FLIGHT TECHNICAL ERROR EVALUATIONS FOR GBAS NAVIGATION REQUIREMENT DEFINITION

Michael Felux*, Jiyun Lee, Florian Holzapfel*****
***German Aerospace Center (DLR), **Korean Advanced Institute of Science and Technology (KAIST), *** Technische Universität München**

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Abstract

Requirements for a CAT-II/III capable service type of the Ground Based Augmentation System (GBAS) were derived from the definition of a safe landing of an aircraft within a pre-defined touchdown zone on the runway. The contributions to the total system error come from the navigation system on the one hand and the autopilot performance on the other hand. In order to quantitatively assess the touchdown performance of a given aircraft and its autopilot, Monte-Carlo simulations are performed and the results modelled. The touchdown performance is usually described by a fitted Gaussian distribution. This distribution is used for deriving the navigation system requirements. This paper argues that fitting the data to a Gaussian distribution is inappropriate and may lead to an underestimation of the residual risks of landing outside the touchdown box. It therefore describes two alternative methods to model the touchdown performance: a Gaussian overbound and a Johnson distribution. Both methods are described and the implications of modelling the performance in these ways are discussed.

1 Introduction

The Ground Based Augmentation System (GBAS) is a precision approach guidance system for aircraft based on Global Navigation Satellite Systems (GNSS). A ground station at an airport generates locally relevant corrections for the navigation signals in order to eliminate errors introduced mainly by ionospheric and tropospheric effects, as well as ephemeris and other errors originating from the satellite. Currently commercially available are GBAS stations supporting operations down to CAT-I minima. In the past years the concept to provide also service under CAT-II and CAT-III conditions was developed and standardized. The main method for the derivation of the required performance of CAT-III capable GBAS is based on the concept of a safe landing and the trade-off between the flight technical error (FTE), i.e. the ability of the autopilot to land the aircraft on a pre-defined spot, and the navigation system error (NSE), i.e. how well the avionics can determine the aircraft's position. In this process a safe landing is characterized by touching down in a predefined area on the runway, the so-called touchdown box. This box is described in the Certification Specifications for All-Weather operations [1] and the Advisory Circular on Criteria for approval of Category III Weather Minima for Takeoff, Landing, and Rollout [2]. It extends from a distance of 200 ft behind the runway threshold to 2700 ft (or 3000 ft in certain cases) and laterally to 5 ft from the edges of the runway and is depicted in Fig. 1. Furthermore, it is assumed that the FTE can be described by a nominal touchdown point (NTDP) on the runway and a dispersion around that point modelled by a Gaussian distribution with a standard deviation σ_{FTE} . In the certification process of aircraft and autoland systems, the FTE performance is determined in Monte-Carlo simulations. It is usually modelled by fitting a Gaussian distribution to the obtained touchdown results.

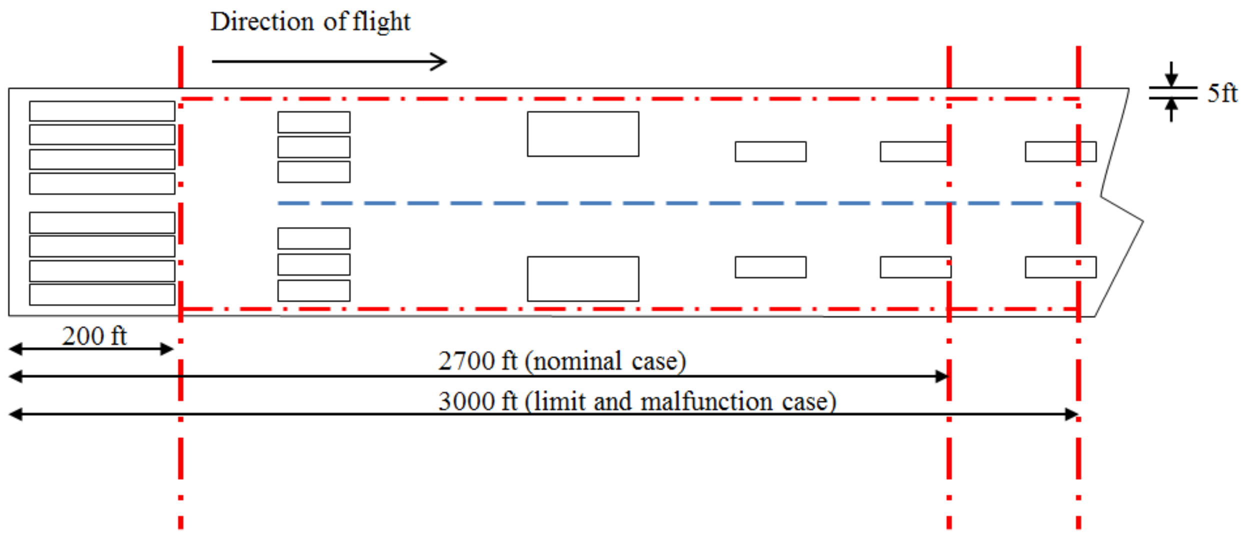

Fig. 1 Illustration of the Touchdown Box on the Runway

The Navigation System Error (NSE) is also considered to follow a Gaussian distribution and is also described as dispersion in along-track direction with zero mean. The along-track error results from a nominal vertical position error NSE_{vert} (e.g. due to multipath and residual ionospheric and tropospheric errors) that translates into an along-track error NSE_{atrk} at touchdown given by

$$NSE_{atrk} = \frac{NSE_{vert}}{\tan(GPA)} \quad (1)$$

where GPA is the glide slope angle of the approach (typically 3°). This assumption is made based as the vertical guidance is switched from the GBAS or ILS to the aircraft's own radar altimeter at a predefined height above ground. Any vertical position error from the navigation system therefore results in an along-track position error at touchdown.

The NSE is considered to be independent of the FTE such that the Total System Error (TSE) can also be described by a Gaussian distribution centered at the NTDP with a standard deviation given as the root sum square of the standard deviations of the NSE and the FTE respectively:

$$\sigma_{TSE} = \sqrt{\sigma_{NSE}^2 + \sigma_{FTE}^2} \quad (2)$$

The problem with the described approach of allocating error budgets is that the Gaussian model does generally not fit, especially to the FTE performance. Fitting the obtained results to a Gaussian distribution may thus lead to an underestimation of the risk of landing outside the touchdown box.

In this paper, we therefore revisit the problem of properly modelling the FTE in a way that it is appropriate for the subsequent derivation of navigation requirements on the basis of a total error budget for automatic landings.

2 Autoland simulations and obtained results

Assessing the touchdown performance of an autopilot is a challenging task as the touchdown behavior is influenced by a number of parameters. The previously mentioned documents [1],[2] give a list of parameters that have to be taken into account when performing these simulations. They include wind, different configurations of the aircraft, weight and balance and airport and runway characteristics, such as slope of the runway or elevation of the airport. In the following an exemplary analysis is performed.

2.1 Autoland simulations

For exemplary simulations an autopilot model of an Airbus A320 was used. The goal of these simulations was not to perform a full qualification of the autopilot model, but rather to obtain a somewhat representative set of touchdown performance data and to analyze the results with respect to the modelling of the data.

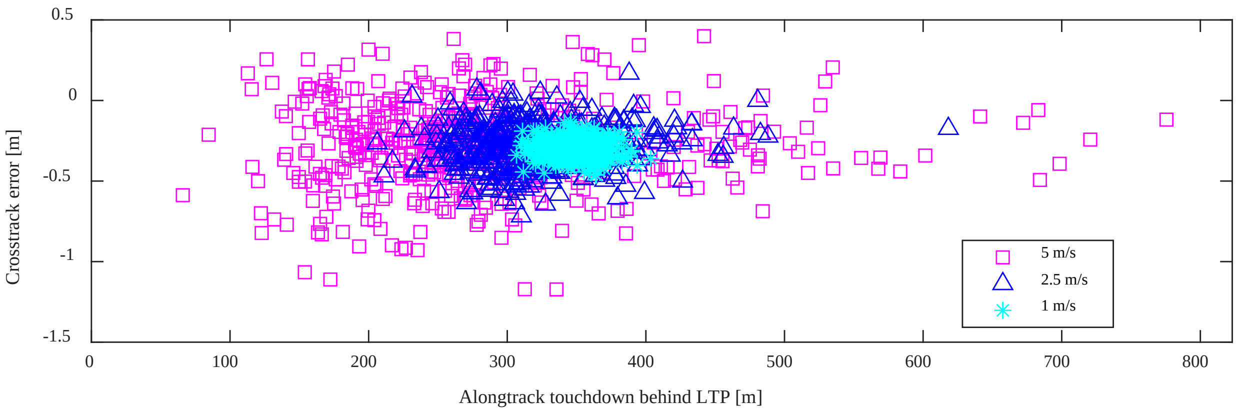

Fig. 2 Touchdown points for 500 landings with light turbulence and steady wind of 1m/s (cyan asterisks), 2.5m/s (blue triangles) and 5m/s (magenta boxes)

The simulations therefore neglected a variety of parameters: 500 approaches using only one approach path to a specific airport (in this case Runway 26 in Braunschweig in Northern Germany at an elevation of 261 ft) was simulated, assuming a perfectly flat runway. Furthermore, only one specific weight (64.5 t, corresponding to the maximum landing weight of the A320) and one specific center of gravity location (22.2% MAC) were chosen. For the wind influence in the initial simulations, the Dryden wind model according to [3] was used simulating a pure headwind (coming from 265° corresponding to the runway orientation) with steady wind speeds of 1 m/s, 2.5 m/s and 5 m/s and different turbulence intensities as described in the following subsection. At a later stage recorded wind speeds from Munich Airport in Germany were used in order to have a realistic distribution of winds. The wind direction and speed relative to the runway were used as input to the Dryden wind model's reference wind.

2.2 General results

In order to show the general nature of the results, the first part of the discussion shows the obtained touchdown points on the runway for the simulations described previously, with the three different steady wind cases and the turbulence intensity set to "light" in the Matlab Aerospace Blockset Dryden wind model block. For each steady wind case 500 landings were simulated. The resulting dispersion of the touchdown points for each case is shown in Fig. 2. On the x-axis the distance from the runway threshold is plotted, on the y-axis the deviation from the centerline of the runway. Recall that the touchdown box limits the area where the aircraft has to land to the area between 200 ft (~60 m) and 2700 ft (823 m, corresponding to the right side plotting limit). The lateral limits are 5 ft (~1.5 m) from the edge of the runway. Assuming a 45 m wide runway this would correspond to a distance of 21 m from the centerline. The obtained results show only very small deviations from the centerline (almost all within 1 m). The reason is mainly that the wind was simulated to be aligned with the runway direction. In a more realistic case the touchdown points would also scatter somewhat more in the lateral direction. As the along-track error (and therefore the vertical NSE according to Eq. (2)) is more constraining, the lateral errors are not further considered in this paper. From the results in Fig. 2 it becomes already apparent that the touchdown points are not really Gaussian distributed, especially for the cases with higher wind speeds. This is also confirmed when the data is tested for Normality e.g. with the Anderson-Darling test [4] or the Kolmogorov-Smirnov test [5] at a 5% significance level. Similar results were also obtained e.g. by Boeing in the certification process of the autoland system of the B757/B767 [6] and in previous work performed at DLR [7]. The next section therefore introduces two different methods how to address the fact that the data is not Gaussian distributed.

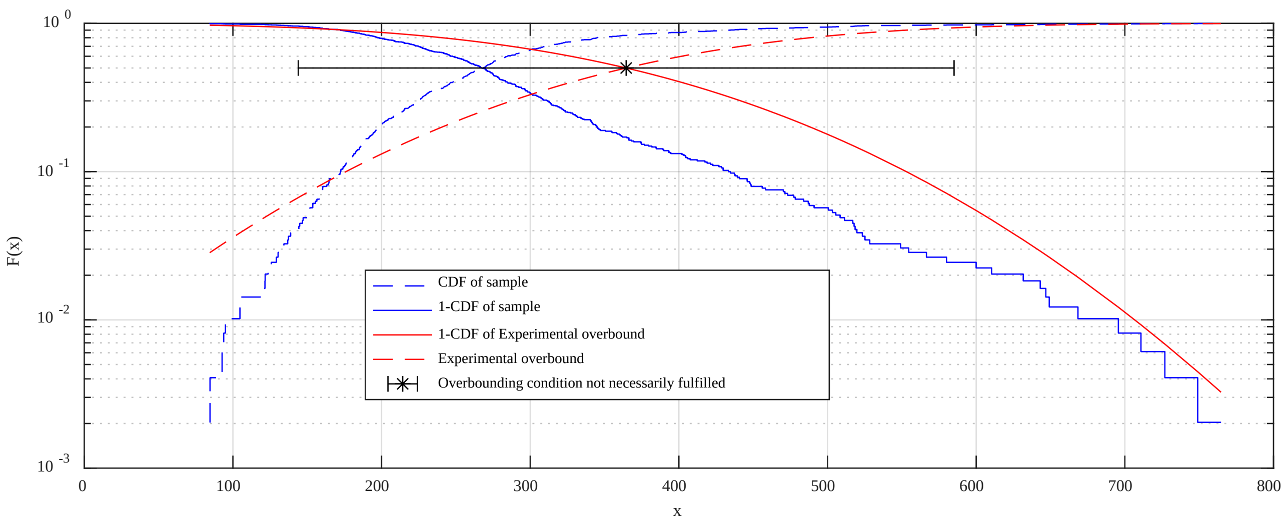

Fig. 3 Experimental distribution (blue) and Gaussian overbound (red)

3 Modelling of obtained FTE performance

In this section the problem of appropriate modelling of the touchdown performance of an aircraft is discussed, based on the obtained data described in the previous section. A general approach that is usually taken and acceptable for many applications is to fit the data to a Gaussian distribution. However, for the purpose of deriving navigation requirements using a Gaussian fit is not a suitable way forward. Fitting the data may describe the data well at the core of the distribution. The risk in the tails of the distribution of the touchdown points may, however, be larger than described by a fitted Gaussian. In order to not underestimate the risk of landing short or long and consequently deriving navigation system requirements not properly bounding the errors, two techniques are described in the following that solve the problem.

3.1 Gaussian Overbounding

The first alternative is called Gaussian Overbounding. The overbound is constructed in such a way that the risk in the tails of the Gaussian overbound is properly bounding the experimental touchdown performance results. Of course, if the distribution is conservative in the tails, the data at the core of the distribution is not properly described (as the integral over all probability density functions has to equal 1). For the purpose of integrity, this is not of concern. As the bounding condition needs to hold in the tails the mathematical formulation of the overbounding condition can then be written as

$$\begin{aligned}\Phi_{overbound}(x) \geq \Phi(x)\ \forall x \leq NTDP - 1.5 \cdot \sigma_{overbound} \\ \Phi_{overbound}(x) \leq \Phi(x)\ \forall x \geq NTDP + 1.5 \cdot \sigma_{overbound}\end{aligned} \quad (3)$$

where $\Phi = \int_{-\infty}^{x} f(x)dx$ is the cumulative distribution function (CDF) of a random variable x with a Gaussian probability density function (PDF), NTDP is the nominal touchdown point on the runway (assumed to be located at 1190 ft behind the runway threshold according to [8]). The $\pm 1.5\ \sigma_{overbound}$ term describes the area in the core of the distribution where the overbounding condition does not have to be fulfilled. The value of $\pm 1.5\,\sigma_{overbound}$ was found to be a value that does not inflate the overbound beyond reasonable values. The choice is, however, not fixed and subject to engineering judgement. Fig. 3 shows the obtained results for the obtained touchdown distribution for a 5m/s reference wind with severe turbulence. The blue curve shows the CDF (the left branch shown as dashed blue line) and 1-CDF (the right branch shown as solid blue line) curves of the sample distribution. The red curve is the CDF (red dashed line) or 1-CDF (red solid line) of the Gaussian overbound. The overbounding condition (Eq. (3)) ensures that the mass in the tails of the overbounding distribution is larger than that of the sample distribution (i.e. the red line stays above the blue line) outside of the $\pm 1.5\sigma_{overbound}$ area that is indicated by the black line and that is centered at the NTDP at 1190 ft or 364 m behind the runway threshold.

In case of a non-symmetric distribution of the touchdown points and a distribution that is not centered at the NTDP (as it is the case in this example and is generally to be expected for different aircraft types and autopilot realizations) it becomes obvious that the overbound is driven by one side of the distribution and may be very conservative for the other side. Thus, while being conservative and ensuring safe error bounding when deriving navigation requirements the result may become very conservative on one side of the distribution. In this case the land-short limit for the overbound is about 1.9e-2 while the land-long probability is just 9.1e-4. It is thus desirable to consider a way of describing the data in a more suitable way, e.g. better describing non-symmetric data. One such way to describe the touchdown performance is by a Johnson distribution.

3.2 The Johnson distribution

The Johnson distribution is a four-parametric distribution obtained by a transformation of a Gaussian distribution. It has the general form

$$z = \gamma + \delta g(y) \qquad (4)$$

z is a Gaussian distributed random variable, $y = \frac{x-\xi}{\lambda}$ and γ, δ, ξ and λ are the four shape parameters of the distribution and $g(y)$ is a transformation function. Depending on the transformation function chosen there are three different resulting types of distributions:

For $g(y) = \ln(y)$ the log normal type (termed SL), for $g(y) = \ln\left(\frac{y}{1-y}\right)$ the bounded type (termed SB), and for $g(y) = \sinh^{-1}(y)$ the unbounded type (SU). For more details on the Johnson distribution the interested reader is referred to [9]. For data evaluation there is a Johnson Curve Toolbox for Matlab available that selects the most appropriate fitting method given the data to be analyzed [10]. It is available for download and was used for data analysis in this work. When comparing the two methods, it should also be noted that the Johnson Curve Toolbox also provides a fit to the data and not an overbound. As with a Gaussian fit this bears the risk of underestimating the actual risk of landing short or landing long. Appropriate inflation to safely bound the distribution in the tails should therefore also be performed if necessary. As compared to the method of choosing a Gaussian overbound that is centered at the NTDP, the Johnson fitting does not use a pre-defined position at which the curve is centered. The fit is therefore also already tighter than when using the previously described Gaussian fitting or overbounding method with a fixed NTDP as is done currently in the requirement derivation process. In order to have a fair comparison between the two methods, however, Fig. 4 shows again the CDF as dashed lines and 1-CDF as solid lines of the experimental touchdown data in blue, the Gaussian overbound now centered at the median of the data in black and the bounding Johnson curve in red. In order to also obtain more realistic touchdown data the wind was simulated according to actual observed winds at Munich Airport during the month of November 2012. The winds directions were rotated by 180° if the tailwind component would have exceeded 4 kts

as it can be assumed that in such a case the opposite approach direction would be used.
As the data is non-symmetric and the core of the median of the distribution is located closer to the beginning of the runway, the land-short case is the more constraining condition. For the given dataset, however, the shape of the bounding distributions is driven by the data on the land-long side. The Johnson curve is generally following the experimental distribution closer than the Gaussian overbound. Both distributions show a significant margin towards the actual data for the land-short case and are thus quite conservative.

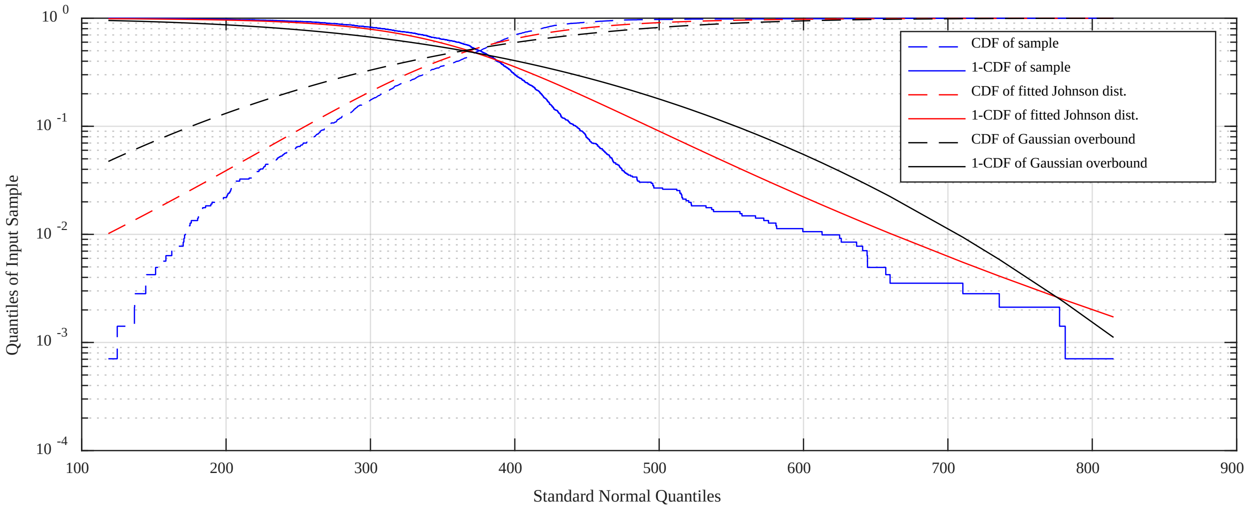

Fig. 4 Experimental distribution (blue) and Gaussian overbound centered at the median of the data (black) compared to Johnson curve overbounding the data (red)

4 Discussion and conclusions

The previous discussions showed that when performing touchdown simulations in order to determine the autopilot performance for a subsequent trade-off between navigation system error and flight technical error budgets it is not appropriate to just use a Gaussian fit to the touchdown data. This would bear the risk of underestimating the probability of landing short or long and may not lead to navigation requirements that ensure the required level of safety.

To solve this problem two alternative methods of describing the touchdown performance were presented: Gaussian overbounding or the use of an overbounding Johnson distribution. The first method inflates the standard distribution until the mass in the tails of the overbound is larger than that of the experimental distribution obtained by the autoland simulations. The other alternative is to use a Johnson distribution that allows for a better description of skewed data. A comparison showed that the fitting is tighter and therefore the conservatism is reduced.

However, in practical applications the Gaussian modelling is generally desired as it makes a trade-off easily understandable and analytically possible. In the case of a four-parametric Johnson distribution no analytical derivation of easy-to-implement requirements is possible. From an implementation perspective it is therefore necessary to judge if the obtained budget for the NSE by using a better fitting method outweighs the potentially more difficult standardization process where the error budget would have to be determined by simple numerical methods.

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8 Contact Author Email Address

michael.felux@dlr.de

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