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COMPOSITE PANELS BASED ON SENSITIVITY ANALYSIS

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Abstract

A reliable engineering analysis technique has been developed to predict the buckling load and bearing capacity of stiffened composite panels. Based on this technique and sensitivity analysis of design parameters related to buckling and post-buckling performance, a structural optimization design method has been proposed for composite panels considering buckling and post-buckling.

1 Introduction

Based on engineering predicting techniques for the buckling load and bearing capacity of stiffened composite panels, an optimization design approach for composite structures which takes post-buckling into consideration is developed and engineering design tools are formed which can be used effectively at the initial design stage of composite structures. The aim of this research is focusing on the stiffened composite panels with different cross sections (such as hat or H et al). Within the proposed optimization method, the design parameters of multilayered stiffened plates are considered as the optimization variables and the structural weight is minimized. The optimization constraints that buckling of structures does not occur and post-buckling failure does not happen under load envelope should be satisfied. On the basis of sensitivity analysis related to the buckling and post-buckling performance, the genetic algorithm is utilized to search the optimization solution. Finally, an effective design scheme for stiffened composite panels is demonstrated and constructed.

2 Optimization approach and stability prediction method

2.1 Load envelope

The stiffened composite panels on planes will experience various complex loads during the flight of planes, therefore the strength and stability of the plane structure under dozens or even hundreds of loading conditions need to be checked. The stability of stiffened composite panels is mainly checked under complex loads such as compression-compression, compression-tension and compression-shear.

The formula for the buckling of flat plates under complex loads is as follows:

$$\begin{aligned} & R_i^n + R_j^m = 1 \\ & R_i = N_i / \ N_i \\ & R_j = N_j / \ N_j \end{aligned} \tag{1}$$

where, R_i is the load ratio per unit width of composite laminate plates, N_i is class I applied load per unit width of laminated plates, N_{icr} is class I buckling load per unit width of laminated plates. The values of n and m are related to the type of composite load.

If $R_i^n + R_j^m < 1$, this indicates that the laminated plate has not undergone buckling, $R_i^n + R_j^m = 1$ indicating that it is in a critical state, and $R_i^n + R_j^m > 1$ indicating that the structure has undergone buckling.

If there are q load conditions to be checked, according to equation(1), so there will be $N_{i1}, N_{i2}, N_{i3} \cdots N_{iq}$ and $N_{j1}, N_{j2}, N_{j3} \cdots N_{jq}$. All loads will be displayed as points (N_{ik}, N_{jk}) (k=1$\cdots q$) in the coordinate system, this is illustrated in Fig.1. According to the shape of the function $R_i^n + R_j^m = 1$, select two points (N_{ip}, N_{jp}) and (N_{il}, N_{jl}), and list the equations:

$$\begin{cases} (\frac{N_{ip}}{N_{icr}})^n + (\frac{N_{jp}}{N_{jcr}})^m = 1 \\ (\frac{N_{il}}{N_{icr}})^n + (\frac{N_{jl}}{N_{jcr}})^m = 1 \end{cases} \qquad (2)$$

Solve the above equations to obtain the values of N_{icr} and N_{jcr}. If the other points (N_{ik}, N_{jk}) (k=1$\cdots q$, k$\neq p$, k$\neq l$) satisfies the inequality $(\frac{N_{ik}}{N_{icr}})^n + (\frac{N_{jk}}{N_{jcr}})^m < 1$, Then the stiffened panels designed according to the loads N_{icr} and N_{jcr} solved by equation (2) will not buckle under the above checking load conditions.

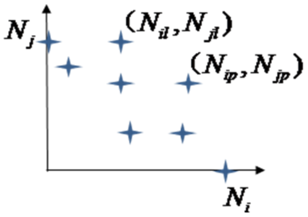

Fig. 1 Checking loads showed in coordinate system

The post-buckling determination curve of flat plates under compression-shear loads is shown in Fig.2. The horizontal axis is $R_c = N_{xext} / N_{cox}$, vertical axis is $R_s = N_{sext} / N_{scr}$. They represent longitudinal compression ratio and shear load ratio under the ultimate load. N_{cox} is post-buckling load under longitudinal compression, N_{scr} is shear buckling load. Check the location of the point (Rc, Rs) whether it is beyond the boundary of the curve in figure 2.If the point is at the internal side of the curve, failure doesn't occur, and vice versa. The calculation method for the post-buckling bearing capacity N_{cox} and the Shear buckling load N_{scr} of stiffened plates is similar to the above calculation method and will not be described in detail here.

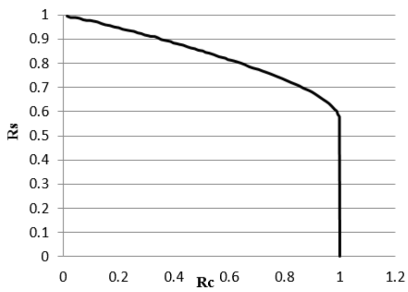

Fig. 2 Compression-shear failure

2.2 Logic expression for optimization

Logic expression refers to the selection of design variables, the definition of objective functions and the determination of constraint conditions in optimization workflow.

2.2.1 Design variables

The structure parameters which have impacts on buckling and post-buckling response are selected as the design variables. It can be partitioned into three categories, including the position parameters such as the stringer spacings, the shape of stringers, the sizes of stringer sections and the ply stacking sequences of all.

The parameters of the first layer are shapes of stringers, such as J section, T section, Hat section and H section, etc. The parameter of the second layer is the number n of stringers on a panel. The parameters of the third layer are sectional dimensions and corresponding composite layers. Taking H-shaped stringer as an example (as shown in Fig.3), the optimization parameters are the width of panel1 b_{f1}, the height of panel2 b_{w1}, the width of panel3 b_{f2} ,the layers of skin layer0, the layers of panel1 layer1, the layers of panel2 layer2 and the layers of panel3 layer3.

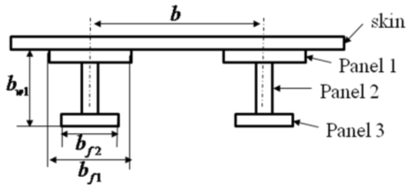

Fig. 3 H cross sections

The constraints of the design variables are as follows：

$$\begin{cases} b=(W-2b_{f1})/(n-1) \\ b>b_{f1} \\ x_1<b_{f1}<x_2 \\ x_3<b_{f2}<x_4 \\ x_5<b_{w1}<x_6 \end{cases} \tag{3}$$

Where, b is the distance between adjacent stringers. W is the width of the bearing load edge of the stiffened panels. x_1, x_2, x_3, x_4, x_5, x_6 are ranges of H-shaped stringers sizes given based on design experience.

2.2.2 Optimization objective

The optimization objective is associated with the buckling and post-buckling performance. Optimization constraints are defined by design and manufacture requirements.

The objective function is described by

$$\min f=\sqrt{\alpha\left(\frac{N_{cr}}{N_{obj.cr}}-1\right)^2+\beta\left(\frac{N_{co}}{N_{obj.co}}-1\right)^2} \tag{4}$$

where α , β represent weight coefficients respectively. N_{cr} is the buckling load of stiffened composite panels and N_{co} means bearing capacity under the current design scheme. Target post-buckling capacity load and buckling load are denoted by $N_{obj.co}$ and $N_{obj.cr}$.

3 Engineering stability analysis method for stiffened composite panels

and its corresponding computer program

Much effort of this research work has been done on this part. On the basis of a certain amount of stability experimental data, engineering stability analysis methods are developed for composite stiffened panels reinforced by a variety of stiffeners. Moreover, the computer codes for the present method are programmed. The reliable prediction tools can ensure optimization results are useable.

Most of the calculation methods can be found in reference 3. The load distribution of panels near the stiffener $\mathrm{N_e}$ is one of the most important parameters for calculating the post-buckling load capacity of stiffened plates. So in this paper, the calculation method of $\mathrm{N_e}$ is mainly discussed.

$\mathrm{N_e}$ is the edge load of the plate where the stiffener support the plate. $\mathrm{N_e} = \eta \mathrm{N_P}$,where η is coefficient, $\mathrm{N_P}$ is the ultimate failure load of laminated plates, which is a key parameter. In this paper, the finite element stiffness reduction method is used to obtain the ultimate failure load per unit width of laminated plates. The 8-node Mindlin element was used to simulate the laminated plates. Some mechanical relation matrices of composite laminated plates obtained by finite element method are as follows:

$$\begin{cases} [K]\{\delta\} = \{F\} \\ [K] = \sum_{e=1}^{n_e} [G]^T [K]_e [G] \\ \{F\} = \sum_{e=1}^{n_e} [G]^T [F]_e \end{cases} \quad (5)$$

where, $[K]$ is the total stiffness matrix. $\{F\}$ is an overall equivalent node force array; $\{\delta\}$ is an array of overall node displacements; $[G]$ is a node displacement array.

In order to analyze the carrying capacity of composite laminates $\mathrm{N_P}$, Hoffman failure theory is chosen as the criterion for analyzing the damage of composite laminates. Hoffman criterion can be expressed as follows:

$$F = \frac{\sigma_L^2}{X_T X_C} + \frac{\sigma_T^2}{Y_T Y_C} - (\frac{1}{X_T X_C})\sigma_L \sigma_T + (\frac{1}{X_T} - \frac{1}{X_C})\sigma_L + (\frac{1}{Y_T} - \frac{1}{Y_C})\sigma_T + (\frac{\tau_{ZL}}{S_{ZL}})^2 + (\frac{\tau_{TZ}}{S_{TZ}})^2 + (\frac{\tau_{LT}}{S_{LT}})^2$$

(6)

Where, X_T (X_C) and Y_T (Y_C) are the tensile (compressive) strength of the composite single-ply fiber in the fiber direction and the vertical fiber direction respectively. S_{ZL} , S_{TZ} and S_{LT} are the shear strength of the single-ply fiber respectively. When the stress values at a single layer in the laminated plate makes $F > 1$, this single layer breaks down. Due to the failure of the composite single-layer, some coefficient values of stiffness matrix in the corresponding failure layer will decrease, thus the overall stiffness of the composite laminated plate will decrease. Therefore, when calculating the ultimate failure load of composite laminates under axial compressive load, the initial load and load step are given first. In each iteration load, the stress values of each layer of the laminates are calculated. Then, according to Hoffman criterion, it is judged whether each layer is damaged. If the damage occurs, stiffness reduction is carried out, and loading continues until it is found that the load and strain no longer maintain a linear relationship. At this time, the load is the ultimate failure load of composite laminates under axial compressive load.

4 Genetic algorithm based on sensitivity analysis

The optimization algorithm is conducted with the following three parts. Firstly, design variables decoupling technology which can realize multi-variable optimization is investigated. Secondly, sensitivity calculation will be done. Thirdly, the genetic algorithm coupled with sensitivity analysis is utilized to perform the optimization task.

4.1 Design variables decoupling

The objective of composite stiffened panel optimization is to make the designed stiffened panel meet the stability requirements under various load conditions, and the structure weight is the lightest. Therefore, the optimization strategy adopted in this paper is divided into two steps: the first step is to generate multiple sample points through various combinations for the optimization parameters of the first layer and the second layer; secondly, for each sample point, optimization is carried out for the third layer optimization parameters. So, initially given the minimum ply and minimum size of a certain sample point, the buckling and post-buckling properties of stiffened plate structures are getting closer to the target value while the weight of stiffened panels structures is increasing by increasing a certain weight of composite fibers each time. At this time, the design variable for the stiffened plate structure is the number N of locations where the composite fibers need to be added, and the location K where the composite fibers need to be added (taking the H - section as an example, $K=[b_{f1},b_{f2},b_{w1},layer0, layer1,layer2,layer3]^T$, each time the length variable increases the unit length, the ply variable increases the corresponding angle one ply at a time.),the angle of the added ply -90°≤θ≤90°.

When the value range of design variables increases, the combination of design variables increases, forming a wider space. In order to solve the coupling problem between multiple design variables, design variables are divided into static and dynamic design variables. In optimization, a sensitivity sequencing process is added to reduce the dimension of design variables.
The number N of design variables that need to be changed, and the ply angle θ [-90,90] are static design variables. In

the optimization process, K(i) is first sorted by some principle to be reduced. In the subsequent genetic algorithm optimization, only the number of K(i) needs to be considered, but not the location added to the composite material. Therefore, K(i) is called a dynamic design variable.

4.2 Sensitivity calculation

Finite difference method is often used to solve sensitivity calculation of design variables in engineering problems. The common finite difference methods are forward difference and center difference. Principle of forward difference method is as follows:

$$\begin{aligned} &\frac{\partial y}{\partial x_i} = \frac{y(X^i) - y(X)}{\Delta x_i} \\ &X^i = [x_1, x_2, \cdots x_i + \Delta x_i \cdots, x_n]^T \\ &X = [x_1, x_2, \cdots x_i \cdots, x_n]^T \end{aligned} \tag{7}$$

Principle of central difference method is:

$$\begin{aligned} &\frac{\partial y}{\partial x_i} = \frac{y(X^{i+}) - y(X^{i-})}{2\Delta x_i} \\ &X^{i+} = [x_1, x_2, \cdots x_i + \Delta x_i \cdots, x_n]^T \\ &X^{i-} = [x_1, x_2, \cdots x_i - \Delta x_i \cdots, x_n]^T \end{aligned} \tag{8}$$

In this paper, the sensitivity of K(i) to the unit structural weight of the objective function is obtained by the finite difference method. The calculation formula is:

$$\frac{\partial f}{\partial K_i} = \frac{f(K^i) - f(K)}{\Delta V} \tag{9}$$

Where, f is an objective function and is listed in the preceding. K is the position variable to increase of composite fiber, and K is an array containing 7 variables for the H-section stiffener. ΔV is the increase in the weight of the stiffened panel structure.

4.3 Optimization by genetic algorithm

This paper uses genetic algorithm as optimization algorithm. Take the H-shaped stiffened plate as an example, the total length of the individual chromosome is divided into five segments. the first segment of chromosome is used to indicate the number N of fiber positions where the composite material needs to be added, because the value range of N is [1,7], the length of this segment of chromosome is 3; The second to fifth segments of chromosomes are used to indicate the angle θ of an additional ply on the skin, panel1, panel2, and panel3 respectively, because the value range of θ is [-90,90], the length of these four segments of chromosomes is 8 respectively. The possible chromosomes are expressed as follows:

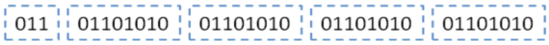

Fig. 4 H cross sections

If the decoding value of the first segment chromosome is 3, it indicates that the number of positions where the composite fiber needs to be added is 3. According to the current sensitivity ranking of K(i), the first three position variables are taken. If the position variable contains a length variable, the length variable needs to increase the unit length on the basis of the previous generation. If the position variable contains a ply variable, a ply is added to the corresponding position, and the ply angle is determined by the second to fifth chromosomes.

The optimization flowchart is depicted in Fig.5 .

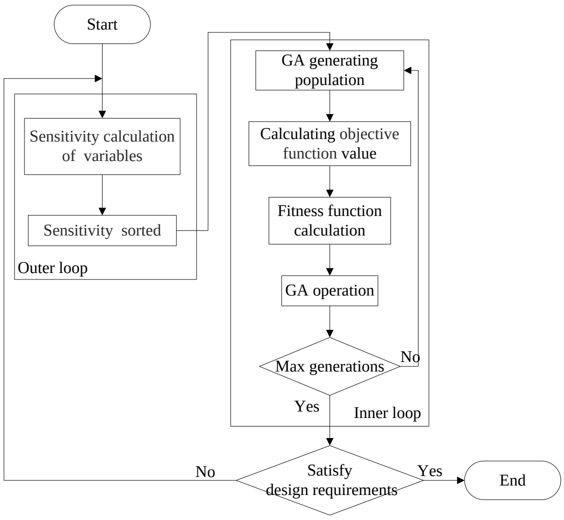

Fig. 5 Optimization flowchart based on sensitivity analysis

5 Optimization workflow

Integrating the methods or tools mentioned above, optimization is carried out as in the following workflow.

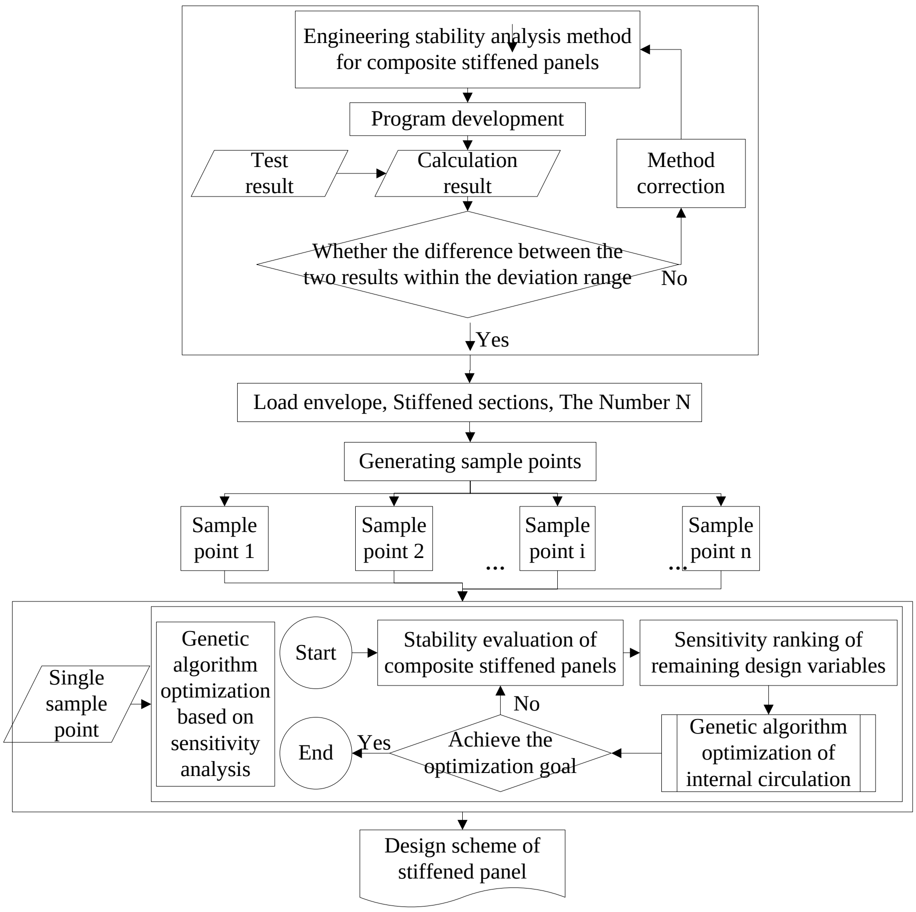

Fig. 6 Optimizing design process

6 Applications

6.1 Stability prediction and validation for analysis method by computer program

In this section, the proposed methods are demonstrated through four plates reinforced with different section sizes of stiffeners. The comparison of calculated loads and experimental data is given in table 1. As shown in Table 1, good agreements between the results of the proposed method and the experimental data are illustrated. Therefore, optimization can be carried out effectively and the results are usable because of the reliable predictions.

Table 1 comparison of calculated loads and experimental data

Panels	M-1		M-2		M-3		M-4	
Load	Buckling load	Bearing capacity	Buckling load	Bearing capacity	Buckling load	Bearing capacity	Buckling load	Bearing capacity
Calculated loads /KN	290	389	208	355	276	380	284	353
Experimental data/KN	292	367	189	384	226	373	277	396
Error /%	-0.7	6.0	10.0	-7.6	22.1	1.9	2.5	-10.9

6.2 The implementation of genetic algorithm based on sensitivity analysis

Application is implemented on some rectangular stiffened plates with specified width and length, whose profiles is H. The sensitivity values of design parameters (such as spacing, dimension of sections and ply stacking sequence) are obtained. After the sensitivity values sorted, chromosomes of genetic algorithm are coded, and then optimizations are carried out.

Conclusion

The stability analysis tool predicts stability performance well and quickly so that the optimization result is valid. The proposed method not only guarantees the loading capacity of stiffened panels but also reduces the structure weight. This method can be integrated into the composite structure design of aircraft at the initial stage.

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