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ITERATIVE GLOBAL/LOCAL NON-INTRUSIVE MODEL COUPLING METHOD FOR STRUCTURE FAILURE ANALYSIS

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Abstract

In this paper, based on the FE non-intrusive coupling framework, an innovative iterative global/local non-intrusive model coupling method was developed to analyze the local crack failure. Several acceleration strategies are studied to further reduce iterations. The FE simulation of a failure experiment of a joint structure is performed to verify the proposed method. Numerical results showed that the coupling results have very good convergence with experimental result.

Keywords: non-intrusive coupling, iterative global/local, local crack failure, numerical simulation

1. Introduction

One of the major challenges of computational solid mechanics in industrial applications is the ability to analyze nonlinear and heterogeneous physical phenomena, such as local plasticity and crack. In order to accurately capture local nonlinear behaviors, traditional FE methods, the meshes of local regions should be refined and a nonlinear analysis should be conducted over the whole domain [1, 2]. However, when the problem size is large or loading history is complex, this approach can lead to impractical computational costs.

Although FE global/local, multi-scale and domain decomposition (DD) methods were proposed to handle local nonlinearity, some major obstacles exist in engineering applications [3-11]. In traditional FE global/local method, the continuity of stress along the interface of global and local model cannot be naturally satisfied because of stiffness difference of local region between global and local model. It can be caused by local nonlinearity or refinement of local model. In addition, most of multi-scale and domain decomposition methods are intrusive with commercial FE solver and cannot be applied to large-scale structural analysis.

In order to reduce the intrusiveness with commercial FE codes, the non–intrusive coupling method has been proposed by Gendre et al. [12, 13] to analyze problems with local nonlinearities. It modifies the traditional global/local method with the basis of non-overlapping DD data exchange scheme. However, the non–intrusive coupling method can only treat compatible mesh interface between global and local model. Based on the non–intrusive coupling method, an iterative global/local method is proposed to consider non-matching interfaces in our previous study [14] through a feasible conservative interface data transfer scheme based on the localized Lagrange multiplier (LLM) method [15, 16] and radial basis function (RBF) interpolation [17].

In this paper, local failure simulation of a joint structure is studied in the non-intrusive iterative global/local framework. The basis of the iterative global/local non-intrusive coupling method is presented in Section 2. In order to improve the convergence rate, interface acceleration methods are introduced in Section 3. A 3D crack simulation of a joint structure is performed in Section 4. Numerical results and discussions are also made in that section. Finally the conclusions are drawn

in Section 5.

2. Non-intrusive algorithm: iterative global/local method

In iterative global/local coupling methods, a global model (coarse mesh) and one or more local models (fine mesh) are defined. Here we first distinguish between three regions: the zone of interest Ω^I, the complementary zone Ω^C and the interface Γ ($\Omega^C \cap \Omega^I$). It should be noted that Ω^I ($\Omega^I = \Omega^{I1} \cup \Omega^{I2} \cup \ldots \cup \Omega^{In}$, n is the number of subdomains) is chosen to include the regions which may contain non-linear solutions with large gradients. The global model governs the entire structure Ω (domain $\Omega = \Omega^C \cup \Omega^I$) and contains the coarse grid, which is assumed to be linear elastic. In contrast, the local model governs Ω^I and contains the fine grid, which is assumed to be nonlinear (see Figure 1). Localized details and behaviors (e.g. cracks, holes, contacts, etc.) are not required in the global model, but these details must be fully resolved in the local model. The interfaces in the global coarse mesh and local fine mesh are respectively defined as global interface and local interface (see Figure 1). Iterative procedures are required between global and local models to eventually reach a prescribed tolerance. Fast convergence will be obtained if acceleration techniques are applied. The flowchart for the iterative global/local non-intrusive coupling algorithm is given in Box 1.

Box 1 Flowchart for the iterative global/local algorithm

Initial conditions and initialization: set *n*=0 and convergence tolerance ε;

1. **Initial global linear computation:** $\mathbf{K}^G \mathbf{u}^G_{(0)} = \mathbf{f}$, (1)

   where $\mathbf{K}^G$ is the global stiffness matrix, $\mathbf{u}^G_{(0)}$ is the global solution and $\mathbf{f}$ denotes the original prescribed external force exerted on the global model.

   Obtain $\mathbf{u}^G_{\Gamma(0)}$ at interface

2. **Local model nonlinear computation:**

   $$\mathbf{K}^L \mathbf{u}^L_{(n)} = \mathbf{f}^L, \text{ with } \mathbf{u}^L_{\Gamma(n)} = \mathbf{U}_{fc} \mathbf{u}^G_{\Gamma(n)} \text{ at interface,} \quad (2)$$

   where $\mathbf{K}^L$ is the local nonlinear stiffness matrix, $\mathbf{f}^L$ denotes the original prescribed external force exerted on the local model and $\mathbf{U}_{fc}$ is the prolongation operator. Subscript *n* indicates the *n*th iteration.

3. **Residual at interface:**

   $$\mathbf{r}_{(n)} = -(\mathbf{T}_{cf} \boldsymbol{\lambda}^L_{(n)} + \boldsymbol{\lambda}^G_{(n)}), \quad (3)$$

   where $\boldsymbol{\lambda}^L_{(n)}$ is the nodal reaction force on interface boundary of Ω^I from local analysis, $\boldsymbol{\lambda}^G_{(n)}$ is the nodal reaction force on interface boundary of Ω^C from global analysis, and $\mathbf{T}_{cf}$ is the corresponding restriction operator. The formulation for $\mathbf{U}_{fc}$ and $\mathbf{T}_{cf}$ are further discussed in Section 3.

4. **Global correction and update:**

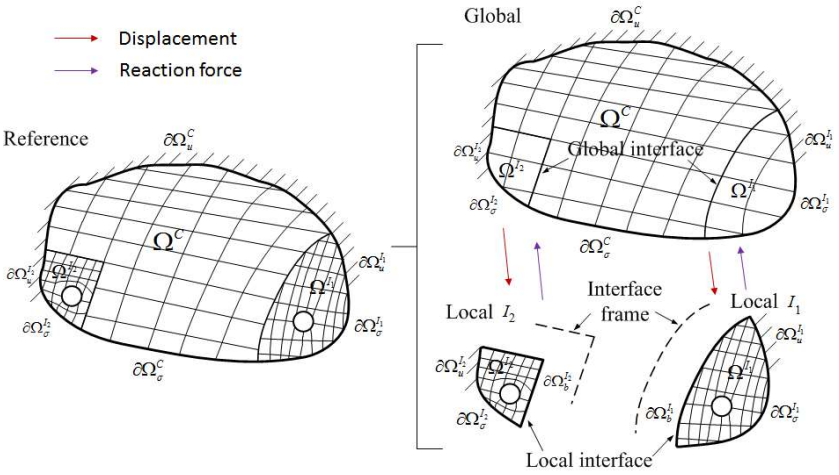

Figure 1 – Demonstration of iterative global/local algorithm.

3. Interface acceleration method

To improve the convergence rate, some acceleration techniques can be adopted. These acceleration techniques can be divided into two categories. One is that the global stiffness $\mathbf{K}^G$ update at each iteration step, such as quasi–Newton method with Symmetric Rank One (SR1) formula. In contrast, the other category of acceleration doesn't change the basic iterative formulation, and the current solution is only modified with these previous solutions without updating global stiffness matrix, such as the Aitken's Δ^2 method. The effects of improving convergence are almost the same between these two types of acceleration when the difference of stiffness between global and local problems is not large enough, such as local plastic problem. However, obvious difference can be observed when large gap of stiffness properties between global and local models exists, such as local crack propagation with long crack length. At that situation of large stiffness gap, the quasi-Newton methods which modify the global stiffness are more efficient in reducing iteration number than the global stiffness non-modified Aitken's Δ^2 acceleration method. The Aitken's Δ^2 acceleration method has been fully introduced in [14]. In this paper, quasi-Newton SR1 method is presented.

The basic purpose of quasi-Newton methods is to improve convergence rate by establishing approximate tangent stiffness matrix through low-rank modification matrix per iteration,

$$\mathbf{K}_{(n+1)} = \mathbf{K}_{(n)} + \Delta\mathbf{K}_{(n)}\,, \tag{7}$$

where $\Delta\mathbf{K}$ is low-rank modification stiffness matrix which has the form of $\rho\mathbf{v}\mathbf{v}^T$ for SR1 formula ($\mathbf{v}$ is a column vector). Suppose $\mathbf{d}_{(n)} = \mathbf{u}^G_{\Gamma(n+1)} - \mathbf{u}^G_{\Gamma(n)}$, $\mathbf{y}_{(n)} = \mathbf{r}_{(n)} - \mathbf{r}_{(n+1)}$, at n_{th} iteration it has

$$\mathbf{K}_{(n)}\mathbf{d}_{(n)} = \mathbf{r}_{(n)}\,, \tag{8}$$

and

$$\mathbf{K}_{(n+1)}\mathbf{d}_{(n)} = \mathbf{y}_{(n)}\,. \tag{9}$$

Relations of (7)-(9) are the basis of quasi-Newton applied in iterative global/local algorithm. And from these relations the detailed modifications of stiffness matrices as well as the corresponding inverse matrices for SR1 formula (according to Sherman-Morrison equation) can be written as

$$\begin{aligned}\mathbf{K}_{(n+1)} &= \mathbf{K}_{(n)} - \frac{\left(\mathbf{K}_{(n)}\mathbf{d}_{(n)} - \mathbf{y}_{(n)}\right)\left(\mathbf{K}_{(n)}\mathbf{d}_{(n)} - \mathbf{y}_{(n)}\right)^T}{\left(\mathbf{K}_{(n)}\mathbf{d}_{(n)} - \mathbf{y}_{(n)}\right)^T \mathbf{d}_{(n)}} \\ &= \mathbf{K}_{(n)} - \frac{\mathbf{r}_{(n+1)}\mathbf{r}_{(n+1)}^T}{\mathbf{r}_{(n+1)}^T\mathbf{d}_{(n)}}\end{aligned}, \tag{10}$$

$$\mathbf{F}_{(n+1)} = \mathbf{F}_{(n)} + \mathbf{F}_{(n)}\frac{\mathbf{r}_{(n+1)}\mathbf{r}_{(n+1)}^T}{\mathbf{r}_{(n+1)}^T\left(\mathbf{d}_{(n)} - \mathbf{F}_{(n)}\mathbf{r}_{(n+1)}\right)}\mathbf{F}_{(n)}\,, \tag{11}$$

where compliance matrix **F** is the inverse of stiffness matrix **K**.

Box 2 quasi-Newton SR1 acceleration algorithm

Input data: $\mathbf{r}_{(n)}$, $\mathbf{u}_{\Gamma(n)}^G$

$\forall n \geq 1$, k=0

FE Global correction and update computation $\mathbf{K}_{(0)}^{-1}\mathbf{r}_{(n)} = \mathbf{F}_{(0)}\mathbf{r}_{(n)}$, where $\mathbf{K}_{(0)} = \mathbf{K}^G$

while $k < n$ do

Compute $\mathbf{F}_{(k+1)}\mathbf{r}_{(n)} = \mathbf{F}_{(k)}\mathbf{r}_{(n)} + \mathbf{F}_{(k)}\mathbf{r}_{(k+1)}\dfrac{\mathbf{r}_{(k+1)}^T\mathbf{F}_{(k)}\mathbf{r}_{(n)}}{\mathbf{r}_{(k+1)}^T\left(\mathbf{d}_{(k)} - \mathbf{F}_{(k)}\mathbf{r}_{(k+1)}\right)}$

k=k+1

4. Local 3D crack simulation of a joint structure

In this section, a failure prediction of a 3D joint structure under static loading through contact

between a lug and rivet shown in Figure 2 is studied. With the left side of structure fixed, a displacement u_d is applied at an angle 5 degree with respect to the x-axis is applied on the rivet along the right side. The material of whole structure is an aluminum alloy AL7075.

Figure 2 – Demonstration of boundary conditions and external load.

4.1 FE modeling

In order to perform a iterative global/local coupling analysis, two FE models - global and local models - are defined as in Section 2. The global model has a coarse mesh and assumes a homogeneous plate without explicitly resolving the contact problem of hole and rivet. The loading of a displacement u_d is directly prescribed on the nodes along the thickness illustrated in Figure 3. In contrast, the local model covers the region of interest in which the lug structure and rivet are fully represented and finely meshed to capture the damage. A load is exerted on the plate through the contact between the hole and rivet. Elements with reduced integration C3D8R are applied in both global and local models. The incompatible meshes of global and local interfaces are demonstrated in Figure 3.

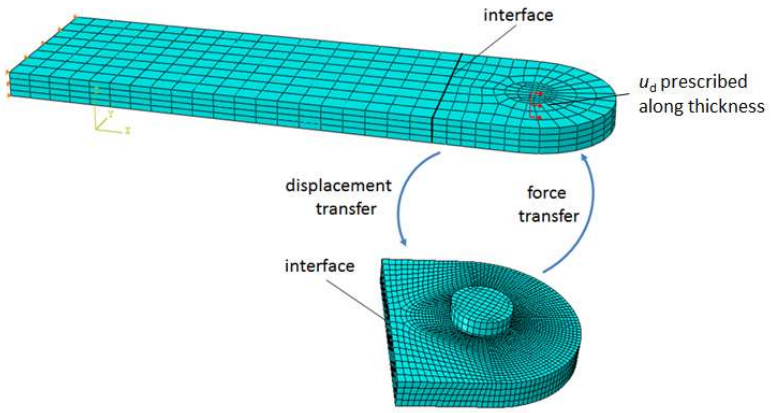

Figure 3 – Details of global and local models of the 3D joint structure.

4.2 Fracture criterion

In this study, the ductile fracture criterion of metal [1, 2] is adopted to simulate progressive failure for ductile material. For this fracture criterion, damage is defined as a relative loss of plasticity and has a general form of invariants: first principal stress invariant I1, second deviatoric stress invariant J2 and third deviatoric stress invariant J3,

$$D(\varepsilon_p) = \int_0^{\varepsilon_p} f(I_1, J_3)\mathrm{d}\varepsilon_p = \int_0^{\varepsilon_p} f(p, \theta_L)\mathrm{d}\varepsilon_p \ , \tag{12}$$

where ε_p is equivalent plastic strain which is related to the invariant J2. The hydrostatic pressure p and the Lode angle parameter θ_L are stress state parameters, in which p is related to the invariant I1 and θ_L is related to the invariant J3,

$$p = -\sigma_m = -\frac{I_1}{3}, \tag{13}$$

$$\theta_L = -\frac{1}{3}\sin^{-1}\left(\frac{27}{2}\frac{J_3}{\sigma_{eq}^3}\right), \tag{14}$$

in which σ_m is mean stress, σ_{eq} is equivalent stress.

The damage accumulation is strongly influenced by the stress states (i.e. hydrostatic pressure and Lode angle) at a material point. The material fails if its accumulated damage D value reaches the unity. A detailed introduction of the fracture model implemented for this analysis is given by Liu and Sun [1], as well as the material constants of AL7075 in fracture criterion.

4.3 Non-intrusive implementations into finite element code

The implementation of the iterative global/local algorithm is simple due to its non-intrusiveness with commercial software. In the present study, we chose the commercial code ABAQUS to run the FE analyses. The iterative global/local framework is coded by Python script to rewrite input file, submit jobs, read output file, compute residual at interface and conduct iterations.

In order to simulate crack initiation and propagation, a local quasi-static calculation is performed using the ABAQUS/Explicit solver combined with the fracture model implemented as a user subroutine VUMAT. Crack propagation is simulated through the element “deletion” technique in FE analysis once its damage value reaches unity. In contrast, the global linear analysis is simply carried out with the implicit ABAQUS/Standard solver. The non-intrusive coupling algorithm is coded in a Python script and interfaced with ABAQUS solver to perform iterative solutions.

Since the plastic deformation and damage depend on the loading history, the loading process after the onset of plasticity is divided into 10 successive steps in the local nonlinear analysis. The analysis during the current step is based on the converged result of the previous step. The flow chart of implementation is illustrated in Figure 4.

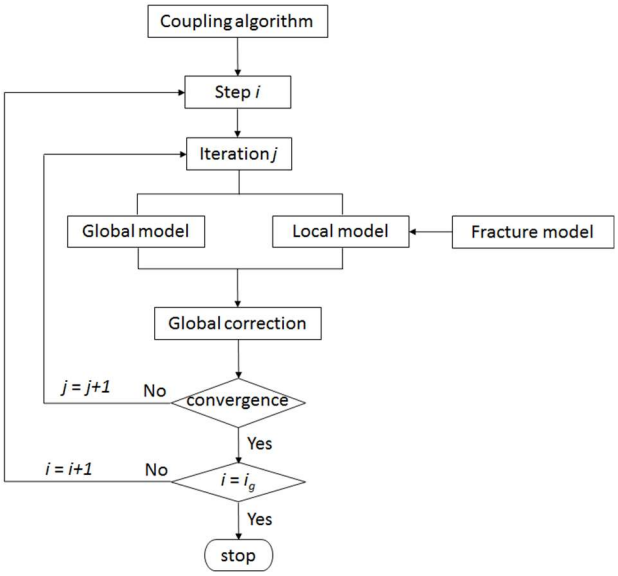

Figure 4 – The flow chart of implementation of iterative global/local algorithm in local failure analysis (where i_g is the number of loading steps).

4.4 Results and discussions

Because of the simplification in global model and local nonlinear effect, it can be seen that an unbalanced stress distributions occur at the interface after the first iteration (Figure 5) which is the traditional global/local method. However, in the iterative global/local algorithm the residual nodal force at interface is fed back into global calculation to diminish this unbalanced interface stress

distribution through the global and local iterative analyses.

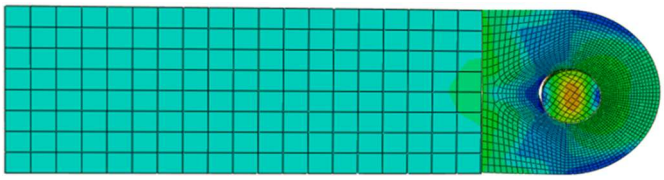

Figure 5 – The unbalanced stress distributions along interface between global and local model in traditional global/local method.

Without acceleration techniques, the original coupling algorithm will have poor convergence rates when significant local nonlinearities or discontinuities (e.g. crack) exist in the local domain. Thus, in order to reduce the iteration number in each loading step, two of the most representative and effective acceleration methods, quasi-Newton SR1 and Aitken's Δ^2, are introduced. The quasi-Newton type method requires an update to the global stiffness in each iteration in order to approximate the current tangent stiffness. On the contrary, the Aitken's method uses the two previous solutions to modify the current calculation instead of updating the global stiffness. In general, the quasi-Newton method is more efficient at increasing the convergence rate, especially for cases with strong local nonlinearities.

In this study both acceleration methods are implemented into the iterative global/local coupling algorithm in each loading step with a convergence tolerance of 10^{-5}. Figure 6 plots the iteration number needed in each loading step with and without accelerations. The convergence rates for the acceleration methods were significantly increased when compared with the non-accelerated algorithm.

In particular, it can be observed that without acceleration the number of iterations required increases with increasing loading steps, suggesting a strong dependence on the loading step or the stiffness difference between local and global models. The iterations significantly increase in the last two loading steps due to the onset of fracture, triggering drastic stiffness degradation in those steps.

In contrast, for the SR1 acceleration the iteration number does not increase with increasing load steps. Aitken's method also shows excellent convergence behavior at the initial load steps, but its efficiency gradually decreases as the number of steps increases, especially after the onset of fracture. In contrast, the quasi-Newton SR1 method maintains excellent convergence behavior even during crack propagation. This indicates that the quasi-Newton method has better performance even with large stiffness differences between local and global models.

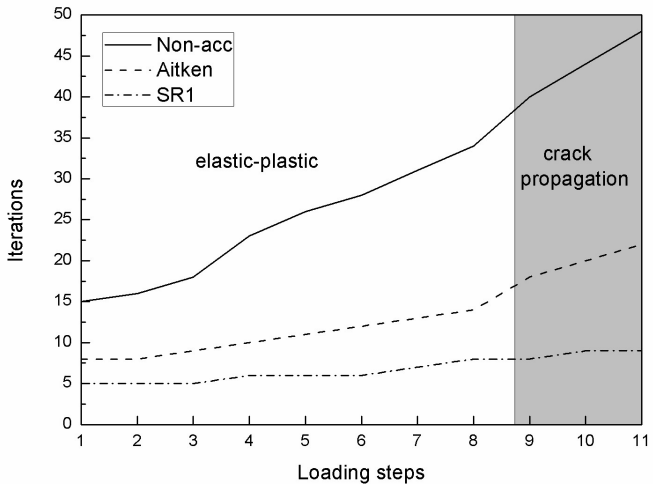

Figure 6 – Iteration number vs. loading steps for algorithms with and without accelerations.

Moreover, an observation at one loading step reveals that the results of the algorithms with and without acceleration techniques show very different convergence evolution over iterations in Figure 7.

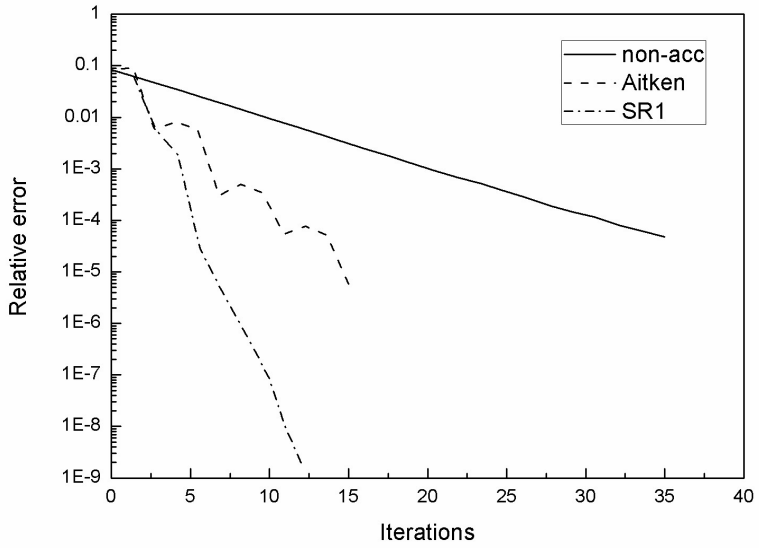

Figure 7 – Demonstration of convergence indicator vs. iteration number.

Analyzing the loading capacity and failure modes of a structure under loading as well as its failure mechanism are also important in engineering applications. Thus, accurate predictions of these aspects in addition to enhanced convergence properties are essential for a reliable numerical method.

The traditional nonlinear FE analysis on the reference FE model is carried out and referred to as the reference result to evaluate the numerical predictions from iterative global/local coupling algorithm. Figure 8 shows the load-displacement curves obtained from the coupling iterative results compared with the reference FE result and experimental data. In comparison to the reference FE result, the non-intrusive coupling algorithm can accurately predict the load capacity in each loading step with a maximum relative error of 0.06% among different loading steps. These numerical results indicate that with a properly adopted fracture model the simulation using the iterative global/local method can provide a load-displacement response that matches experimental result.

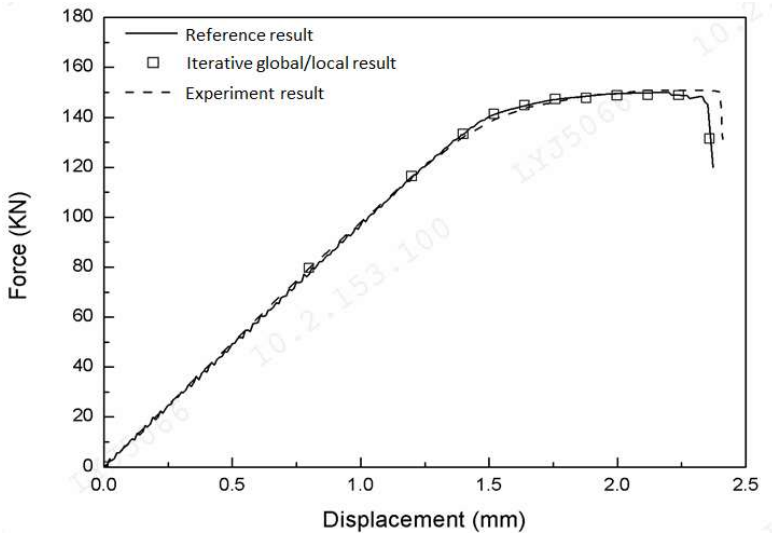

Figure 8 – Force-displacement curves of iterative result, reference result, and experimental data.

The reference FE results indicate that the crack occurs first in the lower part of the hole in the center of the rim when ud=2.095mm and propagates to the surface. From the coupling results, the crack first initiates at the same location of the reference result when ud= 2.102mm, only 0.33% off from the reference result. Moreover, it can be seen from Figure 9 that the resulting damage evolution provided by the iterative global/local coupling algorithm can well predict the reference result at that most critical location. From the damage evolution, both results show that the damage occurs when ud= 0.920mm in the second loading step and the rate of damage accumulation is slow at the beginning and shows an exponential style of growth along with the ud. The maximum difference between reference and coupling curves is less than 0.09%. Furthermore, the crack shape predicted by iterative global/local coupling algorithm agrees very well with the reference result. The final fracture topology observed from simulation and experiment shown in Figure 10 also agrees reasonably well.

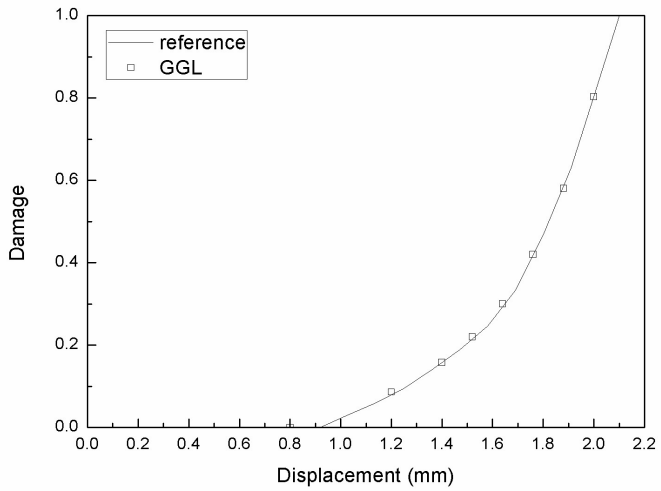

Figure 9 – Damage evolutions of coupling and reference results along with displacement at the most critical location.

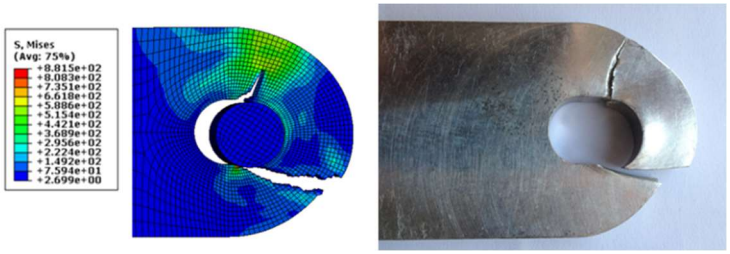

Figure 10 – The comparisons of fracture shapes between iterative global/local simulation and experiment.

5. Conclusions

In this paper, a 3D crack simulation of a joint structure is studied using the iterative global/local non-intrusive coupling method. The iterative global/local method couples the linear global model and nonlinear local model in an iterative way. It converges to reference problem through iterative strategy. In order to improve the convergence rate, quasi-Newton SR1 method is introduced into the iterative global/local method. The iterative global/local algorithm is non-intrusive with solver. In this paper, the iterative global/local algorithm is coded in Python script subroutine and the fracture model is coded in Fortran subroutine in ABAQUS platform.

To verify the fidelity of iterative global/local coupling algorithm in local crack simulation, the comparison of numerical and experimental results is made. It showed that the numerical results

have very good convergence rate and agree very well with experimental result in load-displacement evolution, damage accumulation and crack shapes.

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