DYNAMIC MODEL FOR PLANETARY GEAR SETS OF GEARED TURBOFAN JET ENGINES

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Abstract

A hybrid dynamic model of a planetary gear set has been developed to calculate dynamic loads and analysis of dynamic stresses of gears in aviation engine gearbox. The effect of sun gear support stiffness on the dynamic stresses in the system, as well as the effect of the ratio between satellites mesh phases in planetary gearbox is analyzed. It is shown that the maximum dynamic stresses occur in the ring gear as a result of its natural vibration with bending forms. On the basis of the developed dynamic model, recommendations to decrease dynamic loads in planetary gearboxes are formulated.

1 Introduction

The task of decrease dynamic forces in the gears in aviation transmissions and drives continues to be relevant to develop modern aircraft engines. Its levels largely determine the noise and vibrations in the drive as well as the durability of the gears and other elements of the design, both the gearbox and the engine itself. Due to the dense spectrum of vibrations in the meshing of planetary gearbox, resonant vibrations occur in the compressor blades and other parts resulting in a longer engine development time [1]. Some cases of destruction of the turbofan disks in operation owing to resonant oscillations caused by the over-excitement from the gear set occur. The reasons of the over-excitement were the overestimated parameters of the profile modification of the gear teeth.

As for the aircraft engines, the most loaded gears include the cylindrical and herringbone gears of the planetary gear sets of the geared turbofan engines (such as PW-1100G, PW-1400G, UltraFan) operating at speeds up to 10,000 rpm. The requirements for minimizing the mass necessitate an increase in the accuracy of the strength calculations of aircraft gears and the development of modeling methods for both meshing and the entire planetary gear set [2, 3].

This article describes the developed hybrid dynamic model of the planetary gear set combining the advantages of meshing simulation in FEM and solving dynamics problems by analytical methods.

2 Dynamic model

Two types of dynamic models are usually used to study dynamic processes in planetary gear sets: mathematical models with lumped parameters, in which the meshing is represented in the form of hard disks connected by an elastic-damping coupling [4, 7, 8], and models based on the finite element method (FEM) [5, 6], which allow the most accurate calculation of the compliance of all system elements.

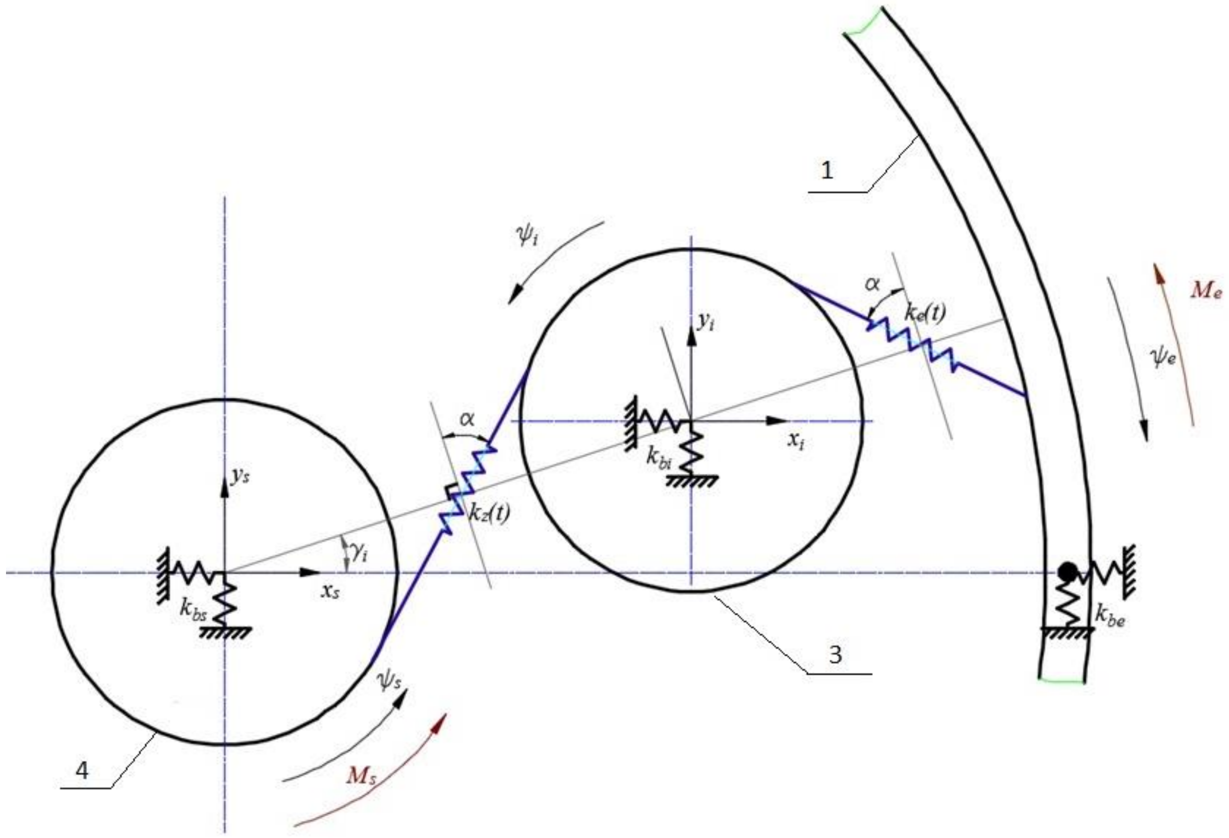

Fig. 1 Model of basic element of planetary gear system – sun-planet and ring-planet gear pairs

This paper studies the hybrid dynamic model combining the speed of solving process of the analytical model and the accuracy of calculating the mesh stiffness characteristics using the FEM.

The planetary set with three satellites is considered to simplify the system. The main subsystem of the planetary gear set dynamic model is a hybrid dynamic model of cylindrical gear pair (fig. 1).

2.1 Model of gear pair

The dynamic model of gear is represented as a 6-degree-of-freedom nonlinear system [9] consisting of:

- gear pair represented by rigid disks with mass m_1 and m_2 and inertia moments J_1 and J_2, respectively, and connected by an elastic-damping coupling with variable mesh stiffness $k_z(t)$, directed along the line of action;

- bearing supports of gear shafts characterized by the stiffness of k_{bx} and k_{by} in accordance with the directions of the selected orthogonal coordinate system for each gear.

The system is balanced by torque applied in opposite directions to the gears (fig. 1).

The governing equations of motion for the model depicted in fig. 1 can be written as follows in the matrix form:

$$[\mathrm{M}]\{\ddot{q}\} + [\mathrm{K}(t,q)]\{q\} + [\mathrm{C}]\{\dot{q}\} = \{\mathrm{F}(t)\} + \{\mathrm{F}_{fr}(t,q)\} \quad (1)$$

where $[\mathrm{M}] = diag[J_1, J_2, m_1, m_1, m_2, m_2]$ is an inertia diagonal matrix, the elements of which are inertia moments and mass of gears; [K(t,q)] is a symmetric stiffness matrix; [C] is a damping matrix obtained by analogy with the stiffness matrix [K(t,q)]; $\{q\} = \{\varphi_1, \varphi_2, x_1, x_2, y_1, y_2\}^T$ is a column matrix of system generalized coordinates which are the angles of rotation, horizontal and vertical movements of the centers of mass of the gears; $\{\mathrm{F}(t)\} = \{M_1(t), M_2(t), 0,0,0,0\}^T$ is a column matrix of external forces; $\{\mathrm{F}_{fr}(t,q)\}$ is a column matrix of frictional forces;

$k_z = f(t, q)$ is a periodically varying mesh stiffness. Time varying mesh stiffness is the main source of kinematic excitation of parametric vibration in a dynamic gear system [9]. The finite element method (FEM) is used to calculate the mesh stiffness function and the transmission error.

Figure 2 shows graphs of mesh stiffness of spur gears with various parameters of profile modification. Gear tooth profile modification is introduced to optimize contact patterns and stresses, to compensate for manufacturing errors, and to reduce gear dynamics. Microgeometry modification of the involute gear teeth dramatically affects the static and dynamic performances of the gear system [11].

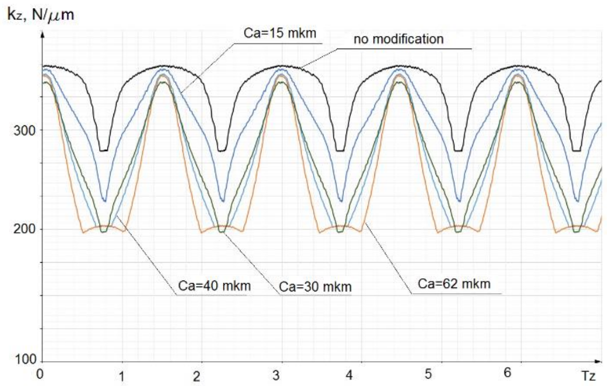

Fig. 2 Finite element calculation of tooth mesh stiffness for varying amount of tip profile modification

Figure 3 shows the comparison of the harmonic amplitudes mesh stiffness functions for various depth parameters of the profile modification. As can be seen, the spectral composition of fig. 3 introduction of the profile modification generally reduces the amplitudes of the harmonics multiples of the tooth. However, if the modification depth is too great, it is possible to decrease the contact ratio ε, which can lead to an increase in the amplitude of the first harmonic in the spectral composition of the polyharmonic mesh stiffness function $k_z = f(t, q)$.

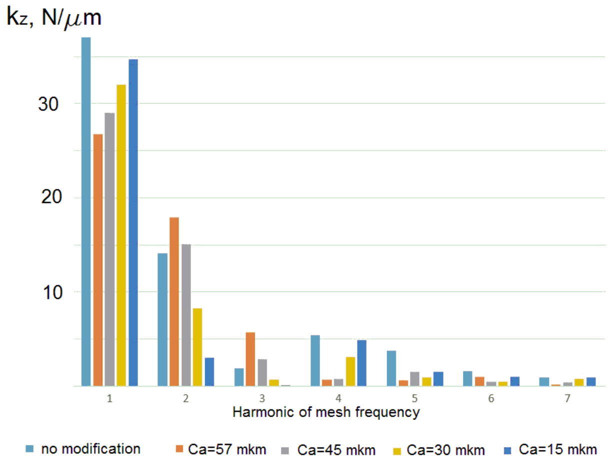

Fig. 3 Comparison of the harmonic amplitudes of mesh stiffness for varying amount of tip profile modification

The time varying mesh stiffness function is non-linear function due to tooth separation (impact) on resonance vibration and for the sampling of the side gap, is taken into account in the stiffness matrix through the additive to the dependences determining the mesh stiffness:

$$k(\boldsymbol{q}, t) = h(\delta_{12})\hat{k}(t), \qquad (2)$$

$$h(\delta_{12}) = \begin{cases} 1, \delta_{12} > 0, \\ 0, \delta_{12} \le 0. \end{cases} \qquad (3)$$

$$\delta_{12} = r_{b1}\varphi_1 + r_{b2}\varphi_2 + e(t) + (y_1 - y_2)cos\alpha + (x_1 - x_2)sin\alpha, \qquad (4)$$

where, $\hat{k}(t)$is a linear, periodically varying mesh stiffness, calculated using FEM; δ_{12} is a value of the relative displacement of spring points imitating the mesh stiffness of the gears; α is an pressure angle.

FEM helps to determine the characteristics of the dependence of bending and contact stresses in meshing, depending on the angle of rotation of the gears during one meshing period in a quasi-static setting.

Figure 4 shows the graphs of the bending stress distribution curves for different profile modification parameters. As can be seen in the figure, the using of the profile modification leads to an increase in the maximum bending stresses in the tooth root.

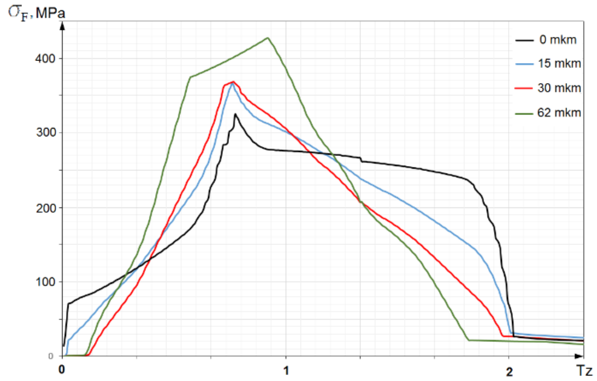

Fig. 4 Finite element calculation of bending root stress for one mesh cycle in gear tooth for varying amount of tip profile modification

Further, the results of the FEM calculations for the static formulation are combined with the results of the solution of the analytical system of the dynamic transmission model (1) - (4). The nonlinear differential equation of motion is solved numerically by using a fourth order, variable step Runge-Kutta (Dormand-Prince pair) numerical integration routine available in MATLAB. The amplitude-frequency response (AFR) of the system can be calculated due to the results of combining the simulation results.

Furthermore, one way to decrease dynamic forces is to use gear pair with a high contact ratio ε>2. Figure 5 shows two AFR of the dynamic response factor $K_v=F_{dyn}/F_{stat}$ for gears with ε=1, 68 (red line) and ε=2,05 (blue line).

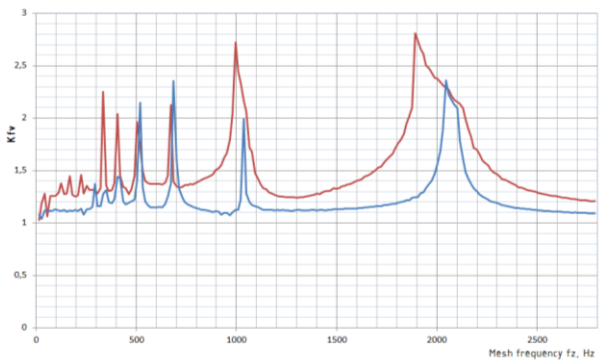

Fig. 5 Dynamic factor as function of rotating speed for varying amount of contact ratio ε

Figure 5 shows that gears with ε>2 operate with lower dynamic response factor in the zone of parametric resonance. There is also a significant narrowing of the zone of nonlinear resonance vibration caused by the loss of contact in the teeth.

2.2 Model of planetary gear set

To estimate the behavior of the nonlinear system of gears of the planetary set, it is necessary to consider a system with N = n + 2 lumped adjoint masses (fig.6) connected by elastic-damping springs, where n is the number of satellites of the planetary gear set. The system is balanced by the torque applied to the sun gear and the ring gear M_e and M_s, respectively.

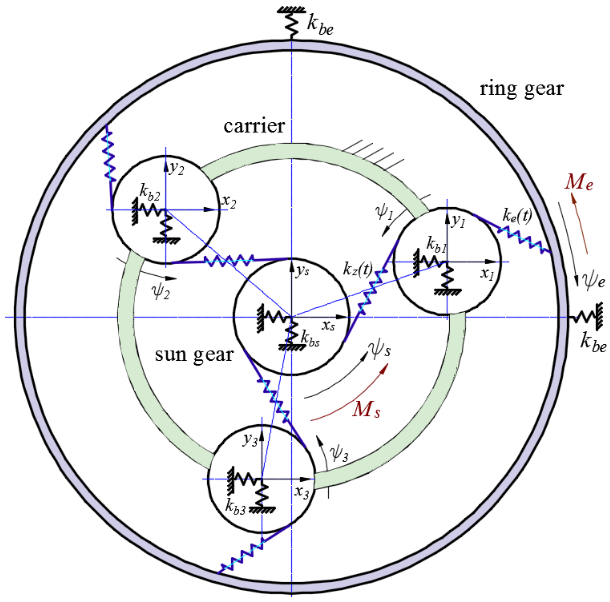

Fig. 6 Dynamic model of planetary gear set

The equation of motion for a planetary gear set with n satellites will have the following form:

$$\boldsymbol{M}\cdot\ddot{q}+\boldsymbol{C}\cdot\dot{q}+\boldsymbol{K}(t,q)\cdot q=\boldsymbol{F}(t), \qquad (5)$$

where column vector $\boldsymbol{q}=[\varphi_s,x_s,y_s,\varphi_1,x_1,y_1,\dots,\varphi_N,\ x_N,y_N,\varphi_e,x_e,y_e]^T$ - describes the movements of the gearbox elements: the sun gear, the epicycle, the satellites, respectively; F (t) - represents a vector of forces from applied externally loads, namely, external torque in the sun and the epicycle; x and y are the horizontal and vertical movements of the elements, φ is the angular displacement in radians relative to the center of the carrier for the central gears and relative to the center of the satellite satellites. The letters s and r refer to the sun gear and the epicycle, respectively, and the numeric characters 1 ... n refer to the satellite number. The total number of degrees of freedom of such a system is 3n + 6.

The use of floating sun gear configuration in the planetary gearbox of jet engine allows to equalize the loads between satellites and to compensate their technological misalignment errors. To consider the influence of the suppoting stiffness on dynamic loads in the gearbox, the most common designs supports of the planetary set elements are simulated (fig.7).

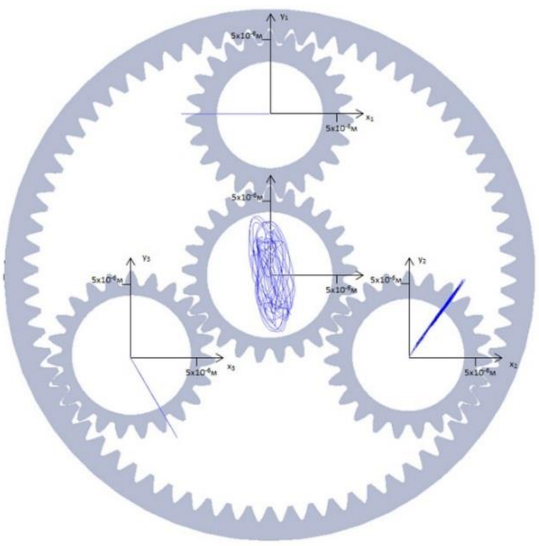

Fig. 7 Radial gear orbit of a three-planet gear set

Figure 8 shows the sun gear center orbit for different types of sun gear supports and the combination of meshing phases of satellites. The coincidence of the "satellite-sun" meshing phases, achieved by the number of teeth of the sun gear by the number of satellites, ensures a minimal displacement of the center of mass of the sun gear even with its floating suspension and the incompatibility of the "satellite-ring gear" phases (Fig. 8(I)). Figure 2 (II) shows the sun gear center locus on a floating support with an occasional phase shift in meshing with the satellites, as a result of which the amplitude of oscillations of its center of mass increases significantly, and the trajectory has an asymmetrical character.

Figure 8 shows the orbit of the sun gear when it is mounted on supports with stiffness exceeding the stiffness of gears (free support of sun fear). The mode amplitude of the center of mass is damped compared to the non-locating bearing, but at the same time the dynamic forces in the gears increase, especially in case of large spacing errors and technology inaccuracy.

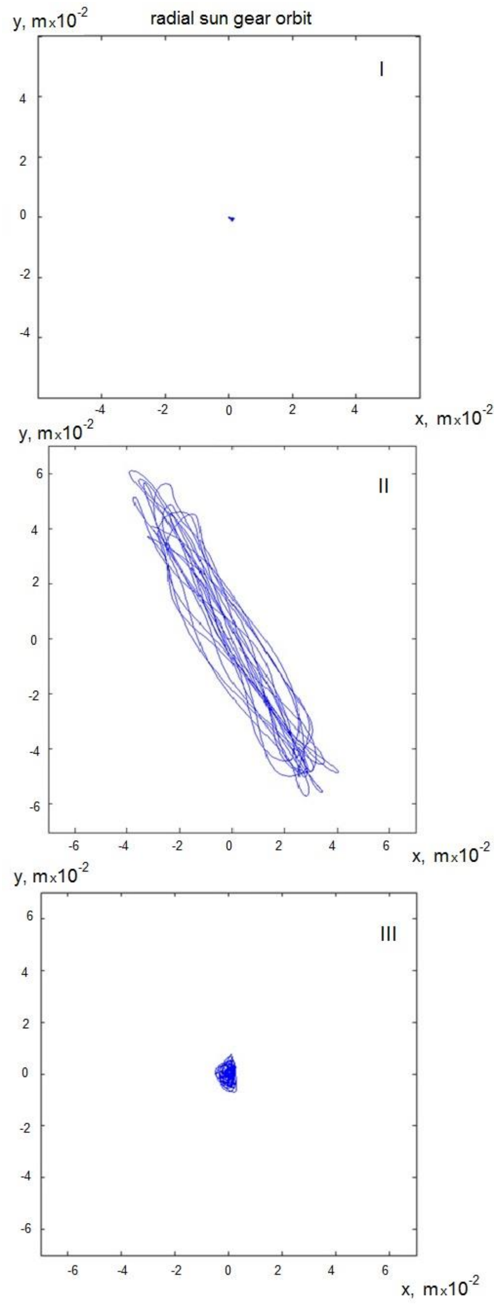

Fig. 8 Radial sun gear orbit of a three-planet gear set with: I – free support of sun gear without phase shift of meshing frequency; II - free support of sun gear with phase shift of meshing frequency; III – fixed support of sun gear

On fig. 9 shows the results of the dynamic simulation of a planetary gearbox with optimal modification teeth of gears (blue line) and for no modification gears (red line).

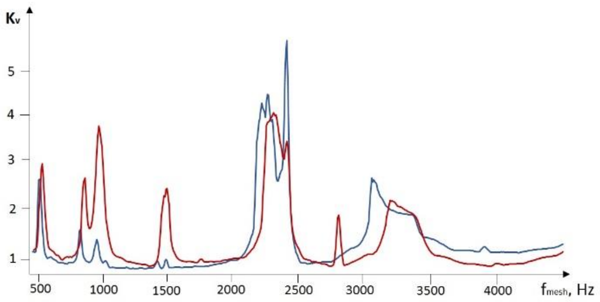

Fig. 9 Dynamic factor as function of rotating speed of sun gear with modification teeth (blue line) and no modification gears (red line)

To analyze the dynamic stress of the planetary gear set parts under the action of dynamic forces as well as to assess the degree of the effect of the ring gear compliance, a dynamic simulation of the planetary set was carried out using a finite element method in the dynamic analysis module using the Newmark method. A three-dimensional model of a planetary set with three satellites mounted on fixed axes was considered. The finite element model is constructed with the use of 20-node 3d finite elements and contact elements [10] with a side size of 0.8 mm. The simulation results are presented in fig. 10.

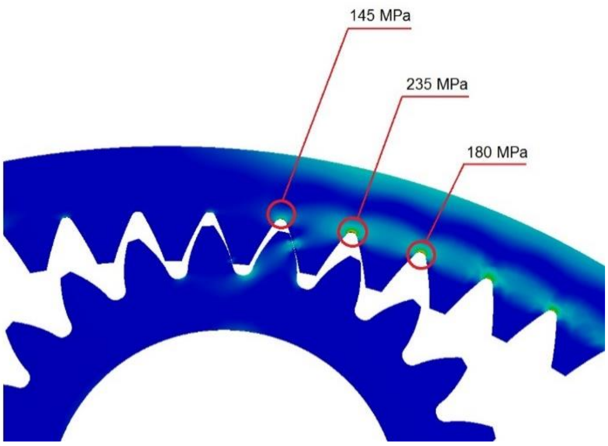

Fig. 10 Finite element calculation of bending root stress in ring gear teeth of planetary gear set model

Modeling with the help of FEM in dynamic formulation by an implicit method (Newmark method) showed that bending stress of a compliant ring gear can cause high values of bending stresses in the tooth root of the ring gear. Different values of thickness of the ring gear rim influence the strain-stress state in which the maximum stresses in the hollow teeth do not arise in the process of meshing with the satellites and after meshing in a position corresponding to the maximum deflection of the ring gear (fig.10).

3 Summary and Conclusion

This paper studies the influence of the sun gear supporting stiffness and the ratio of the phases of the satellite meshing to the dynamic forces in the gear set on the basis of the developed hybrid dynamic model of a planetary gear set. The combination of analytical methods and the finite element method makes it possible to obtain accurate estimates of dynamic forces in gears at low time costs achieved by simulating the process of gear meshing in the FEM and taking into account the possible loss of teeth contact. The results of the simulation show the possibility of significant reduction of dynamic forces in the planetary gear sets by introducing a profile modification of the teeth, the use of gears with a high contact ratio as well as by choosing the optimal combinations of the stiffness of the sun gear and the compliance of the ring gear. Modeling of the meshing of the planetary gear set with the help of FEM in dynamic setting shows that bending stress of the compliant ring gear can cause high values of bending stresses in the tooth root of the ring gear.

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