PROBABILISTIC METHOD OF DETERMINATION OF ALLOWABLE DESIGN PARAMETER SCATTERS ON FLUTTER AND STRENGTH CRITERIA

Viacheslav V. Chedrik (Junior)
Central Aerohydrodynamic Institute (TsAGI), Zhukovsky, Moscow region, Russia

Keywords: *probability analysis, safety factor, structural strength, flutter*

Abstract

This paper proposes a probabilistic method to solve a problem of determination of allowable scatters in structural and load parameters at the assumption that the failure probability and safety factor are given. The method has been applied to evaluation of probabilistic characteristics of the optimized wing structures for civil aircraft.

1 Introduction

The main priority throughout the history of aircraft development is structural safety. To provide it the factors of safety are used at aircraft design. Their application is due to uncertainty in the structural strength and operational loads. At aircraft design the safety factors accommodate uncertainty in material properties, design process assumptions, manufacturing tolerance, structural defects, aircraft maintenance, weather and environmental conditions. Design of the safe structure can be done via two approaches: deterministic and probabilistic. Traditionally, deterministic approach based on aviation regulations is most used.

Structural safety is achieved by combining the safety factors with structural analyses and tests. It is well known that aircraft structures should possess minimum weight at high safety, thus low safety factors are needed in comparison with buildings. To reduce weight the structural optimization with the deterministic approach is widely used. High safety is reached by employing certified materials and production technologies at aircraft manufacturing.

In fact, the application of traditional design practices may miss manufacturing defects in the structure. Possible deterministic design errors can lead to additional costs for aircraft companies since the aircraft will need to be improved in its problem zones, which are detected only during aircraft operation.

The probabilistic approach is known to be based on the variation of the design parameters, so the sensitivity analysis of the structure to the varied parameters in its different zones can be automatically performed. The sensitivity analysis procedure provides information about zones and structural parameters that have the greatest effect on the parameter responsible for the structural safety (stress, strain, critical flutter speed, etc.).

The main advantage of the probabilistic approach over the deterministic one is an honest evaluation of the structural reliability, since the probabilistic approach reflects the true nature of loads and the load-carrying capacity of structure, which the deterministic methods hides in safety factor. One more advantage of the probabilistic approach is that at its using a part of the physical experiments can be replaced by computations.

Recently, the concept of design based on the probabilistic approach began actively to be developed in aviation industry. A few decades ago, it was almost unused in engineering practice due to insufficiency of computer powers. Today, the world community is imposing increasingly aircraft safety requirement due to aggressive commercial competition. Therefore, along with the known approaches, it is necessary to consider new design methods to achieve both structural weight improvement and its increased safety.

In this paper, we propose a probabilistic method of static strength and flutter estimation of wing structure designed by a deterministic approach. The results of the performed studies of two wings are presented. The estimation of allowable design parameter scatter that satisfies the requirements of Russian Aviation Regulation for transport category airplanes (AP-25) in strength and aeroelasticity is given.

2 Main Concepts for Analysis of Failure Probability

This section presents the basic concepts that are used in the development of the probabilistic method. The method is intended to determine the permissible scatter in design parameters at satisfying the flutter and strength requirements.

2.1 Influence of Uncertainty on failure probability

Information about statistical distributions of material properties, geometry, and load is not always available to the extent required. The lack of completeness of information can lead to large errors in the calculation of the structural failure probability. For this reason, many engineers are skeptical about accuracy of failure probability prediction. Nevertheless, in aviation industry probabilistic methods of estimation of structural safety are in intensive development and improvement.

Fig. 1 shows an example of the influence of uncertainty in strength characteristics on the failure probability that is illustrated by two normal distributions of:

1. Loads with expected value μ_L=1.0 and standard deviation σ_L=0.1;
2. Strength with expected value μ_S=1.5 and variable standard deviation σ_S=0.1-0.5.

As the standard deviation σ_S increases, the probability density function (PDF) of the structural strength will become more flat and the area of intersection of the two PDF will increase.

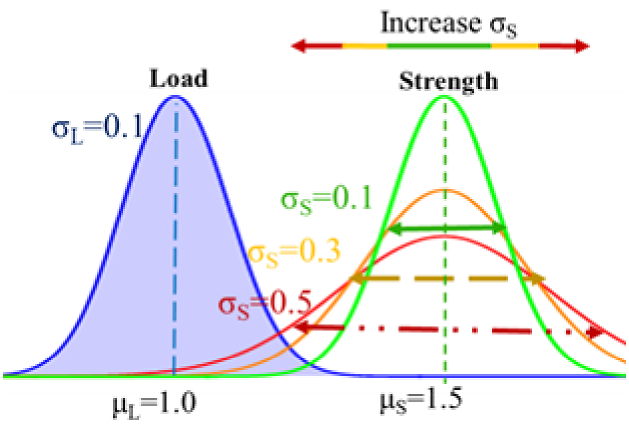

Fig. 1. Probability Density Functions of Load and Strength with variable standard deviation

Dependence of the structural failure probability on the strength standard deviation for this PDF of load and strength is shown in Fig. 2. Structural failure probability P_f is calculated by the well-known Eq. 1 from the theory of probability and mathematical statistics:

$$P_f = \varphi\left[\frac{-(\mu_S - \mu_L)}{\sqrt{\sigma_S^2 + \sigma_L^2}}\right], \qquad (1)$$

where φ is the standard normal distribution function (corresponding to a Gaussian variable with expected value 0 and standard deviation of 1.0).

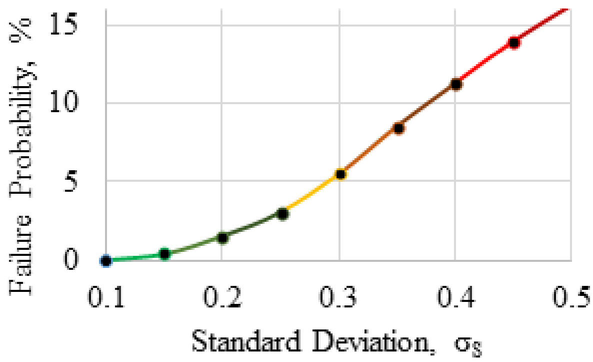

Fig. 2. Dependence of Structural Failure Probability on the Standard Deviation

It can be seen that with an increase in the standard deviation the probability of failure increases quickly. Therefore, it is important to know the scatter in structural parameters and loads.

2.2 Influence of Safety Factor on Failure Probability

The application of the conventional safety factor in the aviation industry has simplified aircraft design. Nevertheless, the use of new materials and non-traditional aircraft layouts puts new requirements on strength, fatigue, aeroelasticity, etc. It gives background for the estimation of safety factors in order to ensure safety at high weight perfection.

The percentage of composite materials in aircraft structures is becoming larger. They are applied in such important components as wing and fuselage. The conventional approach requires the introduction of additional safety factors, which values are chosen as result of analysis of the statistical tests of material samples. It can be noticed that real behavior of composite structure is not always predictable with using deterministic methods. Numerous tests of composite structural elements and aircraft components are too expensive. Application of probabilistic methods for strength analysis of composite structures can play an essential role in design practice to decrease development cost and to improve reliability of obtained results in prediction of real behavior of composite structures.

It is important to understand that the safety factor is closely related to the reliability or structural failure probability. We demonstrate the influence of the safety factor on the failure probability on the example of two normal distributions of load and strength. Unlike the example given in the previous section, the safety factor f=1.2–1.5 and corresponding to it the expected value $\mu_S = f\,\mu_L$ of structural strength PDF are varied here, as shown in Fig. 3.

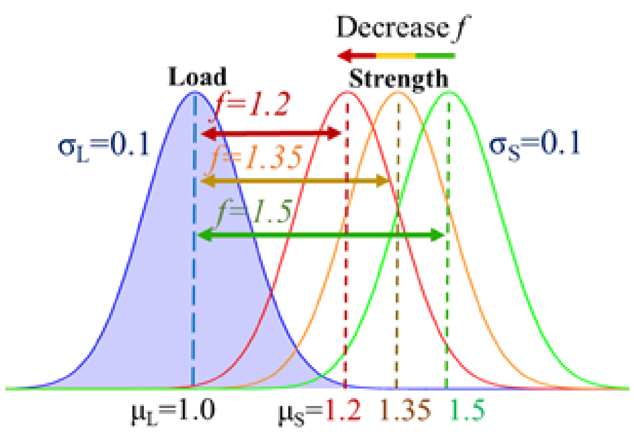

Fig. 3. Probability Density Functions of Load and Strength with variable safety factor

When the safety factor f decreases, the probability density function of the strength will shift to the left and the intersection area of the two PDF will increase. The dependence of the failure probability on the safety factor is shown in Fig. 4.

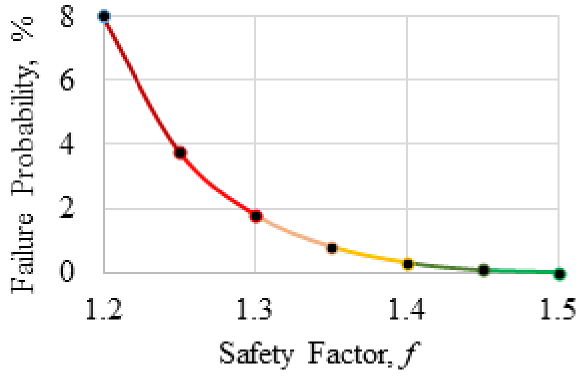

Fig. 4. The Dependence of the Structural Failure Probability on the Safety Factor

It can be seen that while the safety factor decreases, the probability of structural failure increases. The determination of the required safety factor based on the known variations in the structural parameters and loads will allow to design a structure of minimum weight at satisfying the safety requirements.

2.3 Concept of Failure Probability of Aircraft Structure

The founder of the concept was B. Lundberg, who in the middle of the last century defined the flight safety of aircraft structures by the probability of events. He justified the acceptable failure probability of aircraft structure by a probability value of 10^{-9} per one flight hour [1]. At that time, there was an increased interest in the probabilistic and statistical methods in the aircraft industry. The papers [2-4] are made further significant contributions to development of the failure probability in aircraft structural design.

The backgrounds for the application of probabilistic methods to estimate the reliability or structural failure probability are established in the AP-25 in section A-0, paragraph 3 "Probability of special situations" and paragraph 613 "Strength characteristics of materials and their design values." It is written there that the required flight safety should correspond to the probability of an accident not larger than 10^{-6} per flight hour. Although these requirements are not applied to an aircraft structure, they gave rise to the development of probabilistic approach to design of aircraft primary structures. Perhaps, in the future they will become a basis for development of paragraphs of Aviation Regulations in sections C and D, concerning the strength and the structural design in the context of probabilistic approaches.

2.4 Finite Element Method and Probabilistic Analysis

The practical application of the finite element method (FEM) on modern computer clusters makes it possible to carry out a huge number of structural analyses with different characteristics of materials, design parameters and load cases. This can be a basis for developing probabilistic methods for strength and aeroelasticity analysis, since these methods require a huge number of calculations.

Probabilistic analysis involves the use of methods for calculation of a failure probability of structure, which are based on analysis of different strength and aeroelasticity characteristics. The methods are based on the theory of probability and mathematical statistics.

Below in the paper, an algorithm for calculating the failure probability is proposed where the normal distribution law specifies the varied parameters of materials, structures and loads. On the basis of these parameters the data arrays for the structures are generated. Analysis of the structures is accomplished by using modern finite element software. Application of the probabilistic approach together with finite element analysis of aircraft structures allows to create a huge database that can be used at various stages of design.

3 Probabilistic Approach of Determination of Parameters Scatter

The problem statement, basic assumptions and modification of the Monte-Carlo Simulation (MCS) method for the failure probability analysis are described below.

3.1 Problem Statement and Main Assumptions

The problem to be solved in this investigation is to develop a probabilistic method that determines the admissible parameter scatters of aircraft structure and loads under the conditions of static strength and flutter. To solve the problem it is necessary to know the probabilistic parameters of structure such as the safety factor, the uncertainties of the structural and loads parameters (their distribution laws) and the probability of failure.

Safety factor of static strength is equal to 1.5 (§303) and the safety margin of flutter speed is equal to 1.2 (§629) in accordance with AP-25 [5]. It is assumed that the uncertainties of all structural parameters and loads have a normal distribution law. The structural parameters such as thicknesses of the skin and of the spar/rib webs, cross-sectional area of the stringers and the spar/rib caps, material characteristics are considered as the varied parameters. Dynamic pressure, Mach number and normal load factor are the varied parameters for loads calculation. The failure probability of structure must provide the high safety of the aircraft. Choice of its value is discussed below.

To solve the formulated problem for determining the probability of structural failure the developed approach based on the MCS method is used.

3.2 On the Design Probability of Failure

To ensure a high safety level at using probabilistic approaches the value of the probability of failure is specified for one aircraft. The conversion from failure rate to the failure probability of one aircraft is carried out by using the formula:

$$P_f(t) = (1 - e^{-\lambda t}) \qquad (2)$$

where λ – the failure probability per unit time (failure rate), t – aircraft operating time.

The papers [4, 6, 7] are devoted to justification of the probability of failure for aircraft structures. For example, in paper [6] this value is considered to be equal to 10^{-7}, but the method takes into account the full operational scatter of loads and strength characteristics of the aircraft. Statistics of real operating loads acting on the aircraft is not always available, and statistics is absent for new designing aircraft.

In papers [4, 7] to ensure the structural safety under conditions of strength the failure probability is taken to be equal to 10^{-3}, but only the maximum operating loads are used for analysis of the failure probability. Since such loads are used in the design of aircraft structures, we name this probability of failure as "design probability".

The value of 10^{-3} refers to the entire aircraft, while this paper deals with the failure of wing as one of aircraft components. The value of the failure probability for the wing should be significantly less, because aircraft consists of about 10 responsible aggregates.

Here we present considerations similar to those as in the book [8] for the functional system failures in the aircraft. It is known that the total probability of aircraft failure is 10^{-6} per flight hour [5]. Suppose that 10% of all accidents are related to the failure of aircraft structure and that there are about a hundred of potential conditions of failure that can be catastrophic. Thus, dividing the total probability of failure by 10 and 100, we obtain the known value of the failure probability of any aircraft component for all modes of structural failure of 10^{-9} per flight hour [1].

To convert this probability to one aircraft we use Eq. 2 for calculation of the integral failure probability for the whole time of operation. Considering that the design fatigue life of modern civil aircraft is more than 50 thousand flight hours and current trends are aimed to increase fatigue life to 100 thousand flight hours (f.h.), we will take in the Eq. 2 the operating time t=100000 f.h. and the failure rate $\lambda=10^{-9}$ 1/h. The calculated design failure probability is equal to 0.000099995 ($P_f \approx 10^{-4}$) for one of the modes of structural failure (static strength, flutter, etc.) for aircraft component (wing, fuselage, stabilizer, etc.) for the entire fatigue life.

3.3 Scatter of Varied Parameters

In the proposed method, it is supposed that the normal distribution law with the same standard deviation in a percentage of the expected value p is used for all the varied parameters. The probability density function of the normal distribution law is written as follows:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} ,$$

where x is a random variable, μ is an expected value, σ is a standard deviation.

In design practice, the so-called "Six Sigma" is the most commonly used concept [9]. This implies that the distribution of the parameters is limited by the scatter of $\pm 3\sigma$. Then, each varied parameter x_i with the expected value μ_i lies within:

$$\mu_i - 3\sigma_i \leq x_i \leq \mu_i + 3\sigma_i,$$

where $\sigma_i = p\ \mu_i / 100$.

So the problem is to determine the admissible value of p for a given design failure probability of aircraft component.

3.4 Probabilistic Method Based on MCS

A new method based on the Monte-Carlo Simulation method has been developed to determine the structural failure probability. The MCS method is very simple in implementation, but it requires large number of samples of structures for analysis N for confident evaluation of the failure probability that is defined as the ratio of the number of failured structures m to N. For a 95% confidence level analysis in the failure probability the number N of simulations must be three times greater than the inverse of the failure probability [10]. Thus, for 95% confidence level and $P_f=10^{-4}$, the required number of simulations is more than 30000.

The basis of new method is to determine the dependence of the failure criterion $g(x)$ (stresses in the problem of the static strength, the critical flutter speed, etc.) on the design parameters x (elasticity modules, thicknesses, masses, etc.), the magnitude of which is determined by the scatter δ.

Theoretically, structural analyses should be carried out for all values of parameters randomly generated in accordance with the accepted normal distribution law. However, the number of calculations in this case is extremely large. Therefore, to reduce the large number of calculations, it is reasonable to build the dependence of the functions $g(x_i)$ that determine the failure criteria, when changing each single parameter x_i with a certain step and keeping the other parameters constant.

It gives the opportunity to determine gradients of the function with respect to the considered parameters. Knowing the value of the functions $g(x)$ at the nearest calculated point x and the values of the gradients allows to use of approximation to estimate the value of failure criteria. The approximation allows us to answer whether it is necessary to carry out time-cost FEM analysis or this point is obviously acceptable by the failure criterion.

The procedure for reduction of calculations can be simplified by using the parameter reflecting the total influence of all the variations in the parameters on the value of the failure criterion $g(x)$. It is defined as a scalar product of the gradient of function on the scatter vector $\boldsymbol{\delta}$. For convenience, this parameter is normalized via dividing it by the gradient norm, thus value of the normalized scatter Δ is written as follows:

$$\Delta = \frac{\nabla g(\mathbf{x})^T \boldsymbol{\delta}}{\|\nabla g(\mathbf{x})\|}, \qquad (3)$$

where $\boldsymbol{x}$ is a vector of values of varied parameters with components $x_i=\mu_i(1+\delta_i)$, $\boldsymbol{\delta}$ is the scatter vector with components δ_i expressed in percent. The scatters δ_i are determined by the normal distribution law and are limited by "six sigma":

$$-3p \le \delta_i \le 3p.$$

The dependence of the failure criterion on the normalized scatter can be built basing on a small number of calculations compared to what is necessary to estimate the probability of failure. Knowing this dependence, it is easy to determine the critical value of the normalized scatter Δ_F.

The algorithm of the proposed method for estimating the failure probability is shown in Fig. 5. At the initial stage, the maximum number of samples N, standard deviations and expected values for varied parameters (PDF values) are specified. To determine the critical value of Δ_F, a series of calculations are carried out. Number of analyses in them is about 5% of N. The dependence of the function of the failure criterion on the normalized scatter is built on the basis of sensitivity analysis (calculation of gradients). In the main part of the procedure for determining the probability of failure by using the MCS method, the normalized scatter is calculated. If it is less than the critical value Δ_F the structure is to be analyzed, otherwise, the structure is considered as safe. This procedure significantly reduce time of calculating the failure probability compared to the standard MCS algorithm and it does not reduce the accuracy since the number of samples remains the same. The number of failed structures m and all structures n are counted. If n becomes equal to N the algorithm finishes with the calculation of the failure probability.

Analysis of failure probability is performed for different standard deviations p with a certain step. The value of p, at which the probability of failure becomes less than 10^{-4}, defines the maximum allowable design parameter scatter.

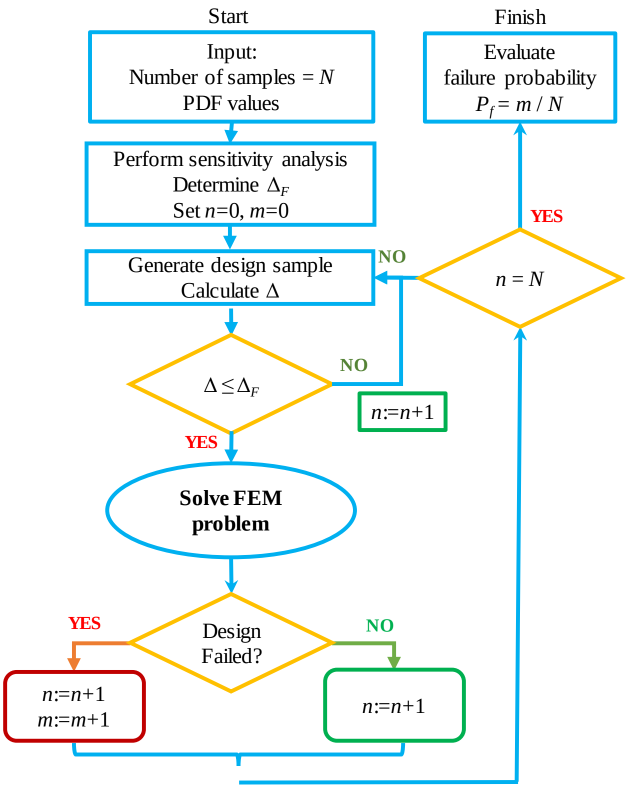

Fig. 5. Flowchart of Probabilistic Method

The numerical results with application of the algorithm are given below.

4 Numerical Examples

The proposed method is demonstrated on the example of medium-range aircraft wings optimized by the minimum weight criterion at satisfying the static strength and flutter constraints.

4.1 Strength Analysis of Wing-box Structure

The wing-box structure was designed with the use of a deterministic approach taking into account the elasticity of the structure for several load cases. The admissible stress is 400 MPa. In Fig. 6 the zones corresponding to the design parameters are shown, such as the thicknesses of the upper and lower skins, the webs of the spars, ribs and the section areas of the wing-box caps. After the optimization, the obtained structure was close to the fully stressed design, as it can be seen in Fig. 7.

When the probabilistic approach is used the obtained design parameters, such as the thickness and cross-sectional area of the structural elements are considered as nominal and the varied parameters are within the given maximum scatter. The modulus of elasticity and the admissible stress of materials are also varied. To reduce the number of calculations the Mach number, dynamic pressure and load factor are varied. They correspond to the extreme load case that are usually indicated by letter “*A*” on the *V-n* diagram.

Since it is not known what value *p* corresponds to the admissible probability of failure, then by successive reduction of *p*=12% with a step of 1%, the admissible value *p* is determined correspondingly to probability of 10^{-4}.

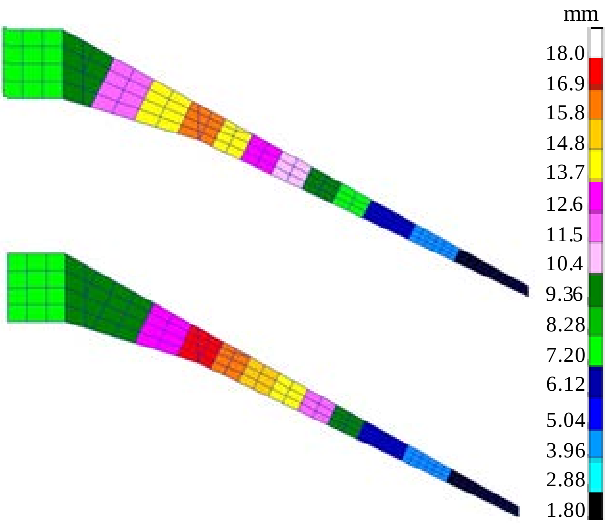

Fig. 6. Optimal Thicknesses of Wing-box Upper and Lower Skins

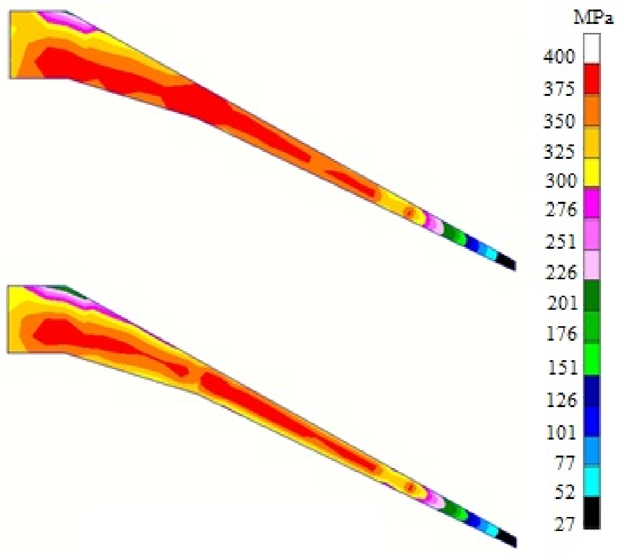

Fig. 7. Von Mises Stresses after Optimization

Probabilistic analysis of the designed medium-range aircraft wing-box under the conditions of static strength with using the deterministic approach, show that at the standard deviation *p*=5% (the scatter is 15%), the probability of structural failure is 0.000093 and it does not exceed the admissible value of 10^{-4}.

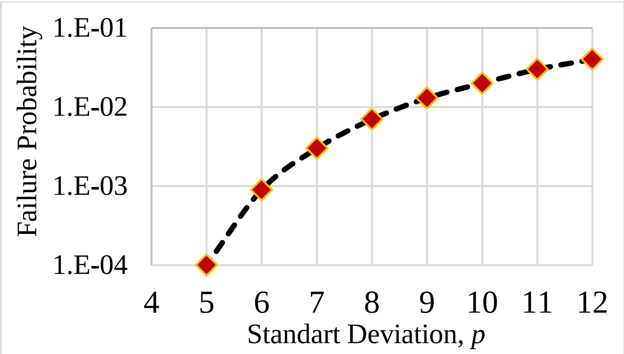

Fig. 8. The Dependence of Failure Probability on Standard Deviation (Under Static Strength Conditions)

It should be noted that when the standard deviation is changed, the structure is sufficiently sensitive to uncertainties. For example, the probabilities of failure at *p*=5% and at *p*=6% shown in Fig. 8 have difference by one order.

4.2 Flutter Probabilistic Analysis

A finite element model of a short-range aircraft wing with an engine under the wing (Fig. 9) is considered to determine the allowable scatter of parameters under the flutter requirement. The stiffness of the center-wing, wing and the engine pylon are simulated in the model. The mass of the fuel and the engine are simulated by the mass point elements, which are fixed to the structure by means of rigid connections. The most critical case from flutter safety conditions is considered. The obtained flutter mode is the interaction of the first bending mode of the wing and the wing torsional mode, due to engine pitch motion. The critical flutter speed is V_F=294 m/s, which satisfies the safety margin of 1.2 with the dive speed V_D=244 m/s.

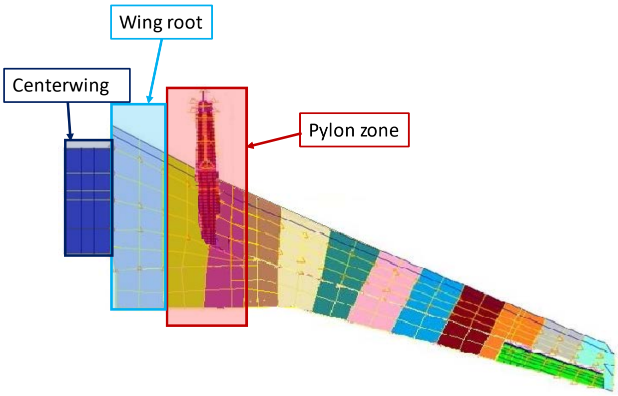

Fig. 9. Wing Finite Element Model with Variable Zones

In this example, the varied parameters are: the thicknesses and areas of the cross sections of the structural elements, the elastic modules of the wing and pylon materials.

The application of the developed method is illustrated in details on the example of determining the probability of wing failure under flutter conditions. It is assumed that the standard deviation is *p*=6%. For the qualitative evaluation of P_f, the number of samples *N*=500 is accepted.

The dependence of the flutter speed on the parameter scatter is shown in Fig. 10. It can be seen that the value of the flutter speed is significantly influenced by the structural stiffness in the pylon zone (red curves): the elastic characteristics of the pylon material, the thickness of its elements and the areas of its attachment to the wing. In addition, the change in the elastic characteristics of the wing materials (black curve) essentially influences on V_F. As for the thickness of the elements in the wing root, the flutter speed V_F is maximum at their nominal values. Cyan curve illustrates this. It is interesting to note that the reduction of the stiffness of the center-wing has a positive effect on the flutter speed (blue curve). Changes in the thicknesses of the rest part of the wing have a slight effect on the speed of the flutter (green curves).

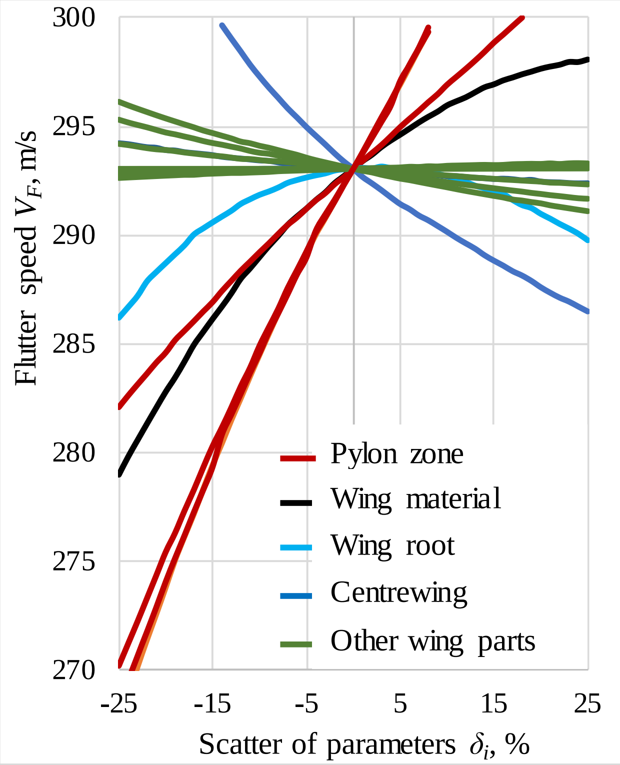

Fig. 10. The Dependence of Flutter Speed on Parameter Scatter

The gradient of the flutter speed is computed for the obtained dependences. It was used for calculation of the normalized scatter Δ by using Eq. 3. To determine the critical value of Δ_F, a small series of calculations is carried out.

In Fig. 11 the calculated points of the flutter speed depending on the normalized scatter are shown. It can be noticed that the trend line can approximate the dependence of the flutter speed on Δ.

Intuitively, the dependence in Fig. 11 shows that Δ_F can be taken to be equal to -7%. The plot shows two areas: the left area ($\Delta \leq \Delta_F$) where the analysis should be carried out and the right area ($\Delta \geq \Delta_F$) where the structure is not failed and obviously there is no need in full structural analysis. Such selection procedure allows in the presented example to reduce the number of calculations by nine times compared to the conventional Monte-Carlo method.

The obtained failure probability is less than 10^{-2} for *p*=6% and it exceeds the allowable value.

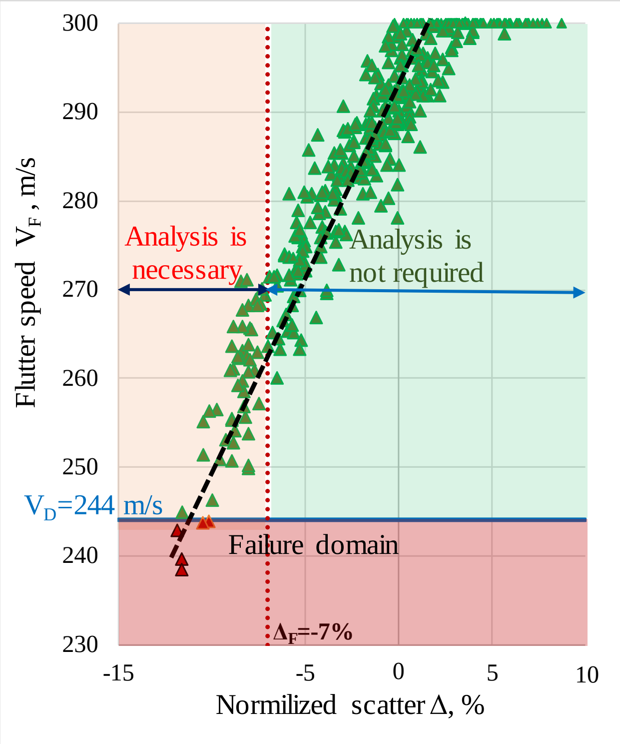

Fig. 11. The Dependence of Flutter Speed on Normalized Scatter

To determine the admissible scatter of the varied parameters, the failure probability is calculated for different *p* in the range from 4% to 12%. It can be seen that the probability of structural failure does not exceed 10^{-4} (it is equal to 0.000093) for the standard deviation of *p*=4% (see Fig. 12).

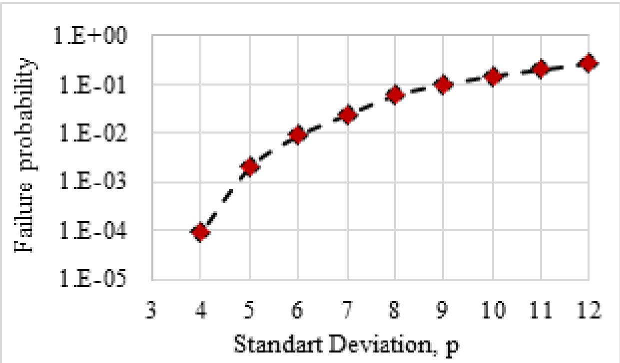

Fig. 12. The Dependence of Failure Probability on Standard Deviation (Under Flutter Conditions)

The probabilistic analysis of the wing under the flutter requirements shows that the wing structure could be considered safe with the safety margin of 1.2, established in paragraph 629 of the part 25 of Aviation Regulation for the scatter of considered varied parameters of ±12%.

5 Conclusions

The method of determination of the admissible scatter of the structural, material and loads parameters for the specified failure probability and safety factor under the requirements of static strength and flutter is proposed. It is shown that the design probability of failure can be considered equal to 10^{-4} as a safety criterion for the aircraft aggregate structure.

The efficient algorithm for the failure probability analysis is implemented. Its advantage is in decrease of the number of full structural analyses by about one order in comparison with the conventional MSC method.

The examples of failure probability evaluation for wing structures of two medium-range aircraft are presented. They show that the standard deviation of the considered varied design parameters and loads should not exceed 4-5% under the requirements of static strength and flutter.

The application of the probabilistic method will allow design engineers to justify more thoroughly the structural safety. It gives an opportunity to estimate the necessary quality level of used materials in terms of the scatter of their characteristics and technological tolerances at manufacturing of load-bearing aircraft components.

6 Contact Author Email Address

vchedrik@tsagi.ru

References

[1] Lundberg B. Fatigue Life of Airplane Structures (Eighteenth Wright Brothers Lecture). *Journal of the Aeronautical Sciences*, Vol 22, No. 6, pp 349-413, 1955.

[2] Lincoln J W. Aging Aircraft Issues in the United States Air Force. *SAMPE Journal*, Vol. 32, No. 5, pp. 27-33, 1996.

[3] Freudenthal A M and Payne A O. The Structural Reliability of Airframes. *Air Force Materials Laboratory*, Dayton, Ohio, Technical Report No. AFML-TR-64-401, 1964.

[4] Selikhov A F and Chizhov V M. *Probabilistic Methods in Aircraft Strength Calculations.* Mashinostroenie, 1987.

[5] *Aviation regulations, Part 25, Airworthiness Standards: Transport Category Airplanes.* Interstate aviation committee, 2015.
[6] Kale A, Acar E, Haftka R T and Stroud W J. Why are Airplanes so Safe Structurally? Effect of Various Safety Measures on Structural Safety of Aircraft. *45th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics & Materials Conference*, Palm Springs, California, AIAA-2004-1629, 2004.
[7] Graham A D, Mallinson G D and Tong Y C. NERF-A Tool for Aircraft Structural Risk Analysis. *Application of Statistics and Probability (ICASP 8)*, Vol.2, pp 1175-1182, 1999.
[8] Wang P. *Civil Aircraft Electrical Power System Safety Assessment: Issues and Practices*. Butterworth-Heinemann, 2017
[9] Navneetha K J, Nandhu M S, Mohammed R A, Ameesh N and Abhijith R. Study of Structural Failure of Aircraft Using Six Sigma Methodology. *International Journal of Research in Mechanical Engineering*, Vol. 4, Issue 3, pp 147-151, 2016.
[10] Melchers R and Beck A. *Structural Reliability Analysis and Prediction*. Wiley, 2018.

Copyright Statement