

## Transfer path analysis of random vibration loads

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**Abstract:** Based on the traditional TPA analysis for periodic signals, the TPA method for random vibration signals is studied. The theoretical research shows that, for random vibration signals, TPA analysis can also be carried out in the form of power spectral density, and the contribution of each transfer path can be obtained. Taking the transfer path analysis of an engine vibration load as an example, the vibration contribution of each excitation source through each transfer path and the main path of vibration transmission are obtained. The fourth excitation source has a larger contribution through path 4. At 90Hz and 130Hz, the large contribution of path 4 is due to the large excitation load, so the installation design of vibration source can be carried out; at 115Hz and 165Hz, the large contribution of path 4 is due to the large transfer characteristics, the structural optimization design or vibration isolation can be used to reduce the vibration response level of the target point.

**Key words:** TPA; random signals; power spectrum density; vibration transfer analysis

### Background

The transfer path analysis (TPA analysis, Transfer Path Analysis) is used to identify and evaluate the energy transfer path from the excitation source to a certain target point. Transmission path analysis can quantitatively analyze different excitation sources and their transmission paths, given which one is the most important path, which has the most important contribution to the vibration and noise, which will cancel each other out. The theoretical formula of traditional TPA analysis is shown in formula (1)

$$U(\omega) = H(\omega) \times F(\omega) \quad (1)$$

Where:  $U(\omega)$  is the total contribution of vibration or noise transmitted to the target point by each path;  $H(\omega)$  is the transfer function from the source to the target point;  $F(\omega)$  is the load at the transmission path, which can be structural load or acoustic load.

Through the above formula, the influence of a certain path on the target point can be determined, and expressed in the form of contribution. The visualization of contribution can help engineers quickly make decisions to find the critical path.

According to formula (1), the unit of transfer function is usually g/N, the unit of load is usually

N, and the unit of total contribution is usually g. Therefore, traditional TPA analysis methods are usually only suitable for the study of deterministic periodic signals [1- 7]. But in many cases, the excitation source is random vibration excitation, the random signal is usually expressed by power spectral density, the unit of load is usually N<sup>2</sup>/Hz, and the unit of response is usually g<sup>2</sup>/Hz. At this time, formula (1) will no longer suitable. It is necessary to develop a transmission path analysis method suitable for random signals.

### TPA analysis method for random signals

For random signals, the relationship between response and load can be expressed as a spectral density function:

$$\mathbf{S}_{UU}(\omega) = \mathbf{H}(\omega) \times \mathbf{S}_{ff}(\omega) \times \mathbf{H}^H(\omega) \quad (2)$$

Where:  $\mathbf{S}_{UU}(\omega)$  is the power spectral density matrix of the response;  $\mathbf{S}_{ff}(\omega)$  is the power spectral density matrix of the load; the superscript H represents the complex conjugate transpose of the matrix.

When carrying out load inversion, it is assumed that the number of loads is  $N_f$  and the number of response points is  $N_u$ . When  $N_f \leq N_u$ , the power spectral density of the vibration source load can be inverted by the power spectral density of the response, as shown in the following formula:

$$\mathbf{S}_{ff}(\omega) = [\mathbf{H}(\omega)]^+ \times \mathbf{S}_{UU}(\omega) \times [\mathbf{H}^H(\omega)]^+ \quad (3)$$

where: "+" means Moore-Penrose pseudo-inverse.

For example, it's assumed that there are two random excitation sources and two target points.

According to formula (2), we can get:

$$\begin{bmatrix} S_{UU11} & S_{UU12} \\ S_{UU21} & S_{UU22} \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \times \begin{bmatrix} S_{ff11} & S_{ff12} \\ S_{ff21} & S_{ff22} \end{bmatrix} \times \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}^H \quad (4)$$

Expand the formula to get:

$$\begin{aligned} S_{UU11} &= H_{11}S_{ff11}H_{11}^* + H_{12}S_{ff21}H_{11}^* \\ &+ H_{11}S_{ff12}H_{12}^* + H_{12}S_{ff22}H_{12}^* \end{aligned} \quad (5)$$

where: superscript \* means to take complex conjugate.

Note that  $S_{ff12}$  and  $S_{ff21}$  are conjugate

each other, so the calculation result of the first term does not contain the imaginary part, which represents the contribution of the first excitation source on the first transmission path; the sum of the second and third terms does not contain the imaginary part, indicating the superimposed crosstalk between the two excitation sources; the fourth item does not contain the imaginary part, which represents the contribution of the second excitation source on the second transmission path. It can be seen that, for random signals, the power spectral density function can also be used for TPA analysis, and to obtain the contribution of each transmission path.

### Application case of random signal TPA analysis

Assuming that the vibration load of an aircraft engine is transmitted to the fuselage structure through 4 excitation points and 4 transmission paths, which are the front and rear beams of the left wing, and the front and rear beams of the right wing. The position of the excitation point is shown in Figure 1. In addition, multiple reference points need to be selected near the excitation point (usually the number of reference points is more than the number of excitation points), a total of 4 reference points are selected, The location is shown in Figure 2.

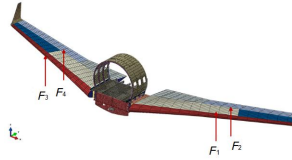


Fig. 1 Position of the excitation point

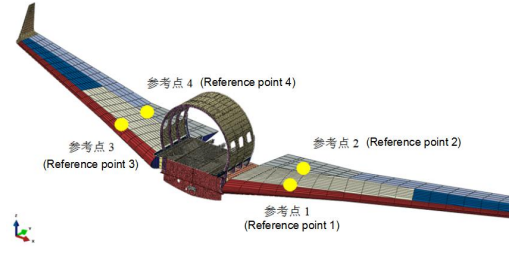


Fig. 2 Selection of reference points

First, dynamic load inversion is performed. According to the random vibration response data of each reference point (Figure 3) and the frequency response function matrix from each excitation point to each reference point, the random load is inversely identified according to formula (3). The self-spectrum and cross-spectrum density functions of each excitation load obtained are shown in Fig. 4.

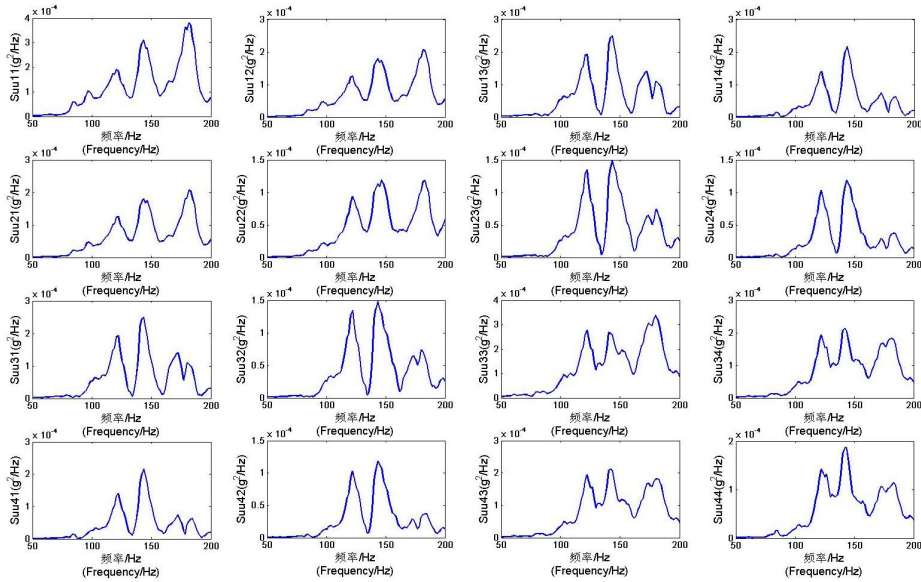


Fig. 3 random vibration response of reference points

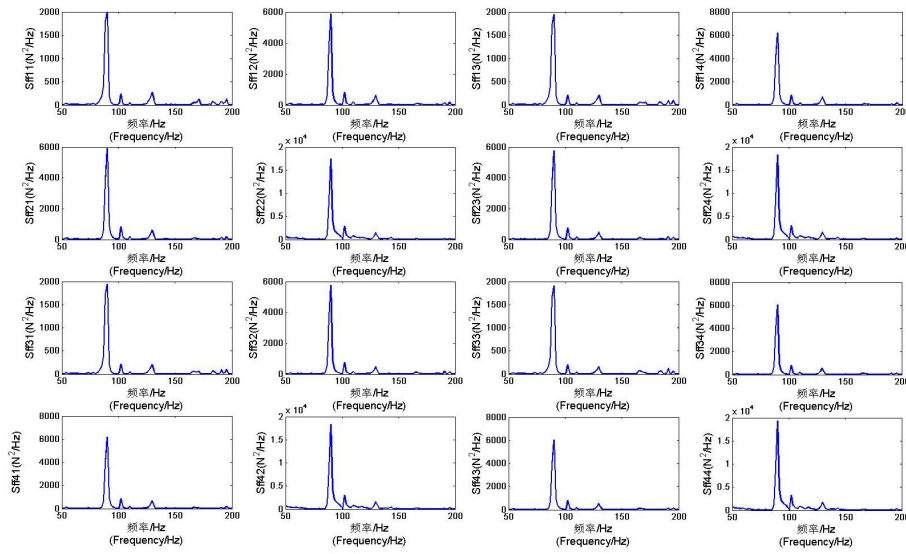


Fig. 4 identified excitation loads

Suppose that we pay more attention to the vibration response level of some certain points on the cabin floor, which is called the target points, as shown in Figure 5.

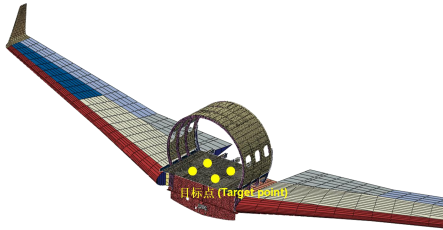


Fig. 5 selection of target points

Based on the identified excitation loads, based on formula (5), the vibration contribution of each excitation source through each transmission path can be

further obtained. The contribution component of the vibration response at a certain target point is shown in Figure 6. It can be seen that, at 90 Hz, 115 Hz, 130 Hz, and 165 Hz, the fourth excitation source has a greater contribution through path 4, which means at these main frequencies, path 4 is the main path for random vibration load transmission. In order to analyze the reason for the larger contribution of path 4, the No. 4 excitation load spectrum and the frequency response function H14 from No. 4 excitation load to the target point are plotted in FIG. 7.

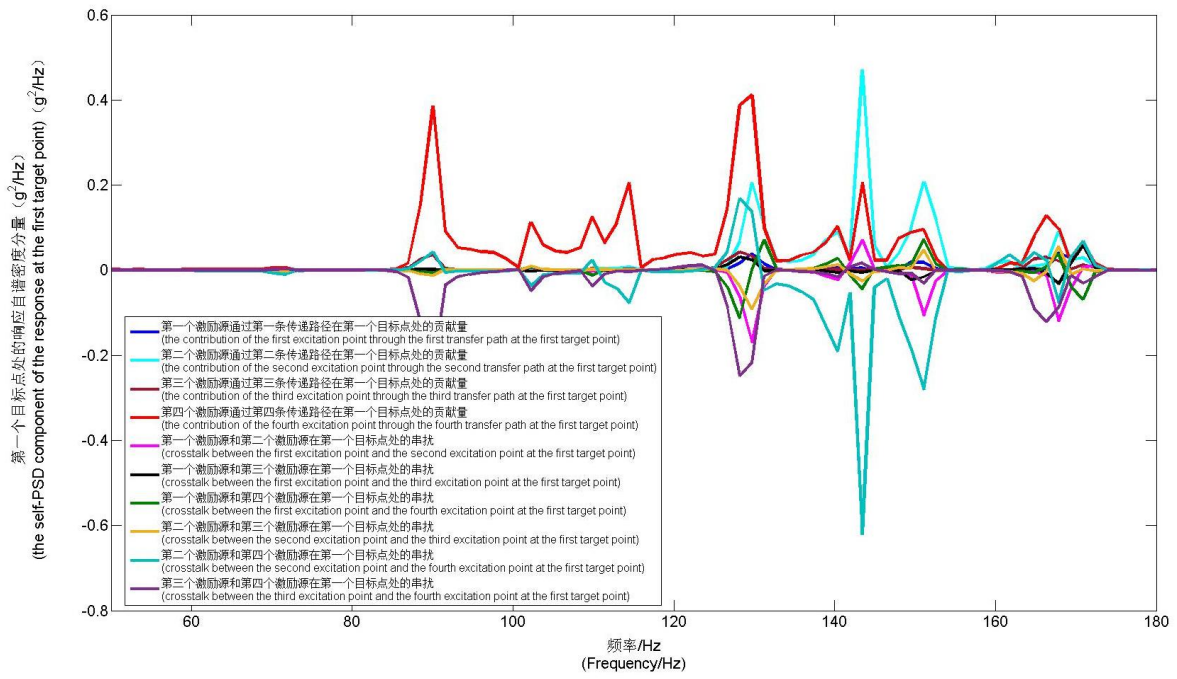
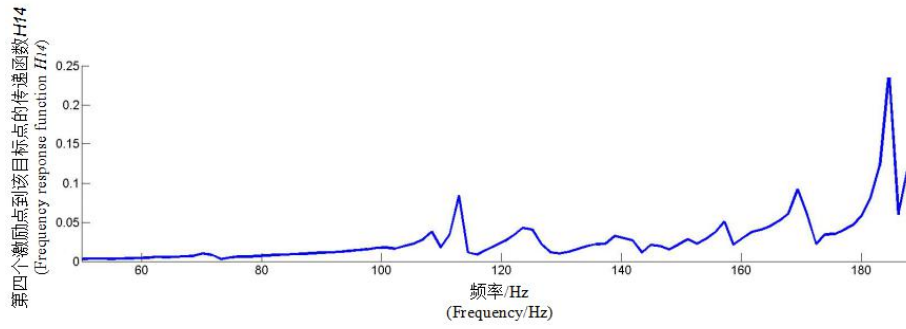
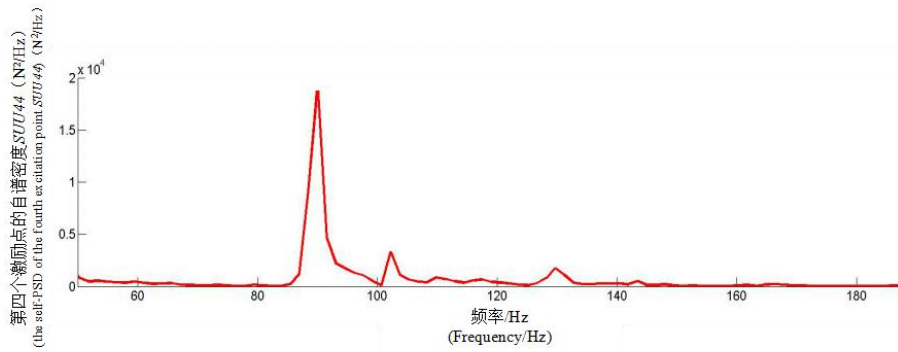


Fig. 6 contribution analysis of each path



(a) Frequency response function  $H_{14}$



(b) the self-PSD of the fourth excitation point  $S_{UU44}$

Fig.7 the data of  $S_{UU44}$  and  $H_{14}$

It can be seen from Figure 7 that, at 90Hz and 130Hz, the reason for the larger contribution of path 4 is due to the larger excitation load, vibration damping installation design can be carried out at the vibration source position; at 115 Hz and 165 Hz, the larger contribution of path 4 is due to the larger transfer characteristics, structural reinforcement or vibration reduction can be performed for this order of vibration mode to reduce the vibration response level of the target point.

## Conclusion

The TPA analysis method of random vibration signals is studied, which is especially suitable for the vibration transmission path analysis caused by non-periodic excitation. Taking the random vibration load TPA analysis of an engine as an example, the main path of vibration transmission is obtained, and the reason for the excessive vibration transmission is analyzed.

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