A METHOD FOR ESTIMATING THE PROBABILITY OF RARE ACCIDENTS IN COMPLEX SYSTEMS

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Boeing Research & Technology

Keywords: *Risk Assessment, Monte Carlo, Rare Events, Algorithms, Aircraft*

Abstract

Estimating the probability of failures or accidents with aerospace systems is often necessary when new concepts or designs are introduced, as it is being done for Autonomous Aircraft. If the design is safe, as it is supposed to be, accident cases are very rare. This feature requires some variance reduction technique in the probability estimation and several algorithms exist for that, however some specific model features may cause difficulties in practice, such as the case of system models where independent agents have to autonomously accomplish missions within finite time, and likely with the presence of human agents. For handling these scenarios, this paper presents a novel estimation approach, based on the combination of the well-established variation reduction technique of Interacting Particles System (IPS) with the long-standing optimization algorithm denominated DIviding RECTangles (DIRECT). When combined, these two techniques yield statistically significant results for extremely low probabilities. In addition, this novel approach allows the identification of intermediate events and simplifies the evaluation of sensitivity of the estimated probabilities to certain system parameters.

1 Introduction

Complex systems such as air traffic management, despite increasing level of automation, still have human individuals responsible for their operations. Such systems, on which individuals interact with the technical subsystems as well as with other individuals participating in the large system, are designated as 'social-technical systems'.

Multi-Agent Dynamic Risk Models (MA-DRM) have been proven successful for analyzing complex socio-technical systems in regard to safety properties [1], [2] as it demonstrated efficiency in identifying hazardous sequences of events which were not identified when using standard risk assessment analyses [3].

MA-DRM methodology combines concepts of distributed artificial intelligence (Multi-Agent Systems) with stochastic estimation methods (Monte Carlo-based algorithms) to model human behavior and interactions, besides technical elements, in order to estimate failure and accident rates. These rates need to be under certain Target Levels of Safety (TLS) on the order of 10^{-9} or below, therefore considered rare or extremely rare events.

For handling these rare events scenarios, a novel estimation approach is presented in this paper, based on the combination of the well-established variation reduction technique of Interacting Particles Systems (IPS) with the long-standing optimization algorithm denominated DIviding RECtangles (DIRECT). Combined, these two techniques yield statistically significant results for extremely low probabilities. The basic principles of this approach were published in [4] and here an in-depth presentation developed.

After this introductory section, the contents of this paper is organized as follows: Section 2 introduces the estimation method aforementioned, Section 3 explains the method in more concrete terms, with a case study on a hypothetical aircraft operation, and Section 4 contains the conclusions.

2 Method for handling rare event scenarios

As this work concerns estimation of rare events arising from the MA-DRM, the model has to be mathematically sound, and the type of mathematical model to be applied depends on the complexity of the system to be modelled. For example, a very elementary system with only one agent (e.g. a bouncing ball) can be modelled as Ordinary Differential Equations (ODE) with continuous variables. However, complex socio-technical systems with multiple agents have large number of disturbances variables.

As the system complexity increases, the system elements may be governed by sets of Stochastic Differential Equations (SDEs) that can be activated or deactivated at any given time. The process of activating and deactivating equations is called switching and, when switching occurs by hitting a boundary or by stochastic jump processes, the system can be mathematically considered a General Stochastic Hybrid System [5]. The execution of such system is a General Stochastic Hybrid Process (GSHP).

The overall method flow of rare event probability estimation for models with SDEs is presented in Fig. 1. The estimation process starts when the system agents become defined, as well as their dynamics and interactions, based on expert knowledge on technical systems and organizations [6]–[8]. This is elaborated in Step 1.

Step 2 of Fig. 1 consists of identifying the set of purely stochastic variables of the model, where "purely" means that each variable must not have in their definitions operations involving other model variables nor a previous value of itself (that is, they must not have a recursive definition). Because of this feature, these variables can be considered as input variables and can be called system parameters or model inputs. These parameters must follow some known probability distributions and this is the reason for performing Step 3, which determines their probability distribution functions. This determination may be based on expert knowledge or data analytics.

The model execution for a given vector value x of the stochastic parameters can be another stochastic process, so the objective function for the search & partition of Step 6 below has to be an aggregate measure of a set of execution instances defined in Step 4.

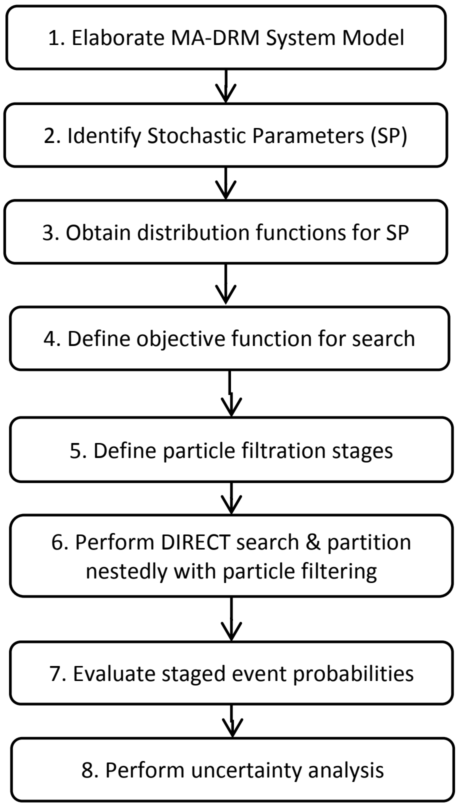

Fig. 1. The method of rare event probability estimation for models with Stochastic Differential Equations (SDEs).

Any aggregate measure is acceptable (e.g. weighted mean, root mean square, etc.) as long as it contributes to finding the regions where the target event occurs and to obtain an acceptable error in the probability estimation of this event. In Step 5, because of the same execution stochasticity, each objective function evaluation may contain a non-null probability of the rare event. A particle filtering technique called Interactive Particle System (IPS) [9], [10] is chosen for this purpose. This technique requires the definition of a filtration criterion and corresponding filtration stages, which make feasible the estimation of very low event probabilities.

In Step 6 the DIRECT-based search and partition is combined with the IPS variance reduction technique. The system execution process has to have the Markov property, which usually holds in physics-based processes outside the sub-atomic domain. In Step 7, the successive filtration stages of IPS allow that the probability not only of the final target event be calculated, but also of the preceding events defined by these filtration stages. This provides a better understanding of the system behaviour before the occurrence of the ultimately critical event, which contributes to design improvement insights and implementation of safeguards.

Finally, Step 8 examines the uncertainty in the knowledge of the moments of the probability distribution functions of the stochastic parameters. This uncertainty raises mainly two questions: i) What is the confidence level of meeting the TLS? And ii) which stochastic parameters of the rare event are most sensitive, and with how much intensity? The answer to the first question determines whether the current system design concept is acceptable, and the answer to the second question helps finding ways to improve the current design concept and is performed by means of sensitivity analysis techniques. Standard statistics has plenty of methods for providing these answers but, in one way or another, these methods require the re-calculation of the rare event probability with different inputs. This goal is greatly facilitated by the partition of the parameter space provided in Step 8, diminishing the necessity of re-executing simulations of the system model.

In order to have a better understanding of how these steps happen in practice, in the next section the method is presented in more concrete terms with a case study on a hypothetical aircraft operation.

3 Case study: hypothetical aircraft operation

This section presents details on the application of the probability estimation method of Fig. 1 for a complex system model. This application case consists in the operation of a transport aircraft in a certain phase of flight, described at high level by a multi-agent system composed by *Environment*, *Aircraft*, and *Pilot*.

Step 1: Elaborate MA-DRM System Model

The *Aircraft* agent is modelled as a point-mass aircraft subject to inputs from the *Environment* and from the *Pilot* [11]. There might be several pilot inputs and input modes to control the aircraft; however, in this example, the only pilot intervention allowed is to command an emergency maneuver of full-thrust climb. The parameter values used to fill the model of aircraft dynamics in this case study correspond to a commercial single aisle jet aircraft.

The programmed flight path for the experiments herein is illustrated in Fig. 2. The aircraft enters the scenario at the upper right corner and follows a predefined route (sequence of waypoints) with a “U-shape,” which descends and passes between two peaks in the terrain.

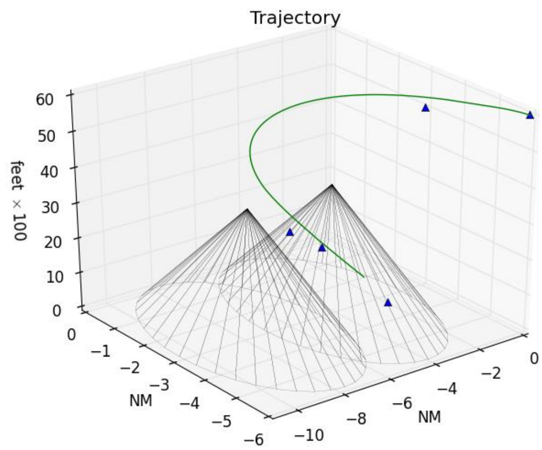

Fig. 2. Illustration of the programmed aircraft operation model. The aircraft enters the scenario at the upper right corner and follows a predefined route, which descends and passes between two peaks in the terrain.

The aircraft flies until either accidentally hitting terrain, going out of the airspace bounding box, or reaching a maximum flight time T, whichever is the first to occur. The idea of such route is to resemble a flight approaching an airport in the proximity of mountains. In nominal conditions, the trajectory terminates at the lowermost waypoint, from which the aircraft will proceed to the final approach. In a risk assessment, which is recommended to happen before the use of a route with similar features in real life, there is interest in the non-nominal conditions, which are triggered by errors or faults that, in extreme cases, lead to an accident, which in this example is hitting terrain. In order to hit terrain, the minimum distance between the terrain and aircraft, d_{min}, has to be 0.

For the purpose of risk assessment, it is assumed that the aircraft has an altimetry fault when it enters in the scenario, which is manifested in a numeric error ϵ_a, which influences the trajectory flown.

The *Environment* agent is composed of terrain and atmosphere. In the example model, the terrain has a base of altitude 0, on which lay two cones with base radius of 3 nautical miles and height of 3600 feet, as shown in Fig. 2. If the distance between terrain and aircraft reaches the minimum value $m = 0$, the simulation is immediately stopped.

The atmosphere is modelled with the Dryden turbulence model [12], requiring an elaborate algorithm for variance reduction in combination with Monte Carlo. This turbulence model will affect the point-mass model of aircraft dynamics. It defines the linear and angular velocity components of air gusts as position-dependent stochastic processes, and is based on the power spectral density of each spatial and angular component. These power spectral densities are rational, so that they can be implemented as exact filters that take a band-limited white noise input and generate a stochastic process output with filters derived from the Dryden power spectral densities.

Thus, if $y_u(s)$ is the power spectral density of the turbulence linear speed component on the dimension u, it is modelled according to Dryden as:

$$y_u(s) = G_u(s)sW(s) \qquad (1)$$

where $G_u(s)$ is the filter or transfer function, and $W(s)$ is a standard Wiener process (a.k.a. Brownian motion) which, when treated as a generalized random process, can have its n-th order derivatives. The first order derivative $sW(s)$ is white noise.

These equations, when transformed to the time domain, result in stochastic differential equations of the form:

$$a_2\ddot{y}(t) + a_1\dot{y}(t) + a_0y(t) + b_2\ddot{W}(t) + b_1\dot{W}(t) = 0 \qquad (2)$$

where the terms a_n and b_m are defined in terms of the turbulence intensity parameters, length parameters , and of the aircraft airspeed $V(t)$ and its derivative $\dot{V}(t)$ [12].

The angular components of the turbulence are not represented because the aircraft model here is only point-mass.

Fig. 3 illustrates the effect of this turbulence model on the aircraft operation:

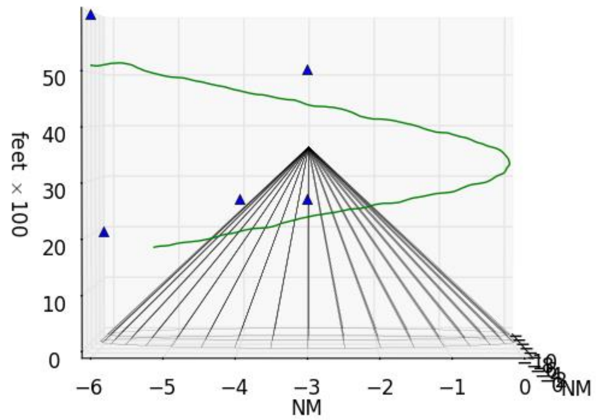

Fig. 3. Illustration of the aircraft operation model with turbulent trajectory.

The *Pilot* agent's role is only to detect the altimetry fault and, in response, initiate the avoidance maneuver. The time that the pilot takes to detect and react to erroneous altitude is stochastic and denoted by t_r.

Aircraft altimetry systems are nowadays very reliable and further are protected by having redundant systems. Still, faults happen in some cases, including icing or other types of sensor obstruction, computing error, etc. Whatever the phenomenon is, it may happen and, in the present model, it is established that, when it happens, the altimetry system will present the above mentioned altitude error ϵ_a, which in turn causes the aircraft to fly with an altitude offset ϵ_h, of same magnitude and opposite value, i.e., $\epsilon_h = -\epsilon_a$. The model assumes that the aircraft enters the scenario with this altitude reading error and that the flight guidance system makes the aircraft fly with the altitude shifted by this error value, until the flight crew detects the altimetry fault by some means.

This completes Step 1. The next step of the estimation approach is to use some of the variables of the system model to perform a search that will lead to the occurrence of the events of interest.

Step 2: Identify Stochastic Parameters

The identified parameters are 4:

- The altitude offset ϵ_h, caused by an altimetry fault;
- The time t_r for crew reaction;
- The horizontal wind components w_x, w_y.

Step 3: Obtain distribution functions for SP

The determination of the probability distribution function of the stochastic parameters should be done by data analytics or by experts in a guided process, such as one described in [13], which has systematic reduction of bias and inconsistency. However, as this case study is hypothetical, the determination here is for illustrative purposes. The time that the pilot takes to react to the altitude error, t_r, is associated to an exponential distribution with the mean μ of 30 seconds; the altitude offset ϵ_h is associated to a normal distribution with moments $\mu = 0$ and $\sigma = 100$ feet; the constant wind velocity components w_x and w_y are normally distributed with moments $\mu = 0$ and $\sigma = 15$ knots.

Step 4: Define objective function for search

The objective function defined here will be used by the search & partition algorithm in next step of the method. When searching for the regions of the parameter space where collision with terrain happens, a natural choice for objective function would be the terrain miss distance in the example model, denoted by $d(x)$, where x is the vector of parameter values. "Miss Distance" means the minimum distance that ever existed between aircraft and terrain in one scenario execution. If $d(x) = 0$, it means that the aircraft collided, so the target event happened. However, this is not enough if the model has SDEs with implicit random variables.

For such complex system models, the approach chosen to handle is to mask its implicit stochasticity under an aggregate measure for the objective function used in DIRECT. Let ξ denote a stochastic instance of the system, also called a particle, which has a set of state variables, including the vector $\boldsymbol{x}$, which stays fixed during the lifetime of ξ. The association of ξ to $\boldsymbol{x}$ is expressed as $\boldsymbol{x} = \boldsymbol{X}(\xi)$. As ξ is here a stochastic process, $\boldsymbol{X}^{-1}(\boldsymbol{x})$ becomes a random variable. For this reason, it is also no longer possible to use the terrain miss distance $d(\boldsymbol{x})$, as previously defined, because it depends on the instantiation of $\boldsymbol{X}^{-1}(\boldsymbol{x})$.

First, $d(\cdot)$ is redefined to the particle (or trajectory) domain, signifying that it evaluates the terrain miss distance of a concrete trajectory of the system model ξ, hence the notation $d(\xi)$. Then, a new function $\bar{d}(\boldsymbol{c}_i) \triangleq E\left[d\left(\xi_j\right)\right]$ is defined, to be used at each hyperbox $\boldsymbol{B}_i$ centered at $\boldsymbol{c}_i$, for $\xi_j \in \boldsymbol{X}^{-1}(\boldsymbol{c}_i), j = 1, \dots, s$. This expresses the expectation or mean of evaluations of the distance function d over a number s of system instances associated to $\boldsymbol{c}_i$. This function maintains the convergence of the DIRECT search and partition algorithm, a fact that was observed in practice but can also be mathematically demonstrated.

The new stochastic features of the model include the fact that, when performing the evaluations $d\left(\xi_j\right)$, for $\xi_j \in \boldsymbol{X}^{-1}(\boldsymbol{c}_i)$, it may happen that some of the obtained values are higher than m (the distance threshold which defines the target event), and others are equal or lower. The ratio between the number of "hits" $\hat{h}(\boldsymbol{B}_i)$, i.e., the number of instance values equal or lower than m, and the total number of instances s at that point, is an estimator of the probability of the system to reach the target region when the input variables assume values in $\boldsymbol{B}_i$, provided $\boldsymbol{B}_i$ is acceptably small. This hit ratio is denoted as $\rho(\boldsymbol{B}_i) = \hat{h}(\boldsymbol{B}_i)/s$ in a crude Monte Carlo definition. Therefore, in the final calculation of the event probability, $\rho(\boldsymbol{B}_i)$ serves as a weighing factor on top of the prior probability of $\boldsymbol{B}_i$.

This means that there is no longer a *border* of the target region, but rather ρ-curves and ρ-regions, with $\rho \in [0,1]$. The higher the ρ, the more frequent the target event being. Thus, the new objective function should promote less subdivision at high plateaus of ρ and concentrate at the slopes that surround the plateaus of $\rho = 1$.

Hence, a definition of objective function f_o was elaborated with a recursive algorithm, in order to obtain this effect. This new objective function received the denomination *Outer* because this feature of concentrating hyperbox subdivision at the outer vicinity of the border of the regions with $\rho = 1$.

Step 5: Define particle filtration stages

The variance reduction technique to be used below, based on particle filtering, needs the definition of a sequence of events which are nested and gradually rarer. In the hypothetical example presented, the final target event is collision with the ground, which happens by definition when $d(\xi) = 0$. However, before this happens, this distance becomes gradually smaller. Starting at $d(\xi) \approx \infty$, the events $d(\xi) \leq m_l$ happen successively with $m_l > 0$ for $l = 1, \ldots, N_F - 1$ and $m_{N_F} = 0$, being N_F the stipulated number of filtration stages. The rationale here is that, given that $d(\xi) \leq m_l$ ($m_0 \approx \infty$) happened, $d(\xi) \leq m_{l+1}$ is not so hard to obtain, thus providing an acceptably large statistical significance. This principle is illustrated in Fig. 7 [19].

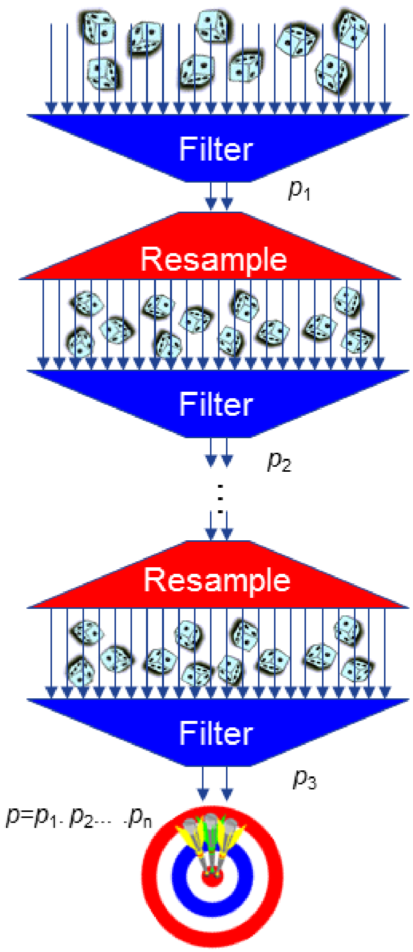

Fig. 7. Illustration of the principle of particle filtering (from [19]).

In this figure, p_l represents the probability of a particle surviving the filter of stage l, and p the final probability of surviving all stages. Usually, the number of stages needed hangs around the decimal order of magnitude of the probability to be estimated, without the sign. If the probability is approximately 1E–15 (1×10^{-15}), the number of stages needed is around 15. The selection of m_l values needs some guessing in the first experiments with a new model, but can be adjusted along preliminary simulation runs. For the hypothetical application example used, the values were set to the ones in Table 1 thus completing Step 5 of the method in Fig.1.

Table 1. Distances used to define particle filtering stages.

Stage index l	Threshold distance m_l (feet)
1	1000
2	900
3	800
4	700
5	600
6	450
7	300
8	225
9	150
10	100
11	75
12	50
13	25
14	0

Step 6: Perform DIRECT search & partition nestedly with particle filtering

Each evaluation of the objective function f_o chosen for DIRECT contains a rare event process and, because of this rarity, we chose to use the Interactive Particle System (IPS) variation reduction technique [9], [10], in order to determine the event probabilities inside each hyperbox. The central mechanism of IPS is, instead of working with just one region of interest, to use a succession of nested regions as illustrated by means of Fig. 7 or, in a formulation of probability theory, a filtration of σ-algebras of outcomes, among which the innermost corresponds to the final region of interest.

This combined algorithm with DIRECT

and IPS is named DIPS, together with the prefix *Outer*, in reference to the newly customized objective function f_o . In this algorithm, a separate IPS run is executed inside each hyperbox $\boldsymbol{B}_i$, where the weights $\omega_k^{\langle l \rangle}$ of the particles inside it must be used to account for the prior probability $P_{\boldsymbol{B}_i}$ of the hyperbox (which in turn is based on the density function g of stochastic parameter values).

At this point, we have all the elements to run the DIPS estimation algorithm. However, because of the high dimensional stochasticity of the system model, we cannot rely on running it a single time. Each time the algorithm runs, a different probability value emerges, and therefore several runs are needed to determine a confidence interval for the target event probability. The histogram of results obtained is highly skewed, due to the high variance of the process modelled, even with the variance reduction technique applied. In this case, using confidence intervals based on normal distributions is not effective, so a different approach is used here, from [18], which is a modified version of the Cox method. Instead of using a standard normal variate z parameter, the t parameter from Student's distribution is used to determine the amplitude of the interval for a given confidence level, in order to better account with small sample sizes. Defining $p = \log(P)$ and σ as the sample standard deviation of p, the limits of the confidence interval for the true mean of P are given by:

$$\exp\left(\bar{p} + \frac{\sigma^2}{2} \pm t\sqrt{\frac{\sigma^2}{N_r} + \frac{\sigma^4}{2(N_r - 1)}}\right) \tag{3}$$

from which the third term is used to define the dispersion measure θ:

$$\theta \triangleq t\sqrt{\frac{\sigma^2}{N_r} + \frac{\sigma^4}{2(N_r - 1)}} \tag{4}$$

The dispersion resulting from successive runs of the estimation algorithm is a tradeoff between the time spent to run each algorithm instantiation and the total number of instantiation used in the sample. After an unstructured trial-and-adjust process, the parameters which define the effort in each algorithm instantiation are settled. In these experiments, the number of particles per hyperbox s was set to 1,000 and the total number of hyperboxes to be generated was set to 16,700. We ran the Outer-DIPS algorithm 32 times, in order to gain some benefit from the law of large numbers, and so we obtained the mean and percentile values of Fig. 8.

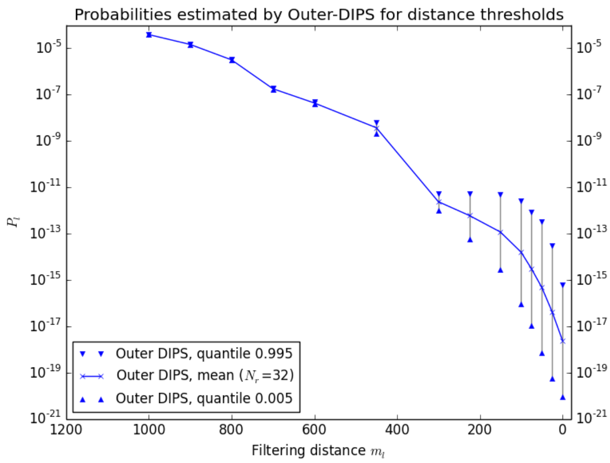

Fig. 8. Probabilities of hitting filtering distances m_l, evaluated by the Outer-DIPS algorithm.

As it can be noted, the confidence intervals, delimited by lower and upper quantiles, are narrow for higher values of m_l, but widen as m_l decreases, as seen in the log scale. At $m_l = 0$, the dispersion of the results, calculated according to Equation 4, is 5.05. From a safety analysis viewpoint, the upper quantile is more important, but the fact of having obtained a statistically significant interval for such low probability event gives more credibility to the estimation. At this point, Steps 5 and 6 of Fig. 1 can be considered complete.

Step 7: Evaluate staged event probabilities

The successive filtration stages of IPS allow that the probability not only of the final target event be calculated, but also of the preceding events defined by these filtration stages. This provides a better understanding of the system behavior before the occurrence of the ultimately critical event, which contributes to design improvement insights and implementation of safeguards.

These probabilities correspond to the different points on the horizontal axis of Fig. 8.

Step 8: Perform uncertainty analysis

The confidence interval that we obtained for the probability of the target event is valid for a unique combination of values of the stochastic parameters of the model. The knowledge of these values is subject to uncertainties, so the uncertainty analysis of Step 8 of Fig. 1 has two main goals: one, reviewing the confidence intervals of the resulting estimate in order to account for these uncertainties; and two, performing parameter sensitivity analysis in order to find the most critical parameters of the system.

The moments (e.g. mean, standard deviation) of the probability distributions of the stochastic parameters are subject to uncertainties. In the case of the hypothetical example under study, the determination of the mean and standard deviation of ϵ_h, and the mean of t_r may have been determined from expert knowledge, which also embodies uncertainties. Usually, the uncertainty of each parameter is summarized as a confidence interval, the worst and best cases of the target event probability occurring at some combination of extremes of those intervals. Re-evaluating the target event probability at these points requires extra computational effort, which may be considerably reduced if the search space is already partitioned.

Instead of fully re-running Step 6, the partition of the search space obtained in that step can be re-used, and only the probabilities of the corresponding hyperboxes are re-calculated. This avoids the re-calculation of the objective function, which is the most expensive part of the overall computation. Finally, by making this calculation faster, the determination of the confidence intervals of the target event probability become proportionally faster, as well as the determination of parameter sensitivities.

Parameter sensitivity analysis produces valuable design information, in the sense that it is possible to identify the stochastic parameters to which the probability of the rare event in the critical system modelled is most sensitive, with the sensitivity rates calculated during this type of analysis [15]-[17]. Therefore, the system design can be changed to decrease the sensitivity of the most sensitive parameters and become more robust and less subject to uncertainties.

4 Conclusions

The approach proposed in this paper for estimation of the probability of rare events is capable of obtaining statistically significant results for probability values lower than any other in the literature, when considering complex socio-technical systems. Our bibliographic search did not find an example of probability estimation below 1E–10 for such models, while here there are reliable results below 1E–17 for a system with considerable complexity, including Stochastic Differential Equations and an elaborate control logic. This shows that the method proposed is ready to be used in models of similar level of complexity, and models with higher complexity can be processed with increasing levels of computing parallelism.

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