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The Vibration Correlation Technique – A Reliable Nondestructive Tool to Predict Buckling Loads

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Abstract

The vibration correlation technique, the VCT, consists of measuring the natural frequencies of a loaded structure, and monitoring their change, while increasing the applied load. Assuming that the vibrational modes are similar to the buckling ones, one can draw a curve, displaying the natural frequencies squared vs. the applied load, and extrapolating the curve to zero frequency would yield the predicted buckling load of the tested structure. The VCT procedure is a nondestructive in-situ method capable of predicting buckling loads of thin walled structures, like beams, plates, panels and shells loaded in compression. The present paper highlights the VCT analytical and experimental state-of-the art by presenting the relevant derivations for columns, plates and shells together with test results stressing the reliable nondestructive nature of the method.

Introduction

The vibration correlation technique , the VCT, a shortcut proposed by the late Prof. Joseph Singer [1] is a nondestructive method to predict buckling loads of compressed thin walled structures, by monitoring the changes of their natural frequences due to the compresive loads.

Besides its capability to nondestructively predict the buckling load of thin walled structures, the approach can also determine the actual in-situ boundary condition of the structures, and therefore the VCT is usually classified in two main groups according to their approach: (1) direct prediction of buckling loads and (2) determination of in situ boundary conditions [1].

The present paper will only deal with the first capability of the method, namely, the nondestructive direct prediction of buckling loads.

Rods, columns and beams

To present the correlation between the natural frequency for a column (or a rod, or a beam) on simply supported boudary condions and the compressive load, Nx (see Fig. 1), as claimed by Lurie [2-4], the following equation can be derived for small vibrations for an isotropic column

$$EI\frac{\partial^4 w(x,t)}{\partial x^4}+N_x\frac{\partial^2 w(x,t)}{\partial x^2}+\rho A\frac{\partial^2 w(x,t)}{\partial t^2}=0 \qquad (1)$$

where L = length, EI= bending stiffness, ρA= mass per unit length with A and E being the cross section of the column and its Young's modulus, respectively, while w(x,t) is the out-of-plane displacement.

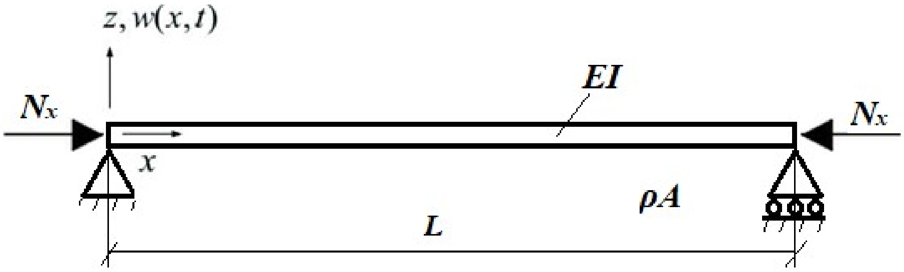

Fig. 1 A compressed column on simply supported boundary conditions

Assuming harmonic vibrationsone can write the solution for Eq.1 as

$$w(x,t)=W\left(x\right)e^{i\omega t} \tag{2}$$

where ω is the angular frequency of the column and $W\left(x\right)$ is a function to be next determined. Substituting Eq. 2 into Eq. 1 while deviding it by EI yields

$$\begin{aligned} &W_{,xxxx}+\lambda^2 W_{,xx}-c^2\omega^2 W=0 \\ &where \\ &\lambda^2 \equiv \frac{N_x}{EI} \qquad c^2 \equiv \frac{\rho A}{EI} \end{aligned} \tag{3}$$

The solution for Eq. 3 is

$$\begin{aligned} &W\left(x\right)=A_1\cosh\left(\alpha_1 x\right)+A_2\sinh\left(\alpha_1 x\right)+A_3\cos\left(\alpha_2 x\right)+A_4\sin\left(\alpha_2 x\right) \\ &where \\ &\alpha_1=\sqrt{-\frac{\lambda^2}{2}+\sqrt{\left(\frac{\lambda^2}{2}\right)^2+c^2\omega^2}}\ , \quad \alpha_2=\sqrt{\frac{\lambda^2}{2}+\sqrt{\left(\frac{\lambda^2}{2}\right)^2+c^2\omega^2}}\ . \end{aligned} \tag{4}$$

The constants A1-A4 in Eq. 4 are to be found by imposing the adequate boundary conditions. For a column with simply-supported boundary conditions , namely lateral displacements and moments vanishing at both ends (x=0,L) leads to an eigenvalue problem.Demanding the vanishing of the determinant of the matrix coefficients A1-A4 yields the following characteristic equation

$$\begin{aligned} &A_4\left(\alpha_1^2+\alpha_2^2\right)\sin\left(\alpha_2 L\right)=0 \\ &\Rightarrow \sin\left(\alpha_2 L\right)=0 \qquad or \qquad \alpha_2 L=n\pi \qquad n=1,2,3.. \end{aligned} \tag{5}$$

Substituting the expression for α_2 in Eq. (4) leads to

$$\frac{\lambda^2}{2}+\sqrt{\left(\frac{\lambda^2}{2}\right)^2+c^2\omega^2}=\frac{n^2\pi^2}{L^2}\Rightarrow\frac{N_x}{2N_{x_{cr}}}+\sqrt{\left(\frac{N_x}{2N_{x_{cr}}}\right)^2+\frac{\omega^2}{\omega_n^2}}=1\Rightarrow\left(\frac{\omega}{\omega_n}\right)^2+\frac{N_x}{N_{x_{cr}}}=1$$

where (6)

$$N_{x_{cr}}=\frac{n^2\pi^2 EI}{L^2},\qquad \omega_n^2=\frac{n^2\pi^4 EI}{\rho A L^4}.$$

Equation 6 depicts that for a column on simply-supported boundary conditions, the square of the natural frequency is inverse linear with the applied axial compression load (see Fig. 2).

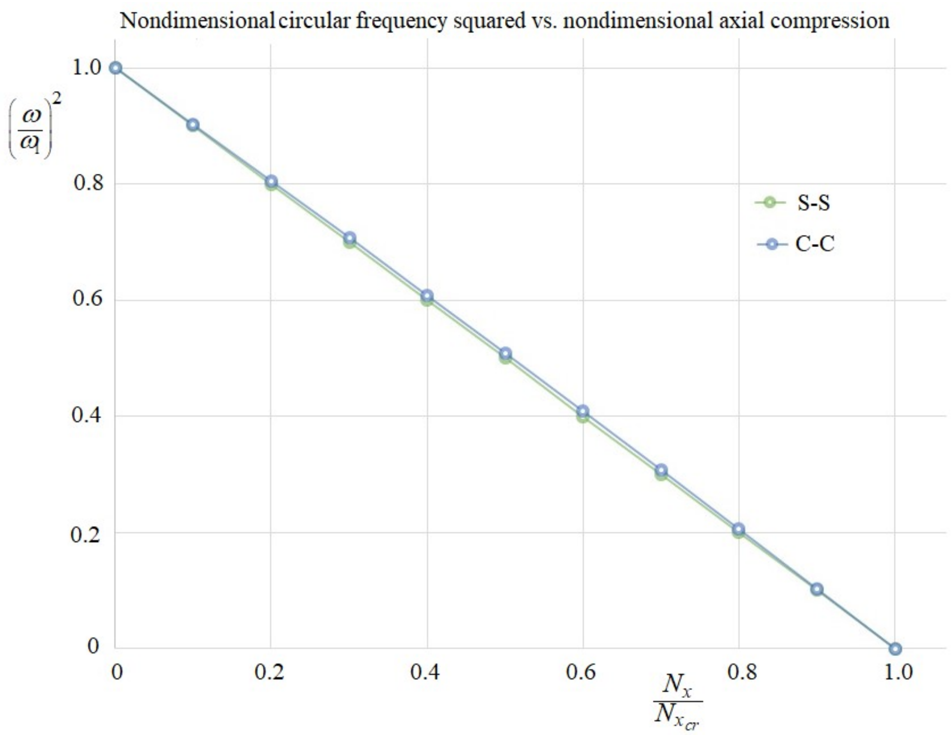

Fig. 2 Nondimensional circular frequency squared vs. nondimensional axial compression- a thin walled column (S-S =simply supported-simply supported, C-C =clamped-clamped)

Note that for a clamped-clamped beam (deflections and slopes vanish at both ends) the characteristic equation has the form of an implicit function, namely

$$2c\omega\left[1-\cosh\left(\alpha_1 L\right)\cos\left(\alpha_2 L\right)\right]-\lambda^2\sinh\left(\alpha_1 L\right)\sin\left(\alpha_2 L\right)=0 \qquad (7)$$

To draw the graph presented in Fig. 2 for the clamped-clamped case, an iterative procedure is applied . An almost inverse linear relation between the natural frequency squared and the compressive load is found also for the clamped-clamped case, similar to the simply supported –simply supported case, dealt above. Therefore, for engineering purposes an inverse linear relation can be assumed for both cases.

It is interested to note that also for tensile loads, the square of the natural frequency is inverse linear with the applied load, yielding

$$1+\frac{N_x}{N_{x_{cr}}}=\left(\frac{\omega}{\omega_n}\right)^2 \tag{8}$$

This means that for tension loads, the stiffness is increasing, due to the axial load, leading to an increase in the bending natural frequencies , while for compression forces, the stiffness is reduced, leading to lower bending natural frequencies.

It can be easily shown that for other cases of compressed columns or beams, like a beam on Winkler type foundation , symmetric or non-symmetric laminated composite beams or a constrained beam under thermal loading (see [5]), the relationship presented in Eq. 6 is viable.

Note that the relationship presented by Eq. 6 was also found experimentaly, as presented in Fig. 3.

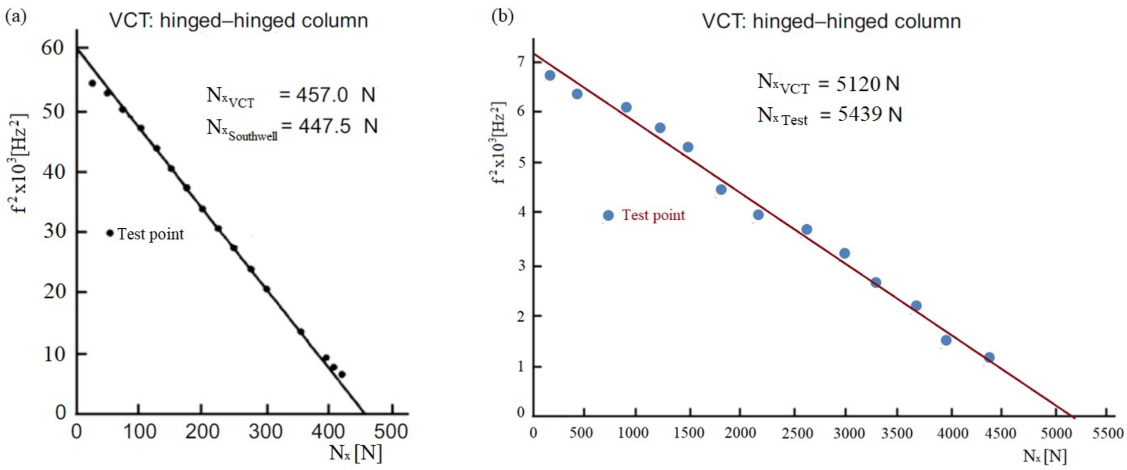

Fig. 3 Hinged-hinged tests (a) laminated composite column [6], (b) iron columns –students test at Aerospace Engineering, Technion, Lab

Note that in Fig.3a the predicted buckling load using the VCT was compared to the results predicted by the Southwell method. The Southwell plot is another graphical method to experimental determine the critical buckling load of a column. This method is considered a non-destructive approach to predict the buckling load, by plotting the lateral deflection, w, as a function of the ratio w/Nx (Nx =the axial compressive load). The slope of the linear curve would be the buckling load, Nx,cr.

Plates and panels

When addressing the issue of plates or panels, it is simply to derive the theoretical relationship between the natural frequencies squared and the applied in-plane load for a rectangular plate on simply supported boundary conditions. Its form is

$$D\left[W_{,xxxx}\left(x,y,t\right)+2W_{,xxyy}\left(x,y,t\right)+W_{,yyyy}\left(x,y,t\right)\right]+$$
$$\rho hW_{,tt}\left(x,y,t\right)=N_xW_{,xx}\left(x,y,t\right)+N_yW_{,yy}\left(x,y,t\right)+2N_{xy}W_{,xy}\left(x,y,t\right)$$
$$where \qquad (9)$$
$$D=\frac{Eh^3}{12\left(1-\upsilon^2\right)}$$

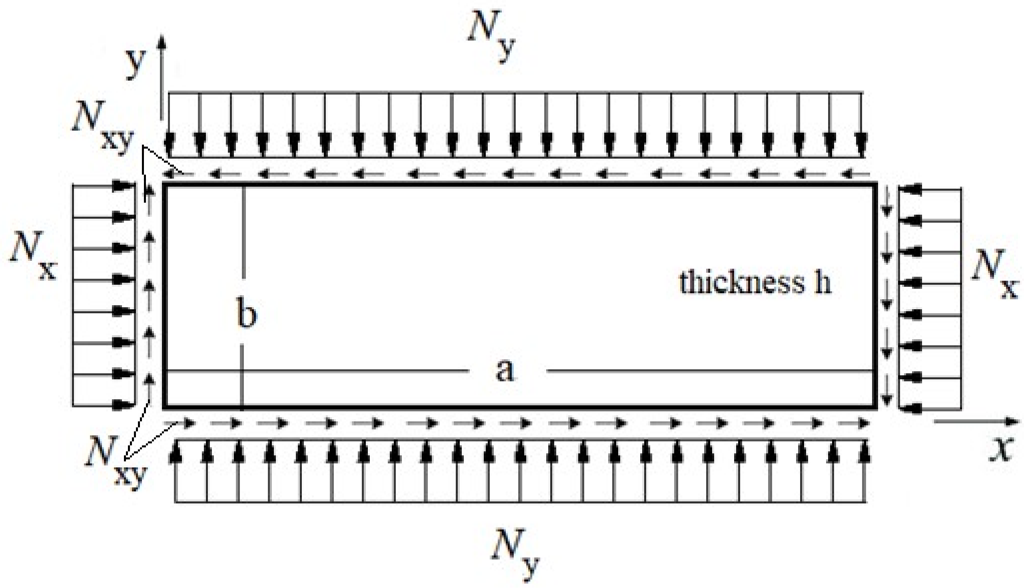

Fig. 4 An isotropic rectangular plate loaded in its plane- a schematic view

Where the varios variable are given in Fig. 4. For the case of a rectangular plate without geometric imperfections, and assuming the shear force is zero (Nxy=0), the solution for the out-of-plane deflection, W(x, y, t), which satisfies the simply supported boundary conditions of the plate, is (ω being the plate circular frequency)

$$W\left(x,y,t\right)=\left(\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\widehat{W}_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\right)e^{i\omega t} \qquad (10)$$

Substituting Eq. 10 back into Eq. 9, yields

$$1-\frac{\rho h\omega^2}{D\pi^4\left[\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2\right]^2}=-\frac{N_x\pi^2\left(\frac{m}{a}\right)^2}{D\pi^4\left[\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2\right]^2}-\frac{N_y\pi^2\left(\frac{n}{b}\right)^2}{D\pi^4\left[\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2\right]^2}$$

or

$$1-\frac{N_x}{N_{x_{cr}}^{mn}}-\frac{N_y}{N_{x_{cr}}^{mn}}=\left(\frac{\omega}{\hat{\omega}_{mn}}\right)^2$$

where

$$N_{x_{cr}}^{mn}=-D\pi^2\frac{a^2}{m^2}\left[\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2\right]^2$$

$$N_{y_{cr}}^{mn}=-D\pi^2\frac{b^2}{n^2}\left[\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2\right]^2$$

$$\hat{\omega}_{mn}^2=D\frac{\pi^4}{\rho h}\left[\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2\right]^2 \qquad (11)$$

A similar expression can be obtained also for a laminate composite perfect plate, using the Classical Lamination Theory (CLT) (see [6] and [7]).

Turning to an isotropic perfect rectangualr plate, loaded in shear it can be stated that no closed-form solutions for an all-around simply-supported plate (or any other boundary conditions) under shear loads are available [8]. Moreover, as it is known and also claimed in [8], the buckling mode shape of the plate under shear would be skewed while the mode shape of the natural frequency under zero load would be sinusoidal, and transformed into a skewed one with the increasing of the shear load [8].

Confirmation of Eq.11 by tests shows that perfect plates, without initial geometric imperfections, obey the linear relation. For a regular plate, with initial geometric imperfections the square of the natural frequecy curls up in the vicinity of the buckling load (see Fig. 5). This was already known to Lurie [2] and was further realized by the calculations performed already in 1948 by Massonnet [9] on circular uniform plates under uniform compression . Note that for very small imperfections, and a structure (stringer stiffened flat plate) resembling the behaviour of a column rather than a plate, like the case presented in Fig. 6, a linear relationship exists.

Similar results are reported in [12], for circular aluminum plates undergoing radial compression, as presented in Fig. 7. Defining the load at which the squared nondimensional frequency changes its path from descent to rising as the predicted buckling load by VCT , one gets Pcr (VCT)=272.9 kN as compared to Pcr (Exp.)=275.8 kN, which is very good nondestructive buckling prediction. Due to its stable postbuckling behavior of a plate, one can record its natural frequency up to the buckling and beyond, without destroying the tested specimen and thus correctly define the postbuckling of a plate having initial geometric imperfections. The test set up is presented in Fig. 8.

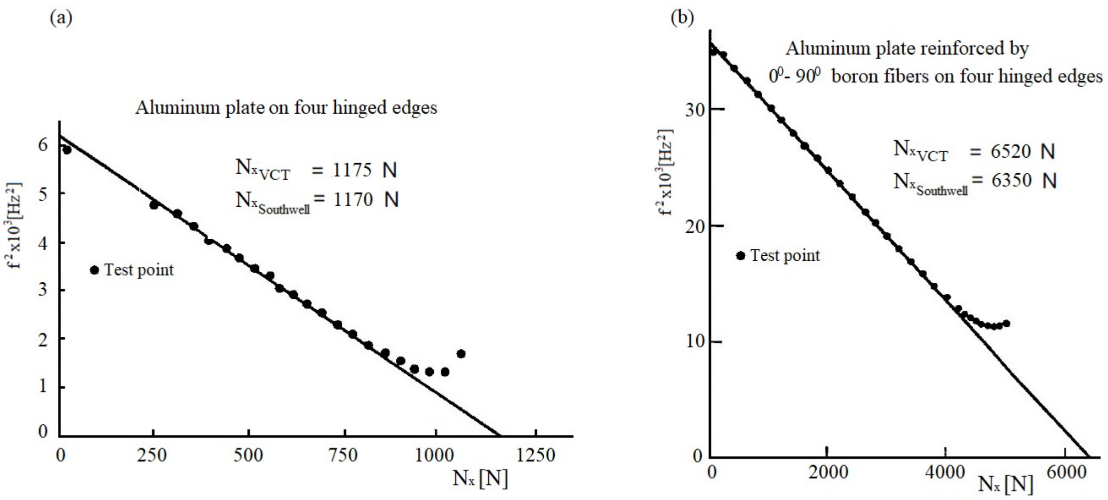

Fig.5 Frequency squared vs. applied axial compression- experimental results for (a) rectangular aluminum plate,(b) aluminum rectangular plate reiforced by 0^0-90^0 boron fibers (reconstructed from [10])

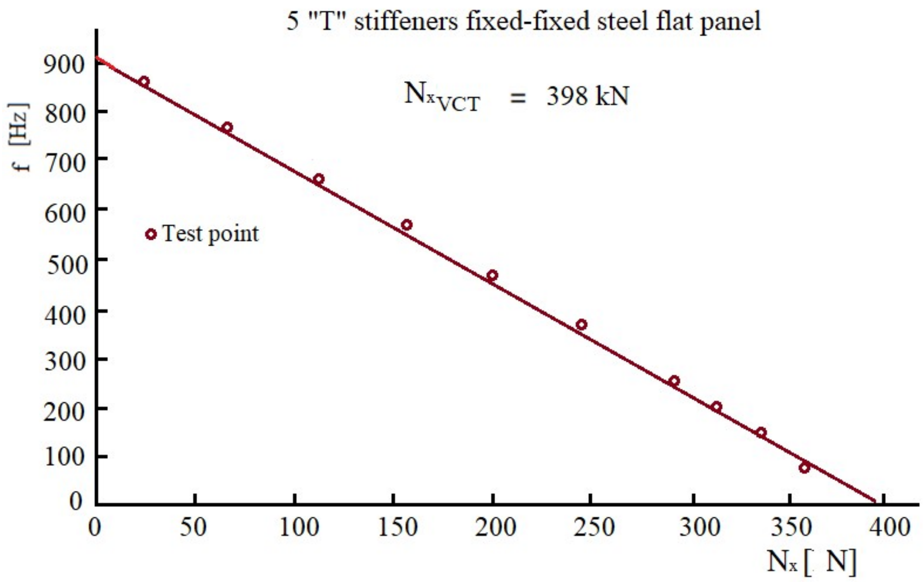

Fig.6 Frequency squared vs. applied axial compression- experimental results for five "T" type stiffeners steel flat panel (reconstructed from [11])

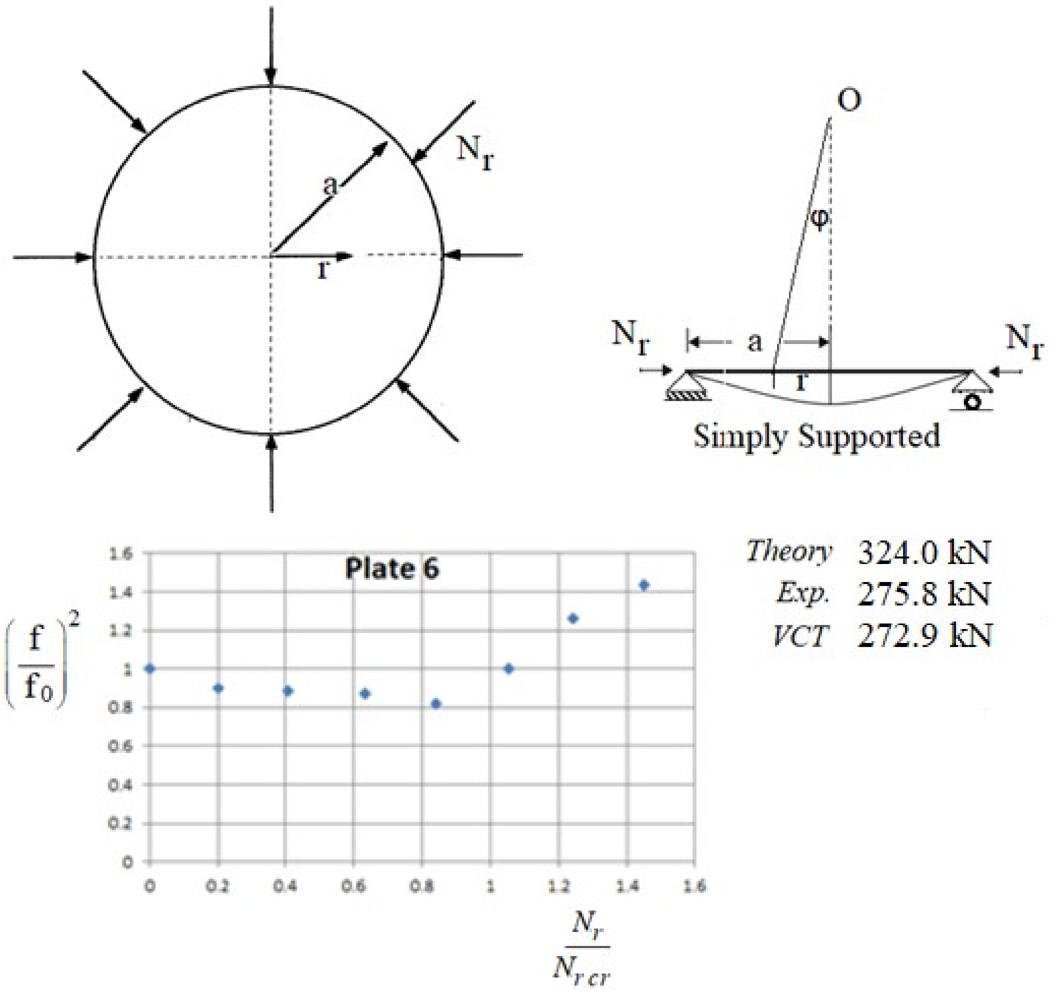

Fig. 7 A simply supported circular plate under radial compression- Application of the VCT approach

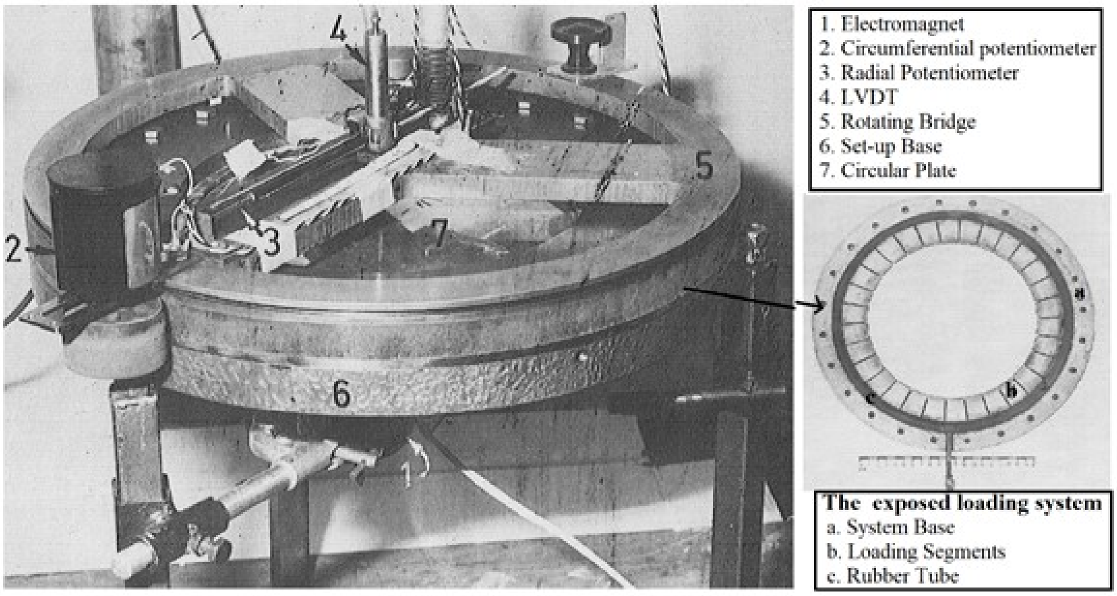

Fig. 8 The test set-up used for circular plates [12]

Cylindrical shells

The VCT method, presented above for columns and plates was also applied to cylindrical shells, using the linear relationship between the square of the natural frequency and the compressive load. Although the experimental relationship was linear [13], its extrapolation to zero frequency yielded erronous buckling predictions.Two models were proposed in the literature, which were able to well nondestructive predicting the buckling of the cylindrical. One is attributed to Souza [14] and has the following form (see Fig. 9a)

$$(1-p)^2+(1-\xi^2)(1-f^4)=1 \tag{12}$$

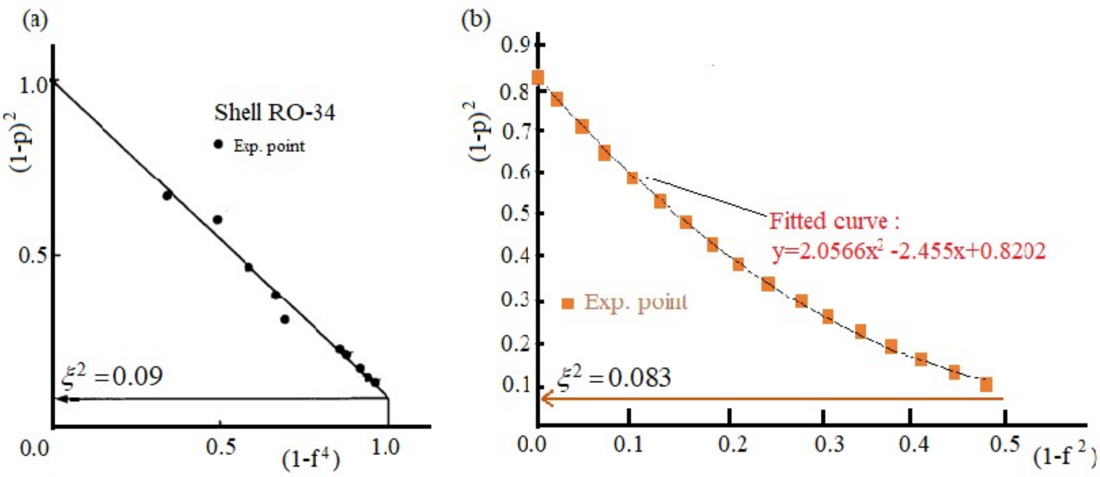

Fig. 9 (a) Souza model [14] application, (b) Arbelo model [15] application

The second model (Arbelo model [15]) has the following form of

$$1-p=f^2=1-(1-f^2)\Rightarrow(1-p)^2=\left[1-(1-f^2)\right]^2 \tag{13}$$

where p=Nx/Nx cr, f=f/f0 , Nx and f being the measured applied axial load and the natural frequency, respectively, fo is the natural frequency at zero load and Nx cr is the theoretical linear buckling load of the shel. ξ2 is the "experimentally" knock-down factor based on the results of the test at relatively low loads (up to 60% from the predicted buckling load) . Once the knock-down factor was experimentally determined, using one of the models, the predicted buckling load is calculated using the following relationship

$$N_{x_{predicted}}=(1-\xi)N_{x_{cr}} \tag{14}$$

The method proposed by Arbelo [15], had been successfully used by various researchers (see [15-27] to predict the buckling of thin wall composite cylindrical shells within 95-100% of the real experimental load.

Excitation methods

A good excitation method would enable to acquire the natural frequencies of a vibrating structure and lead to consistent results when applying VCT.

Normally, an electromagnetic shaker (Fig. 10a) would provide the necessary driving force to excite the specimen, as a function of frequency generator [1]. The response of the specimen can be recorded using a microphone and the resonance can be detected using Lissajous curves(see Fig. 10c), with the excitation voltage and the specimen response voltage being supplied to the two axes of an oscilloscope.

Exciting the specimen using a loudspeaker (see Fig. 10b), would yield an ellipse (Fig.10d), while using a shaker the Lissajous curve would be an eight (Fig.10c), due to the "push-pull" movement of the shaker which is twice the response frequency.

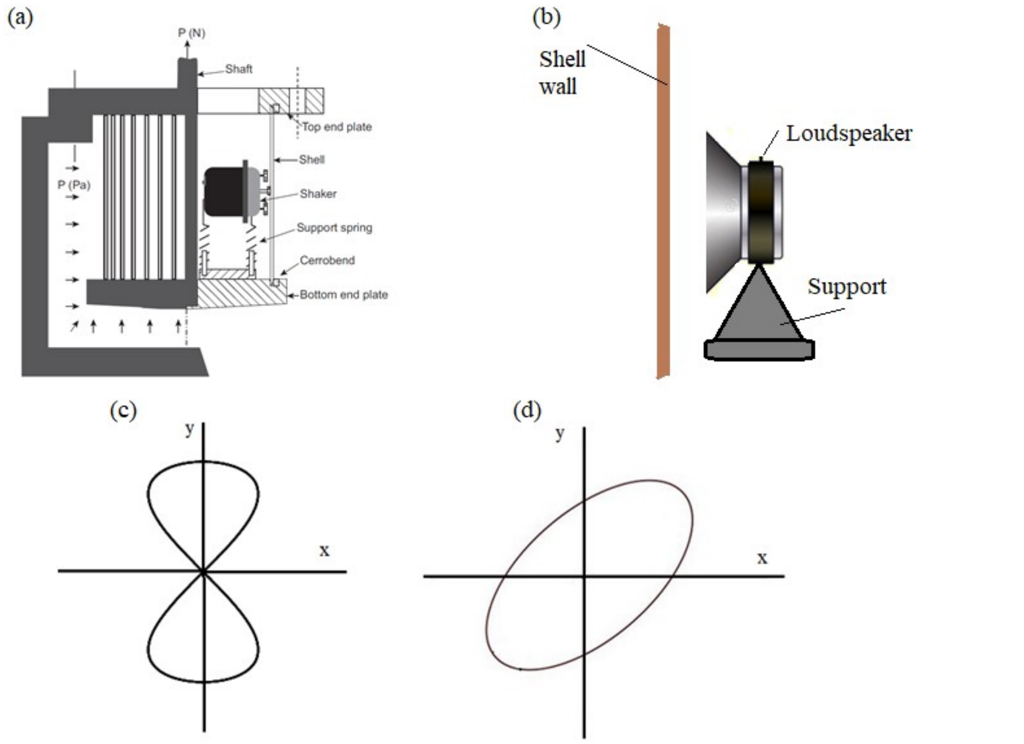

Fig. 10 Excitation methods and their relevant Lissajous curves :(a) a shaker, (b) a loudspeaker, (c) shaker Lissajous curve, (d) loudspeaker Lissajous curve

The modal hammer [28] is onother way for excitation of the structure. The method uses a special dedicated hammer to slightly impact the specimen, and its response is recorded using an accelerometer bonded on the tested specimen and connected to NI DAQ system (see [28]). The excitation system is composed of an impact hammer (from National Instruments-NI), a single accelerometer and a NI DAQ system with a Me Scope Modal Software, to detect the excited natural frequencies and their respective mode shapes of the tested panel. The output is in the form of

Frequency Response Function (FRF) and the determination of the natural frequencies is done by searching peaks on the Imaginary FRF chart .

Another important issue for the successful application of VCT is the mode shapes of the tested specimen. When using a microphone, its voltage can be used to draw the mode shape of the vibrating specimen. Using a single accelerometer to detect the response of the vibrating structure, will not enable the recording of the modes, only the vibrating frequency.

The most sophisticated way to excite a given structure is the use of POLYTEC PSV 500 scanning laser vibrometer as described in [22].. The POLYTEC set-up can measure the natural frequencies and the vibration mode shapes of the tested cylinders. It consists of a laser scanning head, data acquisition unit and a control unit.. The laser vibrometer is capable of covering approximately 1600 of the shell surface with high fidelity grid containing 450 points. After calibration of the scanning process, a typical measurement would include the following steps: (a) the axial compressive load level is applied, (b) the loudspeaker excites the laminated composite shell on a predefined frequency sweep range, while the laser scanning head is measuring the shell response at each predefined grid point, scanning it repeatedly five times (c), application of Fast Fourier's Transform (FFT) on the cylindrical shell response presenting the natural frequencies within the predefined frequency range, (d) saving the values of the natural frequencies and their associated mode shapes on the POLYTEC internal storage.

Conclusions

The vibration correlation technique is a nondestructive tool capable of predicting the buckling loads of thin walled structures with a high precission. In-situ loading a structure up to 60% of the numerically calculated buckling load and monitoring the reduction of the natural frequencies during the loading and then applying the VCT would yield the predicted buckling of the structure to be found at a close proximity of the experimental buckling load.

This tool, the VCT, had been succesfully applied to columns, beams, plates, panels and cylindrical shells showing very good buckling predictions.

Copyright Statement

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