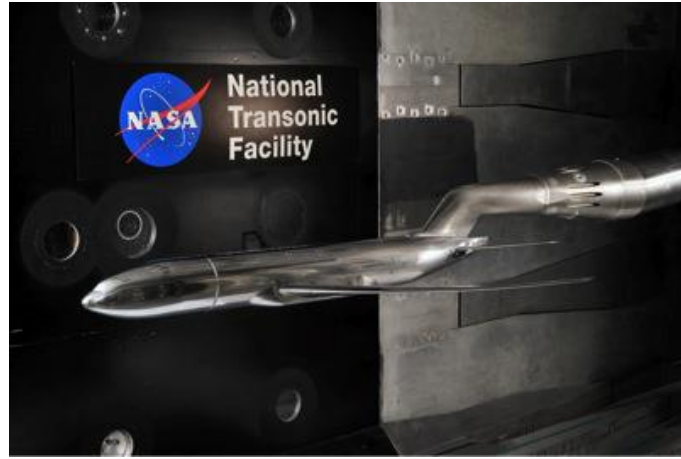


Figure 3 – Wind tunnel model of CRM configuration[21].

In order to predict lift quickly, full potential equation was performed with a constant lift coefficient 0.5, the cruise Mach number was 0.85, and the Reynolds number was 5 million. FEM was solved by Nastran to get the deformation. The aerodynamic shape information and FEM model of CRM wind tunnel model came from the fourth DPW. The nnFM algorithm was used to transmit wing forces from CFD to CSD. RBF algorithm was used to transmit displacements from CSD to CFD. Table 1 lists the key parameters of CRM. Figure 4 shows the aerodynamic and structural grid of wing, including surface grid, CFD grid and FEM grid.

Table 1 – CRM specification

Parameter	Value	Parameter	Value
Cruise Mach number	0.85	Aspect ratio	9
Cruise Lift coefficient	0.5	Reference wing area	167.22 m^2
Reynolds number	5000000	Sweep (leading edge)	37.2°
Dynamic pressure	22700 Pa	Chord length (Root)	11.8658 m
Half span	29.3825 m	Chord length (tip)	2.73376 m

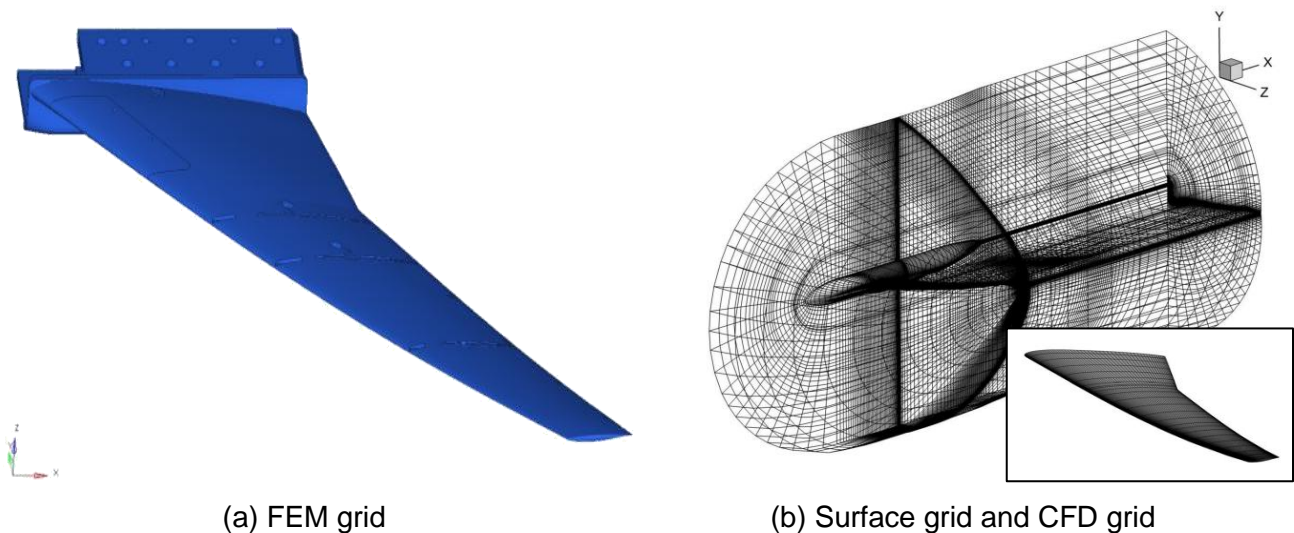
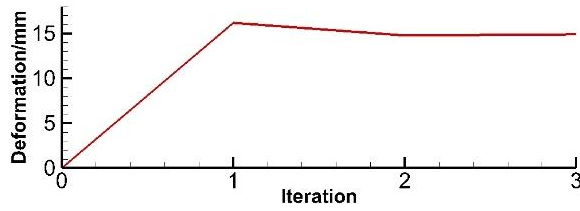


Figure 4 – Grid of CRM.

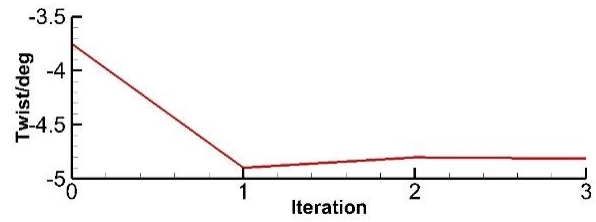
We get the deformed shape of wing after aero/structural analysis. The convergence process is shown in Figure 5. Due to small wing deformation, the process converges after three iterations. Figure 7 shows the pressure coefficient (C_p) of each section along wing span. The C_p obtained after aeroelastic deformation agrees with experimental values well. (a) and (b) in Figure 6 show the displacements and twist angle along the wing span respectively, the curves agree with the experimental data well. The absolute error of displacements at wing tip is 0.55365 mm (experimental

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result is 15.45545mm , and computational one is 14.9009mm) which is less than 1% of wing span, and the absolute error of twist angle at wing tip is 0.03502° (experimental result is -4.77548° , and computational one is -4.8105°) which is less than 0.2 degrees.

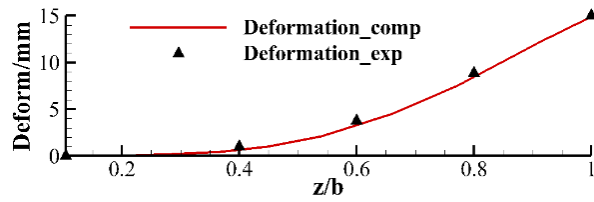


(a) Convergence history of deformation

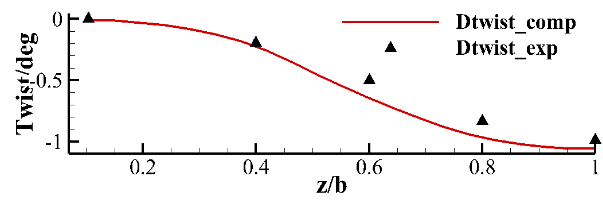


(b) Convergence history of twist

Figure 5 – Convergence history (CRM).



(a) Deformation along wing span



(b) Twist along wing span

Figure 6 – Comparison between the results and experimental value (CRM).

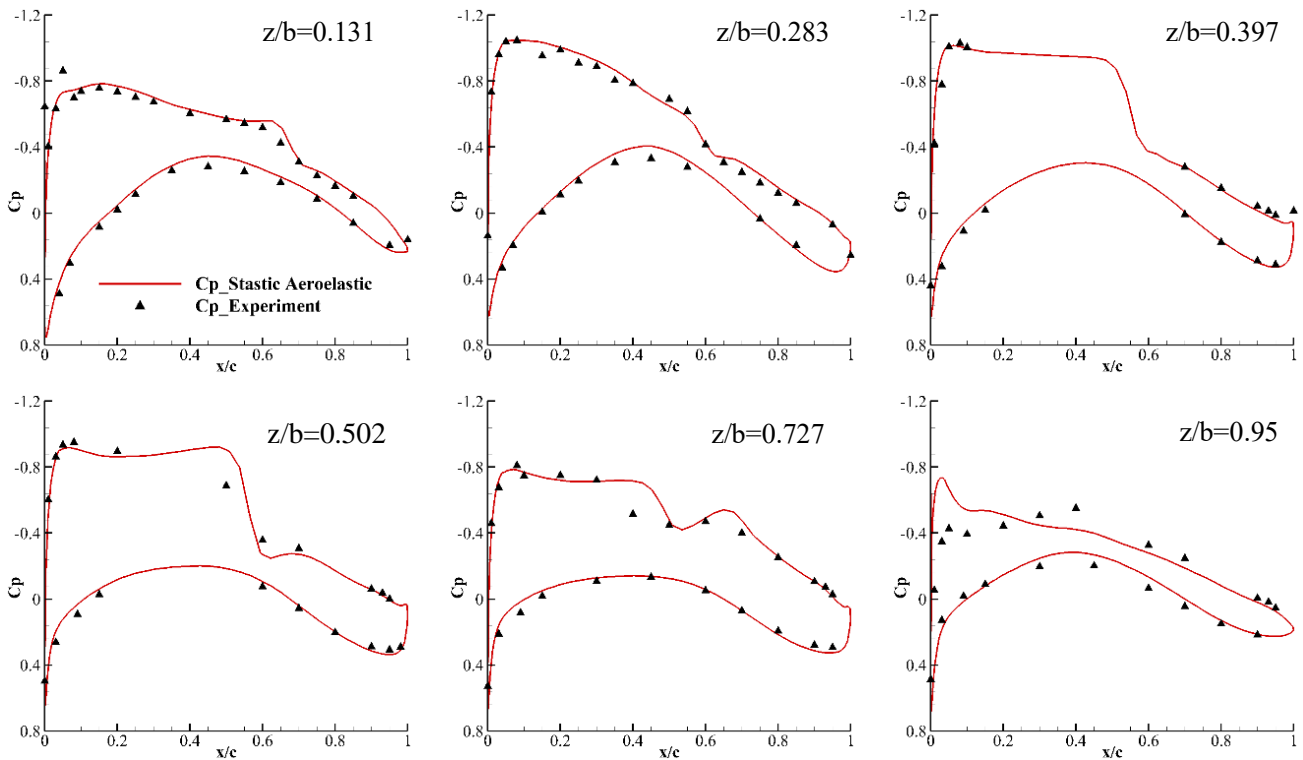


Figure 7 – Comparison of C_p between the results and experimental data on each section (CRM).

3.2 High Reynolds Number Aero/Structural dynamics (HIRENASD) model

The high Reynolds number aero/structural dynamics (HIRENASD) model[22] is taken as the second example to validate the aero/structural code. HIRENASD model is a standard research model published by Aeroelastic Prediction Workshop (AePW), and its wind tunnel experiment was performed by RWTH Aachen University at European Transonic Wind tunnel (ETW). This model contains a wing shape which is commonly used in transonic transport aircraft with high aspect ratio

Evaluation and Validation of Aeroelastic Analysis Program for the “AeroStruct” Wing Optimization Platform and a bump for smooth the wing root. Figure 8 shows the wind tunnel model of HIRENASD configuration.

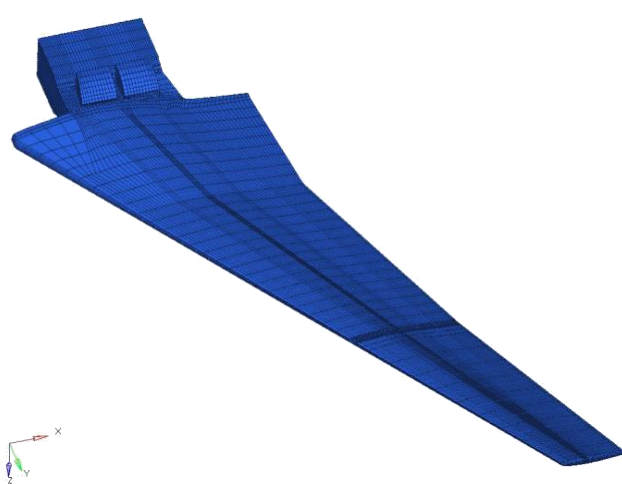


Figure 8 – Wind tunnel model of CRM configuration[22].

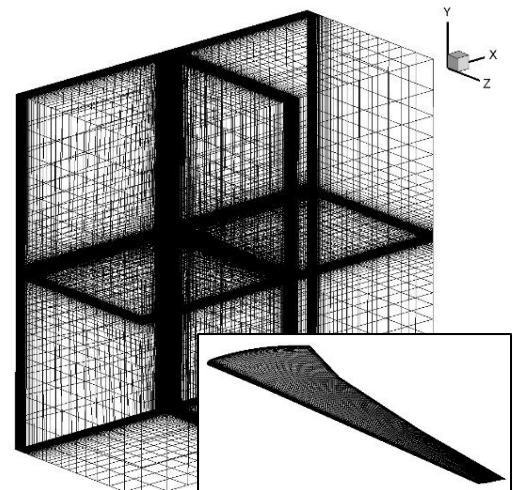
In order to predict lift quickly, Navier-Stokes equations were performed with a constant angle of attack (AoA) 1.5deg, the cruise Mach is 0.8, and the Reynolds number is 7 million. FEM was solved by Nastran to get the deformation. The aerodynamic shape information and FEM model of HIRENASD wind tunnel model came from the AePW [22]. The RBF algorithm was used to transmit both wing forces from CFD to CSD and displacements form CSD to CFD. Table 2 lists the key parameters of HIRENASD. Figure 9 shows the CFD and FEM grid of wing.

Table 2 – CRM specification

Parameter	Value	Parameter	Value
Cruise Mach number	0.7	Aspect ratio	7.98
Angle of attack	1.5°	Reference wing area	0.3926 m^2
Reynolds number	7000000	Sweep(leading edge)	34°
Dynamic pressure	36177.3 Pa	Chord length(Root)	0.54937 m
Half span	1.37571 m	Chord length(tip)	0.14928 m



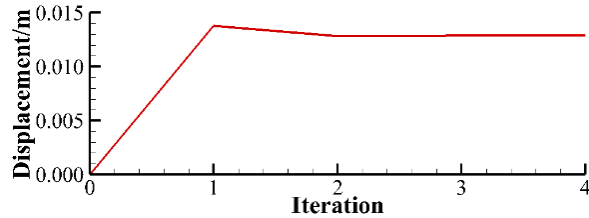
(a) FEM grid



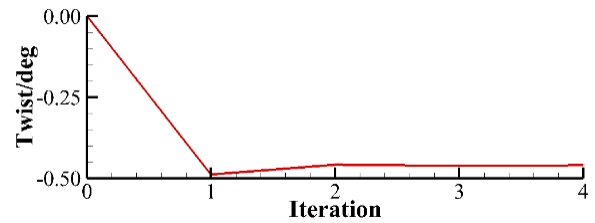
(b) Surface grid and CFD grid

Figure 9 – Grid of HIRENASD model.

We get the deformed shape of wing after aeroelastic analysis. Figure 10 shows the convergence history of aero/structural analysis. Due to small wing deformation, the process converges after four iterations. Figure 11 shows the C_p of each section along wing span. The C_p obtained after aeroelastic deformation agrees with experimental values well. The displacement at the tip of leading edge (0.95 wing span) is 12.8mm, and the absolute error of displacement is 0.3mm (Experimental result is 12.5mm), this value is also less than 1% of wing span.



(a) Convergence history of deformation



(b) Convergence history of twist

Figure 10 – Convergence history (HIRENASD).

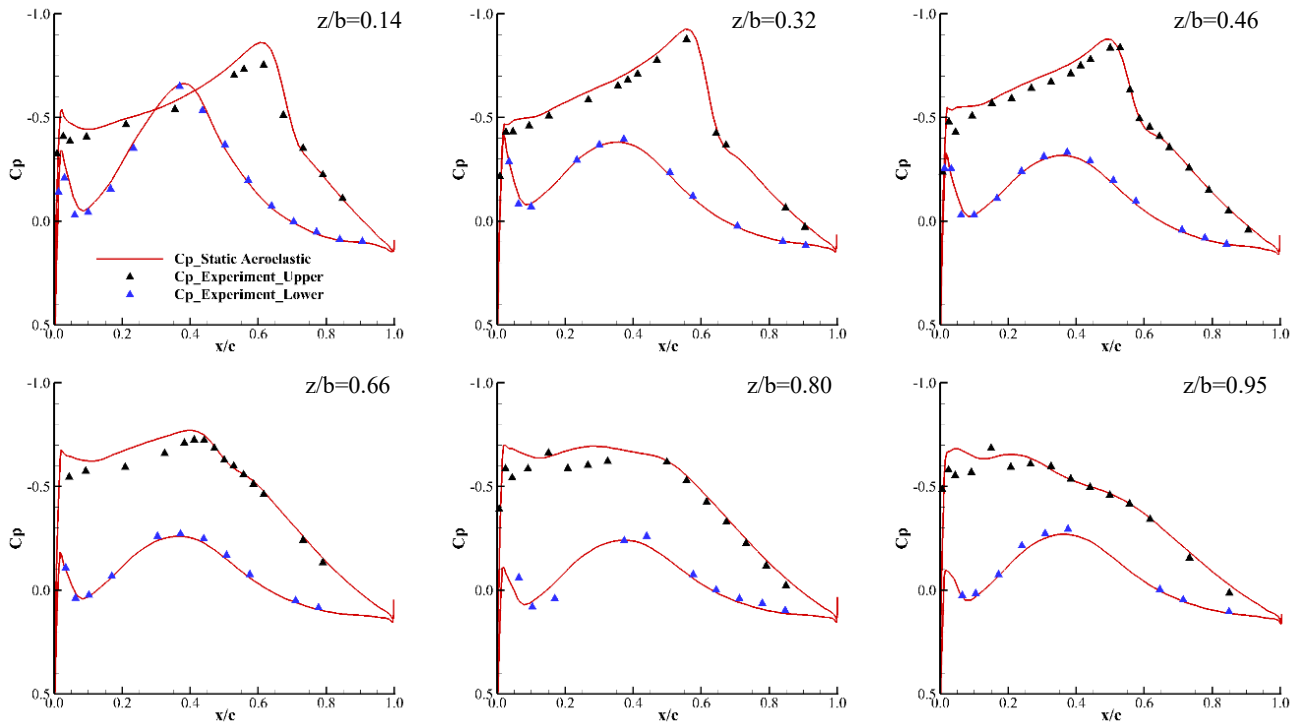


Figure 11 – Comparison of C_p between the results and experimental data on each section (HIRENASD).

3.3 Undeformed Common Research Model-9 (uCRM-9)

Undeformed Common Research Model-9 (uCRM-9) was proposed by Professor Martins based on CRM[23], and uCRM-9 is the jig shape while CRM is the cruise one. The geometry of uCRM-9 is same as that of CRM, except dihedral angles and twist angles along the wing span. Its geometry model is shown on Figure 12.

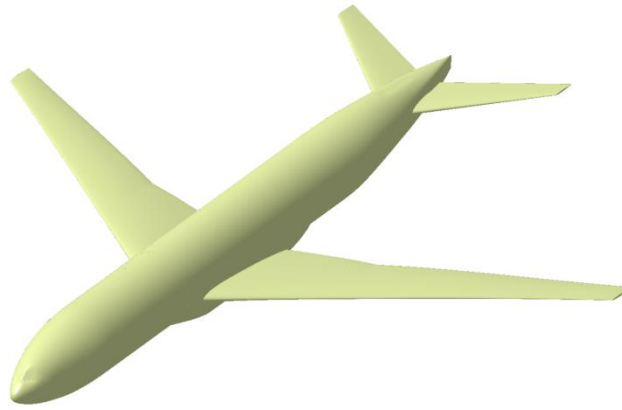


Figure 12 – Wind tunnel model of CRM configuration.

Full potential equation was performed with a constant lift coefficient 0.5, the cruise Mach number was 0.85, and the Reynolds number was 5 million. FEM was constructed and solved by ANSYS, Figure 13 shows the structure grid and CFD grid of uCRM-9. The RBF algorithm was used to transmit both wing forces from CFD to CSD and displacements from CSD to CFD.

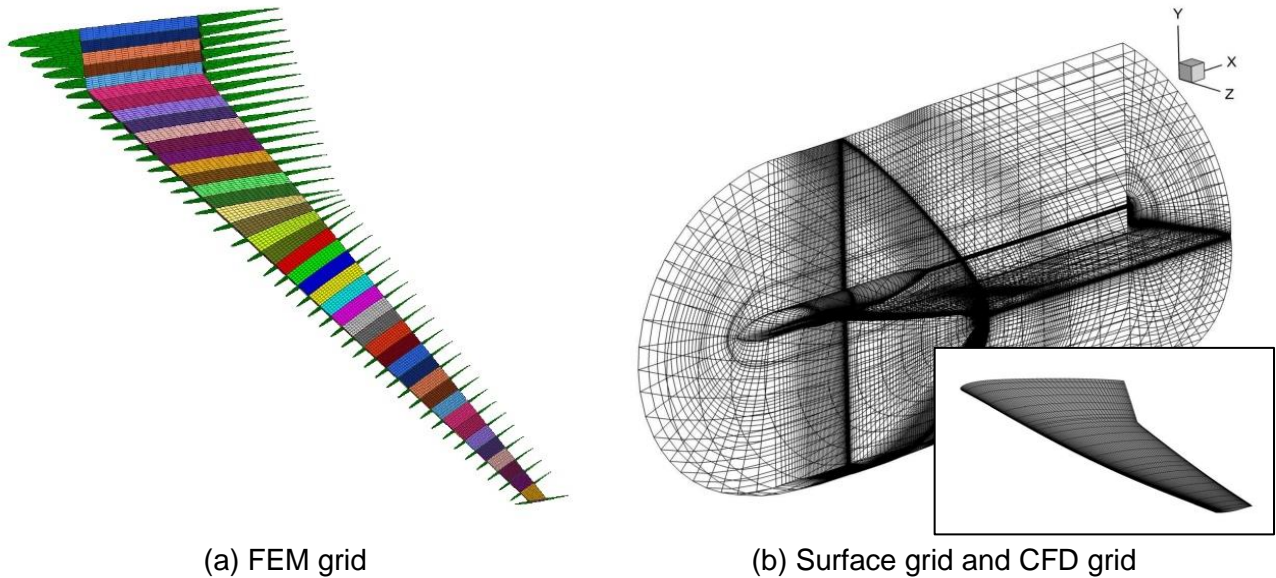
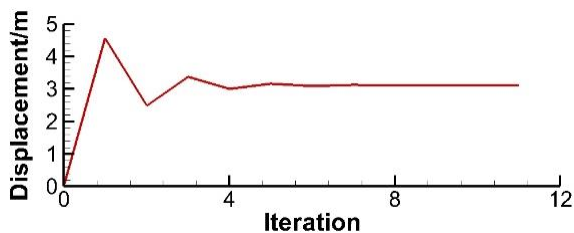
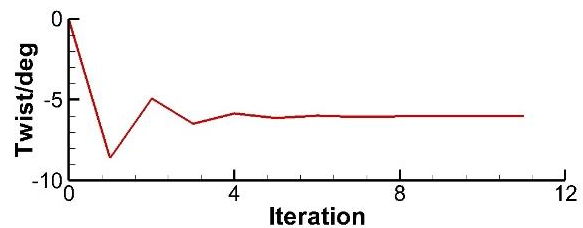


Figure 13 – Grid of uCRM-9 model.

After the aeroelastic analysis of wing, we get the deformed shape of uCRM-9. Figure 14 shows the convergence history of aero/structural analysis. The program converged after 11 iterations, because of the large deformation of the real wing and high accuracy we expect. Figure 15 shows the C_p of each section, the C_p of deformed uCRM-9 agrees well with that of CRM. Figure 16 and Figure 17 compare the displacements and twist of wing between uCRM-9 baseline, uCRM-9 deformed shape and CRM shape. Figure 18 shows the Von Mises stress and C_p contour of deformed uCRM-9 configuration, the distribution of stress and C_p contour is reasonable.



(a) Convergence history of deformation



(b) Convergence history of twist

Figure 14 – Convergence history (uCRM-9).

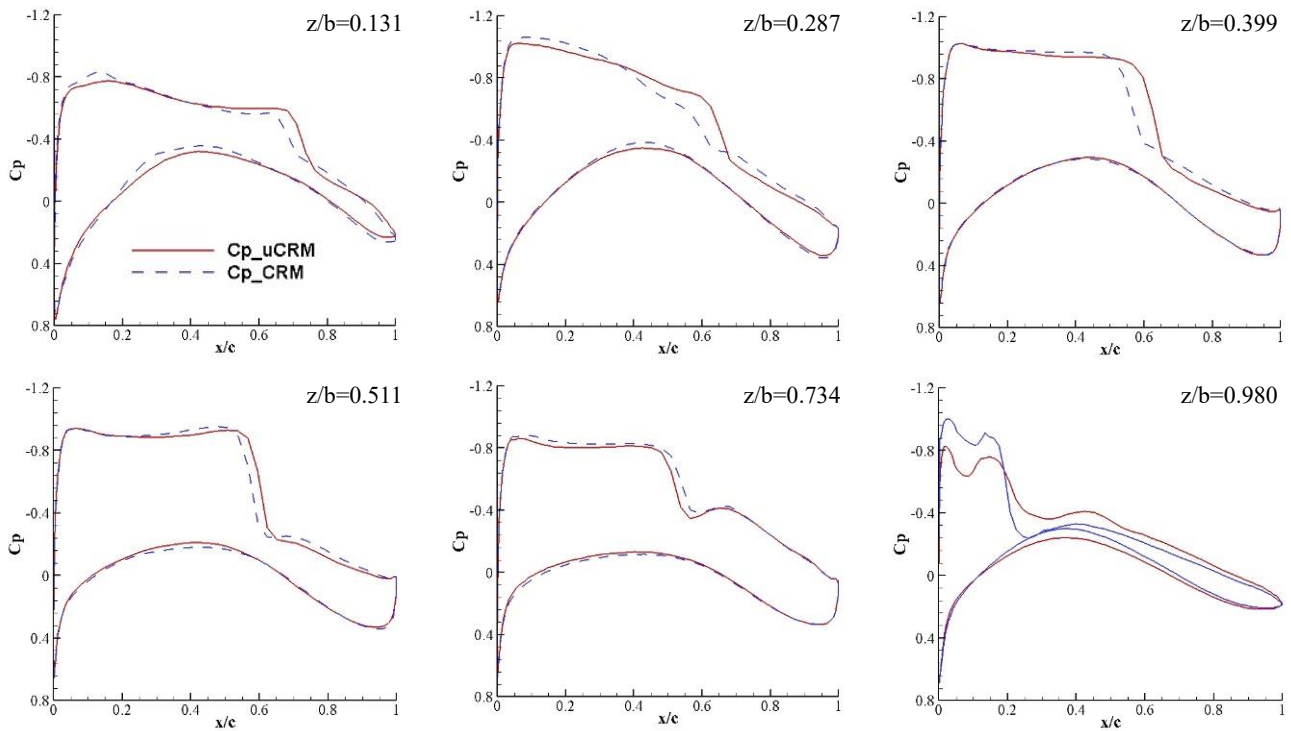


Figure 15 – Comparison of Cp between uCRM and CRM on each section.

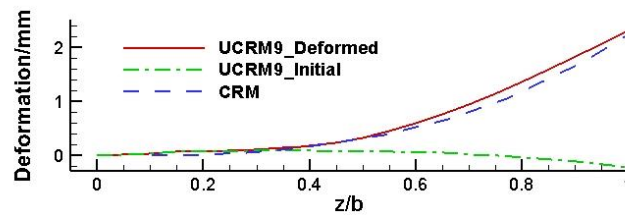


Figure 16 – Comparison of deformation along wing span(uCRM-9).

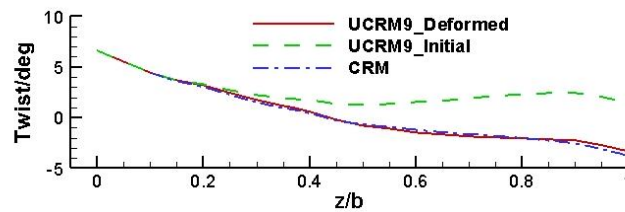


Figure 17 – Comparison of twist angle along wing span(uCRM-9).

As we expected, more iterative steps are required for wings with larger deformations under the weak coupled framework according to Figure 14. In addition, by comparing the Cp, displacements and twist of each section in Figure 15, Figure 16 and Figure 17, we can also find that the deformed shape of uCRM-9 becomes CRM in cruise state, when there is a reasonable structure. The initial vertical coordinate of wing tip of uCRM-9 is $-0.21680m$, and the coordinate of deformed uCRM-9 is $2.30655m$. While the coordinate on the same position of CRM configuration is $2.23505m$. The relative error is only 3.09% between deformed uCRM-9 and CRM. In addition, the wing tip twist of uCRM-9 is 1.46904° , the twist becomes -3.26422° after deformed. The twist of CRM wing tip is -3.75° , this error (12.95%) looks a little big, but it is still a good result. The maximum stress of skin occurs at the middle area along span, this is in line with the actual situation for this kind of wing configuration. It proves

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the effectiveness of our codes once again.

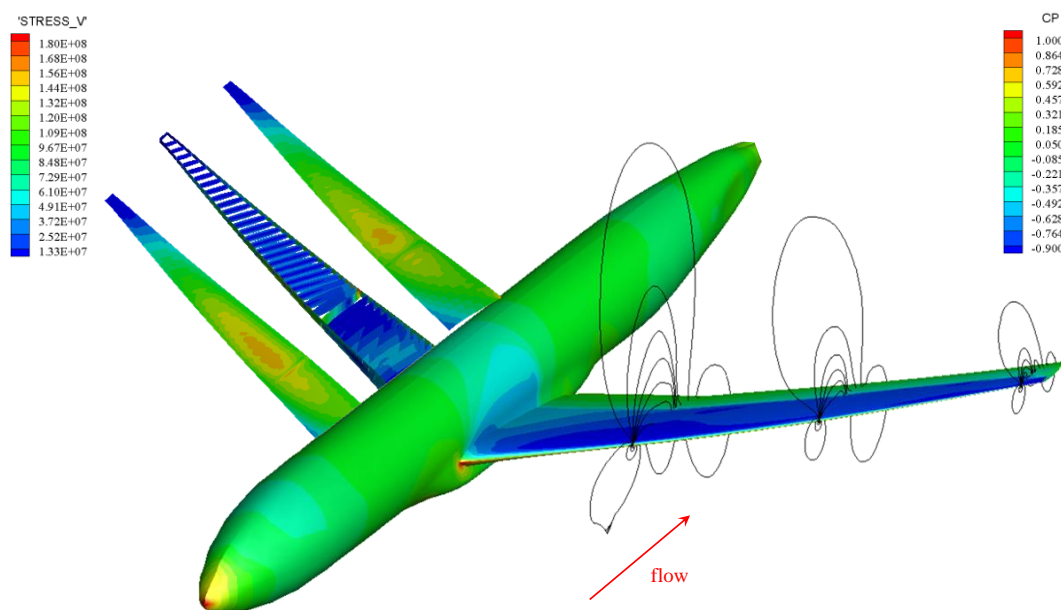


Figure 18 – Stress and Cp contour of deformed uCRM-9.

4. Conclusions

The aero/structural analysis codes for variable-fidelity aeroelastic simulation were developed for the aero/structural design optimization platform “AeroStruct”, which is to address optimization of transonic transport aircraft wing. Full potential equation, Euler equation and NS equation were solved to calculate aerodynamic forces of wing. FEM method were applied to simulate the deformation process through ANSYS or Nastran. RBF interpolation was used for data coupling. CRM and HIRENASD model were performed to validate the codes. And compared with the experimental value, both the validation examples got small error (the error about displacement was less than 1% of wing span, the absolute error about twist was less than 0.2 degrees). Further, the aero/structural codes were applied to uCRM-9 successfully for aeroelastic analysis. It indicates that our codes can work well for the aeroelastic analysis of the high-aspect-ratio transonic wing.

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6. Contact Author Email Address

Shengbo Ling: 2018260252@mail.nwpu.edu.cn

Keshi Zhang: zhangkeshi@nwpu.edu.cn

Zhonghua Han: hanzh@nwpu.edu.cn

Zhongjian Gao: 1048142479@qq.com

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