

AN IMPROVED IDENTIFICATION METHOD FOR ACTUATOR LOOP OF AIRCRAFT ENGINE CONTROL SYSTEM

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Abstract

For low accuracy of Levy frequency domain identification method at low frequency points, a frequency characteristic parameter identification method based on non-linear weight is proposed. In this method, in order to distinguish the influence of different frequencies on identification results, weight coefficients are added to each frequency component. According to the frequency distribution range of swept signal, the weight of each frequency is allocated by non-linear interpolation method. In the semi-physical simulation test-platform, frequency sweep tests were carried out on several main actuator loops of aircraft engine to obtain the dynamic characteristics of each actuator. Based on the dynamic characteristic data, the proposed method is used to identify the loop of the actuator, and a mathematical model is established.

Keywords: aircraft engine, engine model, frequency domain identification

1. General Introduction

Actuator is an important part of control system, and its dynamic characteristics could affect the design of whole control system. However, for a long time, the identification of aircraft engine control system always pays too much attention to the mathematical model of the engine, and ignores the actuator. Due to the integral characteristics of actuators, it is impossible to do the open-loop test, so the test can only be identified under closed-loop conditions. But for a closed-loop system, the output signals reflect the whole system characteristics, including the feedback block. It is very complicated to distinguish the characteristics of the system from those of the feedback block based on the observable output data, and even maybe there is no identifiability. As a special kind of identification problem, the identification of closed-loop systems has become an urgent problem, which has attracted the attention of scholars all over the world. Soderstrom research points out that the noise signal contained in the output of the system will have a certain correlation with the input signal after passing through the feedback block. If the system is identified directly by spectrum analysis method, the result will be a weighted average between the reciprocal of the transfer function of the object to be identified and the feedback block [1]. The research on the identification of controlled object based on frequency characteristics mainly focuses on Levy's work. This method divides the real part and the imaginary part of the transfer function, and the least square method is applied. However the identification accuracy in the low frequency range is not satisfactory. Subsequently, some improved methods have appeared to modify Levy identification method [2-4].

In this paper, based on linear negative feedback systems with identifiability, a frequency characteristic parameter identification method based on non-linear weight is proposed. This method is applied to the dynamic data obtained from the sweep test of the aircraft engine actuator, and the mathematical model with high accuracy is established.

2. Identifiability Analysis

Consider the linear constant negative feedback system in Figure - 1. $r(t)$ is The external excitation signal of the system, $y(t)$ the output signal of the object, $e(t)$ is the error signal, $u(t)$ is the control signal, and $v(t)$ is the output interference signal which is assumed as white noise signals. The

dimensions of these signals are n_r , n_y , n_u and n_v respectively, and $n_u \leq n_r$. Block C is the controller, G is the object to be identified, L is the feed-forward block, F is the feedback block, and H is the white noise filter.

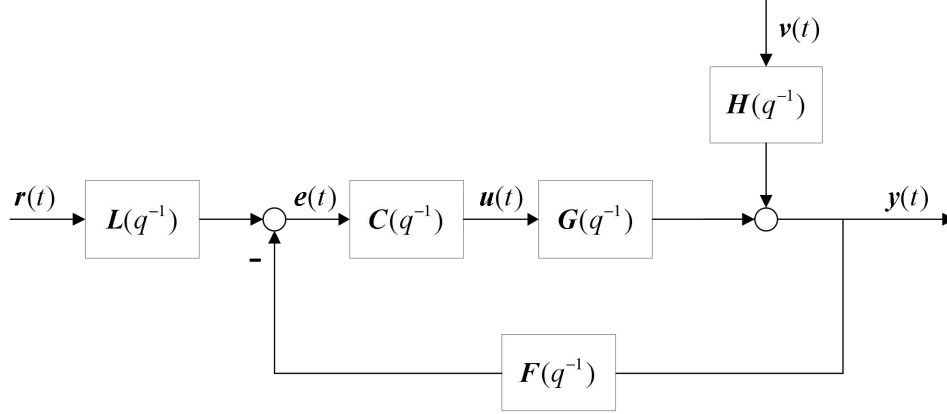


Figure - 1 Structure diagram of the linear constant negative feedback system

For the feedback system in Figure - 1, there are the following relationships between input and output.

$$y(t) = G(q^{-1})u(t) + H(q^{-1})v(t) \quad (1)$$

$$u(t) = C(q^{-1})e(t) \quad (2)$$

$$e(t) = L(q^{-1})r(t) - F(q^{-1})y(t) \quad (3)$$

where, q^{-1} is the back shift operator, $q^{-1}u(t) = u(t-1)$. $\{v(t)\}$ is a sequence of independent random vectors with zero mean and variance σ .

When the value of the system parameter vector to be identified is assumed as θ , $G(q^{-1})$ and $H(q^{-1})$ are marked as $\hat{G}(q^{-1})$ and $\hat{H}(q^{-1})$, then

$$y(t) = \hat{G}(q^{-1})u(t) + \hat{H}(q^{-1})v(t) \quad (4)$$

where, $\{v(t)\}$ is a sequence of independent random vectors with zero mean and variance Λ .

For the closed-loop system in Figure - 1, it is necessary to determine whether it has closed-loop identifiability.

Defining ideal sets of parameters to be identified as $D_T(\theta)$

$$D_T(\theta) = \{\theta \mid \hat{G} = G \text{ and } \hat{H} = H\} \quad (5)$$

Definition 1, system having identifiability means that under a certain model structure and identification method condition, if $N \rightarrow \infty$, the estimated parameters $\theta_N \rightarrow D_T(\theta)$, N is the number of data used to identify.

For the closed-loop system of Figure - 1, based on the definition 1, the following theorem is proposed in this paper:

Theorem 1, for systems Equation (1) and (2), suppose:

- (1) $G(0)C(0)F(0) = 0$;
- (2) The closed-loop system is asymptotically stable;
- (3) There exists a causal asymptotically stable filter, which makes $r(t) = K(q^{-1})v(t) + \bar{r}(t)$, $\bar{r}(t)$ and $v(t)$ are uncorrelated;
- (4) $G(0)C(0)L(0)K(0) = 0$, $\hat{G}(0)C(0)L(0)K(0) = 0$;

The necessary and sufficient condition for the system to be identifiable is

$$\text{rank}(R_T) = n_u + n_y \quad (6)$$

where,

$$R_T = \begin{bmatrix} I & 0 \\ -CF & CL \end{bmatrix} \quad (7)$$

the dimension of I is $n_y \times n_y$, the dimension of 0 is $n_y \times n_r$, the dimension of F is $n_u \times n_y$, the dimension of L is $n_u \times n_r$.

Proof: from Equation (1)~(3), the folloing one can be obtained.

$$y(t) = (I + GCF)^{-1} GCLr(t) + (I + GCF)^{-1} Hv(t) \quad (8)$$

$u(t)$ can be rewritten as follows:

$$u(t) = [CL - CF(I + GCF)^{-1} GCL]r(t) + CF(I + GCF)^{-1} Hv(t) \quad (9)$$

Set $P = (I + GCF)^{-1} G$, then Equation (8) and (9) can be rewritten as

$$y(t) = PCLr(t) + (I + GCF)^{-1} Hv(t) \quad (10)$$

$$u(t) = (I - CFP)CLr(t) + CF(I + GCF)^{-1} Hv(t) \quad (11)$$

Baseon Equation (10) and (11), Equation (4) can be rewritten as

$$\gamma(t) = \hat{H}^{-1} [P - \hat{G}(I - CFP)] CLr(t) + \hat{H}^{-1} (I - \hat{G}CF)(I + GCF)^{-1} Hv(t) \quad (12)$$

According to supposition (3) and (4), K is causal and $G_\theta u(t)$ and $v(t)$ are uncorrelated [5], $\gamma(t)$ can be written as

$$\gamma(t) \equiv v(t) \quad (13)$$

Deeply, the following Equation can be got,

$$\begin{cases} \hat{H}^{-1} [P - \hat{G}(I - CFP)] CL \equiv 0 \\ \hat{H}^{-1} (I - \hat{G}CF)(I + GCF)^{-1} H \equiv I \end{cases} \quad (14)$$

It can be rewritten as follows:

$$\begin{cases} (\hat{H}^{-1} \hat{G} - H^{-1} G)(CFP - I)CL + (\hat{H}^{-1} - H^{-1})PCL \\ \equiv H^{-1} (P - G + GC FP)CL = H^{-1} [(I - GCF)P - G]CL \equiv 0 \\ -(\hat{H}^{-1} \hat{G} - H^{-1} G)CF(I + GCF)^{-1} H + (\hat{H}^{-1} - H^{-1})(I + GCF)^{-1} H \\ \equiv I + H^{-1} (GCF - I)(I + GCF)^{-1} H \equiv 0 \end{cases} \quad (15)$$

The second one of Equation (15) can be further simplified to:

$$-(\hat{H}^{-1} \hat{G} - H^{-1} G)CF + (\hat{H}^{-1} - H^{-1}) \equiv 0 \quad (16)$$

It can be obtained from Equation (15) and (16):

$$\begin{aligned} \begin{bmatrix} 0 & 0 \end{bmatrix} &\equiv \begin{bmatrix} \hat{H}^{-1} - H^{-1} & \hat{H}^{-1} \hat{G} - H^{-1} G \end{bmatrix} \begin{bmatrix} PCL & I \\ (I - CFP)CL & -CF \end{bmatrix} \\ &= \begin{bmatrix} \hat{H}^{-1} - H^{-1} & \hat{H}^{-1} \hat{G} - H^{-1} G \end{bmatrix} \begin{bmatrix} I & 0 \\ -CF & CL \end{bmatrix} \begin{bmatrix} PCL & I \\ I & 0 \end{bmatrix} \end{aligned} \quad (17)$$

In Equation (17), the dimension of the matrix $\begin{bmatrix} PCL & I \\ I & 0 \end{bmatrix}$ is $(n_r + n_y) \times (n_r + n_y)$, so the matrix is nonsingular, then

$$\begin{bmatrix} 0 & 0 \end{bmatrix} = \begin{bmatrix} \hat{H}^{-1} - H^{-1} & \hat{H}^{-1} \hat{G} - H^{-1} G \end{bmatrix} R_T \quad (18)$$

where, the dimension of $R_T = \begin{bmatrix} I & 0 \\ -CF & CL \end{bmatrix}$ is $(n_u + n_y) \times (n_r + n_y)$.

Thus, Equation (6) is satisfied and the following result can be obtained

$$\hat{G} \equiv G, \quad \hat{H} \equiv H$$

It can be concluded that Equation (6) is a sufficient condition for the closed-loop identifiability of the system and the converse is also true, it is also a necessary condition.

3. Aircraft Engine Actuator Loop Identifiability

Generally, the closed loop of the actuator of the aircraft engine control system is shown in Figure - 2. The closed loop to be identified includes the control algorithm of the controller, D/A converter, drive circuit, actuator, sensor, signal processing circuit, A/D converter and software filter.

For the actuator loop of aircraft system in Figure - 2, closed-loop system is asymptotically stable. At time $t=0$, both input and output of the system were zero. If there is no sequence of random vectors $v(t)$, then the input $r(t)$ and $v(t)$ are uncorrelated. There exist $K(q^{-1})=0$ and $K(q^{-1})$ is causal. In summary, the actuator identification system satisfies the suppositions in Theorem 1 and it is a single input and single output system, $n_u=1$, $n_y=1$, then

$$\mathbf{R}_T = \begin{bmatrix} 1 & 0 \\ -CF & CL \end{bmatrix} \quad (19)$$

where, $\text{rank}(\mathbf{R}_T) = n_u + n_y = 2$, according to the Theorem 1, the loop of aircraft engine actuator has closed-loop identifiability.

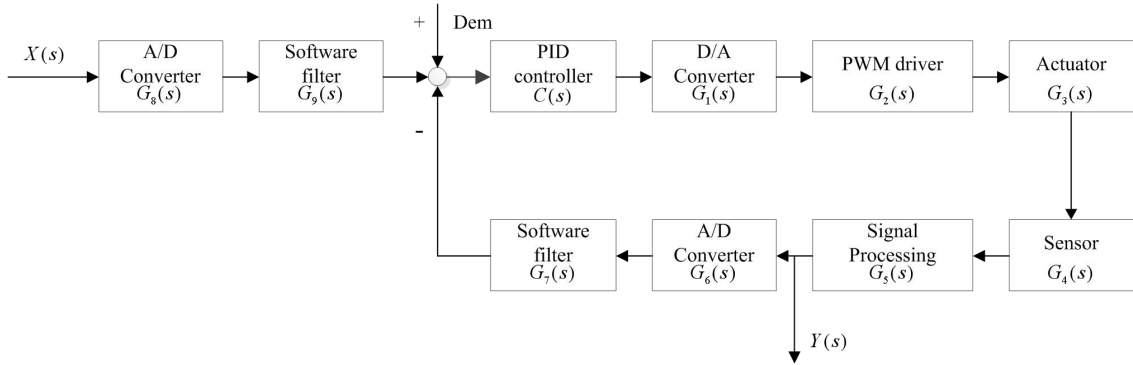


Figure - 2 Closed loop of the actuator of the aircraft engine control system

The transfer function from the input signal $X(s)$ to the output signal is $Y(s)$ in Figure - 2 is

$$K(s) \triangleq \frac{Y(s)}{X(s)} = \frac{C(s)G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)G_8(s)G_9(s)}{1 + C(s)G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)}$$

where, $G_8(s)G_9(s) = G_6(s)G_7(s)$. Let $G(s) = G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)$, then

$$K(s) = \frac{C(s)G(s)}{1 + C(s)G(s)}$$

After the closed-loop is stabilized at a given command, sinusoidal excitation signals of different frequencies are superimposed on the command signal to record the excitation signal and the closed-loop response of the system. The amplitude ratio and phase difference of excitation and steady-state output response can be obtained under different frequency signals, thus the frequency characteristics of the closed-loop can be obtained, and the overall transfer function of the small closed-loop can be identified.

4. Frequency Response Test and Loop Structure Determination

In this paper, experiments are carried out on a semi-physical simulation platform. According to frequency response method, the frequency selection of sinusoidal excitation signal is from low (such as 0.05Hz) to the highest possible frequency.

Through frequency sweep test, the amplitude ratio $A(\omega)$ and phase difference $\varphi(\omega)$ between the sinusoidal excitation input and the steady-state response output can be obtained. Then the logarithmic amplitude-frequency and phase-frequency characteristic curve (Bode diagram) is shown as Figure - 3. Based on Bode diagram, the typical links contained in the transfer function of the system can be determined. Under the structure of the estimated transfer function, the

parameters can be determined by identification algorithm, and the closed-loop transfer function of the actuator to be identified can be obtained.

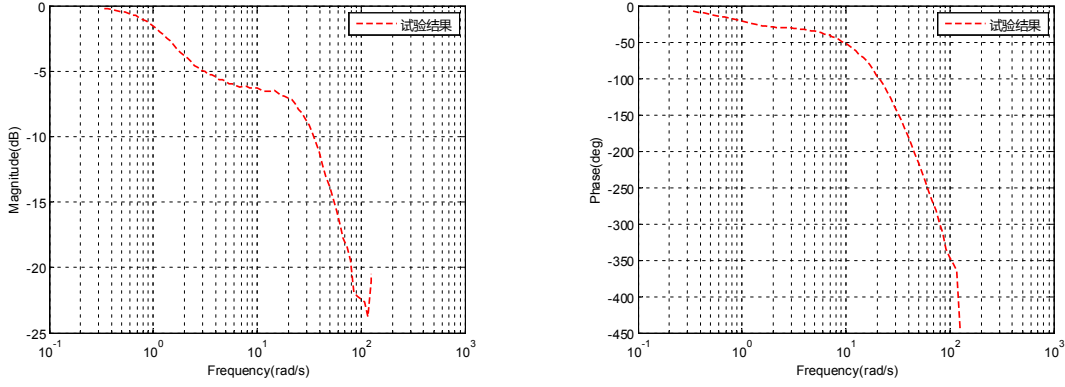


Figure - 3 Logarithmic frequency characteristic

According to the Bode diagram, the typical blocks contained in the transfer function can be determined, and the transfer function of the closed-loop system is obtained preliminarily such as below.

$$K(s) = \frac{T_2 s + 1}{T_1 s + 1} \cdot \frac{1}{1 + 2\zeta \frac{s}{\omega_n} + \frac{s^2}{\omega_n^2}} \cdot (T_3 s + 1) \cdot e^{-\tau s} \quad (20)$$

5. Weighted Frequency Characteristic Parameter Identification Method

For the following linear system,

$$G(s) = \frac{b_0 + b_1 s + b_2 s^2 + \dots + b_m s^m}{a_0 + a_1 s + a_2 s^2 + \dots + a_n s^n} = \frac{\sum_{k=0}^m b_k s^k}{\sum_{k=0}^n a_k s^k} \quad (21)$$

a_k ($k=0, \dots, n$) and b_k ($k=0, \dots, m$) are unknown coefficients to be identified, In general $a_0 = 1$.

For the above linear system, the frequency response under the identified excitation signal is as follows:

$$\hat{G}(j\omega) = \frac{\sum_{k=0}^m \hat{b}_k (j\omega)^k}{1 + \sum_{k=1}^n \hat{a}_k (j\omega)^k} = \frac{N(j\omega)}{D(j\omega)} = \frac{\alpha(\omega) + j\beta(\omega)}{\sigma(\omega) + j\tau(\omega)} \quad (22)$$

where, $\alpha(\omega) = \sum_{k=0}^m \hat{b}_k \text{Re}[(j\omega)^k]$, $\sigma(\omega) = 1 + \sum_{k=1}^n \hat{a}_k \text{Re}[(j\omega)^k]$, $\beta(\omega) = \sum_{k=0}^m \hat{b}_k \text{Im}[(j\omega)^k]$, $\tau(\omega) = \sum_{k=1}^n \hat{a}_k \text{Im}[(j\omega)^k]$

The frequency characteristic errors between the model and the object to be identified are as follows:

$$\varepsilon(j\omega) = G(j\omega) - \frac{N(j\omega)}{D(j\omega)} \quad (23)$$

In this paper, a weighted identification method of frequency characteristic parameters is proposed. By changing the form of above errors and constructing a quadratic form of error function, the identification parameters are obtained.

Define a new frequency characteristic error function

$$E(j\omega) = \varepsilon(j\omega)D(j\omega) = G(j\omega)D(j\omega) - N(j\omega) \quad (24)$$

Define the quadratic form of the new error function

$$|E|^2 = E \cdot E^* = [\text{Re}(G)\sigma - \text{Im}(G)\tau - \alpha]^2 + [\text{Re}(G)\tau + \text{Im}(G)\sigma - \beta]^2 \quad (25)$$

the superscript "*" denotes conjugate transposition.

Set the partial derivative of Equation (25) is zero,

$$\frac{\partial |E|^2}{\partial a_k} = 0 \Leftrightarrow \sigma \left\{ [\text{Im}(G)]^2 + [\text{Re}(G)]^2 \right\} \text{Re}[(j\omega)^k] + \tau \left\{ [\text{Im}(G)]^2 + [\text{Re}(G)]^2 \right\} \text{Im}[(j\omega)^k] \quad (26)$$

$$\alpha \left\{ \text{Im}(G) \text{Im}[(j\omega)^k] - \text{Re}(G) \text{Re}[(j\omega)^k] \right\} + \beta \left\{ -\text{Im}(G) \text{Re}[(j\omega)^k] - \text{Re}(G) \text{Im}[(j\omega)^k] \right\} = 0$$

$$\frac{\partial |E|^2}{\partial b_k} = 0 \Leftrightarrow [\text{Re}(G)\sigma - \text{Im}(G)\tau - \alpha] \text{Re}[(j\omega)^k] + [\text{Re}(G)\tau + \text{Im}(G)\sigma - \beta] \text{Im}[(j\omega)^k] = 0 \quad (27)$$

Combine Equation (26) and Equation (27)

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{b} \\ \mathbf{a} \end{bmatrix} = \begin{bmatrix} \mathbf{e} \\ \mathbf{g} \end{bmatrix} \quad (28)$$

where,

$$\mathbf{A}_{l,c} = -\text{Re}[(j\omega)^l] \text{Re}[(j\omega)^c] - \text{Im}[(j\omega)^l] \text{Im}[(j\omega)^c], \\ l = 0, \dots, m, \quad c = 0, \dots, m$$

$$\mathbf{B}_{l,c} = \text{Re}[(j\omega)^l] \text{Re}[(j\omega)^c] \text{Re}[G(j\omega)] + \text{Im}[(j\omega)^l] \text{Re}[(j\omega)^c] \text{Im}[G(j\omega)] \\ - \text{Re}[(j\omega)^l] \text{Im}[(j\omega)^c] \text{Im}[G(j\omega)] + \text{Im}[(j\omega)^l] \text{Im}[(j\omega)^c] \text{Re}[G(j\omega)] \\ l = 0, \dots, m, \quad c = 1, \dots, n$$

$$\mathbf{C}_{l,c} = -\text{Re}[(j\omega)^l] \text{Re}[(j\omega)^c] \text{Re}[G(j\omega)] + \text{Im}[(j\omega)^l] \text{Re}[(j\omega)^c] \text{Im}[G(j\omega)] \\ - \text{Re}[(j\omega)^l] \text{Im}[(j\omega)^c] \text{Im}[G(j\omega)] - \text{Im}[(j\omega)^l] \text{Im}[(j\omega)^c] \text{Re}[G(j\omega)] \\ l = 1, \dots, n, \quad c = 0, \dots, m$$

$$\mathbf{D}_{l,c} = \left(\left\{ \text{Re}[G(j\omega)] \right\}^2 + \left\{ \text{Im}[G(j\omega)] \right\}^2 \right) \times \left\{ \text{Re}[(j\omega)^l] \text{Re}[(j\omega)^c] + \text{Im}[(j\omega)^l] \text{Im}[(j\omega)^c] \right\} \\ l = 1, \dots, n, \quad c = 1, \dots, n$$

$$\mathbf{e}_{l,1} = -\text{Re}[(j\omega)^l] \text{Re}[G(j\omega)] - \text{Im}[(j\omega)^l] \text{Im}[G(j\omega)], \\ l = 0, \dots, m$$

$$\mathbf{g}_{l,1} = -\text{Re}[(j\omega)^l] \left(\left\{ \text{Re}[G(j\omega)] \right\}^2 + \left\{ \text{Im}[G(j\omega)] \right\}^2 \right) \\ l = 1, \dots, n$$

$$\mathbf{b} = [\hat{b}_0 \quad \dots \quad \hat{b}_m]^T, \quad \mathbf{a} = [\hat{a}_1 \quad \dots \quad \hat{a}_n]^T$$

For frequency ω_i , different weights are given according to the results of frequency response test.

$$\tilde{\mathbf{A}} = \sum_{i=1}^F w_i \mathbf{A}_i, \quad \tilde{\mathbf{B}} = \sum_{i=1}^F w_i \mathbf{B}_i, \quad \tilde{\mathbf{C}} = \sum_{i=1}^F w_i \mathbf{C}_i, \quad \tilde{\mathbf{D}} = \sum_{i=1}^F w_i \mathbf{D}_i, \quad \tilde{\mathbf{e}} = \sum_{i=1}^F w_i \mathbf{e}_i, \quad \tilde{\mathbf{g}} = \sum_{i=1}^F w_i \mathbf{g}_i$$

w_i is the weight of frequency signal ω_i , a piecewise non-linear weight calculation method is adopted in this paper.

$$w_i = \begin{cases} \frac{\omega_2 - \omega_1}{2\omega_1^2} & i = 1 \\ \frac{\omega_{i+1} - \omega_{i-1}}{2\omega_i^2} & 1 < i < F \\ \frac{\omega_F - \omega_{F-1}}{2\omega_F^2} & i = F \end{cases} \quad (29)$$

Because there are delay links in each loop, it is difficult to solve directly. In this paper, an inverse solution method is adopted to determine the unknown parameters. Finally, The actuator loop of aircraft engine control system is identified as

$$K(s) = \frac{0.38s + 1}{0.82s + 1} \cdot \frac{1}{1 + 2 \times 0.73 \times \frac{s}{30} + \frac{s^2}{30^2}} \cdot (0.017s + 1) \cdot e^{-0.045s} \quad (30)$$

Figure - 4 shows some of the identification results compared with the logarithmic frequency characteristics measured in the experiment. From the figure, it can be seen that the identification results coincide with the logarithmic frequency characteristic curve measured by the experiment, and have a high degree of coincidence.

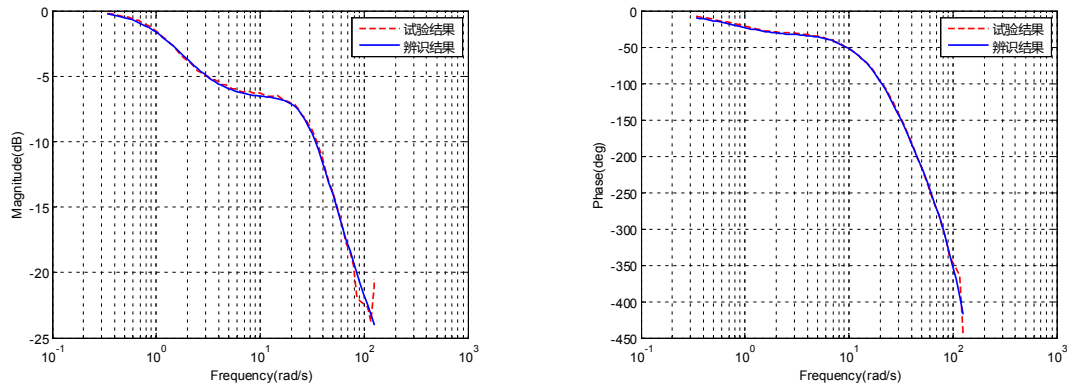


Figure - 4 Comparison of identification results with theory

6. Conclusion

For the problem of low identification accuracy of frequency domain identification method at low frequency, an Improved identification method with non-linear weight for actuator loop of aircraft engine control system is proposed. The method distributes weight coefficients among different frequency through non-linear interpolation, so as to distinguish the influence of different frequency on identification results. By backward solution for the delay, the proposed method is used to identify the actuator closed loop of aircraft engine control system. From the identification results, the proposed method has high accuracy.

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