NEURAL NETWORK DRIVEN ADJOINT SOLVER BETWEEN FLIGHT-PATH OPTIMIZATION AND PANEL METHOD FOR AIRCRAFT DESIGN

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Abstract

A simultaneous design and trajectory optimization tool with adjoint method between flight-path optimization and 3D panel method is developed. To prevent algorithm terminated, Neural Networks is used for distinguish of design feasible/infeasible area. To reconfirm work of the algorithm, the problem of hyper sonic business jets was solved as an example. As the result, the design of hyper sonic business jets which had about 5700km of maximum range was obtained.

1 Introduction

Simultaneous design and trajectory optimization is an effective tool for the first design of new air-transportation system such as solar planes, hyper sonic transporters, or space planes. That is effective because we can obtain new knowledges from the optimized designs of air frames and trajectories. On the other hand, it has large calculating costs because it must estimate aerodynamic coefficients varying depending on air frame geometry. For these costs, it is difficult to increase the number of dimension of design variables.

There are roughly two methods in simultaneous design and trajectory optimization. One is a method using pre-aerodynamic analyses. Once preprocesses are done, trajectory optimization will complete by using the results easily. On the other hand, the calculating time is rapidly increased according to the dimension of design variables. Another is a method using sequential aerodynamic analysis. This method can easily change the geometry of airframe because no preprocess exsists. In addition the calculating costs are reduced because aerodynamic calculations are done only per one trajectory optimization.

Author created a simultaneous design and trajectory optimization tool without pre aerodynamic analyses with coupling between trajectory optimization and 3d Panel methods. This tool calculates the next value of Lagrangian by adjoint method. It takes less iteration for obtaining optimized solution than any other algorithm. In addition of that, there is few programs using 3d panel methods in adjoint methods. As long as we know, this is the first program combined a 3d panel method and trajectory optimization using an adjoint method.

2 Algorithms implementation

This algorithm is divided into the segments to construct Lagrangian, that is the uniform objective function including the original objective function and constraints, and to update the solution. In order to build up the Lagrangian, the adjoint equations must be solved. Moreover, adjoint variables about 3D panel methods have to be calculated after obtaining the solution of trajectory optimization. To update the design, the gradient of the Lagrangian must be calculated by numerical differentiations and the updating vectors for a next design must be searched with satisfying the constraints of design and results of neural network for discrimination of design-feasible areas.

This algorithm's work flow is shown in followings

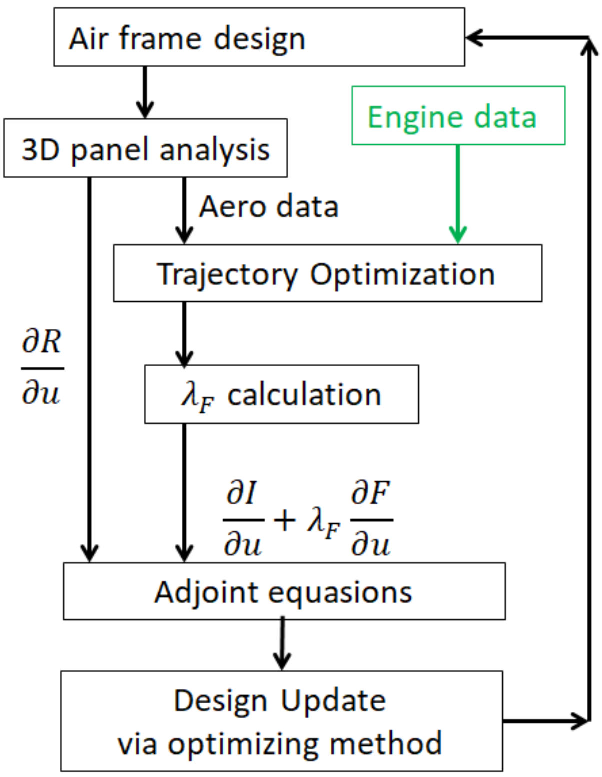

Fig. 1. Design variables

2.1 Coupling between trajectory optimization and 3D panel method

An important point for optimization tools to combine analyses with multi domain is setting a definition of unique Lagrangian and an optimization in the Lagrangian's frame work. In this time we define the Lagrangian as follows.

$$\begin{aligned} L &= I + \boldsymbol{\lambda}_F^T(\boldsymbol{F} - \boldsymbol{s}) + \boldsymbol{\lambda}_R^T\boldsymbol{R} \\ &+\boldsymbol{v}_1^T(\boldsymbol{w} - \boldsymbol{w}_u) + \boldsymbol{v}_2^T(\boldsymbol{w}_l - \boldsymbol{w}) \\ &+\boldsymbol{v}_3^T(\boldsymbol{s} - \boldsymbol{s}_u) + \boldsymbol{v}_4^T(\boldsymbol{s}_l - \boldsymbol{s}) \\ &\boldsymbol{w}_l \le \boldsymbol{w} \le \boldsymbol{w}_u \\ &\boldsymbol{s}_l \le \boldsymbol{s} \le \boldsymbol{s}_u \\ &\boldsymbol{0} \le \boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{v}_4 \end{aligned} \quad (1)$$

The adjoint equations, which are a part of Karush-Kuhn-Tucker condition, are follows.

$$\begin{aligned} \frac{\partial L}{\partial w} &= \frac{\partial \boldsymbol{I}}{\partial \boldsymbol{w}} + \boldsymbol{\lambda}_F^T \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{w}} + \boldsymbol{\lambda}_R^T \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{w}} + \boldsymbol{v}_1^T - \boldsymbol{v}_2^T = \boldsymbol{0} \\ \frac{\partial L}{\partial u} &= \frac{\partial \boldsymbol{I}}{\partial \boldsymbol{u}} + \boldsymbol{\lambda}_F^T \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{u}} + \boldsymbol{\lambda}_R^T \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{u}} = \boldsymbol{0} \end{aligned} \quad (2)$$

When the flow conditions of 3D panel analyses are fixed and the aerodynamic coefficients in arbitral Mach number, angle of attack and deflection angle calculated by interpolated surface are used, the value of gradients for equations of 3D panel method $\frac{\partial R}{\partial w}$ becomes zero. Hence, adjoint variables for trajectory can be calculated by solving linear programming optimization of Wolfe's dual problem.

Now, we consider about the condition of trajectory optimization $\boldsymbol{F}(\boldsymbol{x},\boldsymbol{w},\boldsymbol{u})$. In optimal control with pseudo spectral method, NLP is defined as follows.

$$\begin{aligned} &find. \\ &\qquad \boldsymbol{w}, \boldsymbol{s} \\ &min. \\ &\qquad I(\boldsymbol{x},\boldsymbol{w},\boldsymbol{u}) \\ &such\ that. \\ &\boldsymbol{F}(\boldsymbol{x},\boldsymbol{w},\boldsymbol{u}) = \begin{cases} D\boldsymbol{w} - \dfrac{t_f - t_0}{2} f(\boldsymbol{x},\boldsymbol{w},\boldsymbol{u}) \\ \boldsymbol{Cons}(\boldsymbol{x},\boldsymbol{w},\boldsymbol{u},\boldsymbol{s}) \end{cases} = 0 \end{aligned} \quad (3)$$

t_f and t_0 are included in $\boldsymbol{w}$. $\boldsymbol{F}(\boldsymbol{x},\boldsymbol{w},\boldsymbol{u})$ is constraints of the equations of motion and constraints about trajectory such as boundary, phase-linking, or path. $\boldsymbol{s}$ are the slack variables, which convert inequalities to equalities.

Next, we think of the conditions of 3D panel method, $\boldsymbol{R}(\boldsymbol{x},\boldsymbol{u})$. When the flow condition is fixed, $\boldsymbol{R}(\boldsymbol{x},\boldsymbol{u})$ does not depend on $\boldsymbol{w}$. Used 3D panel methods adopt unstructured mesh and switch analysis methods from boundary element methods to modified Newtonian flow based on Mach number.

When the flow is subsonic, $\boldsymbol{R}(\boldsymbol{x},\boldsymbol{u})$ becomes follow.

$$\begin{aligned} \boldsymbol{u} &= \boldsymbol{\mu} \\ \boldsymbol{R}(\boldsymbol{x},\boldsymbol{u}) &= A(\boldsymbol{x})\boldsymbol{u} - B(\boldsymbol{x})\boldsymbol{\sigma}(\boldsymbol{x}) = \boldsymbol{0} \end{aligned} \quad (4)$$

When the flow is supersonic, $\boldsymbol{R}(\boldsymbol{x},\boldsymbol{u})$ is constructed by modified Newtonian [8] smoothed with interpolation. Before analysis, we calculate Coefficient of pressure for arbitral panel angle in $-\pi \le \delta \le \pi$ in order to prepare radial based function network [9]. $\boldsymbol{R}(\boldsymbol{x},\boldsymbol{u})$ becomes following with this interpolation, $\boldsymbol{RBFN}(\boldsymbol{\delta})$

$$\begin{aligned} \boldsymbol{u} &= \boldsymbol{Cp} \\ \boldsymbol{R}(\boldsymbol{x},\boldsymbol{u}) &= \boldsymbol{u} - \boldsymbol{RBFN}(\boldsymbol{\delta}(\boldsymbol{x})) = \boldsymbol{0} \end{aligned} \quad (5)$$

From the above, we can deal with simultaneous design and trajectory optimization with adjoint method using trajectory optimization and 3D panel method.

2.2 Discrimination of design-feasible region by Neural Network.

If the gradient of the Lagrangian and design-update vector can be calculated, Algorithm should be terminated at the same time that updated solution becomes something infeasible. Hence the function which distinguishes feasible/infeasible area is needed.

The function uses a Neural Network (NN) with sigmoid functions. Neural Networks can use the values of design variable themselves. Then it can reduce memory using. It can be used through the whole iterations of the optimization.

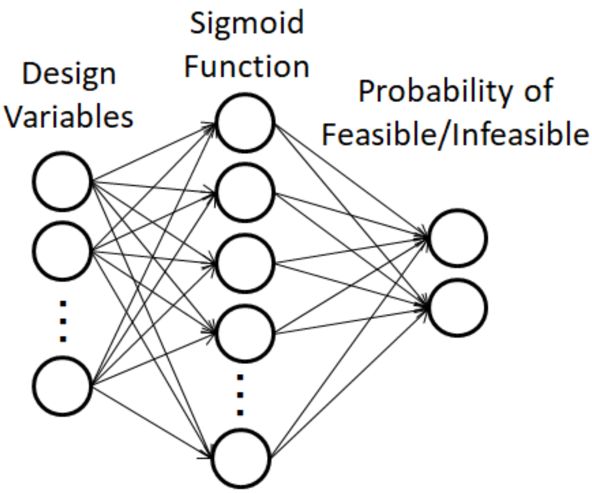

Fig. 2. Neural Network distinguishing feasible/infeasible area

When the trust region method with approximated hessian is used for updating the solution, we can obtain design updating vector by solving optimization problem shown below.

find.

$$\boldsymbol{\Delta x}$$

min.

$$\boldsymbol{\Delta x}^T H \boldsymbol{\Delta x} + \frac{\partial L}{\partial x} \boldsymbol{\Delta x}$$

such that. (6)

$$|\boldsymbol{\Delta x}| - d \leq 0$$

$$crit - NN(\boldsymbol{x_0} + \boldsymbol{\Delta x}) < 0$$

$$\boldsymbol{C_E}(\boldsymbol{x_0} + \boldsymbol{\Delta x}) = \boldsymbol{0}$$

$$\boldsymbol{C_I}(\boldsymbol{x_0} + \boldsymbol{\Delta x}) \leq \boldsymbol{0}$$

We make NN(x) do learning randomly at the region near the current solution x_0. In addition to that, it does learning when the updated solution is infeasible. From this process, we can get information of design-feasible area along the design-updating path through iteration.

3 Design optimization of a business jet

For example, we carried out an optimization for hyper sonic business jets with 10 persons. Assumed engines were 8 the pre-cooled turbo jet engines underdeveloped by JAXA. Its fuel was Liquid Natural Gus (LNG) and filled in fuselage. Cabin was placed in fuselage too. Its diameter was more than 2.5m and its volume was more than 30 m^3. Weight estimation for fuselage, wing, gear and so on is HASA [6], which was statically estimating method.

3.1 Variables of the design

Airframe geometry was combination among circular fuselage and double delta wing with elevons and a vertical fin. Fig.3 shows explanation of deign variables. The vertical fin's area was 13% of main wing area.

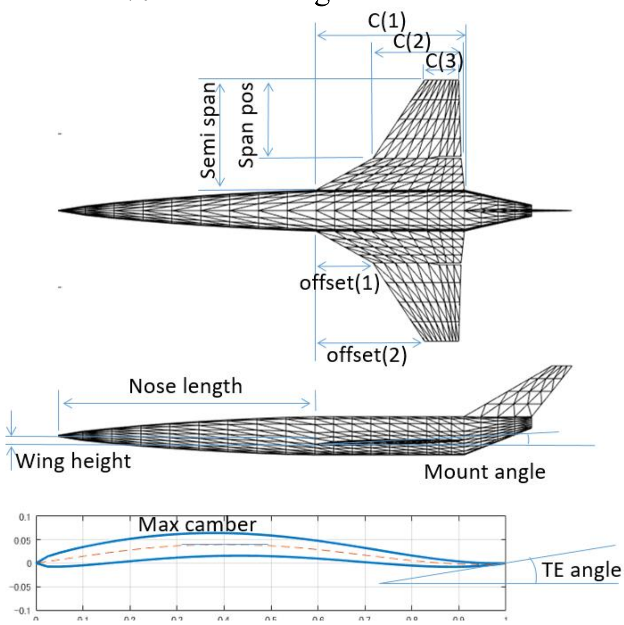

Fig. 3. Design variables

The scaling parameter was included in $\boldsymbol{w}$. Then we designed airframe with certain size and used non-dimensionalized aerodynamic coefficients. When the wing section excluded fuselage, errors was occurred.

Table. 1 Explanation of design variables

Var	Name	Meaning
p(1)	semispan	Semi span in Fig.3
p(2)	spanpos	Span pos in Fig.3
p(3)	chord(1)	C(1) in Fig.3
p(4)	chord(2)	C(2) in Fig.3
p(5)	chord(3)	C(3) in Fig.3
p(6)	twist(1)	AoA of root and inner foils
p(7)	twist(2)	AoA of tip foil
p(8)	offset(1)	offset(1) in Fig.3
p(9)	offset(2)	offset(2) in Fig.3
p(10)	dh	wing height in Fig.3
p(11)	noselength	Nose length in Fig.3
p(12	maxCamY(1)	Max camber of root and inner
p(13)	maxCamY(2)	Max camber of tip foils
p(14	TE angle(1)	TE angle of root and inner
p(15)	TE angle(2)	TE angle of tip

3.2 Objective function and constraints

3.2.1 Objective function and variables

The objective function was maximizing of cruise range. In the actual optimization, negative value of cruise range became an objective function and we minimized its value.

3.2.2 Constraints of design

We imposed constraints that whole geometry entered into Mach cone of Mach 4.5, the actual thickness decreased from root to tip and wing did not be inverse-taper wing on airframe geometry.

3.2.3 Constraints of trajectory

We imposed constraints that dynamic pressure was under 50kPa, acceleration becomes less than ±0.5G and the altitude was under 30 km on trajectory. In ascent phase, positive clime rate was also needed.

3.3 Results

Fig. 4 shows the movement of design variables and Fig.5 shows the movement of objective value and norms of design-updating vector.

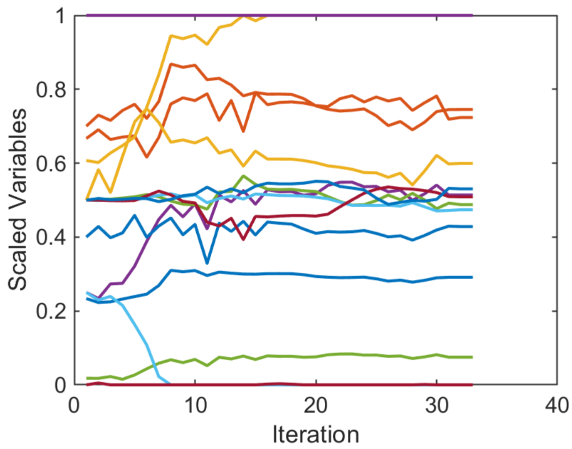

Fig. 4. Movement of design variables

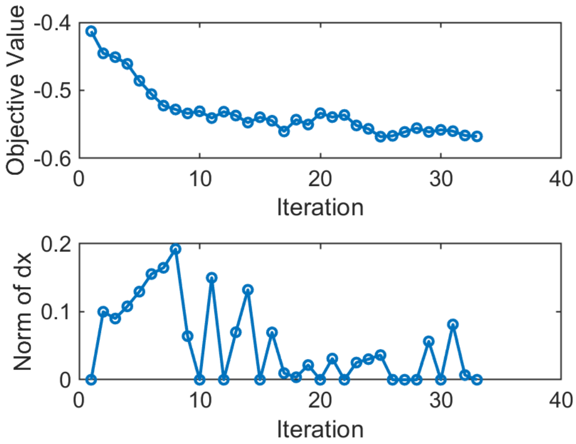

Fig. 5. Movement of objective value and norm of design-updating vector

The movement of design variables was calmed until 35th iteration. Thus, we could say converged result was obtained. The objective value was reached 5500km of cruise range until 10th iteration. The norm of design updating vectors sometimes oscillated and this fact showed that this problem was difficult to find a optimized solution. As a problem of this algorithm, improving on objective value was not confirmed if the Lagrangian value was improved. In this time, objective value sometimes became worse for this reason.

The constraints of take-off and ascent had large influence on the result because LNG fuel, whose density was bigger than liquid hydrogen (LH2), was used. For this reason, The result could carry on Mach 4 cruise only the latter half of the trajectory and the fact had bad influence on the maximum cruise range.

Table. 2. Values of variables

Var	Lower	Initial	Optim	Upper
p(1)	0.13	0.80	0.97	3.00
p(2)	0.00	0.60	0.65	0.90
p(3)	0.20	1.90	3.00	3.00
p(4)	0.20	0.90	1.64	3.00
p(5)	0.20	0.25	0.41	3.00
p(6)	-2.00	-1.00	-2.00	2.00
p(7)	-2.00	-2.00	-2.00	2.00
p(8)	0.00	0.40	0.43	1.00
p(9)	0.00	0.70	0.75	1.00
p(10)	-0.10	0.00	0.02	0.10
p(11)	0.5	2.00	2.00	2.00
p(12	-0.02	0.00	0.00	0.02
p(13)	-0.02	0.00	0.00	0.02
p(14	-5	0.00	0.90	5
p(15)	-5	0.00	0.30	5

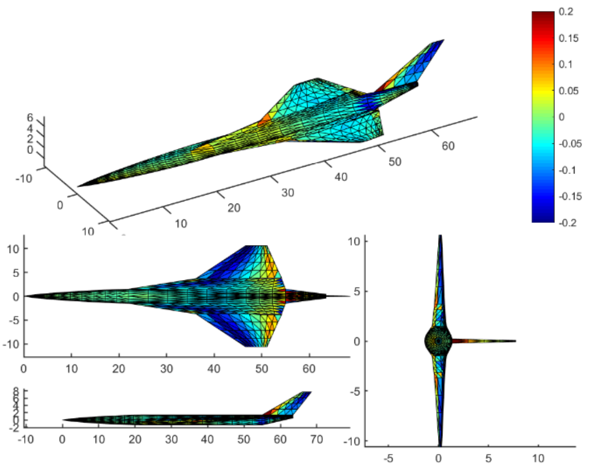

Fig. 6 The geometry of airframe (color shows Cp distribution of Mach 0.3)

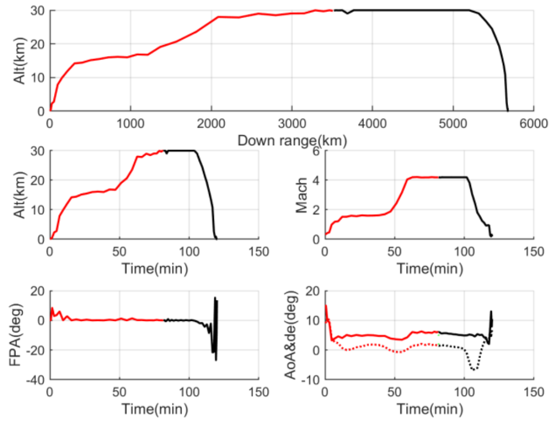

Fig. 7 The optimized trajectory (state variables)

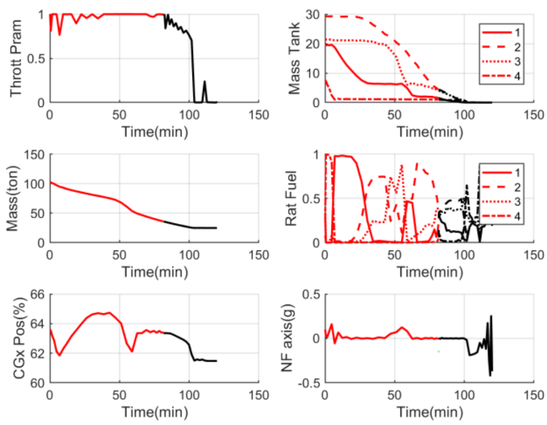

Fig. 8 The optimized trajectory (weight and fuels)

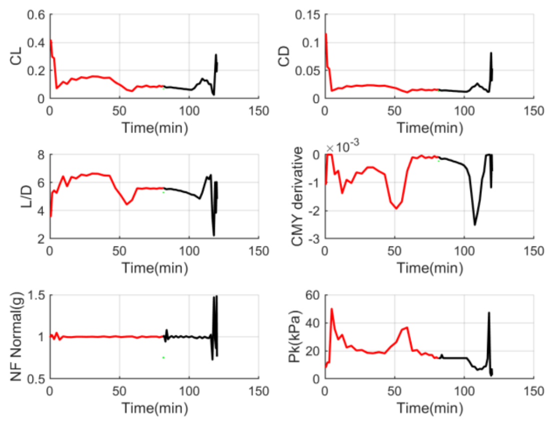

Fig. 9 The optimized trajectory (aerodynamic)

4 Conclusion

We implemented the simultaneous design and trajectory optimization with coupling method between trajectory optimization and 3D panel method. In the updating solution, neural network was used in order to distinguish the design feasible/infeasible area. Using this algorithm, we solved the optimization of hyper sonic business jets and confirmed this algorithm can be effective for new air transportation system.

Multi Domain Optimization (MDO) with adjoint method can be added other analyses such as structure, or sound. This result can be a motivation to develop more effective tools for the system design.

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