ASSESSMENT OF THE AERODYNAMICS AND AEROELASTICITY OF FLAPPING WING USING LINEARIZED COMPUTATIONAL METHOD

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Abstract

The aerodynamics and aeroelasticity of bio-inspired flapping wings in forward flight are simulated. The theoretical geometrical and kinematic bio-inspired baseline flapping wing model was developed based on literature study and observation. The main interest is to compare and assess the dynamics of rigid and flexible flapping wing in forward flight with those obtained using more sophisticated methods. Information on the kinematics and aerodynamics of rigid flapping wing is based on earlier studies.

1 General Introduction

The development of a flapping wing ornithopter mimicking flapping biosystems can be carried out in a progressive manner, by first looking into the geometric, kinematic and aerodynamic characteristics, and may be followed by the incorporation of the flexibility features. These may later on be incorporated into a controllable flapping ornithopter for optimum flight performance to meet the defined mission. In our earlier studies, analytical studies on flapping wings have also been performed to understand the aerodynamic characteristics and flight mechanisms of the flapping wings, which have been discussed in comparison with observation on biosystems and experiments. In view of the complexity of the fluid-structure interaction of a flexible flapping wing, our earlier work has been focussed on rigid wings, deferring the aeroelastic analysis after a comprehensive understanding and model has been gained on the geometric, kinematic and aerodynamic model of the rigid flapping ornithopter. Actually, the biological flapping flyers have flexible wings with anisotropic flexibility in both spanwise and chordwise directions. Using the flexible wings, they can utilize complicated wing motions consisting of flapping, twisting, folding, rotating motions or area expansion and contraction [1-6]. The passive or active deformation of the wing contributes to the generation of appropriate aerodynamic performances according to various flight modes. The artificial flapping flyers inspired from the biological flappers also have thin and flexible passive wings structurally similar to those of insects. Therefore, for the optimal design and the real-time control of flapping-wing flight, an efficient aerodynamic model applicable to general flapping wings is necessary, and an efficient aeroelastic analysis method should be also developed. Wing flexibility may be desirable for improving the flapping wing ornithopter aerodynamic performance and passive and active stability.

Hence, following earlier studies carried out [7-11], a generic approach is first followed to model the geometry, kinematics and aerodynamics of flapping wing ornithopter; considerations are given to the motion of a three-dimensional rigid and thin wing in flapping and pitching motion with and without phase lag. Basic Unsteady Aerodynamic Approach incorporating viscous effect and leading-edge suction is utilized the first baseline approach. The study is focused on a Bi-Wing ornithopter. Parametric study is carried out to reveal the flapping Bi-Wing ornithopter aerodynamic characteristics and for comparative analysis with various selected simple models in the literature. Further analysis is carried out by differentiating the pitching and flapping motion and studying its respective contribution to the flight forces.

Chimakurti [4] mentioned in his work that wing flexibility is found to have a favorable effect on lift generation. However, Zhang et al [5], in their Experimental Investigation on the effects of Flapping Wing Aspect Ratio and Flexibility on Aerodynamic Performance, show that the effect of flexibility reduces the lift. Dai et al [6], in their study on the Aerodynamics and Aeroelasticity of Flapping Wings, give results that indicate that the lift decreases significantly with flexibility. It is then of great interest to understand the overriding factor that influence of the flexibility on the flight performance of the flapping wing biosystem or ornithopter. It is with such motivation that the present study is carried out, starting with basic fundamentals and simplified approaches.

Accordingly, to study the influence of wing flexibility to flapping wing propulsion and lift characteristics, a fundamental representation of unsteady air loads and structural flexibility interaction is developed for the analysis and numerical simulation based on a generic linear aeroelastic analysis using forward speed and oscillatory flapping motion as disturbances. For this purpose, the present approach resorted to two earlier works. The first, as elaborated by Djojodihardjo and Yee [12], the aeroelastic characteristics of the flapping wing ornithopter is investigated. The purpose is to find out, how the flexibility of the wing, here represented by its typical section, influences its aeroelastic stability characteristics and where and how the flexibility could influence the aerodynamic performance. The second is the series of work already carried out in references [7] to [11], which will be used as the baseline for carrying out the aerodynamic performance of a flexible ornithopter wing. Based on the results obtained in the first part of the work, the second part of the work will look at the influence of the flexibility of the wing. For this purpose, the flexibility studies carried out in the first part of the study is used to generate a heuristic model of the influence of the aeroelastic characteristics of the wing on its aerodynamic characteristics, in particular lift and thrust. The results will then be assessed based on physical framework and other available results and approaches in the literature. Further elaborate work will be discussed.

2 General Strip Theory Aerodynamics For Baseline Flapping Rigid Wing Ornithopter

2.1 Aerodynamic Model

The present aerodynamic approach is synthesized using basic foundations that may exhibit the generic contributions of the motion elements of the bio-inspired bi-wing and quad-wing air vehicle characteristics. These are the strip theory and thin wing aerodynamic approach [13-15], and Jones' modified Theodorsen approach [16] which incorporates Garrick's leading-edge suction [17]. The computation of lift and thrust generated by pitching and flapping motion of three-dimensional rigid wing is carried out in a structured approach. Later, the computational model will take into account certain physical parameters that can be identified via observations and established results of various researchers.

To obtain an insight into the mechanism of lift and thrust generation, Djojodihardjo and Ramli [7-9] and Djojodihardjo and Bari [10, 11] analyzed the wing flapping motion by looking into the individual contribution of the pitching, flapping and coupled pitching-flapping to the generation of the aerodynamic forces. Also, the influence of the variation of the forward speed, flapping frequency and pitch-flap phase lag has been analyzed. Such approach will also be followed here through further scrutiny of the motion elements.

The flapping motion of the wing is distinguished into three distinct motions with respect to the three axes; these are: a) *Flapping*, which is up and down plunging motion of the wing; b) *Feathering* is the pitching motion of wing and can vary along the span; c) *Lead-lag*, which is in-plane lateral movement of wing, as incorporated in Fig. 4. For further reference to the present work, the lead-lag motion could be interpreted to apply to the phase lag between pitching and flapping motion, while the fore-and-aft movement can be associated with the orientation of the stroke plane. The degree of freedom of the motion is also depicted in Fig. 4. The flapping angle β varies as a sinusoidal function and pitching angle θ are given by the following equations.

$$\beta(t) = \beta_0 \cos \omega t \qquad (1)$$

$$\theta(t) = \theta_0 \cos(\omega t + \phi) + \theta_{fp} \qquad (2)$$

where θ_0 and β_o indicate maximum value for each variable, ϕ is the lag between pitching and flapping angle and y is the distance along the span of the wing, and θ_{fp} is the sum of the flapping axis angle with respect to flight velocity (incidence angle) and the mean angle of the chord line with respect to the flapping axis, as exhibited in Figure 1.

The present method is exemplified by the use of elliptical planform wing. As a baseline, by referring to Eqs. (1) and (2), β and θ is considered to oscillate following a cosine function; such scheme indicates that these motions start from specified values. A different scheme, however, can be adopted.

Leading-edge suction is included following the analysis of Polhamus [18, 19] and DeLaurier's approximation [13]. Three dimensional effects will later be introduced by using Scherer's modified Theodorsen-Jones Lift Deficiency Factor [20], in addition to the Theodorsen unsteady aerodynamics [15] and its three-dimensional version by Jones [16]. Further refinement is made to improve accuracy. Following Multhopp approach [14], simplified physical approach to the general aerodynamics of arbitrary planform is adopted, i.e. a lifting line in the quarter-chord line for calculating the downwash on the three-quarter-chord line for each strip.

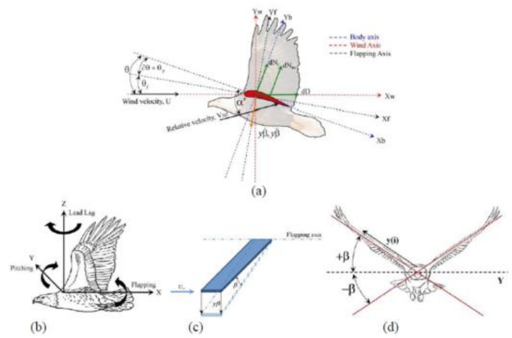

Figure 1: (a) Forces on section of the wing. (b),(c),(d) Flapping and Pitching motion of flapping wing.

In the present analysis no linear variation of the wing's dynamic twist is assumed for simplification and instructiveness. However, in principle, such additional requirements can easily be added due to its linearity. The total normal force acting perpendicularly to the chord line and given by

$$dN = dN_c + dN_{nc} \qquad (3)$$

which consist of the circulatory normal force for each section acts at the quarter chord and also perpendicular to the chord line, given by

$$dN_c = \frac{\rho UV}{2} C_n(y) c dy \qquad (4)$$

and the apparent mass effect that is perpendicular to the wing, and acts at mid chord, and can be calculated as

$$dN_{nc} = \frac{\rho \pi c^2}{4} (U\dot{\alpha} - \frac{1}{4} c\ddot{\theta}) dy \qquad (5)$$

The total chordwise force, dF_x is accumulated by three force components; these are the leading-edge suction, force due to camber, and chordwise friction drag due to viscosity effect. All of these forces are acting along and parallel to the chord line.

$$dF_x = dT_s - dD_{camber} - dD_f \qquad (6)$$

The leading-edge suction, dT_s , following Garrick [17], is given by

$$dT_s = 2\pi\eta_s \left(\alpha' + \theta_{fp} - \frac{1}{4}\frac{c\dot{\theta}}{U} \right) \frac{\rho UV}{2} c dy \qquad (7)$$

while following DeLaurier [13] the chordwise force due to camber and friction is respectively given by

$$dD_{camber} = -2\pi\alpha_o (\alpha' + \theta_{fp}) \frac{\rho UV}{2} c dy \qquad (8)$$

$$dD_f = \frac{1}{2} \rho V_x^2 C_{d_f} c dy \qquad (9)$$

The efficiency term η_s is introduced for the leading-edge suction dT_s to account for viscosity effects. The vertical force dN and the horizontal force dF_x at each strip dy will be resolved into those perpendicular and parallel to the free-stream velocity, respectively. The resulting vertical and horizontal components of the forces is then given by

$$dL = dN\cos\theta + dF_x\sin\theta \qquad (10)$$

$$dT = dF_x\cos\theta - dN\sin\theta \qquad (11)$$

To obtain a three-dimensional lift for each wing, these expressions should be integrated along the span, b; hence

$$L = \int_0^b dL dx \qquad (12)$$

$$T = \int_0^b dT dx \qquad (13)$$

For later comparison with appropriate results from the literature, numerical computations are performed using the following wing geometry and parameters: the wingspan of 40cm, aspect ratio of 6.36, flapping frequency of 7Hz, total flapping angle of 60º, forward speed of 6m/s, maximum pitching angle of 20º, incidence angle of 6º and there is no wing dihedral angle. In the calculation, both the pitching and flapping motions are in cosine function by default, which is subject to parametric study, and the upstroke and downstroke have equal time duration. The wake capture has not been accounted for in the current computational procedure. The computational scheme developed and the aerodynamic forces for bi-wing has been validated and verified satisfactorily in previous work (Djojodihardjo et al [7, 11]).

3 Synthesis of Aeroelastic Approach Philosophy

In the consideration of the aeroelastic effect of flapping ornithoptrer, a simplified two-dimensional approach for the typical section of the flapping wing ornithopter will be resorted following earlier approach elaborated in the Structural Dynamics Equation of typical section in 2D – pitch and heave [12] will be used as a baseline.

Following the Unsteady Aerodynamics elaborated in previous section, the lift L and moment M of the wing as given by Eqs. (12) and (13) could be taken to be acting on the typical section. To start the analysis, one should look at the dynamics of the wing represented by the typical section following the Lagrange equation (by referring to Figure 2):

$$-\frac{d}{dt}\left(m\dot{h} + S_\alpha\dot{\alpha}\right) - K_h h - L = 0 \qquad (14)$$

$$-\frac{d}{dt}\left(S_\alpha\dot{h} + I_\alpha\dot{\alpha}\right) - K_\alpha\alpha + M_y = 0 \qquad (15)$$

which can be reduced to the general governing equation for such two-dimensional pitching and flapping (heaving) aerodynamic section given by:

$$m\ddot{h} + S_\alpha\ddot{\alpha} + K_h h = -L \qquad (16)$$

$$S_\alpha\ddot{h} + I_\alpha\ddot{\alpha} + K_\alpha\alpha = M \qquad (17)$$

where m is the mass per unit span of the typical section, S_α is the static moment of the typical section with respect to the elastic axis, I_α is the polar moment of inertia of the typical section with respect to the elastic axis, K_h and K_α are the bending (heaving) and torsional spring stiffness, respectively, of the typical section. The typical section experiences movement in two degrees of freedom, i.e. h, heaving (bending) displacement in the vertical direction (positive downward), and α, pitching angular displacement (positive nose up). L and M_{AC} are the aerodynamic Lift and Moment, respectively; both L and M_{AC} are acting on the aerodynamic center (L positive upward, M_{AC} positive nose-up)

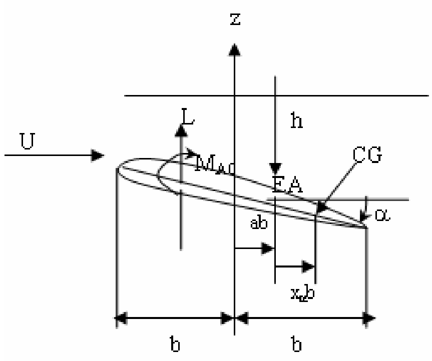

Figure 2: Free body diagram for a typical section

Since the Eqs. (16) and (17) are written for a two degree-of freedom pitching and heaving typical section, one may consider that the ornithopter wing be represented by a typical section. This implies that the inertial and other related properties of the wing be considered to be "collapsed" at the typical section.

Noting that the wing is essentially following pitching and flapping motion for its aerodynamic performance, then the elasticity of the wing could be considered to modify the aerodynamic pitching and flapping. In other words, each of the pitching and flapping motion has two components, the motion based and the elastic based. Hence the dynamics of the flexible pitching and flapping wing can be identified with the parameters:

$$\alpha_{flex-wing} = \alpha_{rigid} + \alpha_{el} \tag{18}$$

$$h_{flex-wing} = h_{rigid} + \alpha_{el} \tag{19}$$

The rigid part can be evaluated using our earlier approach [7-11]:

$$\alpha_{rigid} = \alpha \tag{20}$$

$$h_{rigid} = h \tag{21}$$

then

$$\alpha_{el} = \left(\alpha_{el}\right)_0 e^{i\omega_{el} t} \tag{22}$$

$$h_{el} = \left(h_{el}\right)_0 e^{i\omega_{el} t} \tag{23}$$

where ω_{el} is the harmonic frequency, due to the flexibility of the wing structure (and not to be confused with the flapping or pitching frequency). Substituting into the dynamic equation of motion,

$$m\ddot{h}_{flex} + S_\alpha \ddot{\alpha}_{flex} + K_h h_{flex} = -L\left(\alpha_{flex}, h_{flex}\right) \tag{24}$$

$$S_\alpha \ddot{h}_{flex} + I_\alpha \ddot{\alpha}_{flex} + K_\alpha \alpha_{flex} = M\left(\alpha_{flex}, h_{flex}\right) \tag{25}$$

or

$$\begin{bmatrix} S_\alpha \omega^2 & m\omega^2 + K_h \\ I_\alpha \omega^2 + K_\alpha & S_\alpha \omega^2 \end{bmatrix} \begin{Bmatrix} \alpha_{flex} \\ h_{flex} \end{Bmatrix} = \begin{Bmatrix} -L\left(\alpha_{flex}, h_{flex}\right) \\ M\left(\alpha_{flex}, h_{flex}\right) \end{Bmatrix} \tag{26}$$

The terms on the left-hand side of the equation leads to an Eigenvalue problem, which can be solved to yield the eigenfrequencies and eigenmodes:

$$\begin{bmatrix} S_\alpha \omega^2 & m\omega^2 + K_h \\ I_\alpha \omega^2 + K_\alpha & S_\alpha \omega^2 \end{bmatrix} = 0 \tag{27}$$

Using the dynamic response relationship of the flexible wing due to aerodynamic and other exciting force, one will be able to evaluate the total prevailing aerodynamic angle of the flapping and pitching motion.

4 Typical Section Representation of 3-D Wing for Aeroelastic Analysis

Next three different aerodynamic approximations, with increasing complexity, will be utilized; these are the Quasi-Steady, low frequency oscillation and classical unsteady (harmonic) aerodynamics as described by Theodorsen [15]. In what follows, the simplest approach, the Quasi-Steady Aerodynamic Model, will be followed.

For the quasi-steady aerodynamic model, the aerodynamic Lift L and Moment M_{AC}, as well known in steady linearized aerodynamics, are given by:

$$L(t) = qSC_{L\alpha}\alpha(t) \tag{28}$$

and

$$M_{AC} = 0 \tag{29}$$

Since linearized aerodynamics is used, the airfoil essentially is regarded as a flat plate. From Figure 6, the aerodynamic moment with respect to the elastic axis is given by

$$M_{EA} = 2Leb + M_{AC} = 2qSebC_{L\alpha}\alpha(t) \tag{30}$$

Hence Eqs. (16) and (17) can be rewritten as

$$m\ddot{h} + S_\alpha \ddot{\alpha} + K_h h + qsC_{L\alpha}\alpha = 0 \tag{31}$$

$$S_\alpha \ddot{h} + I_\alpha \ddot{\alpha} + K_\alpha \alpha - 2qSebC_{L\alpha}\alpha = 0 \tag{32}$$

which is known as the flutter equation (since in the form given by Eqs. (31) and (32), or the following equation, Eq. (33), this is an eigen-value or stability equation). In matrix notation, this is given by:

$$[M]\{\ddot{x}\} + ([K] - q[A_o])\{x\} = \{0\} \tag{33}$$

where

$$[M] = \begin{bmatrix} m & S_\alpha \\ S_\alpha & I_\alpha \end{bmatrix} \text{(Inertia)}$$

$$[K] = \begin{bmatrix} K_h & 0 \\ 0 & K_\alpha \end{bmatrix} \text{(Structural stiffness)}$$

(34a, b, c)

$$[A_o] = \begin{bmatrix} 0 & -SC_{L\alpha} \\ 0 & 2sebC_{L\alpha} \end{bmatrix} \text{(Aerodynamic Stiffness)}$$

For convenience, following the practice in aeroelasticity, the analysis of flutter stability can be obtained by assuming a solution of the form:

$$\begin{aligned} h &= \hat{h}e^{pt} \\ \alpha &= \hat{\alpha}e^{pt} \end{aligned} \quad \text{and} \quad \{x\} = \{\hat{x}\}e^{pt} \qquad (35)$$

Solving as eigen-value problem, going through the algebra will results in the flutter stability characteristic equation given by:

$$I_\alpha m p^4 - S_\alpha^2 p^4 + p^2\left[mK_\alpha - 2qSebC_{L\alpha}m + I_\alpha K_h\right]$$
$$-p^2 S_\alpha qSC_{L\alpha} + K_\alpha K_h - 2qSebC_{L\alpha}K_h = 0 \qquad (36)$$

which has a general form of:

$$a_4 p^4 + a_2 p^2 + a_0 = 0 \qquad (37)$$

where:

$$a_4 = I_\alpha m - S_\alpha^2$$
$$\begin{aligned} a_2 &= m\left(K_\alpha - 2qSebC_{L\alpha}\right) + I_\alpha K_h - S_\alpha qSC_{L\alpha} \\ &= mK_\alpha + I_\alpha K_h - \left(2meb + S_\alpha\right)qSC_{L\alpha} \end{aligned} \qquad (38a,b,c)$$
$$a_0 = K_h\left(K_\alpha - 2qSebC_{L\alpha}\right)$$

The characteristic equation is a fourth order polynomial which has four roots;

$$p_{1,2,3,4} = (\sigma + i\omega)_{1,2,3,4}$$
$$= \pm\sqrt{\frac{1}{2a_4}\left(-a_2 \pm \sqrt{a_2^2 - 4a_4 a_0}\right)} \qquad (39)$$

and the solution is given in the form

$$\{x\} = \{\hat{x}\}e^{\sigma t}e^{i\omega t} \qquad (40)$$

where σ is damping, ω is frequency and $\{\hat{x}\}$ Vibration mode representing the displacement vector. Following Done [21, 22], as summarized by Zwaan [23] and Djojodihardjo [24], the solution can be conveniently and comprehensively represented by damping and frequency diagrams as functions of either dynamic pressure q, or reduced frequency (or reduced velocity U_R) $k_R = U/(b.\omega)$, or velocity, as illustrated subsequently, or summarized in a table. The table allows the classification of solution according to the values of the coefficients a_0, a_2, a_4, and the stability categories that result. The stability of motion depends on the value of σ (aerodynamic damping). As can be concluded from Eq. (40), and summarized in Table 1, if $\sigma > 0$, then the displacement vector {x} will oscillate with increasing amplitude in time, and the resulting motion will be unstable. If $\sigma = 0$, a neutrally stable oscillation will result. Only when $\sigma > 0$ the oscillation will subside in time.

$a_2^2 - 4a_4a_0$	>0			<0
	>0		<0	><0
	>0	<0	><0	><0
p^2	$-\omega_1^2, -\omega_2^2$	σ_1^2, σ_2^2	$\sigma^2, -\omega^2$	$-g \pm ih$
p	$\pm i\omega_1, \pm i\omega_2$	$\pm i\sigma_1, \pm i\sigma_2$	$\pm\sigma, \pm i\omega$	$\pm\sigma, \pm i\omega$
Type of motion	Harmonic: 2 pos.freq 2 neg.freq	Aperiodic: 2 diverging 2converging	Aperiodic: 1 diverging 1converging Harmonic: 1 pos.freq 1 neg.freq	Oscillatory: 1 div.pos.freq 1 conv.pos.freq 1 div.neg.freq 1conv.neg.freq
Type of instability	Neutral	Divergence	Divergence	Flutter
Category	I	III	IV	II

Table 1: Flutter stability solution categories.

5 Quasi-Steady Aeroelastic Analysis of Flapping Wing Ornithopter Represented as Typical Section

For the purpose of aeroelastic modeling and assessment using steady model approach, the Ornithopter Flapping Wing Model elaborated in References [7] to [11] is represented by a typical section.

Parameter	Unit	Value
Mass	kg	2.5480e-04
Span	m	0.4
c	m	0.08
b	m	0.04
e		-0.02
Omega_alpha	Rad/sec	5.9178e+04
Omega_h	Rad/sec	5.9178e+04
ρ	Kg/m³	1.225
Cl_alpha		6.2832
y	m	0.2
E	GPa	2.9000e+09
I	m⁴	8.3333e-10
G	GPa	1.1154e+09

Table 2: Flapping wing typical section characteristic of the ornithopter flapping wing model.

Based on the data utilized, baseline sectional properties of the typical section model have been evaluated and tabulated in Table 2. In addition, some parametric study can be carried out to obtain a favorable configuration and aeroelastic configuration.

5.1 Computational Results

The computations are performed using the following wing geometry and parameters: the wingspan of 40cm, chord length of 8cm, and the wing shape is rectangular. The data for the typical section representing the flapping ornithopter wing is tabulated in Table 1.

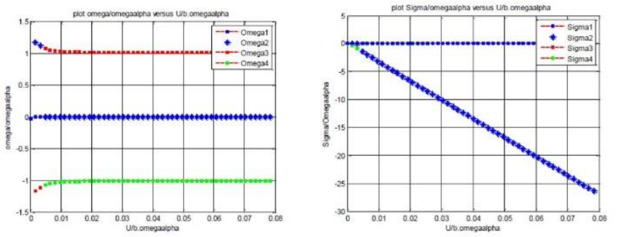

Figure 3: Numerical computation to determine the flutter stability of the Quasi-Steady Model of Ornithopter Wing Typical Section.

Several simplifying assumptions have been made in order to obtain some insight into the flexibility characteristics of the biomimicking ornithopter flapping wing. The elastic properties listed there is based on keratin [25]. The results as shown in Figure 3 and Table 3, indicate characteristics typical of the first column of Table 1, which will not lead to aeroelastic instability. In addition, the results also show that the prevailing eigenfrequencies estimated using the quasi-steady aerodynamics in the operational range of the flapping ornithopter is much smaller than the pitching and flapping frequency of the ornithopter wing. Such conclusion is considered reasonable and in confirmation with observation on biosystem. In addition, the flexural property as represented by K_α shows that, if quasi-steady aerodynamics is assumed, the elastic deflection due to the prevailing aerodynamic force as calculated using the unsteady aerodynamics elaborated in section 2 will produce at most 5% change in θ or α'. Such situation is taken into consideration in establishing a heuristic model as elaborated in succeeding section.

x_alpha	0.5000	r_alpha	0.0015
Mass	2.5480e-04	I_alpha	3.4106e-07
S_alpha	5.0960e-06	K_alpha	1.1944e+03
K_h	906.2500	a0	1.1593e+06
a1	3.4122e+05	a2	0.0561
a3	9.5106e-05	a4	6.0933e-11
Determinant	0.0029	V	-0.9972 -0.9972 0.0747 -0.0747
D	1.0e+29 * -0.1079 0 0 5.5821		
Matrix A	1.0e+30 * 0.2737 3.8000 0.0213 0.2737		

Table 3: Computational Results of Aeroelastic Stability Characteristics – Quasi-Steady Model

6 Incorporation of Quasi-Steady Aerodynamics Flexibility in a Heuristic Model for Aerodynamic Performance Estimation

Based on the findings obtained in previous section, a heuristic aeroelastic model can be established. The simplest one is to incorporate the influence of the aeroelastic properties being reduced to the static flexibility properties. It is also assumed that the flexibility effect acted instantly. Following such rationale, then the effect of aeroelasticity, hence flexibility, is to modify the pitching and heaving angle linearly to a small percentage. The results of such heuristic model assumption to the aerodynamic performance of the flapping wing ornithopter can be calculated using the procedure already outlined in section one. Essentially, a constant Flexibility Coefficient γ_f is introduced to account for flexibility of the wing in pitching and flapping motion.

Then the pitching and heaving motion will be modified as follows

$$\theta(t) = \gamma_f \left(\theta_0 \cos(\omega t + \phi)\right) + \theta_{fp} \quad (41a)$$

$$\dot{\theta}(t) = \gamma_f \left(-\omega\theta_0 \sin(\omega t + \phi)\right) \quad (41b)$$

$$\ddot{\theta}(t) = \gamma_f \left(-\omega^2\theta_0 \cos(\omega t + \phi)\right) \quad (41c)$$

$$h(t) = \gamma_f \left(-y\beta_0 \cos \omega t\right) \quad (41d)$$

$$\dot{h}(t) = \gamma_f \left(y\omega\beta_0 \sin \omega t\right) \quad (41e)$$

$$\ddot{h}(t) = \gamma_f \left(y\omega^2\beta_0 \cos \omega t\right) \quad (41f)$$

The results are exhibited in Figures 4a and 4b, which describe the influence of the flexibility on the lift and thrust produced by the flapping wing, if the flexibility effects are introduced on θ and h. All the results are computed by considering the dynamic stall criterion for attached flow similar to that utilized by DeLaurier [13].

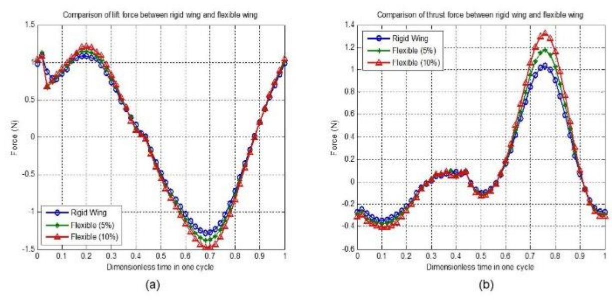

Figure 4: (a) Lift and (b) Thrust variation with rigid wing and flexible wing of 5% and 10% using heuristic model

	Rigid Wing	Flexible Wing (5%)	Flexible Wing (10%)
Average Lift (N)	0.0662	0.0498	0.0386
Average Thrust (N)	0.1110	0.1272	0.1457

Table 4: Comparison of the average Lift and Thrust of rigid and flexible ornithopter wing using heuristic model using θ and has basis of flexible deformation.

Figures 4a and 4b and Table 4 shows the influence of introducing 5% and 10 % flexibility as a representation of the aeroelastic effect using quasi-steady aerodynamics.

If the flexibility factor γ_f is introduced into the apparent angle of attack α', the prevailing equation will be modified as;

$$\alpha' = \gamma_f \left[\frac{AR}{(2+AR)} \left[F(k)\alpha + \frac{c}{2U} \frac{G(k)}{k} \dot{\alpha} \right] - \frac{w_o}{U} \right] \tag{42}$$

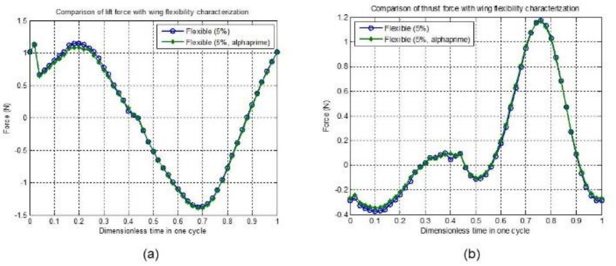

Figure 5: (a) Lift & (b) thrust variation with flexible wing of 5% and 5% from alpha prime (α').

	Flexible Wing (5%)	Flexible Wing (5%, alphaprime)
Average Lift (N)	0.0498	0.0306
Average Thrust (N)	0.1272	0.1395

Table 5 Comparison of the average Lift and Thrust of rigid and flexible ornithopter wing using heuristic model using α' (alpha prime) as basis of flexible deformation.

The results are exhibited in Figures 5a and b, and Table 5. These results show that the effect of static aeroelasticity tends to reduce the lift and increase the thrust. In addition, the introduction of the static aeroelasticity introduced to the primary variables θ and h will produce slightly different values than if the aeroelastic effect is introduced in the derived variable α'. Noting that the heuristic model is a first approximation to the actual state of affairs, such difference may be attributed to many simplifying assumptions, such as the three dimensionality of the flow as represented by α', among others.

Proceeding to the investigation on the static aeroelasticity effects on the individual contribution of pitching and flapping motion components, the results are shown in Figures 6a and 6b, and Table 6. For this particular study, the incidence angle is assumed to be zero. These figures show that the contribution of static aeroelasticity to flapping is more apparent than to pitching.

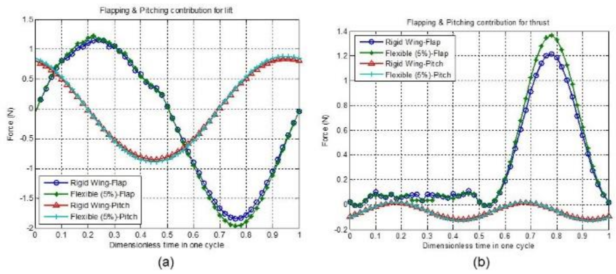

Figure 6: Contribution of flapping and pitching motion individually on (a) lift & (b) thrust forces for rigid wing and flexible wing of 5%.

	Flapping only		Pitching only	
	Rigid Wing	Flexible Wing (5%)	Rigid Wing	Flexible Wing (5%)
Average Lift (N)	-0.1864	-0.2065	0.0161	0.0169
Average Thrust (N)	0.3123	0.3435	-0.0518	-0.0562

Table 6: Comparison of the average Lift and Thrust of rigid and flexible ornithopter wing using heuristic model contributed by pitching and flapping motion

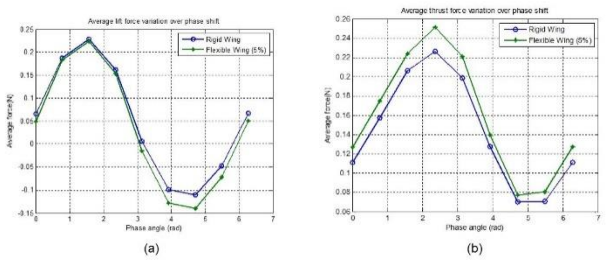

Figure 7: Phase shift influence on (a) lift & (b) thrust forces for rigid wing and flexible wing of 5%.

Next the static aeroelasticity effects on the phase lag between the Pitching and Flapping Motion Components is investigated, and the results are shown in Figures 7a and 7b, and

Table 7. For this particular study, the incidence angle is also assumed to be zero. In this study, a parametric study is carried out by varying the phase lag between flapping and pitching from 0˚ to 360˚ (2π).

The results as exhibited by these figures show the extent the contribution of static aeroelasticity to the influence of the phase lag between the pitching and flapping motion on the lift and thrust generated by the flapping wing ornithopter.

	Rigid Wing		Flexible Wing (5%)	
Phase	Average Lift (N)	Average Thrust (N)	Average Lift (N)	Average Thrust (N)
0	0.0662	0.1110	0.0498	0.1272
0.25π	0.1877	0.1571	0.1823	0.1748
0.5π	0.2289	0.2063	0.2240	0.2238
0.75π	0.1619	0.2263	0.1540	0.2514
π	0.0056	0.1991	-0.0148	0.2212
1.25π	-0.0987	0.1274	-0.1288	0.1399
1.5π	-0.1111	0.0700	-0.1402	0.0775
1.75π	-0.0482	0.0708	-0.0723	0.0808
2π	0.0665	0.1111	0.0502	0.1273

Table 7: Comparison of the average Lift and Thrust of rigid and flexible ornithopter wing using heuristic model due to phase-shift between pitching and flapping motion.

7 Conclusions

The philosophical approach and computation presented in the present paper is based on the utilization of quasi-steady aerodynamics a typical section approximation of the flexible flapping wing ornithopter, based on model utilized in [7-12]. In addition, based on the Aeroelastic stability characteristics obtained using such approach, a heuristic flexibility model has been assumed in estimating the influence of flexibility on the aerodynamic performance of the flexible flapping wing ornithopter. These approaches are considered to be educational. With the introduction of all these simplification, one may expect to obtain a qualitative impression of the influence of flexibility using zeroth order approximation, but yet may gain some insight of the issue using lower cost effort. Further approximation may have to be judged on the cost of the effort compared to the more sophisticated approach using refined model and computational scheme, such as exemplified by [1-4]. Nevertheless, comparing the present results with those of Zhang et al [5] and Dai et al [6], the present results exhibit some similar trend, in the sense that the flapping wing low flexibility exhibits minor influence on the aerodynamic performance. The present approach and model, however, indicate that the flexibility. The present approach and model, however, indicate that the influence of flexibility of the flapping wing improves its capability to produce thrust rather than lift.

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