BAYESIAN APPROACH TO FLIGHT CHARACTERISTICS MODELING WITH REAL FLIGHT DATA

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Abstract

This paper applies a new statistical approach to model aircraft flight characteristics with real flight data. The motivation of this study is to improve reliability of the model as comparable as estimation by using wind tunnel tests and computational fluid dynamics analyses. The reliability will be compensated by not only performing point estimation but also giving distribution, which relies on a Bayesian estimation technique; Markov chain Mote Carlo (MCMC). Application to real flight data to make a statistical model of lift and drag coefficients is performed to demonstrate the capability of the proposed approach. The results shows that the method can plausibly estimate not only coefficients but also confidence interval.

1 Introduction

It is important to decrease inconsistency between estimated and real flight performance of aircraft for cost reduction in new aircraft development. Whenever there is inconsistency, development process must be iterated and more resource is consumed. Therefore, improvement of design tools to make their outputs more realistic is fundamental. According to the background, this study focuses on flight characteristics of aircraft represented by aerodynamics coefficients. Typically, the characteristics are estimated by using results of ground tests represented by computational fluid dynamic (CFD) analyses and wind tunnel tests in design phase. Then, they are validated by using flight test data before production phase. The ground tests, especially CFD analyses, can be substituted for the flight tests in terms of the flight characteristics estimation, when they always output completely realistic results.

In order to improve the accuracy of the ground tests, comparison between the ground and flight test results should be the first choice. However, the typical procedure of the validation between the ground and flight tests is insufficient for the comparison. It is just a simple test whether difference between parameters predicted and obtained by ground and flight tests, respectively, are in an acceptable range, and does not provide quantitative information required for the ground test improvement. The reason why the validation is too simple is mainly constraints of the flight tests, with which incomplete results seem to be obtained compared to the ground tests due to their uncontrollable environment such as unavoidable gust. Therefore, in order to get the quantitative and reliable difference of these multiple tests, a new method to model the flight characteristics by using the flight test data is proposed.

The key of the proposed method is to model the characteristics in a statistical manner. More specifically, it is based on Bayesian estimation framework, which treats probability distribution of parameters. This results in the following two superior points. Firstly, it gives not only point estimation results but also distribution, which enables to discuss the comparison results with appropriate weight. For example, it is difficult to obtain sufficient flight data under condition where flight is unstable. The proposed method will return prospect with less confidence under such condition, which guides us to rely on the ground test results heavier than those of the flight tests. The other is it can utilize knowledge as much as possible by regulating a priori distribution. The knowledge varies from geometrical and physical relation to experience of the former ground tests. The second point contributes to get a reasonable model whose difference from the ground test results is easily interpreted.

In the following, the proposed method are explained in Sec. 2. Firstly, the Bayesian estimation is explained briefly as the basis of the proposed method, and Markov chain Monte Carlo (MCMC) sampling utilized in this study as one of the modeling techniques is introduced. Then, an application of the method to real flight data will be described with its results in Sec. 3. As a demonstration, lift and drag coefficients of one of research aircraft "Hisho" owned by Japan Aerospace Exploration Agency (JAXA) are modeled with real flight data. Finally, conclusion remarks and future works are summarized in Sec. 4.

2 Proposed method

In this section, the fundamental parts of the proposed method, that is, Bayesian estimation and MCMC sampling, will be explained briefly. Although both of them are versatile, the application of them to the flight characteristic modeling by using flight data is intensively described here. In addition, the next section, in which their implementation for this study problem is mentioned, will compensate for the abstractness of this section.

2.1 Bayesian estimation

The proposed method depends on Bayes' theorem

$$p(\vartheta|\mathcal{D}) \propto p(\mathcal{D}|\vartheta)\, p(\vartheta)\,, \tag{1}$$

where $p(x)$ is probability of observing x, and $p(x|y)$ is conditional probability of observing x under observing y. In this study, ϑ and $\mathcal{D}$ represent the flight characteristics to be modeled and flight data, respectively. $p(\vartheta)$ and $p(\vartheta|\mathcal{D})$ is prior and posterior probabilities, respectively. In other words, $p(\vartheta)$ is editable input; it can be no information by applying sufficiently wide uniform distribution, while CFD and wind tunnel test results can be applied if a model balanced between ground and flight tests are preferred. On the other hand, $p(\vartheta|\mathcal{D})$ is output. $p(\vartheta)$ and $p(\vartheta|\mathcal{D})$ are typically represented with probability distribution. Finally, $p(\mathcal{D}|\vartheta)$ is a likelihood function, which governs statistical structure of a target problem. The likelihood function is also designed by a user based on a priori knowledge; in this study, geometrical and physical relation as well as empirical formula are utilized.

2.2 Markov chain Monte Carlo (MCMC) sampling

In order to determine the posterior probability $p(\vartheta|\mathcal{D})$, there are several ways. When the distribution and likelihood function are sufficient simple, for example, representation of generalized linear model (GLM) [1] is adopted, they can be determined analytically. On the other hand, as the modeling of the flight characteristics is so complicated that its structure is not linear and hierarchical, the distribution will be estimated heuristically. The simplest but most inefficient way is to repeat sufficiently random sampling, i.e., Monte Carlo. Thus, MCMC, an improved version of Monte Carlo, is utilized in this study. The MCMC sampling (Chapter 11 of Bishop [2]) enables accurate estimation of the distribution with smaller number of repetition. As its name implies, this is done by generating a next random sample to be correlated with its previous sample.

3 Application to real flight data

This section describes how Bayesian estimation and MCMC sampling are applied to model the flight characteristics with flight data. The target problem is chosen as modeling of lift and drag coefficients of JAXA's research aircraft "Hisho", becasue these coefficients are fundamental parameters representing aircraft perfoirmance. In the following, the target aircraft and flight data used for the modeling are introduced firstly. Then, MCMC tool with statistical model with which the coefficients and flight data are associated are explained. Finally, the results will be shown.

3.1 Target aircraft and flight data

The flight data used for the modeling is collected by “Hisho” shown in Fig. 1. This aircraft was originally Cessna 680 Citation Sovereign, which is fixed wing, twin turbojet powered aircraft, and has been modified for various research purposes [3]. The flight data used for the modeling consists of air data, inertial force, control surface deflection, and engine operating status captured with full authority digital engine control (FADEC). The air data is composed of airspeed, angle of attack, and pressure altitude. The inertial force is acceleration and angular speed measured with inertial measurement unit (IMU). The indirect data such as its total weight and thrust force is secondarily estimated from the direct observed data. With the data and physical properties of the aircraft, the aerodynamic coefficients are calculated by using equation of motion:

$$\begin{bmatrix} C_L \\ C_D \end{bmatrix} = \frac{1}{qS}\left[\mathrm{Rot}(\alpha,\beta)\left(m\vec{a}-\vec{T}\right)\right]_{z,x} \tag{2}$$

is applied for calculation of the lift C_L and drag C_D coefficients, where q, S, m, $\vec{a}$ and $\vec{T}$ are dynamic pressure, wing area, accelerometer outputs, and thrust force, respectively. $\mathrm{Rot}(\alpha,\beta)$ is a coordinate transformation matrix from body to wind axes. It is noted that the data recorded at the maximum frequency of 50 Hz is down sampled to 1 Hz for the modeling for reduction of computational load.

In order to get a practical model of the lift and drag coefficients, the flight data used for the estimation is selected to include various flight conditions. Actually, shown in Fig. 2, the various combination of altitude and Mach number, which mostly covers the flyable envelop of “Hisho”, is

Fig. 1 Research aircraft “Hisho”, which means fly higher in Japanese

included in the flight data. In addition, for simplicity, the flight data is arranged to have only clean configuration and trimmed condition parts, in which the flaps are 0°, the landing gears are retracted, and the active maneuvering is not performed.

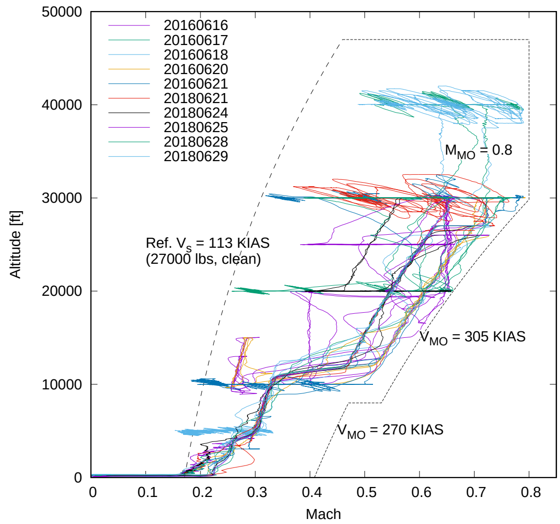


Fig. 2 Altitude and Mach number variation of used flight data

3.2 Model definition and MCMC tool

The lift coefficient C_L is supposed to be modeled well with a commonly used linear model, in which the total lift can be separated into parts varied with changes of angle of attack α, stabilizer position δ_{stab} and elevator deflection δ_e:

$$C_L \approx C_{L0} + C_{L\alpha}\alpha + C_{L\delta_{\mathrm{stab}}}\delta_{\mathrm{stab}} + C_{L\delta_e}\delta_e \tag{3}$$

where C_{L0} and C_{Lx} are baseline and contribution of x, respectively. On the other hand, the drag coefficient C_D are assumed to be

$$C_D \approx C_{D0} + C_{D\,Re} \log_{10} Re + C_{Dc}\left(\frac{1}{\sqrt{1-M^2}} - 1\right) + C_{Di} C_L^2 \qquad (4)$$

based on Chapter 12 of Raymer [4]. The last of the left hand side is known as the induced drag, and the other terms compose the parasitic drag. In this assumption, the skin friction drag affected by Reynolds number Re is extracted from the parasitic drag as the second term. In addition, the transonic compressibility drag (wave drag), whose contribution is measured by Mach number M, is also separately considered as the third term.

The MCMC sampling will estimate the distribution of the goal variables appeared as the symbol C in Eqs. (3)-(4). Before the estimation, the flight data must be checked because if there is collinearity between several explanatory variables, the results are deteriorated. Figure 3 shows pair of scatter, frequency, and correlation plots. Especially, the upper triangle elements are the correlation plots, whose values are scaled Spearman's rank correlation coefficients [5]. 100 and -100 of the coefficients mean full positive and negative correlated pairs, respectively. It is noted that absolute values of the coefficients are intentionally omitted.

Except for the combinations with the goal variables, i.e., C_L and C_D, the angle of attack α and stabilizer position δ_{stab} recorded in the flight data are strongly correlated as Spearman's value is -96. Therefore, in order to perform better estimation, the stabilizer position is dropped from the model, then, Eq. (3) is

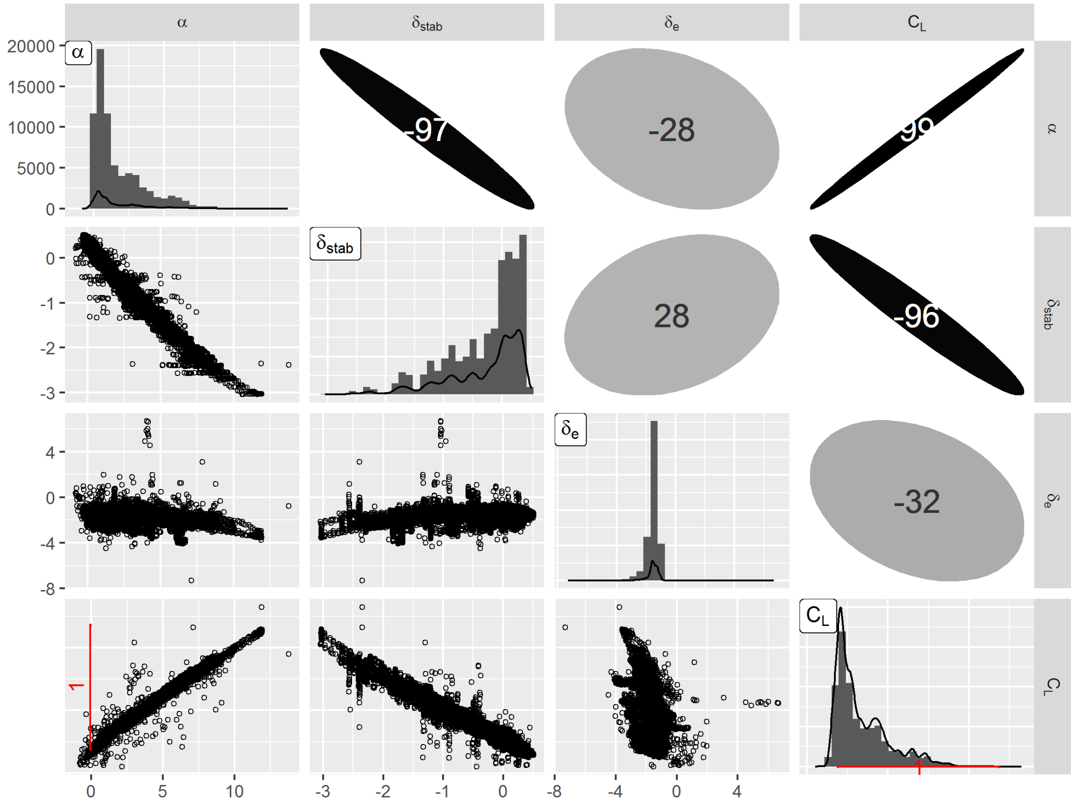
$$C_L \approx C_{L0} + C_{L\alpha}\alpha + C_{L\,\delta_e}\delta_e\,. \qquad (3')$$

It is noted that this phenomena is probably derived from autopilot of "Hisho", which is mostly utilized to perform its trimmed flight.

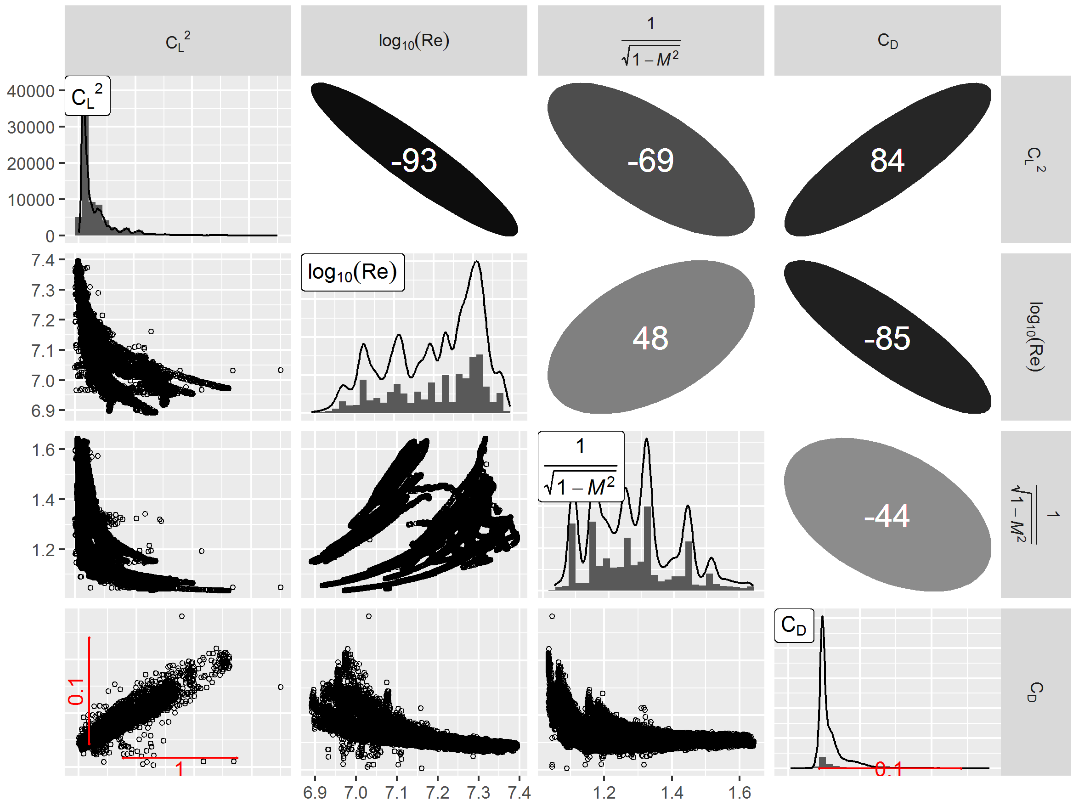
For the MCMC sampling, a software named Stan [6] is utilized. Stan has an effective MCMC implementation, No-U-Turn Sampling (NUTS) [7], which enables us to get plausible distribution with relatively small number of samples compared to other algorithm when a problem is complicated. Stan requires an input model to define a problem, and the model described in Listing 1 is used in this study . The model declares that the observed values are governed by Eqs. (3') and (4), and their residuals are assumed to be normally distributed with their standard deviation depicted by $\sigma(C_L)$ and $\sigma(C_D)$, respectively. Other configuration for Stan is summarized in the followings: initial distribution is non-informative prior. Both the number of iteration for warm up and sampling are 1000. Thining, which truncates samples to get less self correlated distribute, is 3, and 4 chains are used to check the estimated distribution.

Listing 1 Model for Stan

```
data {
  int<lower=1> N; // Number of samples
  vector[N] alpha_deg; // α
  vector[N] de_deg; // δ_e
  vector[N] cL; // C_L
  vector[N] re; // Re
  vector[N] mach; // M
  vector[N] cD; // C_D
}
parameters {
  real cL0; // C_L0
  real cLa; // C_Lα
  real cLde; // C_Lδe
  real<lower=0> sigma_cL; // σ(C_L)
  real cD0; // C_D0
  real inv_epar; // C_Di
  real cDre; // C_DRe
  real cDc; // C_Dc
  real<lower=0> sigma_cD; // σ(C_D)
}
model {
  cL ~ normal(cL0
        + (cLa * alpha_deg)
        + (cLde * de_deg),
        sigma_cL); // Eq. (3')
  cD ~ normal(cD0
        + ((cL .* cL) * inv_epar)
        + (cDre * log10(re))
        + (cDc * ((1 ./ sqrt(1 − (mach .* mach)))
                − 1)),
        sigma_cD); // Eq. (4)
}
```



(a) For lift coefficient (C_L). The symbols correspond to those in Eq. (3).



(b) For drag coefficient (C_D). The symbols correspond to those in Eq. (4).

Fig. 3 Pair plots for lift and drag coefficient modeling. For each set, the diagonal elements show frequency plots. The elements in the lower and upper triangle areas are scatter and correlation plots of each two variables, respectively.

3.3 Results

The estimated distribution of the parameters consisting the model are summarized in Fig. 4 and Table 1. It is noted that while the baseline of the lift and drag coefficients have been estimated, their relative differences from the sample mean values are indicated as ΔC_{L0} and ΔC_{D0}, respectively. Before discussion of the results, validity of the estimated distribution will be checked with $\hat{R}$ values. According to Chapter 11 of Gelman [8], when $\hat{R} < 1.1$ for all parameters subject to three or more chains, sampling is successfully converged to get accurate distribution. The $\hat{R}$ values shown in Table 1 satisfy this condition, and the distribution results in being sufficiently accurately estimated. The convergence is also checked with the trace plots in Fig. 5, which shows that the samples are sufficiently random.

There are some interesting details of the results which can be revealed with the distribution estimation. Firstly, the estimation of the baseline lift is easier than the baseline drag because standard deviation of $\sigma(C_{D0})$ is larger than that of $\sigma(C_{L0})$. It is typical that the drag is smaller force than the lift and this result is reasonable. The analogy can be applied to the relation between the overall and elevator driven lift; standard deviation of $C_{L\delta_e}$ is comparably larger. The compressibility drag C_{Dc} is well estimated because the distributed values are always positive, which means the drag increases as Mach number goes higher. Although the skin friction drag in association with Reynolds number C_{DRe} is estimated with relatively less confidence as the scale of mean and standard deviation equal, it seems to be plausible because its contribution is sufficiently smaller than the induced drag C_{Di}.

Fig. 6 shows lift curve and drag polar. The observed values are depicted by small circles. The shallow and dense gray areas are 80 % and 50 % Bayesian prediction intervals, respectively. In both the lift curve and drag polar, most of observed values are covered by the intervals. Therefore, it is concluded that the distribute estimation driving the prediction is performed appropriately. In Fig. 7, the observed and predicted coefficients are indicated. It also supports the appropriate estimation, because most of them are located on or near the diagonal line, which corresponds to match the prediction with the observation. It is noted that for the prediction of C_D, the altitude and airspeed are fixed as their mean values of the flight data, which causes small prediction error.

4 Conclusion

This paper described the new modeling method of aircraft flight characteristics with real flight data. The method was intended to make the comparison of the results of CFD, wind tunnel tests, and flight tests easier by using the framework of Bayesian estimation. Its significant difference from the typical method is that its model is represented with probability distribution, which enables us to gain more information than the conventional point estimation. For the demonstrative purpose, the relation of the lift and drag coefficients of the fixed wing aircraft was successfully modeled with the real flight data by using the proposed method. Actually, the method argued how accurate the estimated aerodynamic coefficients are in addition to the point estimation. Moreover, the reasonable changes of skin coefficient and transonic compressive drag in association with Mach and Reynolds number were successfully extracted. The method can be applied to more complex modeling, for instance, dynamic effects, which is one of future works of this study. It is noted that this work was supported by JSPS KAKENHI Grant Number 15K06612.

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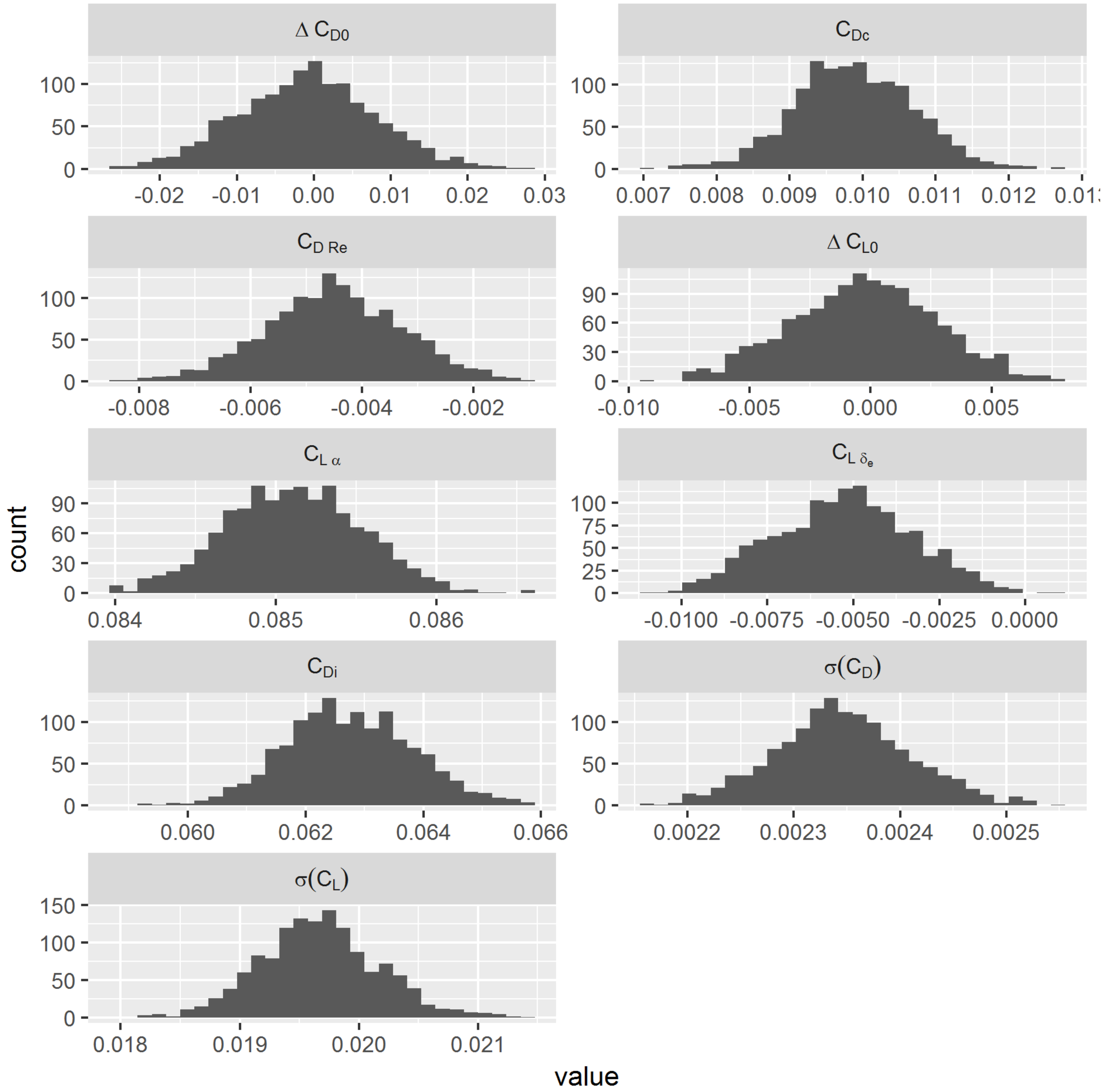


Fig. 4 Histogram of samples corresponding to the estimated distribution

Table 1 Summary of estimated distribution

Item	Mean	Standard deviation	2.5%	Median (50%)	97.5%	$\hat{R}$
ΔC_{L0}	$(0.000e+00)$	$8.126e-05$	$-1.777e-02$	$1.407e-04$	$1.790e-02$	$1.001e+00$
$C_{L\alpha}$ *	$8.515e-02$	$4.272e-04$	$8.432e-02$	$8.515e-02$	$8.598e-02$	$1.000e+00$
$C_{L\delta_e}$ *	$-5.087e-03$	$2.010e-03$	$-8.950e-03$	$-5.039e-03$	$-1.231e-03$	$9.981e-01$
$\sigma(C_L)$	$1.973e-02$	$5.076e-04$	$1.877e-02$	$1.972e-02$	$2.081e-02$	$1.002e+00$
ΔC_{D0}	$(0.000e+00)$	$2.970e-03$	$-5.748e-03$	$5.598e-05$	$5.620e-03$	$9.984e-01$
C_{Dc}	$9.971e-03$	$8.215e-04$	$8.404e-03$	$9.949e-03$	$1.152e-02$	$1.004e+00$
C_{DRe}	$-4.376e-03$	$1.233e-03$	$-6.843e-03$	$-4.395e-03$	$-1.943e-03$	$1.000e+00$
C_{Di}	$6.287e-02$	$1.086e-03$	$6.084e-02$	$6.283e-02$	$6.507e-02$	$1.006e+00$
$\sigma(C_D)$	$2.353e-03$	$6.438e-05$	$2.230e-03$	$2.351e-03$	$2.482e-03$	$1.001e+00$

* [1/deg]

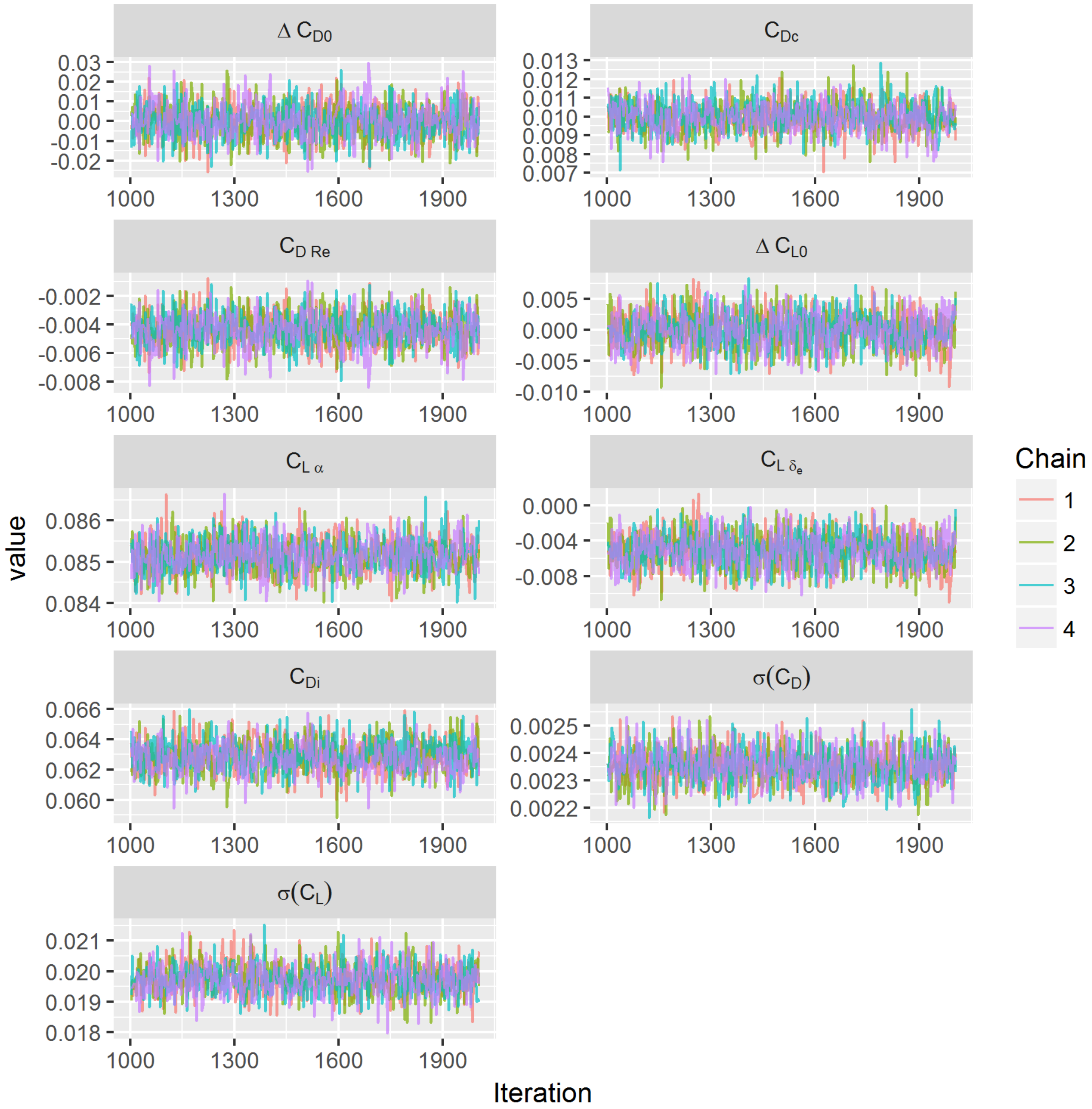
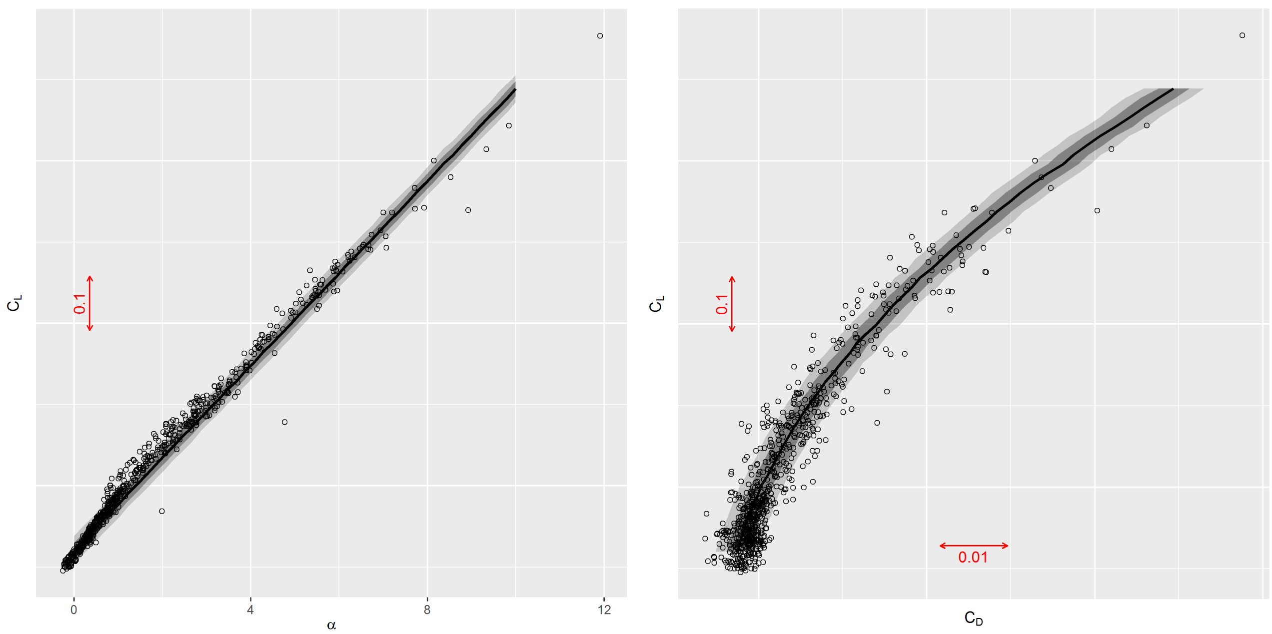
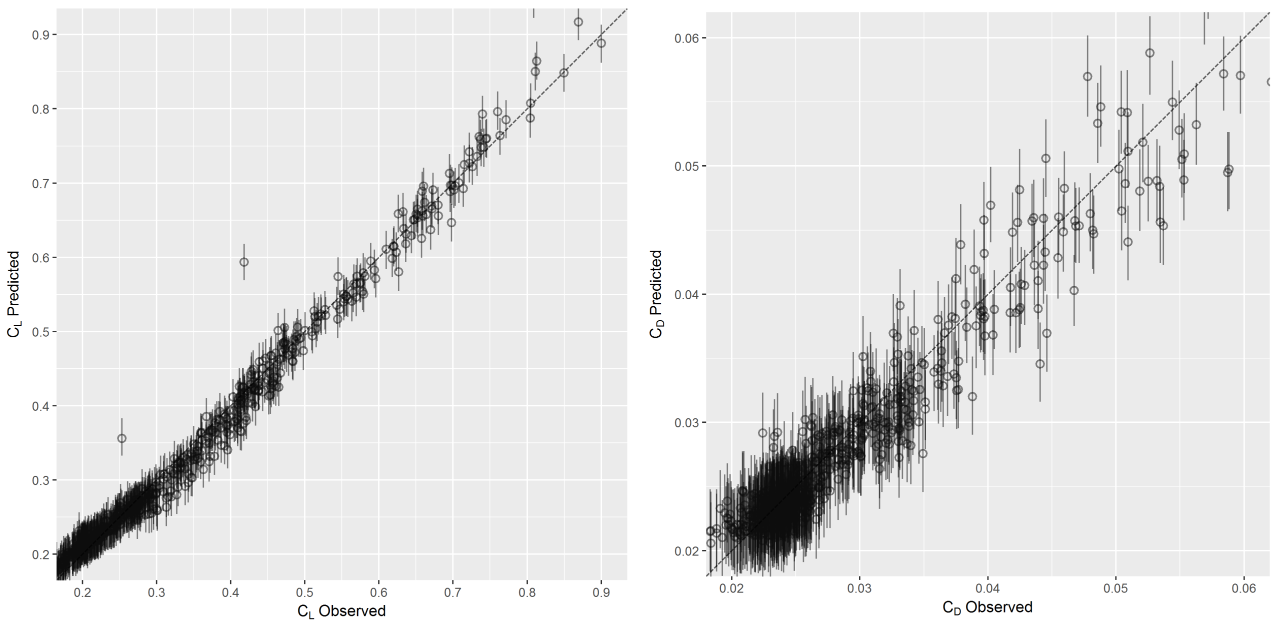


Fig. 5 Observed and predicted coefficients



(a) Lift curve (b) Drag polar

Fig. 6 Observed values and Bayesian prediction intervals



(a) Lift coefficient (b) Drag coefficient

Fig. 7 Observed and predicted coefficients. The vertical bars represent 80 % prediction interval.

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