ICAS 2020 SHANGHAI
International Council of the Aeronautical Sciences
32nd Congress of the International Council of the Aeronautical Sciences
September 6-10, 2021
Pudong Shangri-La, Shanghai, China

MODEL CURVES: A SIMPLE WAY TO ACCOUNT FOR COMPLEX EFFECTS

Andrey Bautin[1], Yury Svirsky[1], Valentin Basov[1], Yakov Melnikov[1]

[1]Central Aerohydrodynamic Institute (TsAGI), Structural Integrity department,

1 Zhukovsky St., 140180, Russian Federation

Abstract

An approach is presented that improves fatigue life estimates and at the same time reduces the requirements for the volume of experimental data, which is based on determining the parameters of the fatigue resistance model from tests with typed loading programs for the structure under consideration. The versions of the developed approach usage are shown for the nominal stress and local stress-strain approaches to fatigue life estimates of aircraft structures.

Keywords: fatigue life, *S-N* curve, calculation of service life based on local stresses, finite element method, cyclic hardening (stress-strain) curve

Introduction

The aim of the work is to improve the accuracy of the fatigue life estimates and, at the same time, to reduce the requirements for the volume of necessary experimental data based on the determination of the parameters of the fatigue resistance models under the typified loading conditions of the structure under consideration. Fatigue life of the structure depends on many factors. Usually, fatigue properties are determined under regular cyclic loading (with constant stress limits) to determinefatigue life and then special models are used to schematize the actual irregular loading as a set of cycles with constant parameters. The main disadvantage of this approach is the need to experimentally obtain a large amount of data required by these models. For example, our research showed that the power κ in the generalized Oding formula (also called the Walker formula), which takes into account for the asymmetry of loading (1), varies from 0.1 to 0.9 under different loading conditions, that is, the equivalent value of κ depends on the combination of loading modes in the program loading and/or in service:

$$s_{eqv} = \left(2s_a\right)^{1-\kappa}\left(s_a + s_m\right)^{\kappa} = \left(2s_a\right)^{1-\kappa}\left(s_{max}\right)^{\kappa} \quad (1)$$

Thereby, we need to experimentally obtain this value for all loading modes. The same is true for the m - slope of the fatigue curve (2):

$$N_f = \left(C_0 / \sigma_{eqv}\right)^m \quad (2)$$

We propose another way, which is to use the selected models as a basis for determining theparameters of the fatigue resistance of the structure. In order to calculate the required parameters, special procedures for determining the so-called "model curves" have been developed. Their description is given below for two cases: determination of the fatigue properties of aircraft structuresin nominal stresses and calculation of fatigue life based on local stresses and strains.

1. Determination of the characteristics of fatigue properties of aircraft structures by nominal stresses

The fatigue properties of aircraft structures under regular and irregular loading can differ significantly.

This change can be considered, for example, by constructing the fatigue curves of typified panels loaded with typified spectra for the structure under consideration with different scaling factors of these spectra. The disadvantages of this method include its high cost, as well as the difficulty of applying a large number of loads with low-amplitude in these spectra. The latter is due to the fact that panel testing requires large-scale testing machines or stands, the amplitude-frequency characteristics of which do not allow providing high loading frequencies and, therefore, reproducing a large number of low-amplitude loads in an acceptable time frame.

In the proposed approach, the fatigue properties under irregular loading are proposed to be determined for typified stress concentrators in an aircraft structure, and the fatigue life of the structure under consideration should be evaluated by considering the effect of nominal stresses in the zone in which this concentrator is located. Implementation of this approach requires testing samples with a typified stress concentrator, for example, a plate with an open hole, with several variants of typified loading programs at nominal stresses for the structure under consideration. As such a program, in many cases, we propose to use modifications of typified programs such as TWIST [1], FALSTAFF [2] and others. The modification is carried out in order for the integral distribution of completed cycles to correspond to the range of integral distribution of the loading of the aircraft structure under consideration.

Let us demonstrate the performed procedures for the case when the characteristics of fatigue strength are modeled by equations (1) and (2), as well as by the hypothesis of linear summation of damages - (3) (Miner's rule):

$$\sum_{i=1}^{M} \frac{n_i}{N_{f\,i}} = 1 \tag{3}$$

where *i* – is the index of the loading mode, that is, the index of the completed cycle with the same values σ_{max} and σ_a; *n* – the number of completed cycles in a given loading mode; *M* – number of completed cycles in a given loading mode number of loading modes in a typified program.

Testing with these programs with different scaling factors gives the constants C_0 and *m* in equation (2), variation of the asymmetry of their “Ground-Air-Ground” cycle gives the constant *κ* in equation (1).

The loading program is represented according to the rainflow method [3] according to the original algorithm [4]. As a result of representation, we get a table of completed cycles (Figure 1).

i	σ_{max}	σ_a	n_i
1			
2			
…			
M			

Figure 1 – View of the original table of completed cycles

Then, for each tested specimen, we bring all loading modes to one *j*-th mode. The most frequently encountered mode is usually selected as such a mode. The reduced number of cycles in the *j*-th mode is determined according to the linear summation hypothesis of fatigue failures by the formula (4):

$$N_j = \sum_{i=1}^{M} \frac{n_i \cdot N_{f\,j}}{N_{f\,i}} = \sum_{i=1}^{M} n_i \left(\frac{\sigma_{eqv\,i}}{\sigma_{eqv\,j}} \right)^m \tag{4}$$

Thus, we get a set of points on the *S-N* curve $\left(N_j, \sigma_{eqvj}\right)$, from which we can determine the parameters of the *S-N* curve (in the general case, using the least squares method). For the selected model of fatigue resistance, to obtain a system of linear equations, finding the logarithm of formula (2) is performed, if we want to determine the constants C_0 and *m*. If we want to determine the constant *κ*, we must additionally find the logarithm of equation (1). Since the reduction to the *j*-th mode requires the presence of fatigue resistance characteristics, which are determined as a result of this procedure, the problem of their determination is solved by the iterative method, in which the characteristics determined from the test results under regular loading are usually taken as the initial values

For carbon fiber reinforced polymer (CFRP), a similar procedure is applied, which differs in that in the compression region the generalization of the Oding formula (1) proposed by A. Pankov [5] and fatigue curves for the areas of tension (left branch) and compression (right branch) are applied. Thus, the following formulas are used to take into account the influence of cycle asymmetry:

- for the right branch - the formula (1);
- for the left branch - the formula (5):

$$\sigma_{eqv} = (2\sigma_a)^{1-\kappa}(\sigma_a - \sigma_m)^{\kappa} \tag{5}$$

fatigue life at the parameters of the cycle of the *i*-th stage according to the expressions (6):

$$\begin{cases} N_f = \left(-\dfrac{C_{-\infty}}{(2\sigma_a)^{1-\kappa}(\sigma_a-\sigma_m)^{\kappa}}\right)^{m_{-\infty}}, \quad \text{for } \sigma_m+\sigma_a=\sigma_{max}\le 0; \\ N_f = min\left[\left(\dfrac{C_0}{(2\sigma_a)^{1-\kappa}(\sigma_m+\sigma_a)^{\kappa}}\right)^{m_0}; \left(-\dfrac{C_{-\infty}}{(2\sigma_a)^{1-\kappa}(\sigma_a-\sigma_m)^{\kappa}}\right)^{m_{-\infty}}\right], \quad \text{for } \quad \sigma_a>|\sigma_m|; \\ N_f = \left(\dfrac{C_0}{(2\sigma_a)^{1-\kappa}(\sigma_m+\sigma_a)^{\kappa}}\right)^{m_0}, \quad \text{for } \sigma_a \le \sigma_m. \end{cases} \tag{6}$$

The procedures for determining the constants for CFRP and metal from a mathematical point of view coincide only for cases when all loading modes lie only in the compression region (the first expression in (6)) or in the tension region (the third expression in (6)). Otherwise, more complex mathematical methods must be used (see below).

2. Calculation of fatigue life based on local stresses and deformations

Numerous studies of the fatigue process have shown that this process is very localized, that is, it is determined not by the general stress-strain state, but by stresses and strains acting in a very small area. Fatigue analysis based on local stresses and strains consists in determining the sequence of local stresses and strains at the point of failure and using these sequences to determine fatigue life. This method makes it possible to determine the residual stresses at the point of failure after highloads and thus take into account the interaction of loads under random loading. Usually, the use of this method requires the determination of the *S-N* curves of "smooth" samples, that is, samples with a minimum stress concentration factor, and cyclic stress-strain curves. In industrial use, obtaining these data is difficult, since the bulk of experimental data on fatigue are obtained on specimens in the form of a plate with an open hole, joints, and other types of structural elements. In accordance with the proposed approach, the *S-N* curves of "smooth" samples are obtained from the *S-N* curves of structural elements, which additionally allows taking into account stress gradients, manufacturing technology, and other factors. Our analysis has shown that cyclic deformation of a material in aircraft structures can be approximated by a combination of elastoplastic static deformation curves and cyclic stress-strain curves if we use a material “memory” model to describe the transition from static to cyclicstress-strain curves. Unfortunately, the need to take into account for plasticity and other nonlinear effects associated with it requires calculating half cycle after half cycle. Considering that the fatigue life of modern structures is measured in thousands and even millions of cycles, the use of the finite element method (FEM) for

these calculations becomes almost unreal. For example, the calculation time for 1000 loading cycles of finite element models designed to determine the parameters of the elastoplastic stress-strain state using the ABAQUS/CAE 6.13 4 complex varies from 38 hours to 5 months. Methods for calculating fatigue life, taking into account local elastoplastic stress-strain state, began to develop intensively starting from the end of the 60s of the last century and by the beginning of this century were widely used in industry [6-10]. A number of approximate solutions to the elastoplastic problem have been proposed to assess the fatigue life taking into account for the local elastoplastic stress-strain state.
Stress-strain curve is usually approximated in a power-law form (Ramberg-Osgood equation [11]):

$$e_a = \frac{s_a}{E} + \left(\frac{s_a}{C_a}\right)^{C_n} \tag{7}$$

where e_a - amplitude of local strains; s_a - amplitude of local stresses; E - elastic modulus; C_n – exponent in power-law approximation of the stress-strain curve; C_a – constant of the material in the power-law approximation of the stress-strain curve in terms of stress amplitudes.
We propose another version of the approximation of the stress-strain curve for calculations:

$$\begin{cases} \Delta e = \dfrac{\Delta s}{E}, \quad \text{for} \quad \Delta s \le S_e; \\ \Delta e = \dfrac{\Delta s}{E} + \left(\dfrac{\Delta s - S_e}{C_d}\right)^{C_n}, \quad \text{for} \quad \Delta s > S_e; \\ e_a = \dfrac{s_a}{E} + \left(\dfrac{s_a - S_e}{C_a}\right)^{C_n}, \quad \text{for} \quad s_a > S_e. \end{cases} \tag{8}$$

where Δe – strain range; Δs – range of stresses; E – elastic modulus; S_e – proportional limit with plastic deformation tolerance 0.01%. C_n – power in power-law approximation of the stress-strain curve; C_a – constant of the material in the power-law approximation of the stress-strain curve in terms of strain amplitudes; C_d – constant of the material in the power-law approximation of the stress-strain curve in terms of strain ranges (static curve).
The second expression in (8) is used for calculations along the static stress-strain curve, the third expression in (8) is used for the cyclic curve.
Advantages of the used approximation variant:

- in the elastic region, a purely elastic solution is used;
- the stress-strain curve will have a linear section, which does not require solving a nonlinear equation, and at the same time continuous, which will eliminate problems with its use in an iterative process;
- constants in formulas (6) and (7) can be determined by reference values $S_{0.2}$ - yield stress, S_b - ultimate strength and δ - elongation at fracture:

$$C_n = \frac{lg\left(\delta - \dfrac{S_b}{E}\right) - lg(0.002)}{lg\left(S_b - S_e\right) - lg\left(S_{0.2} - S_e\right)}; \qquad C_a = \frac{S_{0.2} - S_e}{\sqrt[C_n]{0.002}}$$

where $S_{0.2}$ – is the yield point with a tolerance for plastic deformation 0.2%.
Accordingly with Masing's principle [12] (a cyclic stress-strain curve is obtained from a static one if it is considered recorded in amplitudes):

$$C_a = C_d / 2^{C_n - 1/C_n} \tag{9}$$

(according to the Masing's principle, the value becomes the proportional limit for the amplitudes).

The determination of local stresses and strains by approximate methods is based on the use of hypotheses about the relationship of an elastic solution with an elastoplastic solution. Two ways are available:

Application of Neuber's formula [13], that relates elastoplastic stresses and strains at the maximum loaded point with nominal stresses – (10):

$$(e_a \cdot s_a) = \frac{(K_t \cdot \sigma_a)^2}{E} \tag{10}$$

where K_t – elastic coefficient of stress concentration (for joints, the difference in stress concentration coefficients in tension and compression should be taken into account); σ_a – amplitude of nominal stresses (of completed cycle, determined using the method of completed cycles or "rainflow" [3]).

Application the Molski-Glinka approach [14], which is similar to the theory of the *J*-integral in fracture mechanics, it is assumed that the elastic strain energy in the fracture zone is equal to the energy expended during elastic-plastic strains. In this case, the formula can be obtained by integration using the approximation of the strain-stress curve (9). We obtain an equation with respect to the s_a in the form (11):

$$\frac{s_a^2}{E} + \frac{2 \cdot C_n}{C_n + 1} \cdot \frac{s_a^{C_n+1}}{C_a^{C_n+1}} - \frac{\left(K_t \cdot \sigma_a\right)^2}{E} = 0 \tag{11}$$

For comparison, a similar equation for the Neuber formula is written in the form (12):

$$\frac{s_a^2}{E} + \frac{s_a^{C_n+1}}{C_a^{C_n+1}} - \frac{\left(K_t \cdot \sigma_a\right)^2}{E} = 0 \tag{12}$$

Determination of the elastoplastic stress-strain state is reduced to solving the nonlinear equation (11) or (12) with respect to s_a and finding the deformations from the used approximation of the strain-stress curve. As a numerical method, it is recommended to use Newton's iterative method [15], taking as a zero approximation the elastic solution. The end of the iterative process occurs when the specified computation accuracy is reached or when the specified maximum number of iterations is exceeded.

A comparison of these formulas with the solution obtained by the FEM for a plate with an open hole is shown in Figure 2.

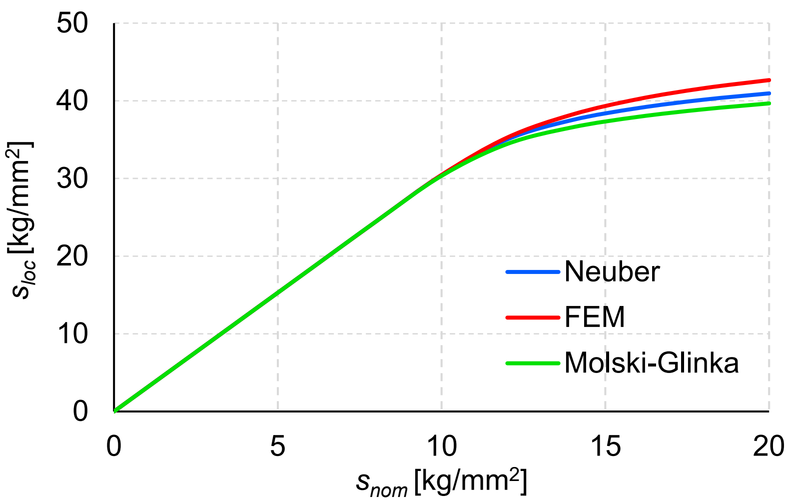
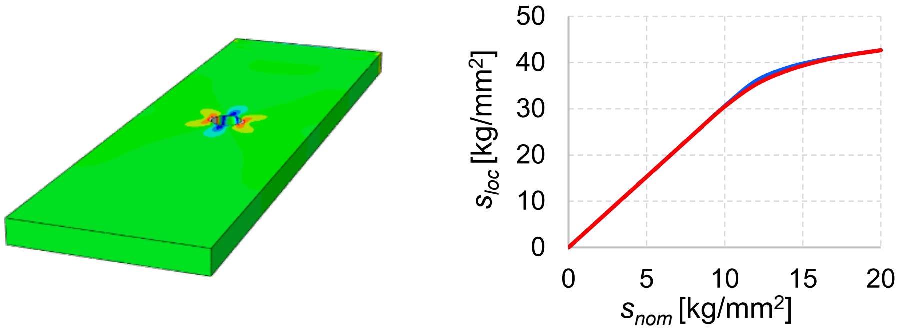
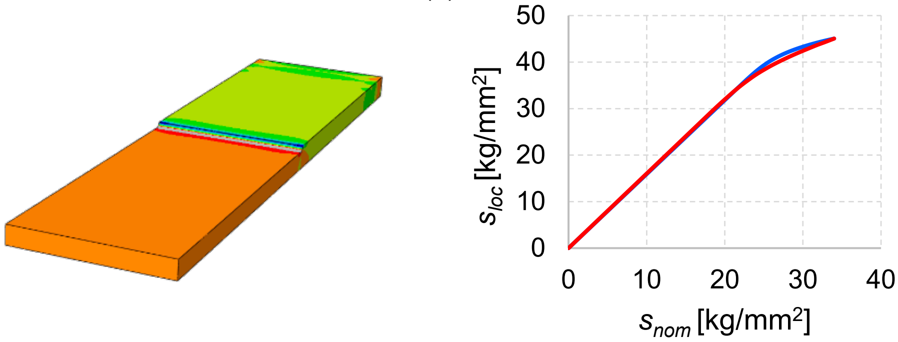


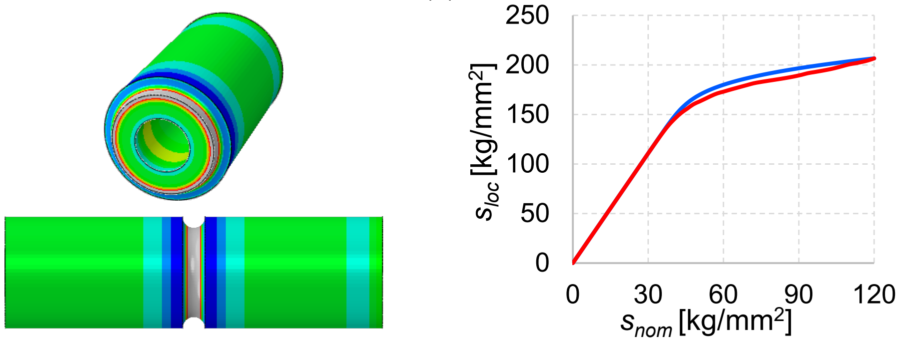
Figure 2 - Dependence of local stresses s_{loc} on nominal stresses s_{nom}



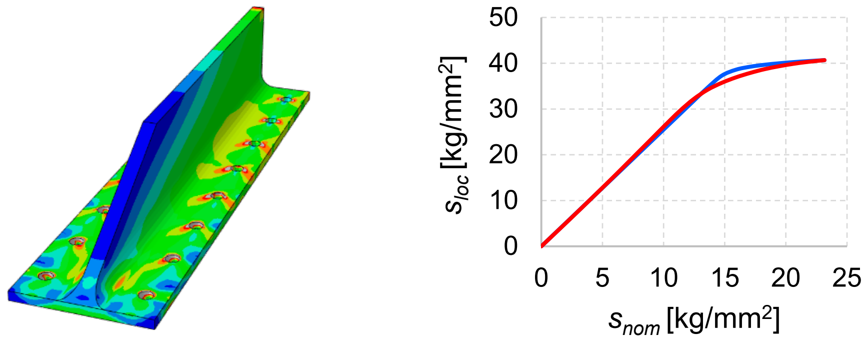
(a)



(b)



(c)



(d)

Figure 3 – s_{loc} - s_{nom} dependences under static load for various concentrators encountered in aircraft structure. Dependencies were obtained using the FEM - red line and according to the proposed method - blue line ((a) – plate with an open hole; (b) - fillet transition; (c) – cylindrical specimen with a circular groove; (d) – end part of the stringer with an optimized wall shape (the most loaded point is selected at the edge of the hole)

For a closer approximation to a more accurate FEM solution, we proposed a generalization of the Neuber and Molski-Glinka approaches for approximating the strain-stress curves (8) in the form (13):

$$\frac{s_a^2}{E} + \rho \cdot \frac{\left(s_a - S_e\right)^{C_n+1}}{C_a^{C_n+1}} - \frac{\left(K_t \cdot \sigma_a\right)^2}{E} = 0 \qquad (13)$$

where the constant $-\rho$ is determined by comparison with the FEM solution. The Neuber and Molski-Glinka formulas can be obtained from formula (13) as special cases.

A comparison was made between the calculation of the FEM and the result obtained by the proposed method for various typified stress concentrators found in aircraft structures [16]. The elastic-plastic nature of the material deformation in the calculations of the FEM was modeled using ABAQUS tools, and the approximation of the strain-stress curve by points was used. To obtain the intermediate points of the strain-stress curve, equations (8) were used. The values of all used mechanical characteristics were taken from the reference book [17].

The calculation and comparison were carried out for four variants of geometric concentrators: plate with an open hole, fillet transition, cylindrical specimen with a circular groove, and holes in the structure of the end part of the stringer with an optimized wall shape. The constant - ρ in formula (5) was selected according to the maximum stress values. Figure 3 shows the s_{loc} - s_{nom} dependences for the calculated variants obtained by the proposed method (in blue) and by processing the FEM calculation (in red). The average discrepancy between the results in the area of plastic deformations is about 2%.

In our approach, local stresses and strains are determined using formulas (6) and (5), while in formula (5) the amplitudes of completed cycles σ_a are used. The obtained history of local stresses and strains is schematized using the "rainflow" method, resulting in a table of completed cycles, similar to Table 1 for nominal stresses (Figure 4).

i	s_{max}	e_a	n_i
1			
2			
…			
M			

Figure 4 – View of the table of completed cycles for the proposed method

The fatigue resistance model of "smooth" samples is taken similar to that used for testing samples with concentrators at nominal stresses. The asymmetry is taken into account using equation (14):

$$e_{eqv} = \left(2e_a\right)^{1-\kappa} \left(s_{max}\right)^{\kappa} \qquad (14)$$

where e_{eqv} – is the equivalent strain, s_{max} - is the maximum stress, e_a – amplitude of strains. Equation (19) is used for the *S-N* curve:

$$N_f = \left(\frac{C_e}{e_{eqv}}\right)^{m_e} \qquad (15)$$

Therefore, further procedures for determining the characteristics of the fatigue resistance of "smooth" samples are similar to those used to determine the parameters of fatigue resistance for calculations using nominal stresses.

3. Mathematical methods

Since the number of samples is usually greater than the number of parameters to be determined, the least squares method can be used to obtain the fatigue resistance characteristics. Using this method requires determining the minimum of a function of several variables. In the simplest case, to find the minimum, we can obtain a system of linear equations from an experiment with scaling loads and load asymmetry variations, using the linear summation hypothesis of fatigue failures (3) to obtain the basic

equation. Using the iterative method, we can compute the parameters *m*, C_0 and *κ* of the fatigue life model for a typified service load.
For more complex models of fatigue life (for example, with a fatigue limit), a method developed at TsAGI is used, which makes it possible to determine the global minimum and avoid local minima. In very complex cases, the parameter space exploration method is used. This method was developed to solve engineering optimization problems [18].

4. Conclusions

The proposed approximation of the static and cyclic strain-stress curves, obtained on the basis of standard material constants and the Masing principle, will reduce the cost of obtaining the initial data for calculating local stresses and strains under cyclic loading. The proposed formula for determining the dependences of local stresses and strains on the nominal stresses for typified concentrators provides obtaining the necessary dependences close in accuracy to the results determined by the finite element method. Considering the fact that the calculation speed according to formula (5) is several orders of magnitude higher than the calculation speed using FEM, and the complexity of the proposed approach is several times less, the use of this formula will allow us to develop effective methods for calculating fatigue life based on taking into account for local elastic-plastic stresses and deformations under multi-cycle loading, typified for aircraft structures.

5. Contact Author Email Address

andrey.bautin@gmail.com

6. Copyright Statement

7. References

[1] De Jonge J.B., Shutz D., Lowak H., Schijve J. *A standardized load sequence for flight simulation tests on transport aircraft wing structures*. LBF Bericht Nr. FB-106, 1973.

[2] FALSTAFF. Description of a Fighter Aircraft Loading STAndard For Fatigue evaluation. (Flugzeugwerke, Emmen). *Proceedings of ICAF*, doc. No 839, 1976.

[3] Standard Practices for Cycle Counting in Fatigue Analysis. ASTM Standard E 1049 – 85 (Reapproved 1997), 1997.

[4] Svirskiy Yu.A. Method of reduced registration of load distributions at completed-scale fatigue tests and in health monitoring systems. *Proceedings of ICDQM-2018*, pp. 90-93, 2018.

[5] Konovalov V.V. and Pankov A.V. Verification of CFRP Components Fatigue Evaluation Procedure under Irregular Cycling Loading. *Proceedings of* Conference *ICAS*, 2016.

[6] Crews J.H., Hardrath, H.F. A Study of Cyclic Plastic Stresses at a Notch Root. *Proceedings of SESA*, pp. 313-320, 1966.

[7] Wetzel R.M. Smooth Specimen Simulation of Fatigue Behaviour of Notches. *Journal of Materials, JMLSA*, Vol. 3, No. 3, 1968.

[8] Martin, J.F., Topper, T.H. and Sinclair, G.M. *Computer ased simulation of cyclic stress–strain behavior. Report No. 326. Department of Theoretical and Applied Mechanics* 1969. Also published in ASTM Materials Research and Standards, 1971.

[9] Conle A., Nowack H. Verification of a Neuber-based Notch Analysis by the Companion – specimen Method. *Experimental Mechanics*, Vol.17, No.2, 1977.

[10] Lee Y., Taylor D. *Strain-Based Fatigue Analysis and Design, Chapter 5 of "Fatigue Testing and Analysis (Theory and Practice)", ed. by Lee Y., Pan J., Hathaway R.B., Barkey M.E.*, Elsevier Inc., pp. 181-251, 2005.

[11] Ramberg W., Osgood W.R. Description of stress-strain curves by three parameters. *NACA TN-902*, 1943.

[12] Masing G. Eigenspannungen und Verfestigung beim Messing. *Proceedings of the Second International Congress of Applied Mechanics*, 1926.

[13] Neuber H. Theory of Stress Concentration for Shear-Strained Prismatical Bodies with Arbitrary Nonlinear Stress-Strain Law. *Journal of Applied Mechanics,* pp. 544-550, 1961.

[14] Molsky K., Glinka G.A Method of Elastic-plastic Stress and Strain Calculation at a Notch Root. *Material Science and Eng*., Vol. 50, No. 1, pp. 93-100, 1981.

[15] Volkov E.A. *Chislennye metody*. Fizmatlit, 2003.

[16] Svirskiy Yu.A., Bautin A.A., Basov V.N., Lukjanchuk A.A. An approximate method for solving a local elastic-plastic problem. *Aerospace MAI Journal*, Vol. 27, No 2, pp. 61-70, 2020.

[17] UAC. *Raschetnye znacheniya kharakteristik aviatsionnykh metallicheskikh konstruktsionnykh materialov*. Handbook ed.3. pp. 305. 2011.

[18] Sobol I.M., Statnikov R.B. *Vybor optimalnyh parametrov v zadachah so mnogimi kriteriyami*. Nauka, 198