NONLINEAR MODEL PREDICTIVE CONTROL FOR OPTIMAL AIRCRAFT SEQUENCING

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Abstract

A real-time, bi-level feedback control algorithm for assigning arrival times is presented, which are optimal with regard to the cumulated fuel consumption of inbound aircraft. The bi-level structure implements the solution of optimal control problems for each individual aircraft to determine the minimum fuel consumption for the assigned arrival time. Additional constraints are included in the upper level to assure safe time separations at the merge point. After obtaining the solution of the original problem at the initial time point, the bi-level problem is approximated by a blended quadratic program. This program provides a prediction and correction of the optimal arrival times for the next time point and the respective change in initial states of all aircraft. Furthermore, exact solutions for the lower level optimal control problems are calculated for the updated arrival times. The algorithm is successfully validated for a example of nine aircraft and a frequency of six updates per minute for the arrival time assignment.

1 Introduction

Global air traffic has grown steadily over the last decades and is expected to continue growing at an exponential rate [1]. Consequently, challenges arise concerning the performance of the air traffic system, which has already reached its limits in some areas, such as runway capacity at highly frequented airports. Several research programs, e.g. the Single European Sky ATM Research Joint Undertaking (SESAR JU) [2] and Next Generation Air Transportation System (NextGen) in the USA, have been initiated to modernize the air traffic system, thus increasing efficiency and air space capacity.

A special focus has been drawn to the terminal maneuvering areas (TMA) of large aerodromes, which are a bottleneck of the air traffic system [3]. SESAR's User Preferred Routing and i4D trajectories as well as Extended Arrival Managers (E-AMAN) facilitate challenges by increasing flexibility and predictability. Similarly, programs like System Wide Information Management (SWIM) increase the amount of data available to all stakeholders of air traffic. [2]

Regarding challenges of the future air traffic system, it is imperative to focus research on smart algorithms, which can make use of the combined potential of modern technologies. This study is concerned with the solution of the sequencing problem under the conditions that safety constraints, such as temporal and spatial separations of aircraft, are fulfilled and each aircraft is operated optimally.

The problem of calculating an optimal sequence for arriving aircraft along with optimized trajectories has been formulated in different ways: BITTNER et al. [4, 5] combine several aircraft in a large scale optimal control problem (OCP), which is solved using a direct collocation method. TORATANI et al. [6] proposed a simultaneous optimization of the trajectory and sequence by calculating a criterion function using indirect optimal control methods and varying the final time. Subsequently, a mixed integer linear program (MILP) including constraints for the time separation at the runway is formulated and solved. A similar approach is utilized in [7]. A bi-level algorithm is proposed in [8], where the upper level sequencing problem is formulated as a combinatorial program and solved using a genetic algorithm. The embedded lower level OCPs are solved sequentially by applying a direct collocation method. Other formulations can be found in [9, 10].

A similar problem is addressed by HULT et al. [11] and ZANON et al. [12]: The problem of coordinating traffic at an intersection is solved using a primal decomposition and a subsequent solution of the distributed problem. These are embedded within an upper level problem that provides occupancy time slots for each car.

In this study, the bi-level formulation of the optimal arrival time assignment is solved in a first step to generate nominal arrival times and corresponding optimal trajectories. Furthermore, the algorithm is extended by a quadratic approximation of the optimal solution over all variations of fixed parameters. Specifically, these are the current position of each aircraft, which acts as the initial boundary condition for the OCPs. In order to achieve a good approximation of the optimal solution, the optimization vector is updated using the blended solution of two problems [13]:

- a linear prediction of the optimal solution for a parameter variation ("predictor")
- a Newton step of the problem at the new parameter ("corrector").

The remaining paper is structured as follows: The original parametric optimization problem for the optimal arrival time assignment is presented in section 2. As outlined above, the problem is constrained by optimal control problems, which are described in section 3. Furthermore, the quadratic approximation and respective calculation of the updated problem is detailed in section 4. Finally, a case study is presented in section 5 before concluding the paper in section 6.

2 Arrival Time Assignment

The optimization problem associated to the sequencing problem considered here is defined as follows: Find the optimal final times $\hat{t}^*_{f,i}$ for each aircraft i, which minimizes the overall fuel consumption by maximizing the final mass of the aircraft:

$$\hat{\mathbf{t}}^*_{\mathrm{f}}\left(\mathbf{x}_0, \mathbf{t}_0\right) = \underset{\left(\hat{\mathbf{t}}_{\mathrm{f}}, \hat{\mathbf{m}}_{\mathrm{f}}\right)}{\arg\min} \sum_{i=1}^{n} -\hat{m}_{\mathrm{f,i}} \tag{1}$$

where n represents the number of aircraft. The optimization variables are the arrival times $\hat{t}_{\mathrm{f,i}}$ and the optimal final mass $\hat{m}_{\mathrm{f,i}}$ of each aircraft, which implicitly depend on the initial states $\mathbf{x}_0$ and times $\mathbf{t}_0$ of each trajectory.

To respect operational constraints imposed by the safety regulations within the air traffic system,

$$n_c = \frac{n}{2}\left(n-1\right) \tag{2}$$

algebraic inequalities as follows are added to the optimization problem:

$$\begin{aligned} c_{\hat{t},\mathrm{ij}} &= \left(\hat{t}_{\mathrm{f,i}} - \hat{t}_{\mathrm{f,j}} - \Delta \underline{t}_{\mathrm{ij}}\right)^2 - \left(\Delta \bar{t}_{\mathrm{ij}}\right)^2 \geq 0,\, i \neq j \\ \Delta \underline{t}_{\mathrm{ij}} &= \frac{\Delta t_{\mathrm{WTC,ji}} - \Delta t_{\mathrm{WTC,ij}}}{2} \\ \Delta \bar{t}_{\mathrm{ij}} &= \frac{\Delta t_{\mathrm{WTC,ji}} + \Delta t_{\mathrm{WTC,ij}}}{2} \end{aligned} \tag{3}$$

These inequalities ensure a pairwise time separation $\Delta t_{\mathrm{WTC,ij}}$ or $\Delta t_{\mathrm{WTC,ji}}$ between aircraft i and j, depending on which of the aircraft arrives at the final approach fix first, as depicted in Fig. 1. The required time separation as defined in ICAO Doc. 4444 [14] in dependence on the aircrafts' wake turbulence category is displayed in Table 1a.

To facilitate the solution of the bi-level problem and the calculation of real-time updates, the distances separation constraints specified in Table 1b are approximated by a time separation. The maximum value of this estimation and the required time separation stated in Table 1a is considered within the respective constraints given in equation (3).

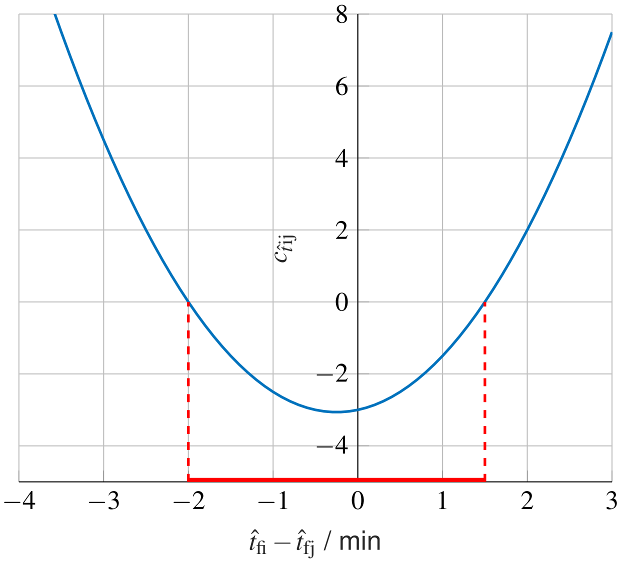


Fig. 1 : Time separation constraint function between aircraft i (H) and j (M).

Table 1: Separation based on the *wake turbulence categories* L(ight), M(edium), H(eavy), and S(uper Heavy) of preceding (row) and following aircraft (column). [14]

(a) Time separation in *min*

	L	M	H	S
L	0	0	0	0
M	3	2	0	0
H	3	2	0	0
S	3	3	2	0

(b) Horizontal distance separation in *nm*

	L	M	H	S
L	3	3	3	3
M	5	3	3	3
H	6	5	4	4
S	8	5	4	4

Finally, the lower level OCPs are added as additional equality constraints for each aircraft:

$$c_{\hat{m},\mathrm{i}} = m^*_{\mathrm{f,i}}\left(\mathbf{x}_{0\mathrm{i}}, t_{0\mathrm{i}}\right) - \hat{m}_{\mathrm{f,i}} = 0 \tag{4}$$

where $\hat{m}_{\mathrm{f,i}}$ is a slack variable for the optimal final mass introduced in equation (1). Furthermore, $m^*_{\mathrm{f,i}}$ represents the value function of the respective optimal control problem, which is a function of the initial state $\mathbf{x}_{0,\mathrm{i}}$ and time $t_{0,\mathrm{i}}$ of the aircraft.

3 Optimal Control Problems

The lower level problems receive the assigned arrival times from the upper level problem in each iteration. Furthermore, additional quantities are fixed, such as the current states of the aircraft and the current time. This determines the duration until reaching the fixed final boundary condition, i.e. the final approach fix.

However, the problem of finding a trajectory respecting the above mentioned constraints has infinite dimensions. In order to obtain an optimal solution it is assumed, that each pilot tries to find the optimal solution regarding the operating cost. In this paper, these are assumed to be only dependent on the fuel consumption. The calculation of the described trajectories leads to the OCP:

$$\begin{aligned}
m^*_{\mathrm{f,i}}\left(\mathbf{x}_{0,\mathrm{i}}, t_{0,\mathrm{i}}\right) &= \min_{\mathbf{u}\in\mathbf{U}} -m_{\mathrm{f,i}} && (5)\\
s.t. & && \\
\dot{\mathbf{x}}_\mathrm{i} &= \mathbf{f}\left(\mathbf{x}_\mathrm{i}, \mathbf{u}_\mathrm{i}\right) && (6)\\
\boldsymbol{\Psi}\left(\mathbf{x}_{0,\mathrm{i}}, \mathbf{x}_{\mathrm{f,i}}, t_{0,\mathrm{i}}\right) &\leq 0 && (7)\\
\mathbf{c}_\mathrm{i}\left(\mathbf{x}_\mathrm{i}, \mathbf{u}_\mathrm{i}\right) &\leq 0 && (8)\\
\hat{t}_{\mathrm{f,i}} - t_{\mathrm{f,i}} &= 0 && (9)
\end{aligned}$$

Besides the objective function given in equation (5), several constraints have to be respected, such as the aircraft's dynamics (6), the initial and final boundary conditions (7), as well as path inequality constraints (8). Finally, the arrival time $\hat{t}_{\mathrm{f,i}}$ of each aircraft i is determined by the upper level problem as denoted in equation (9).

Each aircraft is modeled as a point mass considering a quasi static decrease in the aircraft mass due to the fuel consumption according to

[15]. The resulting seven states are listed in Table 2. The propagation in WGS84 coordinates and consideration of a round and rotating earth model in the translation equations of motion [16] provide the possibility to extend the problem to a larger AMAN horizon:

$$\begin{aligned} & \dot{\vec{\mathbf{V}}}_K + \left(\vec{\boldsymbol{\omega}}^{OK}\right)_K \times \left(\vec{\mathbf{V}}_K\right)_K^E \\ + \; & \underbrace{\left(\vec{\boldsymbol{\omega}}^{EO}\right)_K \times \left(\vec{\mathbf{V}}_K\right)_K^E}_{\text{round earth}} \\ + \; & \underbrace{2 \cdot \left(\vec{\boldsymbol{\omega}}^{IE}\right)_K \times \left(\vec{\mathbf{V}}_K\right)_K^E - \left(\vec{\boldsymbol{\omega}}^{IE}\right)_K \times \left[\left(\vec{\boldsymbol{\omega}}^{IE}\right)_K \times \left(\vec{\mathbf{r}}^G\right)_K\right]}_{\text{rotating earth}} \\ = \; & \tfrac{1}{m} \Sigma \left(\vec{\mathbf{F}}\right)_K \end{aligned} \tag{10}$$

The forces acting on the aircraft as well as the fuel consumption are modeled according to the Base of Aircraft Data family 3 (BADA 3) published by EUROCONTROL [17]. Hence, the control variables are chosen as the kinematic angle of attack, bank angle, and the thrust lever position as displayed in Table 2.

While the final boundary conditions in equation (7) are fixed to the final approach fix (FAF), we introduce slack variables for the initial boundary conditions to respect the current state of each aircraft:

$$\boldsymbol{\Psi}_{IBC}\left(\mathbf{x}_{0,\mathrm{i}}, t_{0,\mathrm{i}}\right) = \left[\begin{array}{c} \mathbf{x}_i\left(t_0\right) - \mathbf{x}_{0,i} \\ t_{current} - t_{0,i} \end{array}\right] = 0 \tag{11}$$

where the current state of the aircraft $\mathbf{x}_{0,i}$ as well as the current time $t_{current}$ are fixed parameters to the optimization problem. The value function $m^*_{\mathrm{f,i}}\left(\mathbf{x}_{0\mathrm{i}}, t_{0\mathrm{i}}\right)$ of the OCP, i.e. the optimal objective, is a function of these parameters.

The OCPs are solved using a direct collocation transcription method implemented by the optimal control toolbox FALCON.m [18]. The resulting nonlinear program (NLP) is solved by utilizing the interior point solver IPOPT [19]. Furthermore, the gradients

$$\nabla_{\left(\mathbf{x}_{0\mathrm{i}}, t_{0\mathrm{i}}\right)} m^*_{\mathrm{f,i}}\left(\mathbf{x}_{0\mathrm{i}}, t_{0\mathrm{i}}\right)$$

and hessian matrices of the value functions

$$\nabla^2_{\left(\mathbf{x}_{0\mathrm{i}}, t_{0\mathrm{i}}\right)} m^*_{\mathrm{f,i}}\left(\mathbf{x}_{0\mathrm{i}}, t_{0\mathrm{i}}\right)$$

with respect to the current state of the aircraft have to be calculated and provided to the upper level problem. To this end, post-optimal sensitivities [20, 21] are employed. The derivatives of the upper level problem are constructed according to [15].

4 Near-Optimal Feedback Control

Solving the bi-level optimization problem including all lower level OCPs may require a significant amount of time, depending predominantly on the initial guess and the solver settings. Hence, an exact solution is not feasible for a real-time application. Instead, information contained in the nominal solution is utilized to approximate the optimization problem for parameter variations in the neighborhood of the nominal solution. In a first step this can be achieved by obtaining a piecewise linear prediction of the solution for a change in the parameters. Furthermore, the quadratic approximation can be utilized to perform a correction.

4.1 Prediction

For the prediction of the optimal solution for a variation of parameters, a quadratic program (QP) is formed using the Hessian matrix of the optimization problem at the current point with respect to the optimization variables and the fixed parameters:

$$\nabla^2_{(\mathrm{z,p})^T} \mathcal{L} = \left[\begin{array}{cc} H & \nabla^2_{\mathrm{pz}} \mathcal{L}^T \\ \nabla^2_{\mathrm{pz}} \mathcal{L} & \nabla^2_{\mathrm{pp}} \mathcal{L} \end{array}\right] \tag{12}$$

The solution of the resulting QP

$$\begin{aligned} \min_{\Delta z} \quad & \tfrac{1}{2} \Delta z^T H \Delta z + \Delta p^T \nabla_{\mathrm{pz}} \mathcal{L} \Delta z \\ s.t. \quad & \mathbf{c} + \nabla_{\mathrm{z}} \mathbf{c}^T \Delta z + \nabla_{\mathrm{p}} \mathbf{c}^T \Delta p \leq 0 \end{aligned} \tag{13}$$

approximates the optimal deviation of the optimization variables with respect to a change in the parameters. Hereby, the constraints of the original problem are respected in a neighborhood of the current parameters values, specifically changes in the active set are detected [13].

Table 2: States and controls for the point mass aircraft model along with their min and max values.

	Description	Min	Max
φ	geodetic latitude	$-90°$	$90°$
λ	geodetic longitude	$-180°$	$180°$
h	altitude above reference ellipsoid	$5000\,ft$	∞
V_K	kinematic speed	$V_{k,min}$	$V_{k,max}$
χ_K	flight path azimuth angle	$-180°$	$180°$
γ_K	flight path climb angle	$-45°$	0
m	aircraft mass	ZFW_k	m_0
α_K	kinematic angle of attack	$-8°$	$12°$
μ_K	kinematic bank angle	$-30°$	$30°$
δ_T	thrust lever position	0	1

4.2 Correction

Despite the fact that the QP approximation is able to detect active set changes and thus "corners" in the optimal solution, the linear prediction of the solution of the perturbed optimization problem may include errors depending on the perturbation magnitude of the disturbed parameters p. Hence, the obtained solution may also be seen as an initial guess to the perturbed problem, which can be solved to optimality in a next step. As this may be computationally expensive, a correction can be achieved by performing a Newton step instead [13], which can be calculated by the following QP:

$$\begin{aligned} \min_{\Delta z} \quad & \tfrac{1}{2}\Delta z^T H \Delta z + \nabla J^T \Delta z \\ s.t. \quad & \mathbf{c} + \nabla_z \mathbf{c}^T \Delta z \leq 0 \end{aligned} \tag{14}$$

The correction step provides accurate results as the prediction increases the accuracy of the linearization at the current point [13].

4.3 Real-time Iteration

Reducing the computational effort further, the problem of finding prediction and correction steps can be blended into a single QP as follows [13]:

$$\begin{aligned} \min_{\Delta z} \quad & \tfrac{1}{2}\Delta z^T H \Delta z + \nabla J^T \Delta z + \Delta p^T \nabla_{pz} \mathcal{L} \Delta z \\ s.t. \quad & \mathbf{c} + \nabla_z \mathbf{c}^T \Delta z + \nabla_p \mathbf{c}^T \Delta p \leq 0 \end{aligned} \tag{15}$$

It is important to notice that the evaluation of the Hessian matrix and gradients requires information of the lower level problems. Therefor, the solution of all lower level problems under the updated constraints is required, which can be obtained in different ways, e.g. by performing similar updates for the lower levels. For the present application it has proven to be sufficient to resolve the lower level problems utilizing the warm start functionality of the interior point solver IPOPT [19]. Please note that the proposed problem formulation allows for a fully independent solution of the lower level problems. Specifically, the problems can be solved in parallel or on distributed hardware.

5 Case Studies

A validation of the performed algorithm is conducted by a two case studies: The first study concerns an ideal case, where the aircraft are able to follow the optimal trajectory perfectly. Secondly, a more realistic case is considered by introducing uncertainties. For this case, a normally distributed noise term is added to the current position of the aircraft, which is provided to the feedback control. The standard deviation for the noise terms are set to ten meters in the north, east, and down direction, with a mean value of zero.

For both case studies, nine aircraft are considered approaching the final approach fix of run-

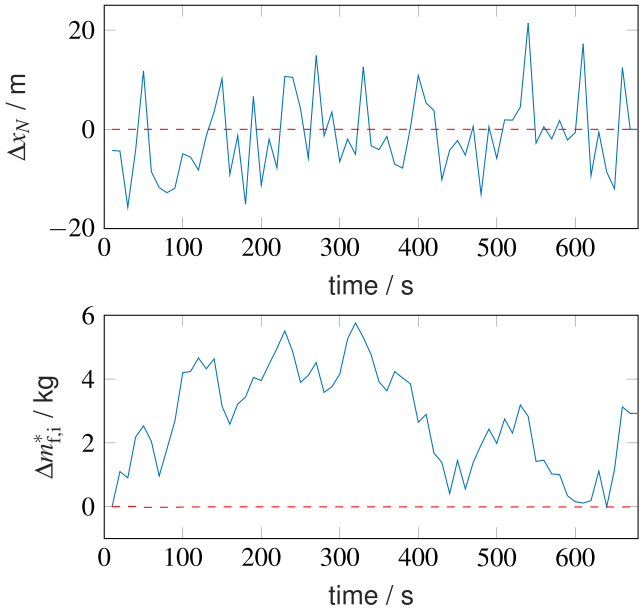


Fig. 2 : Variation of the optimal final mass of an aircraft due to a disturbance in the current position. The red, dashed curves represent the ideal case, the blue, solid lines show the case subject to deviations from the optimal trajectories.

way 08L of Munich Airport. The aircraft enter the TMA at a uniformly distributed time difference with a mean value of 2 minutes and a standard deviation of 30 seconds, i.e. aircraft have to be delayed/accelerated to meet the separation constraints at the final approach fix. This can be observed for the nominal solution displayed in Fig. 3a, where the yellow aircraft takes a slight detour.

The algorithm is run on a Core i7-4770 CPU@3.40 GHz, 16 GB RAM desktop computer running Windows 10. The nominal solution is obtained by an exact bi-level optimization in 230.13 seconds. Subsequently, the solution is updated at a frequency of 0.1 Hz using the algorithm proposed in section 4 for the current states of all aircraft. The solution for the case of disturbed trajectories is shown in Fig. 3b.

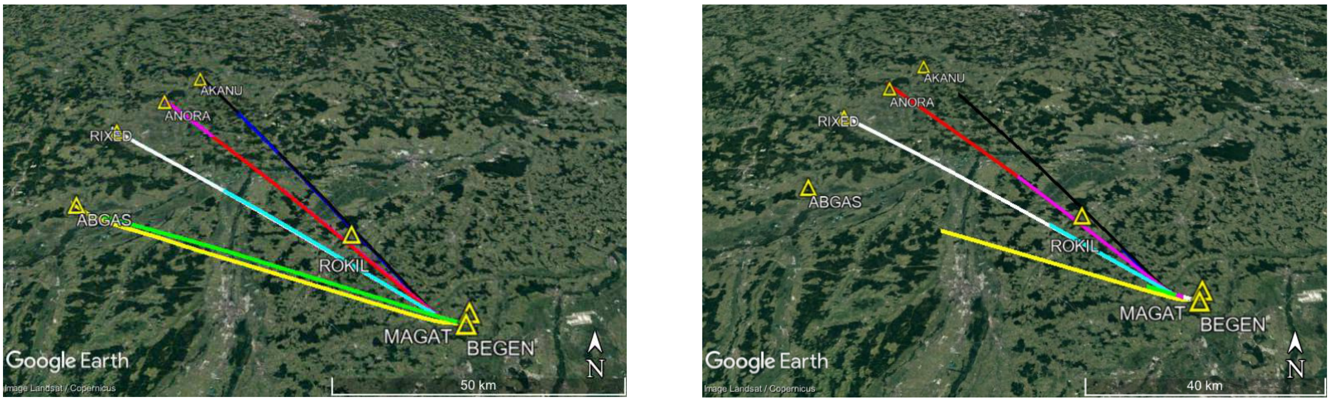
Figure 2 shows the deviation of the position of aircraft 1 with respect to the optimal trajectory, which is zero for the ideal case. Below, the current estimation of the optimal final mass is shown. Again, the dashed graph depicts the ideal case and represents the accuracy of the algorithm. The solid line illustrates the updated solution for the current state of the aircraft.

Finally, Fig. 4 depicts the CPU times required to solve the lower level OCPs for each aircraft, which feature a perturbed initial boundary condition according to the current position of each aircraft. The maximum CPU time for the solution of an optimal control problem is 5.23 seconds (mean: 1.06 s, std: 0.94 s), which is well below the required 10 seconds to achieve an update rate of 0.1 Hz.

6 Conclusion

A bi-level algorithm for finding optimal arrival times and trajectories for multiple aircraft approaching an airport was presented. The algorithm is extended by an approximation of the perturbed solution by a quadratic program, accounting for the current position of each aircraft. The validation of the algorithm shows the feasibility of a real-time application for an approach scenario, if the solution of the lower level optimal control problems is performed in parallel.

Future work will include the implementation of a quadratic approximation of the perturbed solutions of the lower level optimal control problems, which may be implemented at a higher frequency to act as an optimal guidance algorithm for the individual aircraft. Furthermore, the development of the implementation on distributed hardware will enable the application in inbound aircraft. Finally, the algorithm may be combined with global optimization techniques to provide an optimal initial guess and help to overcome the drawbacks of gradient-based optimization algorithms, such as convergence to a local minimum.



(a) Nominal solution before entry of aircraft into the TMA.

(b) Updated solution at 830 seconds, shortly before the first aircraft reaches the FAF.

Fig. 3 : Optimal trajectories at different time points.

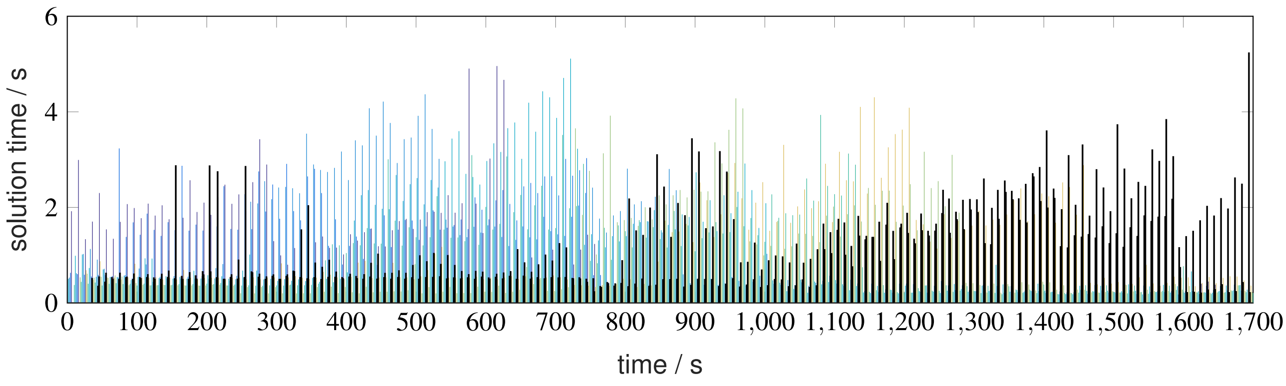


Fig. 4 : CPU times required for the solution of the perturbed optimal control problem.

7 References

[1] *Flightpath 2050: Europe's vision for aviation ; maintaining global leadership and serving society's needs ; report of the High-Level Group on Aviation Research*. Policy / European Commission. Publ. Off. of the Europ. Union, Luxembourg, 2011.

[2] SESAR Joint Undertaking. European ATM master plan, 2012.

[3] Nigel Dennis. Developments of hubbing at European airports. *Air & Space Europe*, 3(1-2):51–55, 2001.

[4] Matthias Bittner, Matthias Rieck, Benedikt Grüter, and Florian Holzapfel. Optimal Conflict Free Approach Trajectories for Multiple Aircraft. In *ENRI Int. Workshop on ATM/CNS (EIWAC2015)*. Tokyo, Japan, 2015.

[5] Matthias Bittner, Matthias Rieck, Benedikt Grüter, and Florian Holzapfel. Optimal Approach Trajectories for Multiple Aircraft Considering Disturbances and Configuration Changes. In *ICAS 30th International Congress of the International Council of the Aeronautical Sciences*, 2016.

[6] Daichi Toratani, Daniel Delahaye, S. Ueno, and T. Higuchi. Merging Optimization Method with Multiple Entry Points for Extended Terminal Maneuvering Area. In *EIWAC 2015, 4th ENRI International Workshop on ATM/CNS*, 2015.

[7] Daichi Toratani, Navinda K. Wickramasinghe, and Eri Itoh. Merging Optimization Method Application to Arrival Scheduling Algorithm for Parallel Runways. In *AIAA Modeling and Simulation Technologies Conference*, Reston, Virginia, 2017. American Institute of Aeronautics and Astronautics.

[8] Benedikt Grüter, Matthias Bittner, Matthias Rieck, Johannes Diepolder, and Florian Holzapfel. Optimal Sequencing in ATM Combining Genetic Algorithms and Gradient Based Methods to a Bilevel Approach. In *ICAS 30th International Congress of the International Council of the Aeronautical Sciences*, 2016.

[9] Florian Fisch, Matthias Bittner, and Florian Holzapfel. Optimal Scheduling of Fuel-Minimal Approach Trajectories. *Journal of Aerospace Operations*, 2014.

[10] Hang Guo. Application of Genetic Algorithms in the New Air Traffic Management Simulation System. *Physics Procedia*, 33:604–611, 2012.

[11] R. Hult, M. Zanon, S. Gros, and P. Falcone, editors. *Primal decomposition of the optimal coordination of vehicles at traffic intersections: 2016 IEEE 55th Conference on Decision and Control (CDC)*, 2016.

[12] Mario Zanon, Sebastien Gros, Henk Wymeersch, and Paolo Falcone. An Asynchronous Algorithm for Optimal Vehicle Coordination at Traffic Intersections. *IFAC-PapersOnLine*, 50, 2017.

[13] Moritz Diehl, Uslu Ilknur, Findeisen Rolf, Schwarzkopf Stefan, Allgöwer Frank, Bock H. Georg, Bürner Tobias, Gilles Ernst Dieter, Kienle Achim, Schlöder Johannes P., and Stein Erik. Real-Time Optimization for Large Scale Processes: Nonlinear Model Predictive Control of a High Purity Distillation Column. In Martin Grötschel, Krumke Sven O., and Rambau Jörg, editors, *Online Optimization of Large Scale Systems*, pages 363–383. Springer Berlin Heidelberg, Berlin, Heidelberg, 2001.

[14] International Civil Aviation Organization. Air traffic management: Doc 4444.

[15] Benedikt Grüter, Matthias Bittner, Matthias Rieck, Johannes Diepolder, and Florian Holzapfel. Bi-Level Homotopic Aircraft Sequencing Using Gradient-Based Arrival

Time Assignment and Direct Optimal Control. In *57th Israel Annual Conference on Aerospace Sciences*, 2017.

[16] Rudolf Brockhaus, Wolfgang Alles, and Robert Luckner. *Flugregelung*. Springer, Dordrecht, 2011.

[17] A. Nuic. User Manual for the Base of Aircraft Data (BADA) Revision 3.11. *Atmosphere*, 2010:001, 2010.

[18] Matthias Rieck, Matthias Bittner, Benedikt Grüter, and Johannes Diepolder. FALCON.m: User Guide. 2016.

[19] Andreas Wächter and Lorenz T. Biegler. On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming. *Mathematical programming*, 106(1):25–57, 2006.

[20] Anthony V. Fiacco. *Introduction to sensitivity and stability analysis in nonlinear programming*, volume 165 of *Mathematics in science and engineering*. Acad. Press, New York, 1983.

[21] Christof Büskens. *Optimierungsmethoden und Sensitivitätsanalyse für optimale Steuerprozesse mit Steuer- und Zustands-Beschränkungen: Münster (Westfalen), Univ., Diss., 1998*. 1998.

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