AN EFFICIENT FRAMEWORK FOR AERODYNAMIC CONFIGURATION EVOLUTION OF REENTRY VEHICLES

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Abstract

In this paper an efficient framework is developed for aerodynamic database generation and management of reentry vehicles during aerodynamic configuration evolution and design phase. For reentry vehicles with wide range of flight envelope, a large number of data is required to full-fill the aerodynamic multi-dimensional tables. To reduce the number of high-fidelity analyses without considerable accuracy loss, a proper combination of sampling, interpolation and data fusion methods are required. To show the capabilities of the developed framework, Orion reentry capsule with wide flight envelope is assumed as a test case. Orion aerodynamic database is generated efficiently and the obtained results are in good agreement in compare with experimental data. It is demonstrated that if the reentry vehicle geometry undergoes some modifications, it is possible to update whole aerodynamic database with minimum computational effort using the presented computational framework.

1 Introduction

Atmospheric reentry flight is inevitable when we want to return a valuable payload from space. Mostly, reentry vehicles are containing astronauts or important objects, and consequently their design and development must be reliable and accurate. One of the key aspects of reentry vehicle development is aerodynamic design and simulation as an input for other aspects of design e.g. structure, thermal, stability and control [1]. For reentry vehicles, aerodynamic database generation process is challenging, time consuming and expensive because of wide-ranging flight envelope including hypersonic, supersonic, transonic and subsonic flow regimes. Typically, aerodynamic forces and moments of these vehicles are presented in multi-dimensional tables and a large number of aerodynamic data ($\sim 10^6$) is required to full-fill the table. For aerodynamic database construction, there are several sources and methods. The real and costly data are obtained from flight tests. Wind tunnel test is cheaper but has limitations regards scaling, blockage and measurements. Computational Fluid Dynamics (CFD) is another source of aerodynamic data that can predict non-linear flow physics. Another approach and the cheapest one is semi-empirical methods combined with approximate and linear aerodynamic theories. It is obvious that full-filling whole aerodynamic table with high fidelity data is very expensive and nearly impossible and therefore employing different fidelity methods is unavoidable. Also, for the fusion of multi-fidelity data and improvement of the aerodynamic table accuracy, surrogate models can be used [2-5].

The main idea of this work is to extend the application of surrogate-based frameworks to the reentry vehicles' aerodynamic database development and to update an available database of a base geometry with some modifications. Due to extensive range of flight envelope, database generation for reentry vehicle is more time consuming and expensive, in compare to aerial vehicles like aircrafts.

2 Computational Framework

2.1 General Algorithm

For reentry vehicle aerodynamic database generation, some special solvers like Newtonian solvers and also efficient combination of sampling, interpolation and data fusion methods must be used to reduce cost of computations without loss of accuracy. Moreover, different grid generation and solver settings should be optimally set for each regimes of flight i.e. hypersonic, supersonic, transonic and subsonic. On the other hand, to overcome the lack of fast solvers for generating initial trends of aerodynamic coefficients in all regimes of reentry flight, implementing more efficient sampling techniques is necessary in compare to the previous studies. In the current work, mean squared error (MSE) and maximum expected improvement (MEI) criteria are used for efficient selection of new samples locations. The process for generating aerodynamic data is started by initial sampling via LHS methodology. Low-fidelity solvers such as semi-empirical codes and Euler solver are implemented for producing aerodynamic data on these points. Once low-fidelity aerodynamic coefficients are produced by using solvers, Kriging interpolation is used to interpolate data and fulfill the gaps. Then, MSE of the dataset is determined and a sample point is created in maximum of MSE to calculate new aerodynamic data. The process continues similarly until the MSE criteria is met. Now the trends of all aerodynamic coefficients are calculated. To increase accuracy of the trends, some high-fidelity samples must be considered in whole flight envelope with Latin Hypercube Sampling (LHS). The minimum of these samples is 16 due to possible permutations for including extreme bounds of each flight envelope parameters (Mach, angle of attack, sideslip angle and altitude). Considering 16 samples in this manner, avoids the need of extrapolation. For new sample points of database, Navier-Stokes solver is used as an accurate aerodynamic data generator. Then, Co-Kriging data fusion method is used to combine low and high-fidelity datasets. Hence, Maximum of Expected Improvement (EI) function is calculated and new sample points are created and accurate aerodynamic data are calculated by Navier-Stokes solver further. The procedure will be pursued until the EI criteria is met. Eventually, the final database is prepared in form of 5-D matrices. Five-dimensional matrices are composed from different combinations of variables e.g. (M, α, β, Re, C_i) and they will form the complete aerodynamic database of the reentry vehicle.

If an initial database of a similar geometry is available, the database of a new geometry (with some modifications in comparison to base geometry) can be constructed with only a few new sample points to update the basic database by using co-Kriging method (Fig. 1).

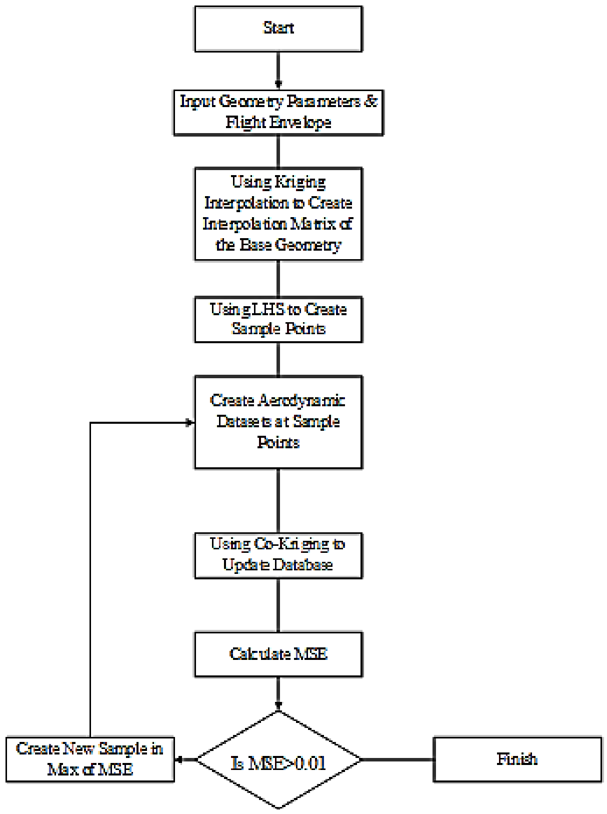

Fig 1. The surrogate-based framework for updating an initial aerodynamic database to a new similar geometry database

2.2 Theoretical Foundations

If f(x) is defined by a k-vector of design variables x, all the information that we can obtain from f(x) is through discrete samples,

$$\{\mathbf{x^i} \rightarrow \mathbf{y^i} = \mathbf{f}(\mathbf{x^i}) | \mathbf{i} = \mathbf{1}, \dots, \mathbf{n}\} \quad 1$$

where n is the number of sample points. The samples could be shown in matrix form,

$$\mathbf{X} = \{\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \dots, \mathbf{x}^{(n)}\} \quad (2)$$

that x itself is a k-dimensional vector of design variables. To achieve uniform accuracy in all of the design sites, the samples must be dispersed uniformly. Latin hypercube is a method of sampling that can spread samples uniform in all of the design space. Building a Latin Hypercube can be done by splitting the design space into equal size bins. Samples are placed in the bins in a way that no other sample should exist in any direction of bins axis [6, 7].

According to the sampling plan, the samples and their relative outputs can be represented in the following way [8, 9]:

$$\mathbf{X} = \{\mathrm{x}^{(1)}, \mathrm{x}^{(2)}, \dots, \mathrm{x}^{(n)}\}^{\mathrm{T}}, \quad \mathbf{y} = \{\mathrm{y}^{(1)}, \mathrm{y}^{(2)}, \dots, \mathrm{y}^{n}\}^{\mathrm{T}} \quad (3)$$

A radial basis function of $\hat{f}$ can be considered in a fixed form as below [24, 25]:

$$\hat{f}(\boldsymbol{x}) = \boldsymbol{w}^T\boldsymbol{\psi} = \sum_{i=1}^{n_c} \omega_i \psi(\|\boldsymbol{x} - \boldsymbol{c}^{(i)}\|) = y^{(j)}, \quad j = 0,1,2,\dots,n \quad (4)$$

that can be represented in a matrix form like $\mathbf{\Psi w} = \mathbf{y}$ where $\Psi_{i,j} = \psi(\|\mathbf{x}^{(i)} - \mathbf{x}^{(j)}\|)$. In Equation 4, c(i) denotes the ith of the nc basis function center, $\boldsymbol{w}$ denotes weighting matrix and $\boldsymbol{\psi}$ is the basis function that can be assumed as below [8]:

$$\psi^{(i)} = exp\left(-\sum_{j=1}^{k} \theta_j \left|x_j^{(i)} - x_j\right|^{p_j}\right) \quad (\theta_j \geq 0\,;\ p_j \in [1,2]) \quad (5)$$

This basis function is known as Kriging. The parameter θ_j determines the significance of jth component and exponent p_j is related to the smoothness of the function. Y matrix is a set of random vectors that can be expressed as:

$$\mathbf{Y} = \begin{pmatrix} Y(x^{(1)}) \\ \vdots \\ Y(x^{(n)}) \end{pmatrix} \quad (6)$$

This random field has a mean of 1μ and the variables are correlated with each other using the Kriging basis function expression:

$$\boldsymbol{cor}[\boldsymbol{Y}(\boldsymbol{x}^{(i)}), \boldsymbol{Y}(\boldsymbol{x}^{(l)})] = \mathbf{exp}\left(-\sum_{\mathbf{j=1}}^{\mathbf{k}} \boldsymbol{\theta}_{\mathbf{j}} \left|\mathbf{x}_{\mathbf{j}}^{(\mathbf{i})} - \mathbf{x}_{\mathbf{j}}^{(\mathbf{l})}\right|^{\mathbf{p}_{\mathbf{j}}}\right) \quad (7)$$

Now we can construct an n×n correlation matrix of all the observed data [25]:

$$\mathbf{\Psi} = \begin{pmatrix} \mathrm{cor}[\mathrm{Y}(\mathrm{x}^{(1)}), \mathrm{Y}(\mathrm{x}^{(1)})] & \cdots & \mathrm{cor}[\mathrm{Y}(\mathrm{x}^{(1)}), \mathrm{Y}(\mathrm{x}^{(n)})] \\ \vdots & \ddots & \vdots \\ \mathrm{cor}[\mathrm{Y}(\mathrm{x}^{(n)}), \mathrm{Y}(\mathrm{x}^{(1)})] & \dots & \mathrm{cor}[\mathrm{Y}(\mathrm{x}^{(n)}), \mathrm{Y}(\mathrm{x}^{(n)})] \end{pmatrix} \quad (8)$$

and a covariance matrix (see Appendix),

$$\mathrm{Cov}(\mathbf{Y}, \mathbf{Y}) = \sigma^2 \mathbf{\Psi} \quad (9)$$

where σ^2 is the standard deviation of Y. These corrections depend on the absolute distance between sample points and the parameters θ_j and p_j. To compute the Kriging model, the parameters μ, σ^2, θ_j and p_j can be estimated using the Maximum Likelihood Estimates (MLEs).

Once the model parameters found, the Kriging correlation can be used to predict new values based on the observed data. A new prediction of $\hat{y}$ at x, should be consistent with the observed data and therefore with the correlation parameters we have found. So, the observed data y should be augmented with the new prediction $\hat{y}$ to give the vector $\widetilde{\boldsymbol{y}} = \{\boldsymbol{y}^T, \hat{y}\}^T$. So a vector of correlations between the observed data and the new predictions can be defined as Equation 10:

$$\boldsymbol{\psi} = \begin{pmatrix} \mathrm{cor}[Y(\mathrm{x}^{(1)}), Y(\mathrm{x})] \\ \vdots \\ \mathrm{cor}[Y(\mathrm{x}^{(n)}), Y(\mathrm{x})] \end{pmatrix} = \begin{pmatrix} \psi^{(1)} \\ \vdots \\ \psi^{(n)} \end{pmatrix} \quad (10)$$

Now, the augmented correlation matrix can be written as:

$$\widetilde{\mathbf{\Psi}} = \begin{pmatrix} \mathbf{\Psi} & \boldsymbol{\psi} \\ \boldsymbol{\psi}^T & 1 \end{pmatrix} \quad (11)$$

Note that the last element of $\widetilde{\boldsymbol{\Psi}}$ is 1 which represent the correlation of a point with itself is 1. So the MLE for $\hat{y}$ will be,

$$\hat{y}(\mathbf{x}) = \hat{\mu} + \boldsymbol{\psi}^T \boldsymbol{\Psi}^{-1}(\mathbf{y} - \mathbf{1}\hat{\mu}) \qquad 12$$

Finally, the mean squared error (MSE) and the expected improvement function of the Kriging predictor are derived as [8],

$$\hat{s}^2(\mathbf{x}) = \sigma^2 \left[1 - \boldsymbol{\psi}^T \boldsymbol{\Psi}^{-1} \boldsymbol{\psi} + \frac{1 - \mathbf{1}^T \boldsymbol{\Psi}^{-1} \boldsymbol{\psi}}{\mathbf{1}^T \boldsymbol{\Psi}^{-1} \mathbf{1}}\right] \qquad 13$$

To correlate multiple sets of data from different sources, Co-Kriging can be used. If the most accurate high-fidelity data has values y_e at points X_e and the less accurate low-fidelity data has values y_c at points X_c. The combined set of sample points and their values can be assumed as below [8, 10]:

$$\mathbf{X} = \begin{pmatrix} \mathbf{X}_c \\ \mathbf{X}_e \end{pmatrix} = \begin{pmatrix} x_c^{(1)} \\ \vdots \\ x_c^{(n_c)} \\ x_e^{(1)} \\ \vdots \\ x_e^{(n_e)} \end{pmatrix}, \quad \mathbf{Y} = \begin{pmatrix} \mathbf{Y}_c(\mathbf{X}_c) \\ \mathbf{Y}_e(\mathbf{X}_e) \end{pmatrix} = \begin{pmatrix} \mathbf{Y}_c(x_c^{(1)}) \\ \vdots \\ \mathbf{Y}_c(x_c^{(n_c)}) \\ \mathbf{Y}_e(x_e^{(1)}) \\ \vdots \\ \mathbf{Y}_e(x_e^{(n_e)}) \end{pmatrix} \qquad 14$$

So the complete covariance matrix is,

$$\mathbf{C} = \begin{pmatrix} \sigma_c^2 \boldsymbol{\Psi}_c(\mathbf{X}_c, \mathbf{X}_c) & \rho \sigma_c^2 \boldsymbol{\Psi}_c(\mathbf{X}_c, \mathbf{X}_e) \\ \rho \sigma_c^2 \boldsymbol{\Psi}_c(\mathbf{X}_e, \mathbf{X}_c) & \rho^2 \sigma_c^2 \boldsymbol{\Psi}_c(\mathbf{X}_e, \mathbf{X}_e) + \sigma_d^2 \boldsymbol{\Psi}_d(\mathbf{X}_e, \mathbf{X}_e) \end{pmatrix} \qquad 15$$

Consequently, in co-Kriging we have more parameters to estimate, i.e. $\theta_c, \theta_d, P_c, P_d$, ρ (a constant scaling factor). Also, cheap data parameters, i.e. μ, σ_c^2, θ_c, P_c can be estimated by maximizing likelihood function [8, 10].

Once the co-Kriging parameters estimated, $\hat{y}_e(\mathbf{x})$ i.e. final prediction can be obtained:

$$\hat{y}_e(\mathbf{x}) = \hat{\mu} + \mathbf{c}^T \mathbf{C}^{-1}(\mathbf{y} - \mathbf{1}\hat{\mu}) \qquad 16$$

where, c is a column vector of the covariance of X and x. The corresponding MSE can be calculated as below too,

$$\hat{s}^2(x) \approx \rho^2 \hat{\sigma}_c^2 + \hat{\sigma}_d^2 - \mathbf{c}^T \mathbf{C}^{-1} \mathbf{c} + \frac{1 - \mathbf{1}^T \mathbf{C}^{-1} \mathbf{c}}{\mathbf{1}^T \mathbf{C}^{-1} \mathbf{1}} \qquad 17$$

3 Results and Discussion

Using the designed framework, it can be supposed that if the aerodynamic coefficients' trends of an assumed geometry are available, they can be used for other similar geometry. These trends can be updated only with a few number of accurate data and the updated aerodynamic database can be generated efficiently via Co-Kriging method. For example, Orion aerodynamic database [11] is generated and available (Fig. 2). If the aerodynamic configuration is required to be changed to another one like ARD geometry (Fig. 3), the aerodynamic database of new geometry (i.e. ARD) can be generated by updating Orion aerodynamic database with a few number of accurate CFD data. High fidelity data should be generated on some sample points that can be determined by sampling methods like LHS. In just few hours, aerodynamic database of the modified geometry can be updated entirely. As shown in Fig. 4, the results are compared with experimental data of ARD configuration [12]. It can be observed that not only the aerodynamic database is updated successfully using a few sample points but also the accuracy of the trend is in good agreement with experimental results. In Fig. 5, Lift coefficients of Orion reentry vehicle in angle of attack of 14 degree are considered as a trend of ARD lift coefficient in its trim angle of attack. With the samples, Orion lift coefficient trend is updated for ARD vehicle lift coefficients successfully. Moreover, this process can reduce the total time of aerodynamic database generation and modification about 80 percent in compare to recalculation of whole aerodynamic database. In conclusion, the proposed framework can be implemented well in aerodynamic configuration evolution and development of reentry vehicles.

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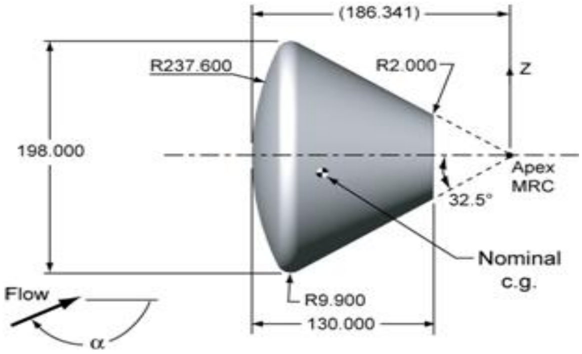

Fig 2. Orion configuration

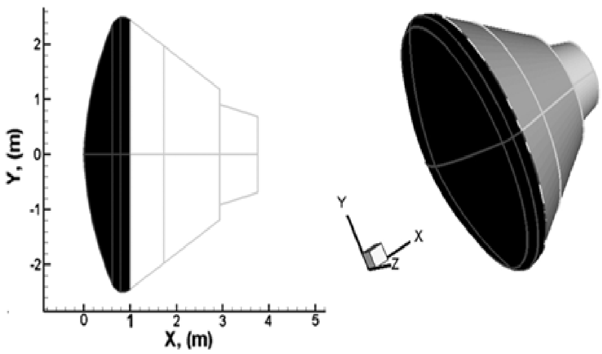

Fig 3. ARD configuration

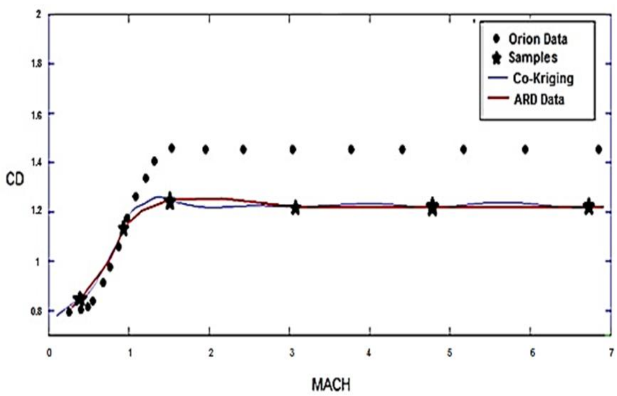

Fig 4. Drag coefficient vs. Mach number for Orion and ARD reentry vehicles

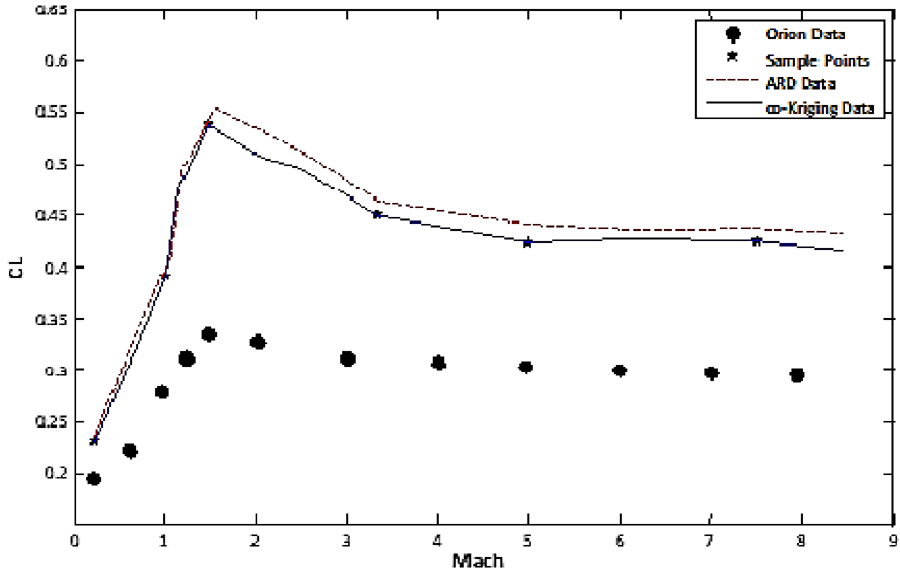

Fig 5. Lift coefficient vs. Mach number for Orion and ARD reentry vehicles (co-Kriging data is comparable with ARD Experimental data)

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